# On the spontaneous breakdown of Lorentz symmetry in matrix models of superstrings

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In string or M theories, the spontaneous breaking of 10D or 11D Lorentz symmetry is required to describe our space-time. A direct approach to this issue is provided by the type IIB matrix model. We study its 4D version, which corresponds to the zero volume limit of 4D super $SU(N)$ Yang-Mills theory. Based on the moment of inertia as a criterion, spontaneous symmetry breaking (SSB) seems to occur, so that only one extended direction remains, as first observed by Bialas and Burda *et al.* However, using Wilson loops as probes of space-time we do not observe any sign of SSB in Monte Carlo simulations where $N$ is as large as 48. This agrees with an earlier observation that the phase of the fermionic integral, which is absent in the 4D model, should play a crucial role if SSB of Lorentz symmetry really occurs in the 10D type IIB matrix model.

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## I. INTRODUCTION

Matrix models [1,2] are considered the most promising candidate for a nonperturbative definition of string or M theories. They may play an analogous role as lattice gauge theory does in quantum field theory. One of the most fundamental questions that can be addressed using these models is the issue of spontaneous breakdown of Lorentz invariance, which is required to occur in order for these theories in 10 (or 11) dimensions to describe our four-dimensional space-time. For early works which address this issue using string field theory, see Refs. [3].

The type IIB matrix model [or Ishibashi-Kawai-Kitazawa-Tsuchiya (IKKT) model] [2], which is conjectured to describe type IIB superstrings nonperturbatively, is a supersymmetric matrix model composed of 10 bosonic matrices and 16 fermionic matrices, which can be obtained formally by taking the zero-volume limit of 10D super SU($N$) Yang-Mills theory.[1] This model is particularly suitable for the study of spontaneous symmetry breaking (SSB) of Lorentz invariance,[2] since it is manifestly invariant under SO(10) transformations, which transform the bosonic and fermionic matrices as a vector and as a Majorana-Weyl spinor, respectively. The bosonic matrices represent the dynamically generated space-time. A $d$-dimensional space-time is described by configurations with $d$ bosonic matrices having much broader eigenvalue distributions than the other $(10-d)$ matrices, up to some SO(10) transformation.[3] Our four-dimensional space-time may be accounted for, if the $d=4$ configurations (in the above sense) dominate the integration over the bosonic matrices. It was found recently that the type IIB matrix model is indeed endowed with a natural mechanism that may realize such a scenario [5,6].

The realization of our space-time as a "brane" in a higher-dimensional space-time has attracted much attention as an alternative to the more conventional approach in string theory using compactification (see Ref. [7] and references therein). It turned out that such a setup has many phenomenological advantages, including possible mechanisms which may solve the cosmological problem and the hierarchy problem. However, the dynamical origin of the brane has not been discussed so far. The type IIB matrix model enables us to investigate whether a four-dimensional space-time emerges *dynamically* as a brane in ten-dimensional type IIB superstring theory through some nonperturbative effects.[4]

The spontaneous breakdown of Lorentz symmetry in matrix models has been addressed first in the bosonic case [9], where fermionic matrices are omitted (for recent work on the bosonic model, see Refs. [10,11]). There the absence of SSB has been established by both an analytical method (to all orders in a $1/D$ expansion) and by Monte Carlo simulations. The same numerical result was obtained in the 6D and 10D supersymmetric (SUSY) matrix models [12], albeit with some simplifications to enable simulations at large $N$. The fermion integrals are complex in general in these cases, and the simulations were carried out including only the modulus,

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[1]Dimensionally reduced Yang-Mills theories were first studied in Refs. [4].

[2]When one defines the type IIB matrix model nonperturbatively, a Wick rotation to Euclidean signature is needed. Hence by Lorentz symmetry we actually mean SO(10) symmetry.

[3]Throughout this paper, we denote the initial dimensionality by $D$ and the dimension after a possible SSB as $d$.

[4]A possible obstacle may be that gravitons propagate in ten-dimensional space-time, and hence one fails to reproduce the observed four-dimensional Newton's law. In Ref. [8], it was demonstrated that this obstacle can be avoided in the case of D3-brane backgrounds due to the mechanism of Ref. [7].

 

but omitting the phase. In addition, a low-energy effective theory was used in order to further reduce the computational effort.

In Ref. [13] we presented Monte Carlo simulations of the 4D version of the type IIB matrix model, which is a supersymmetric matrix model obtained from the zero-volume limit of 4D super SU($N$) Yang-Mills theory. We were able to study the model with $N=16,24,32,48$ without any simplifications. These values of $N$ turned out to be sufficiently large to extract the large $N$ behavior of the space-time structure and to reveal the large $N$ scaling for a number of Wilson loop correlators.

Recently, it has been reported for the 4D SUSY model up to $N=8$ that the space-time is observed to be one-dimensional, if one selects configurations with large extent from the ensemble [14]. In the $D$-dimensional SUSY models in general, $D=4,6,10$, configurations with large extent are suppressed only by the power $-(2D-5)$, independent of $N$ [15]. Therefore, the observed anisotropic configurations may play some role in the large $N$ limit, and such effects may also be relevant in other SUSY models, including the type IIB matrix model.

In this paper, we reconsider the issue of SSB of Lorentz invariance in the 4D SUSY model. If we adopt the conventional criterion based on the moment of inertia tensor, then the space-time appears one-dimensional, as suggested by the observation in Ref. [14]. This would mean that the SSB does occur at large $N$. However, this conclusion depends on the definition of the order parameter, as we shall see. Thus we have to address the question which criterion for the SSB of Lorentz symmetry is actually physical.

We recall that in the interpretation of the type IIB matrix model as a string theory, the Wilson loops are identified with the string creation operators [16]. Physical observables—such as scattering amplitudes—should be extracted from correlation functions of Wilson loops, which were observed to have well-defined large $N$ limits in $D=4$ [13]. We therefore propose a "physical criterion" of SSB using Wilson loops as a probe, and we study it by Monte Carlo simulations of the full 4D $SU(N)$ SUSY model. As a result, we find no trend of SSB up to $N=48$.

## II. THE 4D TYPE IIB MATRIX MODEL

The model we investigate is a supersymmetric matrix model obtained from the zero-volume limit of 4D SU($N$) super Yang-Mills theory. Its partition function is given by

$$Z=\int dA\, e^{-S_b}\int d\psi d\bar{\psi}\, e^{-S_f},$$

$$S_b=-\frac{1}{4g^2}\mathrm{Tr}\,[A_\mu,A_\nu]^2,$$

$$S_f=-\frac{1}{g^2}\mathrm{Tr}(\bar{\psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu,\psi_\beta]), \tag{2.1}$$

where $A_\mu$ ($\mu=1,\ldots,4$) are bosonic traceless $N\times N$ Hermitian matrices, and $\psi_\alpha$, $\bar{\psi}_\alpha$ ($\alpha=1,2$) are fermionic traceless $N\times N$ complex matrices. The $2\times 2$ unitary matrices $\Gamma_\mu$ are gamma matrices after Weyl projection; they can be given for example by

$$\Gamma_1=i\sigma_1,\quad \Gamma_2=i\sigma_2,\quad \Gamma_3=i\sigma_3,\quad \Gamma_4=\mathbf{1}. \tag{2.2}$$

This model is invariant under 4D Lorentz transformations, where $A_\mu$ transforms as a vector and $\psi_\alpha$ as a Weyl spinor. The model is manifestly supersymmetric, and it also has a SU($N$) symmetry

$$A_\mu\to VA_\mu V^\dagger;\quad \psi_\alpha\to V\psi_\alpha V^\dagger,\quad \bar{\psi}_\alpha\to V\bar{\psi}_\alpha V^\dagger, \tag{2.3}$$

where $V\in\mathrm{SU}(N)$. All these symmetries are inherited from the super Yang-Mills theory before the zero-volume limit. The model can be regarded as the four-dimensional counterpart of the type IIB matrix model.

The model is well-defined for arbitrary $N\geqslant 2$ without any cutoff. This was first conjectured based on numerical results at small $N$ [17], confirmed further at larger $N$ [13] and finally proved by Ref. [11]. Therefore, the parameter $g$—which is the only parameter of the model—can be absorbed by rescaling the variables,

$$A_\mu=g^{1/2}X_\mu;\quad \psi_\alpha=g^{3/4}\Psi_\alpha. \tag{2.4}$$

Therefore, $g$ is a scale parameter rather than a coupling constant, i.e. the $g$ dependence of physical quantities is completely determined on dimensional grounds. The parameter $g$ should be tuned appropriately as one sends $N$ to infinity, so that each correlation function of Wilson loops has a finite large $N$ limit. This issue has been studied numerically in Ref. [13]. The conclusion is that the product $g^2N$ has to be kept constant when taking the large $N$ limit. The tuning of $g$ was also discussed in terms of analytical arguments in Refs. [16,18–20].

The integration over fermionic variables can be done explicitly and the result is given by $\det\mathcal{M}$, $\mathcal{M}$ being a $2(N^2-1)\times 2(N^2-1)$ complex matrix which depends on $A_\mu$. The matrix $\mathcal{M}$ is given by

$$\mathcal{M}_{a\alpha,b\beta}\equiv(\Gamma_\mu)_{\alpha\beta}\,\mathrm{Tr}(t^a[A_\mu,t^b]), \tag{2.5}$$

where $t^a$ are generators of SU($N$), and we consider $(a\alpha)$ respectively $(b\beta)$ as one index. Hence the system we want to simulate can be written in terms of bosonic variables,

$$Z=\int dA\, e^{-S_b}\det\mathcal{M}. \tag{2.6}$$

A crucial point is that the determinant $\det\mathcal{M}$ is *real positive* [13]. This property was demonstrated in Ref. [13], and it had been suspected earlier [17]. (It also holds in other 4D SUSY models, see Ref. [21] and second Ref. in [14].) Due to this property, we can simulate the model using a standard algorithm for dealing with dynamical fermions (the so-called Hybrid R algorithm [22]). In the framework of this algorithm, each update of a configuration is done by solving a Hamiltonian equation for a fixed "time" $\tau$. This algorithm is plagued by a systematic error due to the discretization of $\tau$ that we used to solve the equation numerically. We performed simulations at three different values of the "time step" $\Delta\tau$ and we extrapolate to $\Delta\tau=0$.

## III. SSB OF LORENTZ SYMMETRY?

In the type IIB matrix model, the eigenvalues of the bosonic matrices $A_\mu$ are interpreted as the space-time coordinates [2,18,23]. We adopt this point of view in the 4D model as well. Since the matrices $A_\mu$ are not simultaneously diagonalizable in general, the space-time is not classical. To extract the space-time structure we first define the space-time uncertainty $\Delta$ by [13]

$$\Delta^2=\frac{1}{N}\mathrm{Tr}(A_\mu{}^2)-\max_{U\in \mathrm{SU}(N)}\frac{1}{N}\sum_i \{(UA_\mu U^\dagger)_{ii}\}^2, \quad (3.1)$$

which is invariant under Lorentz transformations and under the SU($N$) transformations (2.3). Formula (3.1) has been derived in Ref. [9] based on the analogy to quantum mechanics, considering $A_\mu$ as an operator acting on a space of states. As a natural property, $\Delta^2$ vanishes if and only if the matrices $A_\mu$ are diagonalizable simultaneously. For each configuration $A_\mu$ generated by a Monte Carlo simulation, we maximize $\Sigma_i\{(UA_\mu U^\dagger)_{ii}\}^2$ with respect to the SU($N$) matrix $U$. We denote the matrix which yields the maximum as $U_{\max}$, and we define

$$x_{i\mu}=(U_{\max}A_\mu U^\dagger_{\max})_{ii} \quad (3.2)$$

as the space-time coordinates of $N$ points $x_i$ $(i=1,\dots,N)$ in four-dimensional space-time.

In order to search the spontaneous breakdown of Lorentz symmetry, we first consider the moment of inertia tensor of $N$ points $x_i$. It can be defined as

$$T_{\mu\nu}=\frac{2}{N(N-1)}\sum_{i<j}(x_{i\mu}-x_{j\mu})(x_{i\nu}-x_{j\nu}), \quad (3.3)$$

which is a $D\times D$ real symmetric matrix.[5] The $D$ eigenvalues $\lambda_1>\lambda_2>\cdots>\lambda_D>0$ of the tensor $T$ represent the principal moments of inertia. We take the average $\langle\lambda_\mu\rangle$ over all configurations generated by the Monte Carlo simulation.

In Fig. 1 we plot the results for $\langle\lambda_\mu\rangle/g$ against $\Delta\tau$ at $N=16$, 24 and 32. (In all the figures in this section, we plot dimensionless quantities, so that they do not depend on the choice of the scale parameter $g$.) As $\Delta\tau$ vanishes, $\langle\lambda_1\rangle/g$ is observed to diverge as

$$\langle\lambda_1\rangle/g\sim -c_1\ln\Delta\tau+c_0 \quad (3.4)$$

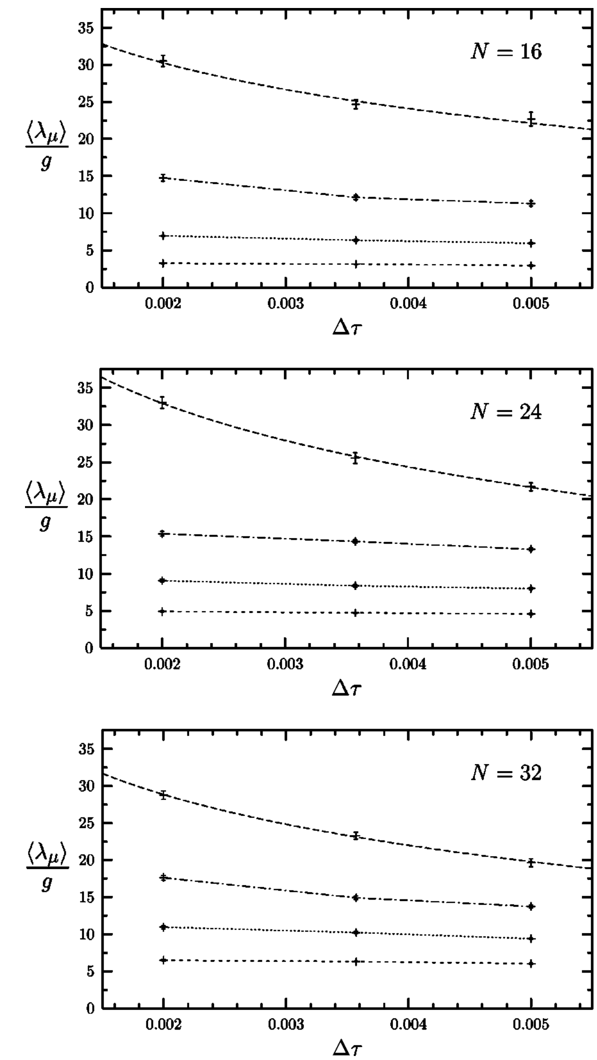


FIG. 1. The eigenvalues of the tensor of inertia $T$ as functions of the algorithmic "time step" $\Delta\tau$ at $N=16$, 24 and 32. The dashed lines on top represent the fits to Eq. (3.4), confirming that the largest eigenvalue diverges logarithmically in the limit $\Delta\tau\to 0$. (The other lines are drawn to guide the eye.)

(where $c_0$, $c_1$ are constant in $\Delta\tau$), while the other eigenvalues converge.[6] We recall that the extent of the space-time $R$ defined by

$$R^2=\frac{2}{N(N-1)}\left\langle\sum_{i<j}(x_i-x_j)^2\right\rangle \quad (3.5)$$

was found to be divergent [13]. Since $R^2=\Sigma_\mu\langle\lambda_\mu\rangle$, at least $\langle\lambda_1\rangle$ has to diverge as $\Delta\tau\to 0$. What we observe here is that $\langle\lambda_1\rangle$ is indeed the only divergent eigenvalue. This is consistent with the observation in Ref. [14].

[5] This quantity has also been studied in Refs. [19,12]. Alternatively, one may define the moment of inertia tensor by $I_{\mu\nu}=(1/N)\mathrm{Tr}(A_\mu A_\nu)$, as it was done in Refs. [9,14]. We have also measured the eigenvalues of that tensor, and the spectra are in qualitative agreement with the eigenvalues that we present here [based on definition (3.3)].

[6] We note that based on our data, a logarithmic divergence of $\langle\lambda_2\rangle$ is unlikely, but it cannot be absolutely excluded. This slight uncertainty is still a little stronger if we use the definition $I_{\mu\nu}=(1/N)\mathrm{Tr}(A_\mu A_\nu)$ for the moment of inertia tensor.

Let us introduce the probability distribution for the distance of two space-time points as

$$\rho(r) = \frac{2}{N(N-1)} \left\langle \sum_{i<j} \delta\left(r - \sqrt{(x_i - x_j)^2}\right) \right\rangle. \tag{3.6}$$

Then $R^2$ can be written as

$$R^2 = \int_0^\infty dr\, r^2 \rho(r). \tag{3.7}$$

The observed logarithmic divergence of $R^2$ is consistent with the asymptotic behavior

$$\rho(r) \sim r^{-3}, \tag{3.8}$$

which was predicted analytically [15]. Based on this observation, a modified definition for the extent of the space-time has been introduced in Ref. [13],

$$R_{\text{new}} = \frac{2}{N(N-1)} \left\langle \sum_{i<j} \sqrt{(x_i - x_j)^2} \right\rangle = \int_0^\infty dr\, r \rho(r), \tag{3.9}$$

which turned out to be finite—as expected from relation (3.8). The large $N$ behavior of this quantity has been observed to amount to $R_{\text{new}}/\sqrt{g} = 3.30(1) \times N^{1/4}$, which is consistent with the prediction based on the low-energy effective theory [18].

This motivates us to define analogously a new tensor

$$T^{(\text{new})}_{\mu\nu} = \frac{2}{N(N-1)} \left\langle \sum_{i<j} \frac{(x_{i\mu} - x_{j\mu})(x_{i\nu} - x_{j\nu})}{\sqrt{(x_i - x_j)^2}} \right\rangle. \tag{3.10}$$

Let us denote the $D$ eigenvalues of the tensor $T^{(\text{new})}$ as $\lambda_1^{(\text{new})} > \lambda_2^{(\text{new})} > \cdots > \lambda_D^{(\text{new})} > 0$. Due to the relation $\Sigma_\mu \lambda_\mu^{(\text{new})} = R_{\text{new}}$, all the eigenvalues are expected to converge. In Fig. 2 we show the results for $\langle \lambda_\mu^{(\text{new})} \rangle / \sqrt{g}$ again at $N = 16$, 24 and 32. We carried out an extrapolation to $\Delta\tau = 0$ assuming the observable $\mathcal{O}(\Delta\tau)$ at small $\Delta\tau$ to behave as [13]

$$\mathcal{O}(\Delta\tau) - \mathcal{O}(\Delta\tau = 0) \propto (\Delta\tau)^2 \cdot |\ln \Delta\tau|. \tag{3.11}$$

In Fig. 3 we plot the extrapolated and normalized eigenvalues $\langle \lambda_\mu^{(\text{new})} \rangle / (\sqrt{g} N^{1/4})$ against $1/N$. (This is the normalization needed for a finite large $N$ limit.) We observe that they move closer together as $N$ increases. Therefore we cannot recognize any trend for SSB.

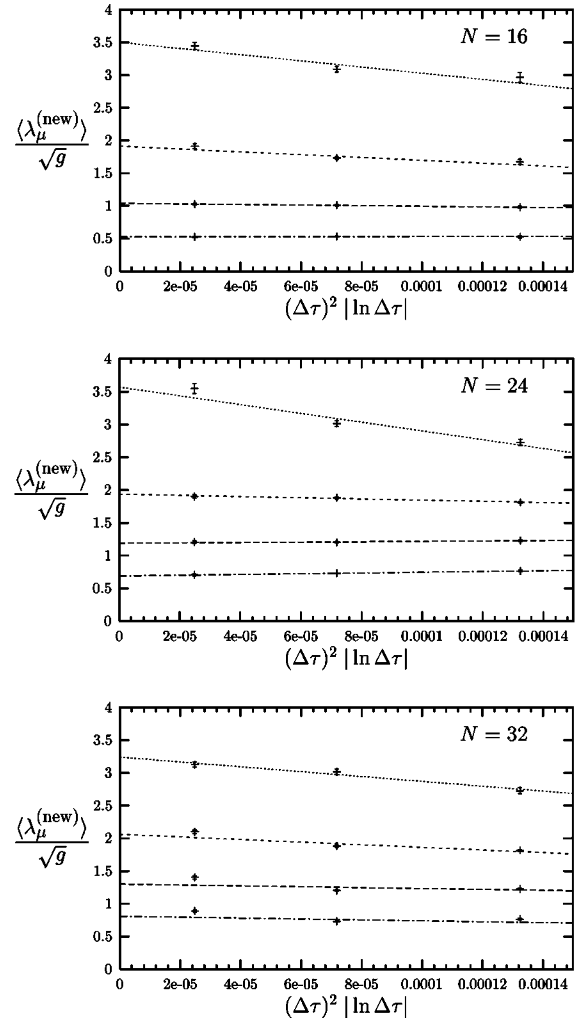


FIG. 2. The eigenvalues of the new tensor $T^{(\text{new})}_{\mu,\nu}$ as functions of the algorithmic "time step" $\Delta\tau$ at $N = 16$, 24 and 32. The straight lines are the fits to Eq. (3.11). We see that now all eigenvalues converge in the limit $\Delta\tau \to 0$.

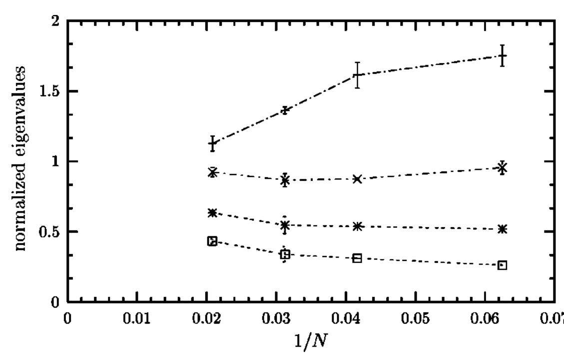


FIG. 3. The eigenvalues of the new tensor $T^{(\text{new})}_{\mu,\nu}$, extrapolated to $\Delta\tau = 0$ and normalized by the factor $(\sqrt{g} N^{1/4})^{-1}$. We also include the results for $N = 48$, which are obtained at $\Delta\tau = 0.002$ (our statistics at other values of $\Delta\tau$ is insufficient for a sensible extrapolation in this case).

## IV. A PHYSICAL CRITERION IN TERMS OF WILSON LOOPS

The results in the previous section reveal a subtlety in the issue of SSB in the 4D SUSY model. The crucial question is whether any signal of SSB can be probed by physical quantities, such as scattering amplitudes. Therefore we have to reconsider how the type IIB matrix model is interpreted as a string theory. In Ref. [16] it has been demonstrated that Wilson loops in matrix models can be identified with string creation operators in string theory. Hence Wilson loop correlation functions are the only objects with a direct physical interpretation in string theory. So we should ask whether any signal of SSB can be probed by Wilson loops.

We recall that the extent of the space-time can be probed by the vacuum expectation value (VEV) of the "Polyakov line" [13]

$$P(\vec{p}) = \frac{1}{N} \mathrm{Tr} \exp(i p_\mu A_\mu), \tag{4.1}$$

where $p_\mu$ represents the total dimensionful (and hence "physical") momentum carried by the string.[7] The VEV $\langle P(\vec{p}) \rangle$ depends only on $p = \sqrt{p_\mu p_\mu}$ due to the SO(4) invariance. It starts at 1 for $p = 0$ and drops down to zero at some value $\bar{p}$. Then $1/\bar{p}$ is a measure for the extent of the space-time. In Ref. [13], it was shown that the one-point function $\langle P(\vec{p}) \rangle$, as well as other Wilson loop correlators, converge to a certain function of $p$ in the large $N$ limit, if the scale parameter $g$ is taken to be proportional to $1/\sqrt{N}$. This means in particular that $\bar{p} \propto (\sqrt{g} N^{1/4})^{-1}$. [If we set $g = (N/48)^{-1/2}$, as we do in Figs. 4 and 5, we find $\bar{p} \approx 0.7$ for $N = 16, \ldots, 48$.] Therefore, the large $N$ behavior of the space-time extent probed by the Polyakov line is $1/\bar{p} \propto \sqrt{g} N^{1/4}$, which is consistent with the result obtained from $R_{\rm new}$ defined in Eq. (3.9).

Let us formulate the SSB of Lorentz symmetry by using the Wilson loops as a probe. For each configuration $A_\mu$ we perform a SO(4) transformation $\tilde{A}_\mu = \Lambda_{\mu\nu} A_\nu$ so that $\tilde{I}_{\mu\nu} = (1/N) \mathrm{Tr}(\tilde{A}_\mu \tilde{A}_\nu)$ becomes diagonal: $\tilde{I} = \mathrm{diag}(\omega_1, \omega_2, \omega_3, \omega_4)$, where $\omega_1 > \omega_2 > \omega_3 > \omega_4 > 0$. Then we define

$$\tilde{P}_\mu(p) = \frac{1}{N} \mathrm{Tr} \exp(i p \tilde{A}_\mu). \tag{4.2}$$

As a consequence of the diagonalization, the VEVs $\langle \tilde{P}_\mu(p) \rangle$ $(\mu = 1, \ldots, 4)$ do depend on $\mu$ for finite $N$. If these functions are different even in the large $N$ limit, we may conclude that the SSB of SO(4) symmetry occurs.

This method is analogous to the Ising model, where the magnetization $\langle |M| \rangle$ ($M$ is the sum of spins divided by their number), serves as an order parameter for the SSB of $Z_2$ symmetry. Taking the absolute value of $M$ corresponds to making an appropriate SO(4) transformation from $A_\mu$ to $\tilde{A}_\mu$. Note, on the other hand, that $\langle M \rangle = 0$ for any finite lattice volume even if the SSB takes place (for infinite lattice volume). Similarly, the VEV $\langle P(\vec{p}) \rangle$ in Eq. (4.1) depends only on $p = \sqrt{p_\mu p_\mu}$, even if the SO(4) symmetry is spontaneously broken.

[7]This momentum $p$ is identical to $k_{\rm phys} (= k/\sqrt{g})$ introduced in Ref. [13]. We take this opportunity to correct a typo in Fig. 5 of Ref. [13]. The label for the horizontal axis should be $k/\sqrt{g}$ instead of $k^2/g$. This typo has propagated to our review articles [24,25] as well. Figure 3 of [24] and Fig. 3 of [25] should have $p$ instead of $p^2$ as the label.

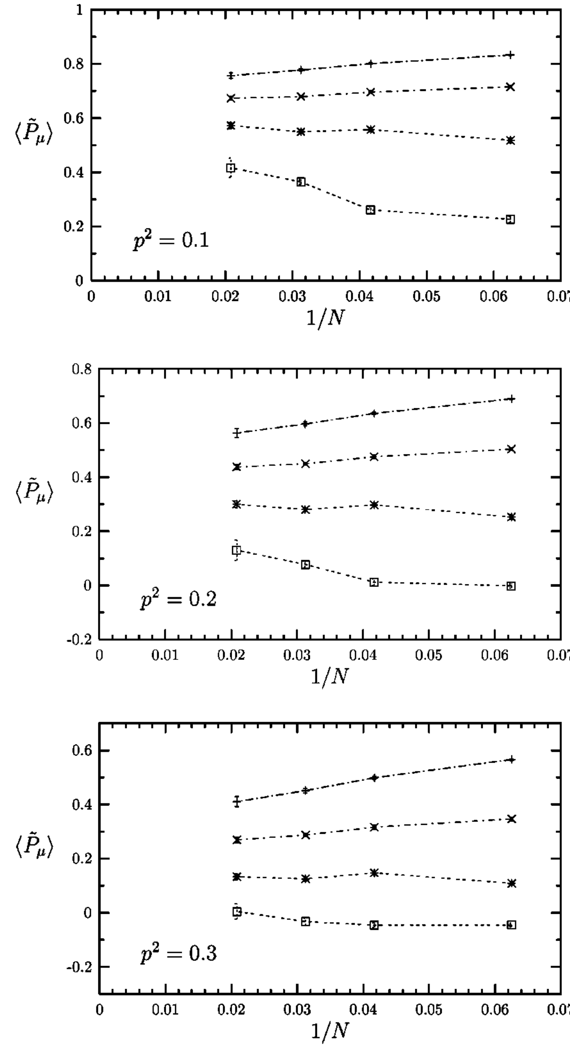


FIG. 4. The four Polyakov lines $\langle \tilde{P}_\mu(p) \rangle$ $(\mu = 1, \ldots, 4)$ at $p = 0.316,\ 0.447,\ 0.548$, where we take the scaling parameter $g$ to be $g = (N/48)^{-1/2}$. The Polyakov lines become approximately equidistant and they move closer together as $N$ increases; hence we do not see any signal for SSB of Lorentz symmetry. (The lines are drawn to guide the eye.)

Our results are shown in Fig. 4. (In this case we just present the results obtained at $\Delta\tau = 0.002$, which appears to be sufficiently small.) Here we set $g = (N/48)^{-1/2}$ and plot $\langle \tilde{P}_\mu(p) \rangle$ against $N$ at three different values of the momentum $p$, all of them below $\bar{p}$. We observe no trend of SSB up to $N = 48$. To illustrate this observation in yet another way, we show in Fig. 5 the functions $\langle \tilde{P}_\mu(p) \rangle$ for $N = 16$ and for $N = 48$. We see that the results for the four Polyakov lines move closer together as $N$ increases.

Finally, we clarify the relation of the above result to the previous result (3.4). Let us denote the eigenvalues of $\tilde{A}_\mu$ as $\alpha_{\mu i}$ $(i = 1, \ldots, N)$ and introduce the probability distribution of the $\alpha_{\mu i}$ as

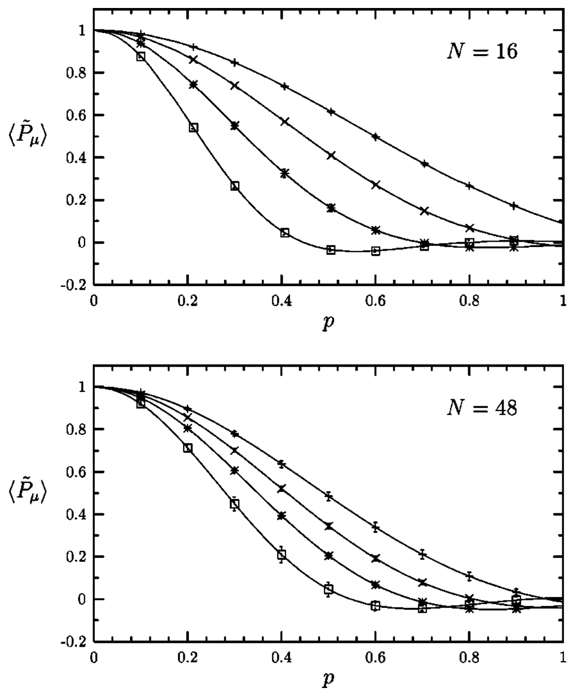


FIG. 5. The Polyakov lines $\langle \widetilde{P}_\mu(p)\rangle$ $(\mu=1,\ldots,4)$ at $N=16$ and at $N=48$. They move closer together as $N$ increases, in agreement with the proposed scenario that they coincide in the large $N$ limit. (The curves were measured quasi-continuously, and we show a few error bars as examples.)

$$f_\mu(x)=\left\langle \frac{1}{N}\sum_{i=1}^{N} \delta(x-\alpha_{\mu i})\right\rangle . \tag{4.3}$$

Then we can write $\langle \widetilde{P}_\mu(p)\rangle$ as

$$\langle \widetilde{P}_\mu(p)\rangle=\left\langle \frac{1}{N}\sum_{i=1}^{N} e^{ip\alpha_{\mu i}}\right\rangle=\int_{-\infty}^{\infty} dx f_\mu(x) e^{ipx}. \tag{4.4}$$

Thus the VEV of the Polyakov line is just the Fourier transform of the distribution $f_\mu(x)$. Our observation that $\langle \widetilde{P}_\mu(p)\rangle$ for $\mu=1,2,3,4$ approach the same function of $p$ at $N=\infty$ implies that their inverse Fourier transforms $f_\mu(x)$ for $\mu=1,2,3,4$ also approach the same function of $x$ at $N\to\infty$. On the other hand, our result in Fig. 1 suggests that $f_1(x)\sim C(N)/|x|^3$ at large $x$, but the $f_\mu(x)$ with $\mu=2,3,4$ have a sharper fall-off at large $x$. This suggests that the coefficient $C(N)$ vanishes in the large $N$ limit (at fixed $g^2N$).

We can rephrase the above statement in the following way. Note that $\langle \omega_\mu\rangle=\langle (1/N)\mathrm{Tr}(\widetilde{A}_\mu)^2\rangle$ (here we do not sum over the index $\mu$ on the right-hand side) can be written as

$$\begin{aligned}\langle \omega_\mu\rangle&=\left\langle \frac{1}{N}\sum_{i=1}^{N} (\alpha_{\mu i})^2\right\rangle=\int_{-\infty}^{\infty} dx x^2 f_\mu(x)\\ &=-\frac{d^2}{dp^2}\langle \widetilde{P}_\mu(p)\rangle\bigg|_{p=0}. \end{aligned}\tag{4.5}$$

The logarithmic divergence of $\langle \omega_1\rangle$ found in the previous section (consider footnote 5) implies that $\langle \widetilde{P}_1(p)\rangle$ has a non-analyticity at $p=0$ of the form

$$\langle \widetilde{P}_1(p)\rangle \sim 1+\widetilde{C}(N)p^2\ln p+\cdots. \tag{4.6}$$

On the other hand, $\langle \widetilde{P}_\mu(p)\rangle$ with $\mu=2,3,4$ do not have this non-analyticity, since $\langle \omega_2\rangle$, $\langle \omega_3\rangle$, $\langle \omega_4\rangle$ are finite. Our observation that the four Polyakov lines $\langle \widetilde{P}_\mu(p)\rangle$ $(\mu=1,\ldots,4)$ converge to a single function of $p$ suggests that the coefficient $\widetilde{C}(N)$ in Eq. (4.6) vanishes in the large $N$ limit (at $g^2N$ fixed).

These statements can be checked by extracting the coefficient of the logarithmic divergence in Eq. (3.4). If we fit the data for $\langle \lambda_1\rangle/g$ to the expected asymptotic function $-c_1(N)\ln\Delta\tau+c_0(N)$, we find that the normalized coefficient $c_1(N)/\sqrt{N}$ amounts to 2.2(3), 2.52(7), 1.75(7) for $N=16$, 24 and 32, respectively. Hence the behavior $C,\widetilde{C}\to 0$ as $N\to\infty$ is conceivable.

To summarize, our observation in this section implies that the quantities

$$\int_{-\infty}^{\infty} dx\, x^2[\lim_{N\to\infty} f_\mu(x)]=-\frac{d^2}{dp^2}[\lim_{N\to\infty}\langle \widetilde{P}_\mu(p)\rangle]\bigg|_{p=0} \quad (\mu=1,\ldots,4) \tag{4.7}$$

are all finite and equal. Notice that in Eq. (4.7), the limit $N\to\infty$ is taken *before* setting $p=0$. The point is that one should first take the large $N$ limit of the Wilson loop to make it actually physical, and *then* its derivatives at $p=0$ inherit a physical meaning, too. This does not need to be true for derivatives at $p=0$ for finite $N$. In fact, our results suggest that the $p\to 0$ limit and the large $N$ limit do not commute.

## V. DISCUSSION

In this paper, we wanted to clarify the issue of spontaneous Lorentz symmetry breakdown in supersymmetric matrix models, which was raised by Refs. [14]. We propose a physical criterion for SSB, which we consider as a solution to this problem. In the particular case of the 4D SUSY model, configurations with only one-dimensional extent dominate when one adopts a conventional criterion for the SSB using the moment of inertia tensor, as was suggested by Refs. [14]. However, contributions of those configurations to physical quantities such as Wilson loop correlators seem to be strongly suppressed in the large $N$ limit. Indeed, if we rely on our physical criterion using Wilson loops as a probe, we do not observe any trend of SSB; the space-time probed by Wilson loops appears to be four-dimensional.

Let us comment on the cases $D=6$ and $D=10$. We recall that in the $D$-dimensional SUSY models in general, $D=4,6,10$, the eigenvalue distribution of $A_\mu$ has a slow fall-off with the power $-(2D-5)$ independent of $N$ [15]. Therefore, as pointed out also in Ref. [14], a problem may arise if one considers a tensor such as $(I^n)_{\mu\nu}$, where $I_{\mu\nu}=(1/N)\mathrm{Tr}(A_\mu A_\nu)$ and $n\geqslant D-3$. The VEVs of the corresponding eigenvalues can be written as

$$\langle(\omega_\mu)^n\rangle = \left\langle \frac{1}{N}\sum_{i=1}^{N}(\alpha_{\mu i})^{2n}\right\rangle = \int_{-\infty}^{\infty} dx\, x^{2n} f_\mu(x)$$

$$= (-1)^n \frac{d^{2n}}{dp^{2n}}\langle \tilde{P}_\mu(p)\rangle\bigg|_{p=0}. \tag{5.1}$$

However, one has to take the large $N$ limit before setting $p=0$, in order to have a correct interpretation in string theory. The possible divergence of $\langle(\omega_1)^n\rangle$ for finite $N$ does not immediately imply an appearance of the SSB of Lorentz symmetry in physical quantities, as we have seen in $D=4$. On the other hand, the conventional moment of inertia tensor $I_{\mu\nu}$ (or $T_{\mu\nu}$ in Sec. III) does not have such a problem in $D=6,10$, since the $p\to 0$ limit and the large $N$ limit commute. If one observes an SSB with the conventional moment of inertia tensor, it immediately implies an SSB in physical quantities.

The absence of SSB in 4D SUSY model (with the physical criterion) is consistent with the conjecture that the phase of the determinant in the fermionic partition function plays a crucial role in a possible SSB of Lorentz symmetry. This conjecture is supported by the following results:

(1) The bosonic model does not show SSB [9].

(2) The SUSY model in $D=6$ and $D=10$ (using a low-energy effective theory) does not exhibit SSB if one omits the phase of the determinant [12].

(3) SSB does appear in the $\nu$-deformed SUSY model in $D=6$ and $D=10$ in the large $\nu$ limit, where $\nu$ couples to the phase of the fermionic partition function [5].

(4) No SSB occurs in the 4D SUSY model (where the fermionic partition function is real positive), as discussed in the present work, where we do not use any simplification of the full model.

When one integrates over the bosonic matrices in the 6D or 10D type IIB matrix model, the phase of the fermion integral fluctuates rapidly except for the vicinity of configurations, for which the phase becomes stationary. In fact, this happens for any $d$-dimensional configurations with $3\leqslant d \leqslant D-2$ [5]. Those configurations are therefore considerably enhanced compared to the case where the phase is omitted. By using a saddle-point approximation, it was found that only the dimensionalities within the above range are possible. Note in particular that $d=4$ is not excluded.

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[1] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
[2] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. **B498**, 467 (1997).
[3] J. Greensite and F. R. Klinkhamer, Nucl. Phys. **B304**, 108 (1988); V. A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989).
[4] G. K. Savvidy, Phys. Lett. **159B**, 325 (1985); M. Claudson and M. B. Halpern, Nucl. Phys. **B250**, 689 (1985).
[5] J. Nishimura and G. Vernizzi, J. High Energy Phys. **04**, 015 (2000).
[6] J. Nishimura and G. Vernizzi, Phys. Rev. Lett. **85**, 4664 (2000).
[7] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[8] N. Ishibashi, S. Iso, H. Kawai, and Y. Kitazawa, Nucl. Phys. **B583**, 159 (2000).
[9] T. Hotta, J. Nishimura, and A. Tsuchiya, Nucl. Phys. **B545**, 543 (1999).
[10] W. Krauth and M. Staudacher, Phys. Lett. B **435**, 350 (1998); ''Statistical Physics Approach to M-theory Integrals,'' cond-mat/0010127. S. Horata and H. Egawa, Phys. Rev. Lett. **86**, 4455 (2001); S. Oda and F. Sugino, J. High Energy Phys. **03**, 026 (2001); K. N. Anagnostopoulos, J. Nishimura, and P. Olesen, *ibid.* **04**, 034 (2001); P. Austing and J. F. Wheater, *ibid.* **02**, 028 (2001).
[11] P. Austing and J. F. Wheater, J. High Energy Phys. **04**, 019 (2001).
[12] J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 011 (2000).
[13] J. Ambjørn, K. N. Anagnostopoulos, W. Bientenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 013 (2000).
[14] Z. Burda, B. Petersson, and J. Tabaczek, Nucl. Phys. **B602**, 399 (2001); see also P. Bialas, Z. Burda, B. Petersson, and J. Tabaczek, *ibid.* **B592**, 391 (2001).
[15] W. Krauth and M. Staudacher, Phys. Lett. B **453**, 253 (1999).
[16] M. Fukuma, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. **B510**, 158 (1998).
[17] W. Krauth, H. Nicolai, and M. Staudacher, Phys. Lett. B **431**, 31 (1998).
[18] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, and T. Tada, Prog. Theor. Phys. **99**, 713 (1998).
[19] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, Prog. Theor. Phys. Suppl. **134**, 47 (1999).
[20] N. Kitsunezaki and S. Uehara, Prog. Theor. Phys. **105**, 515 (2001).
[21] J. Ambjørn and S. Varsted, Phys. Lett. B **257**, 305 (1991).

[22] S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. D **35**, 2531 (1987).
[23] S. Iso and H. Kawai, Int. J. Mod. Phys. A **15**, 651 (2000).
[24] J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, Nucl. Phys. B (Proc. Suppl.) **94**, 685 (2001).
[25] J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, ''Monte Carlo Studies of the Dimensionally Reduced 4D SU(N) Super Yang-Mills Theory,'' hep-th/0101084.