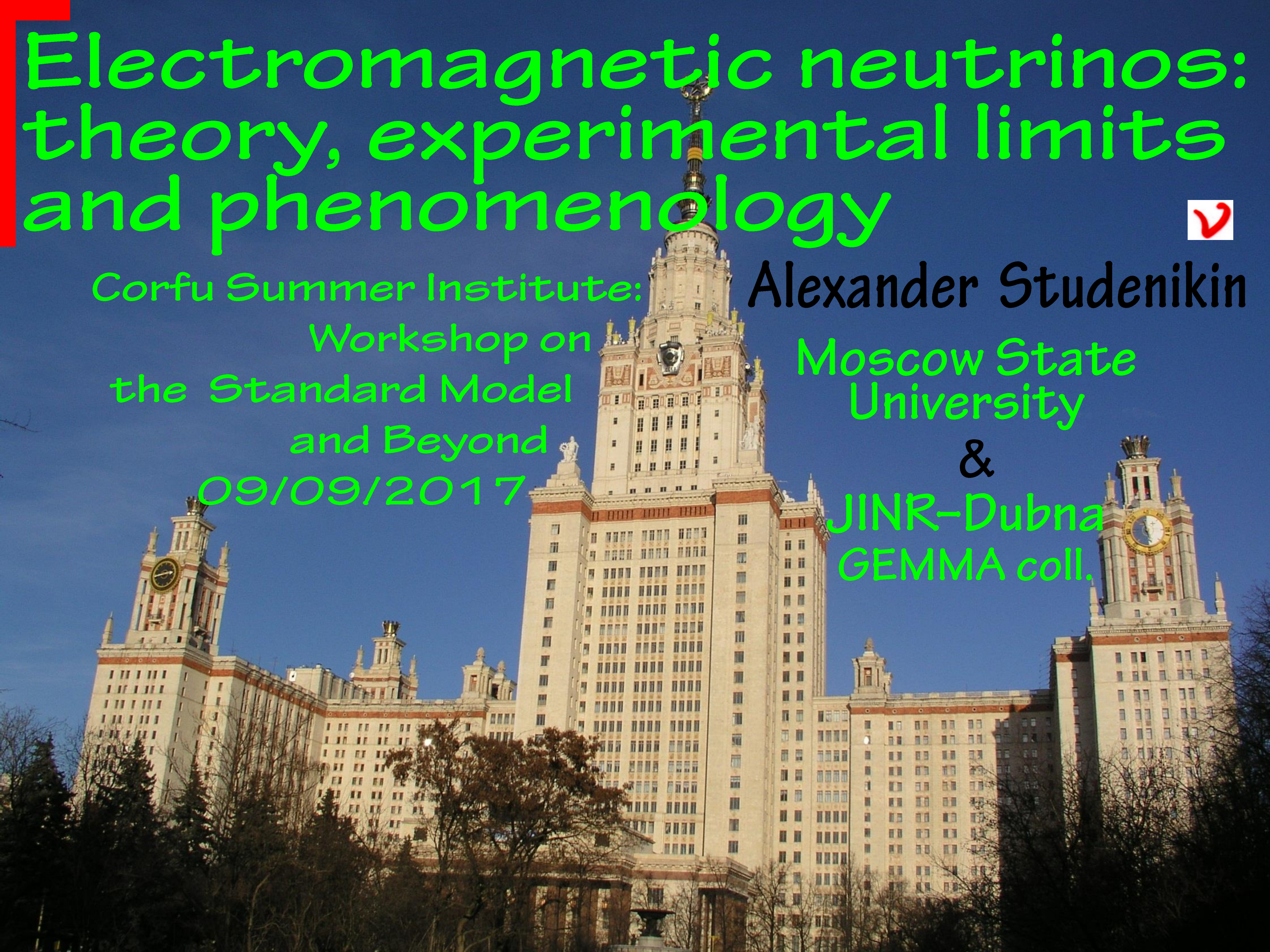


Electromagnetic neutrinos: theory, experimental limits and phenomenology

Corfu Summer Institute:
Workshop on
the Standard Model
and Beyond
09/09/2017

Alexander Studenikin

Moscow State University
&
JINR-Dubna
GEMMA coll.

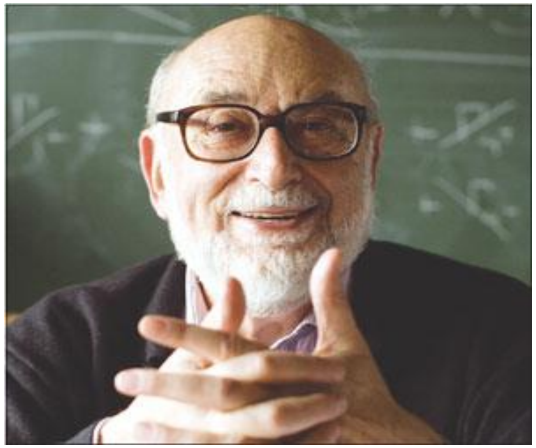
Discovery of Higgs boson at LHC

is one of the most important results in

Particle Physics

has ever obtained

Robert Brout



François Englert

Peter Higgs

Observation of Higgs boson confirms the symmetry breaking mechanism by Brout-Englert-Higgs (BEH)

- provides final glorious triumph of

Standard Model

that was crowned by

Nobel Prize 2013

… since 2013 studies of ν properties is the most promising way in search for NEW Physics

Beyond Standard Model

ν electromagnetic properties

REVIEWS OF MODERN PHYSICS, VOLUME 87, APRIL–JUNE 2015

Neutrino electromagnetic interactions: A window to new physics

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(published 16 June 2015)

A review is given of the theory and phenomenology of neutrino electromagnetic interactions, which provide powerful tools to probe the physics beyond the standard model. After a derivation of the general structure of the electromagnetic interactions of Dirac and Majorana neutrinos in the one-photon approximation, the effects of neutrino electromagnetic interactions in terrestrial experiments and in astrophysical environments are discussed. The experimental bounds on neutrino electromagnetic properties are presented and the predictions of theories beyond the standard model are confronted.

DOI: 10.1103/RevModPhys.87.531 PACS numbers: 14.60.St, 13.15.+g, 13.35.Hb, 14.60.Lm

CONTENTS

Outline

① review of ν electromagnetic properties

② experimental constraints on μ_ν and q_ν
(including GEMMA and Borexino collabs. results)

③ ν electromagnetic interactions (new effects)

④ new ν spin (flavour) oscillations

Studenikin (2004, 2017)

Бруно Понтекорво

Staff member at
Faculty of Physics of
Moscow State University,
1966 - 1986

if $m_\nu \neq 0$

then $\nu \leftrightarrow \bar{\nu}$

in vacuum

Bruno Pontecorvo,
«Inverse β processes and nonconcervation of leptonic charge»,
JINR Preprint P-95, Dubna, 1957,
3 pages :

60 years of mixing and oscillations !

«Neutrinos in vacuum can transform themselves into antineutrino and vice versa. This means that neutrino and antineutrino are particle mixtures… So, for example, a beam of neutral leptons from a reactor which at first consists mainly of antineutrinos will change its composition and at a certain distance R from the reactor will be composed of neutrino and antineutrino in equal quantities».

Бруно Понтекорво

Bruno Pontecorvo,
«Мезоний и антимезоний»,
ЖЭТФ 33 (1957) 549-551 :

«Выше предполагалось, что имеет место закон сохранения нейтринного заряда… Этот закон пока не установлен… Если теория двухкомпонентного нейтрино оказалась бы неверной… и если бы не имел места закон сохранения нейтринного заряда, то в принципе переходы

нейтрино → антинейтрино
в вакууме возможны».

if

$m_\nu \neq 0$

then

$\boxed{\nu \leftrightarrow \bar{\nu}}$

In vacuum

… problem and puzzle …

ν electromagnetic properties

up to now nothing has been seen

… in spite of reasonable efforts …

- results of terrestrial lab experiments on μ_ν (and ν EM properties in general)
- as well as data from astrophysics and cosmology

are in agreement with "ZERO" ν EM properties

… However, in course of recent development of knowledge on ν mixing and oscillations,

ν exhibits unexpected properties (puzzles)

W. Pauli, 1930

- neutral "neutron" $\Rightarrow$ ν E.Fermi, 1933
- probably $\mu_\nu \neq 0$!?

Pauli himself wrote to Baade:

"Today I did something a physicist should never do. I predicted something which will never be observed experimentally..."

H. Bethe, R. Peierls,

«The ‘neutrino’»

Nature 133 (1934) 532

- «There is no practically possible way of observing the neutrino» !

... puzzles ...

- ... what about electromagnetic properties of ν ?

Arthur McDonald

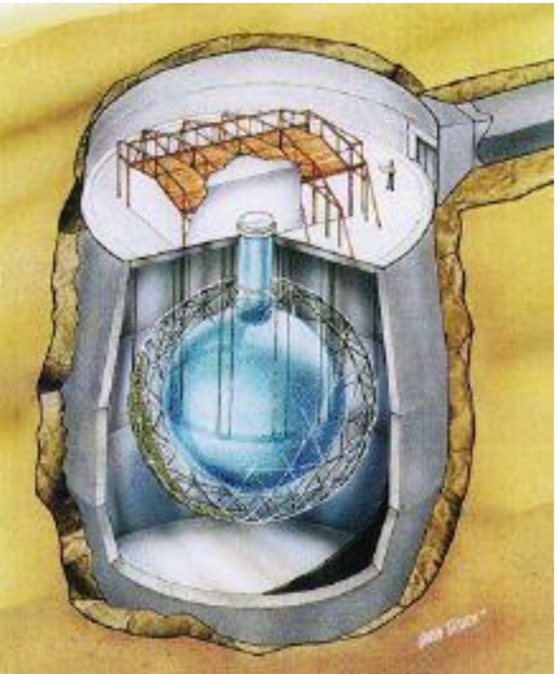
Sudbury Neutrino Observatory

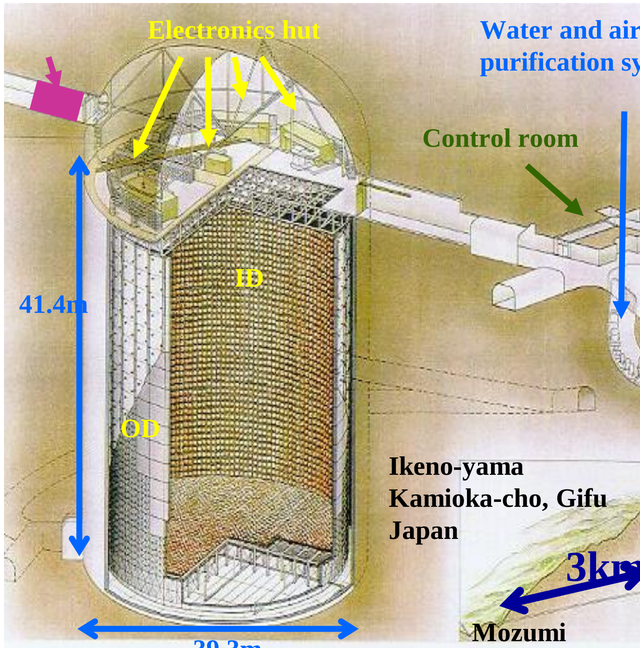
The Nobel Prize in Physics 2015

Takaaki Kajita

Super-Kamiokande Experiment

« for the discovery of neutrino oscillations, which shows that neutrinos have mass »





$m_\nu \neq 0$ … a tool for studying physics

Beyond Extended Standard Model…

Theory (Standard Model with ν_R)

$$\mu_\nu = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \sim \boxed{3\cdot 10^{-19}} \mu_B \left(\frac{m_\nu}{1\,\mathrm{eV}}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Shrock, 1977; Fujikawa Shrock, 1980

magnetic moment

$$a_e = \frac{\alpha_{QED}}{2\pi} \sim 10^{-3}$$

… much greater values are desired

for astrophysical or cosmology

visualization of μ_ν

new physics

… hopes for physics BESM …

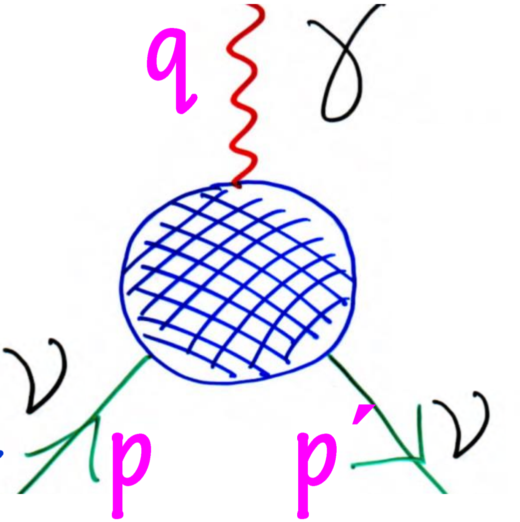
①

ν electromagnetic properties

(flash on theory)

$$m_\nu \neq 0$$

✓ electromagnetic vertex function



$$< \psi(p')|J_\mu^{EM}|\psi(p) >= \bar{u}(p')\Lambda_\mu(q,l)u(p)$$

Matrix element of electromagnetic current is a Lorentz vector

$\Lambda_\mu(q,l)$ should be constructed using ●

matrices $\hat{1}, \quad \gamma_5, \quad \gamma_\mu, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu},$

tensors $g_{\mu\nu}, \quad \epsilon_{\mu\nu\sigma\gamma}$

vectors q_μ and l_μ

$$\boxed{q_\mu = p'_\mu - p_\mu, \; l_\mu = p'_\mu + p_\mu}$$

● Lorentz covariance (1) and electromagnetic gauge invariance (2) ⇒

Matrix element of electromagnetic current between neutrino states

$$\langle\nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p),$$

where vertex function generally contains 4 form factors

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
$$+f_A(q^2)(q^2\gamma_\mu-q_\mu \not{q})\gamma_5$$

1. electric

2. magnetic

dipole

3. electric

4. anapole

- Hermiticity and discrete symmetries of EM current J_μ^{EM} put constraints on form factors

Dirac ν

1) CP invariance + Hermiticity $\Longrightarrow$ $f_E = 0$,

2) at zero momentum transfer only electric Charge $f_Q(0)$ and magnetic moment $f_M(0)$ contribute to $H_{int} \sim J_\mu^{EM}A^\mu$

3) Hermiticity itself $\Longrightarrow$ three form factors are real: $Imf_Q = Imf_M = Imf_A = 0$

Majorana ν

1) from CPT invariance (regardless CP or $\not{C}\not{P}$).

$$f_Q = f_M = f_E = 0$$

...as early as 1939, W.Pauli...

- EM properties $\Longrightarrow$ a way to distinguish Dirac and Majorana ν

In general case matrix element of $J_\mu^{\rm EM}$ can be considered between different initial $\psi_i(p)$ and final $\psi_j(p')$ states of different masses

$$\boxed{< \psi_j(p')|J_\mu^{EM}|\psi_i(p) >= \bar{u}_j(p')\Lambda_\mu(q)u_i(p)}$$

$p^2 = m_i^2, \ p'^2 = m_j^2:$

… beyond SM…

and

$$\boxed{\Lambda_\mu(q) = \Big(f_Q(q^2)_{ij} + f_A(q^2)_{ij}\gamma_5\Big)(q^2\gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i\sigma_{\mu\nu}q^\nu + f_E(q^2)_{ij}\sigma_{\mu\nu}q^\nu\gamma_5}$$

form factors are matrices in ν mass eigenstates space.

Dirac ν (off-diagonal case $i \neq j$) Majorana ν

1) Hermiticity itself does not apply restrictions on form factors,

2) CP invariance + Hermiticity

$$\Downarrow$$

$f_Q(q^2), \ f_M(q^2), \ f_E(q^2), \ f_A(q^2)$

are relatively real (no relative phases) .

… quite different EM properties …

1) CP invariance + hermiticity

$$\boxed{\mu_{ij}^M = 2\mu_{ij}^D \ \ and \ \ \epsilon_{ij}^M = 0}$$

or

$$\boxed{\mu_{ij}^M = 0 \ \ and \ \ \epsilon_{ij}^M = 2\epsilon_{ij}^D}$$

3

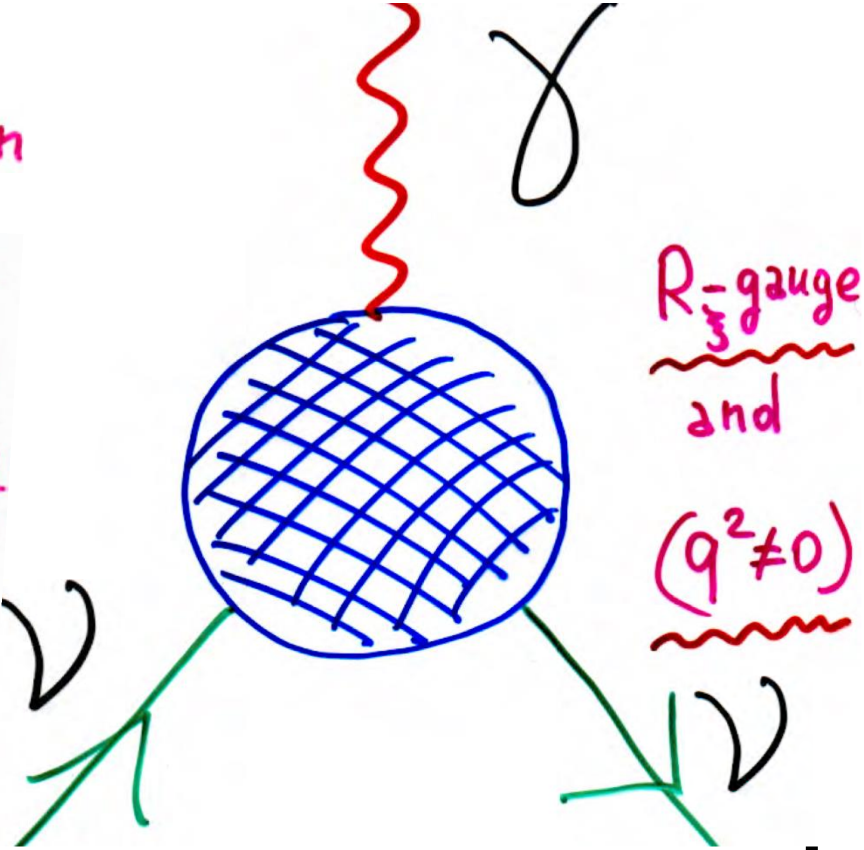
. . . a bit of ν electromagnetic properties theory

3.1 ν vertex function

The most general study of the **massive neutrino** vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

M. Dvornikov, A. Studenikin

⊛ Phys. Rev. D 63, 073001, 2004,
"Electric charge and magnetic moment of massive neutrino";
JETP 126 (2004), N8, 1
⊛ "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge
and
$(q^2 \neq 0)$

$$\Delta_\mu(q) = \overset{\text{charge}}{\underline{f_Q}}(q^2)\gamma_\mu + \overset{\text{magnetic moment}}{\underline{f_M}}(q^2)\, i\sigma_{\mu\nu}q^\nu - \underset{\text{electric moment}}{\underline{f_E}}(q^2)\, i\sigma_{\mu\nu}q^\nu\gamma_5 - \underset{\text{anapole moment}}{\underline{f_A}}(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

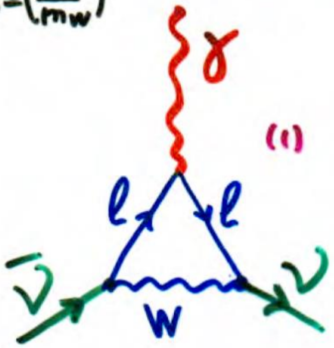
$a = \left(\frac{m_\ell}{m_W}\right)^2,$

$b = \left(\frac{m_\nu}{m_W}\right)^2$

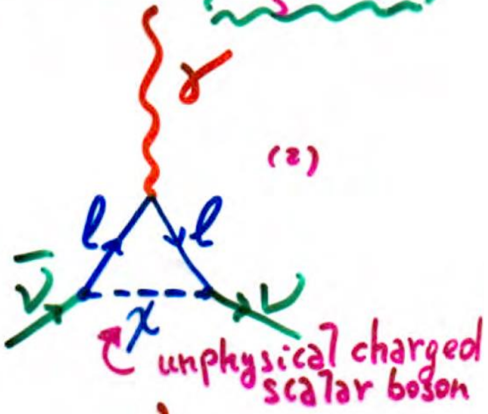
Proper vertices

R_ξ-gauge

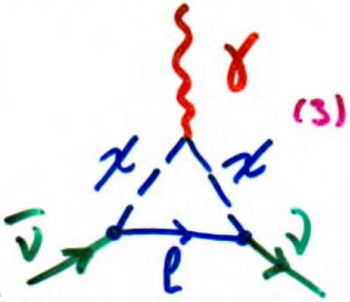
(1)



(2)

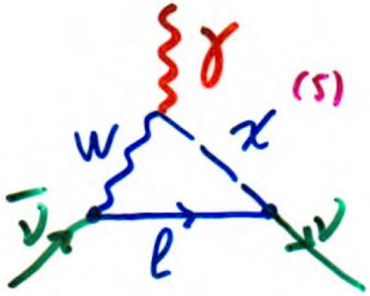


(3)

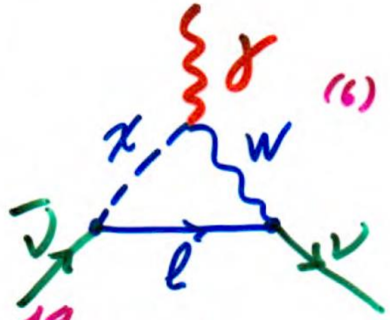


(4)

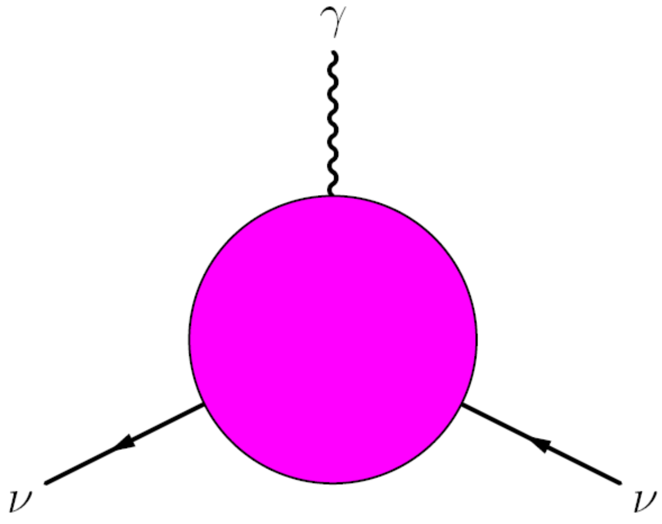
(5)



(6)



$$\Lambda_\mu(q) = \sum_{i=1}^{14} \Lambda_\mu^i(q)$$

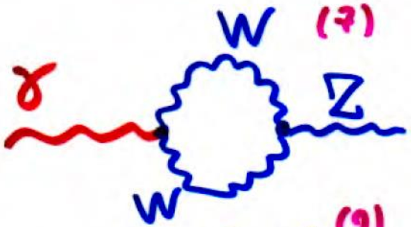


$$\Lambda_\mu(q)$$

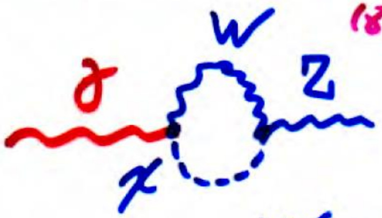
$$\Lambda^j_\mu(q) = \frac{g}{2\cos\theta_W}\,\Pi^{(j)}_{\mu\nu}(q)\,\frac{1}{q^2-M_Z^2} \times \left\{ g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2-\alpha_Z M_Z^2} \right\} \gamma^L_\alpha, \; j=7,\dots,14$$

γ-Z self-energy diagrams

(7)



(8)



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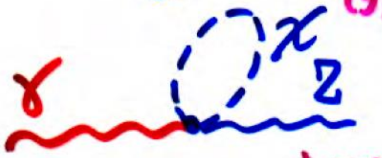
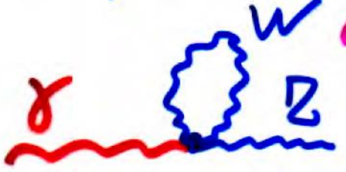


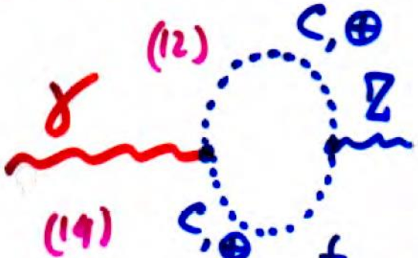
(10)



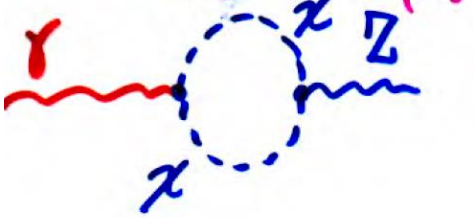
charged ghost

(11)

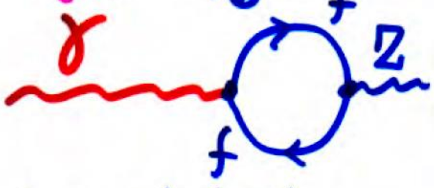
(12)



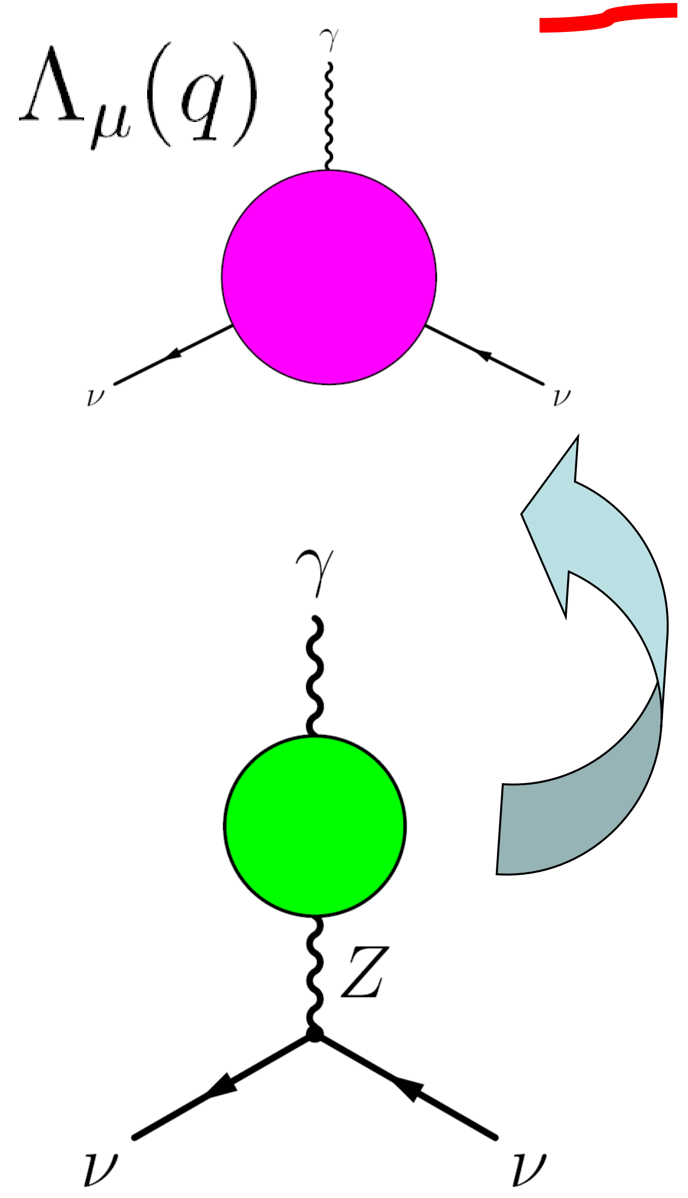
(13)



(14)



$f = u, c, t, d, s, b$
quarks



$\gamma - Z$ self-energy diagrams

Dipole magnetic $\boxed{f_M(q^2)}$ and electric $\boxed{f_E(q^2)}$ are most well studied and theoretically understood among form factors

…because in the limit $\boxed{q^2 \to 0}$ they have nonvanishing values

$\boxed{\mu_\nu = f_M(0)}$ ⟸ ν magnetic moment ●

$\boxed{\epsilon_\nu = f_E(0)}$ ⟸ ν electric moment ???

3.2

Calculation of ν magnetic moment
(massive ν , arbitrary R_ξ - gauge)

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\underline{\sigma_{\mu\nu}q^\nu}-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5+f_A(q^2)(q^2\gamma_\mu-q_\mu\not{q})\gamma_5$$

magnetic moment

Dvornikov, Studenikin, PRD 2004

$$\mu(a,b,\alpha)=f_M(q^2=0)$$

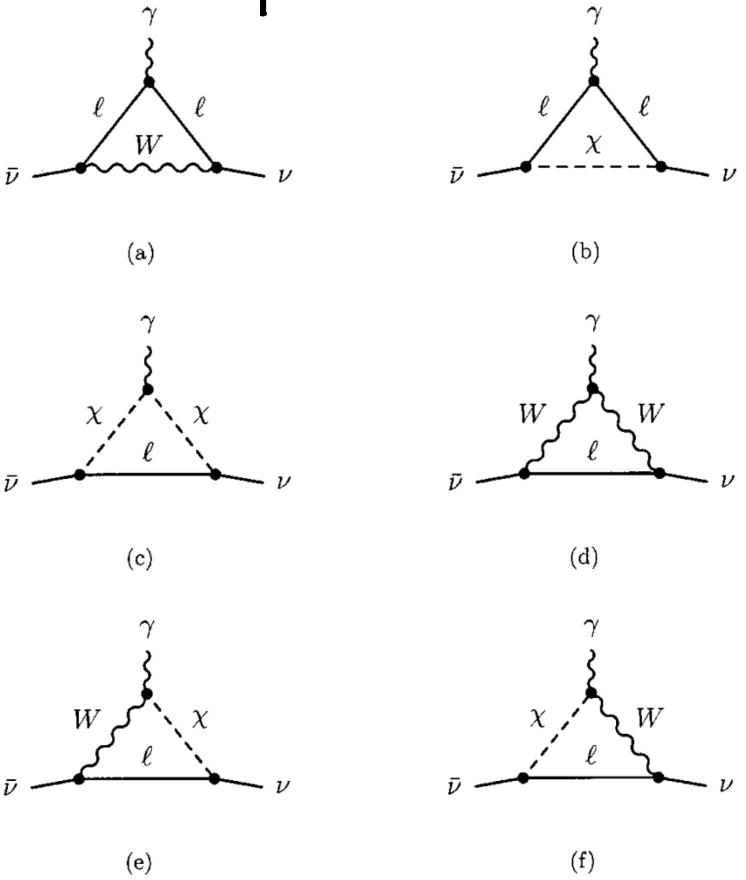
two mass parameters ● $a=\left(\frac{m_\ell}{M_W}\right)^2$ $b=\left(\frac{m_\nu}{M_W}\right)^2$ ●

$$\mu(a,b,\alpha)=\sum_{i=1}^{6}\mu^{(i)}(a,b,\alpha)$$

and gauge-fixing parameter $\alpha=\frac{1}{\xi}$ ●

$\xi=0$ - unitary gauge, $\xi=1$ - ‘t Hooft-Feynman gauge

Proper vertices



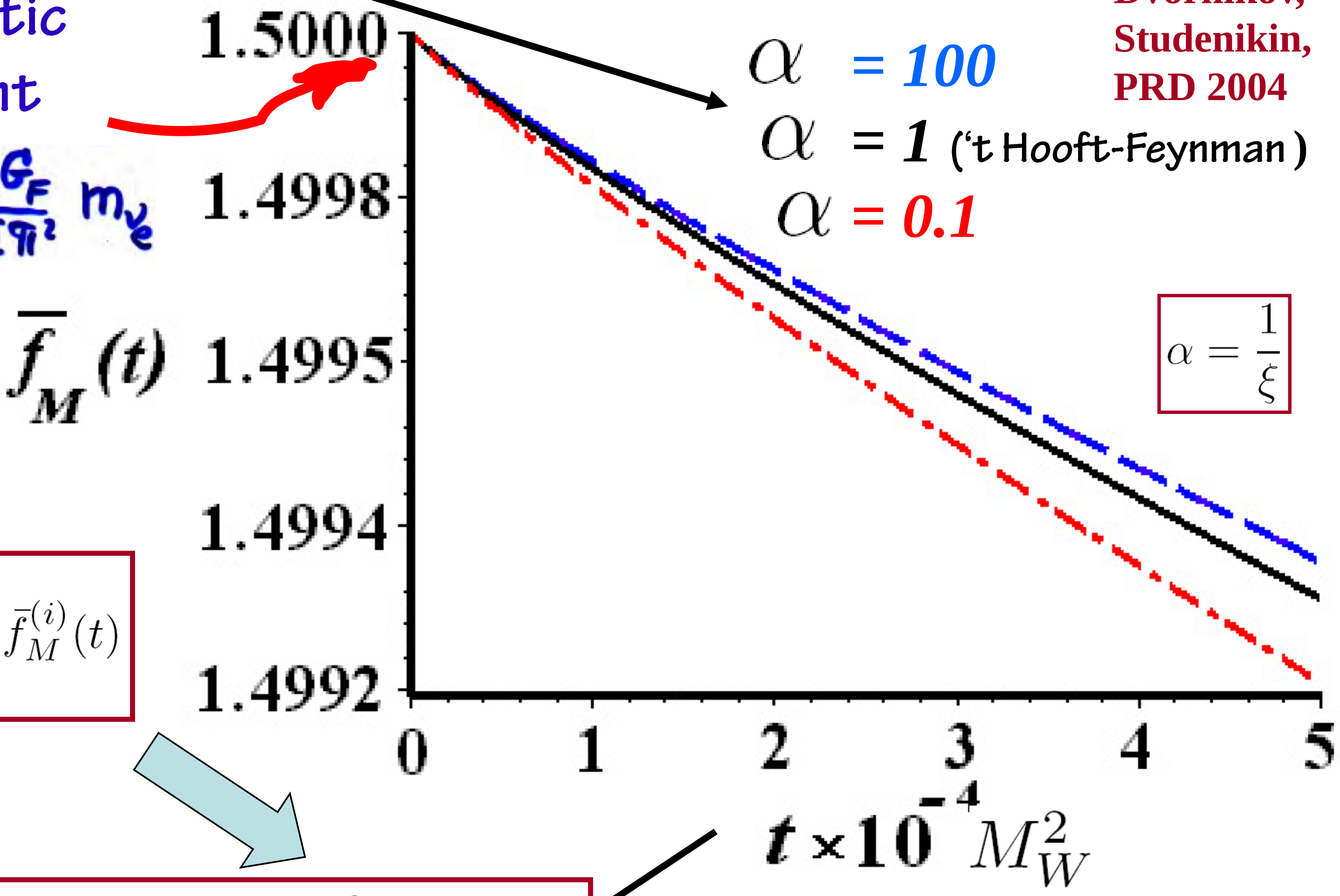
(a) (b) (c) (d) (e) (f)

Gauge and qxq dependence …

Dvornikov, Studenikin, PRD 2004

ν magnetic moment

● $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$



$$\alpha = \frac{1}{\xi}$$

$$\bar{f}_M(t) = \sum_{i=1}^{6} \bar{f}_M^{(i)}(t)$$

$$f_M(q^2) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \sum_{i=1}^{6} \bar{f}_M^{(i)}(q^2)$$

ν dipole magnetic form factor

- ● $m_\nu \ll m_e \ll M_W$ light ν → $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$

$$\mu_\nu = \frac{eG_F \boxed{m_\nu}}{4\pi^2\sqrt{2}} \frac{3}{4(1-a)^3}(2 - 7a + 6a^2 - 2a^2 \ln a - a^3) \qquad a = \left(\frac{m_e}{M_W}\right)^2$$

Gabral-Rosetti, Bernabeu, Vidal, Zepeda, Eur.Phys.J C 12 (2000) 633

Dvornikov, Studenikin, Phys.Rev.D 69 (2004) 073001; JETP 99 (2004) 254

- ● $m_e \ll m_\nu \ll M_W$ intermediate ν

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} \boxed{m_\nu} \left\{1 + \frac{5}{18} b\right\} \qquad b = \left(\frac{m_\nu}{M_W}\right)^2$$

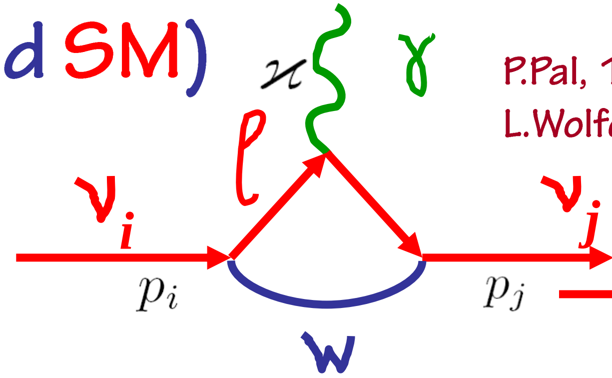
- ● $m_e \ll M_W \ll m_\nu$ heavy ν

$$\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} \boxed{m_\nu}$$

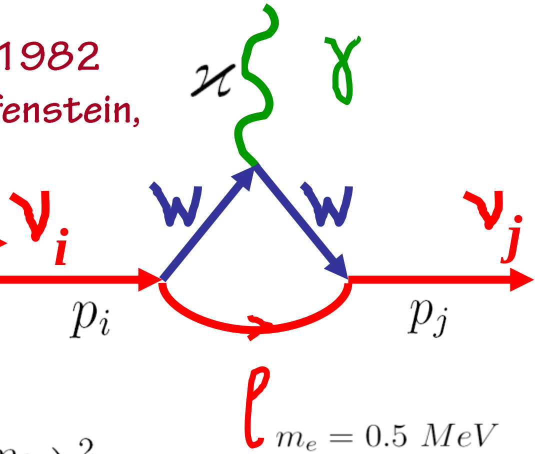
$\sim 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1eV}\right)$

… μ_ν in case of mixing … ➡ 0

Neutrino (beyond SM) dipole moments (+ transition moments)



P.Pal, 1982
L.Wolfenstein,



- **Dirac neutrino**

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\sum_{l=e,\,\mu,\,\tau} f(r_l)U_{lj}U^*_{li}$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

$m_e = 0.5\ MeV$
$m_\mu = 105.7\ MeV$
$m_\tau = 1.78\ GeV$
$m_W = 80.2\ GeV$

- $m_i, m_j \ll m_l, m_W$

$$f(r_l) \approx \underline{\frac{3}{2}}\left(1 - \underline{\frac{1}{2} r_l}\right), \quad r_l \ll 1$$

transition moments vanish because unitarity of U implies that its rows or columns represent orthogonal vectors

- **Majorana neutrino only for** $i \neq j$

$$\mu^M_{ij} = 2\mu^D_{ij} \quad and \quad \epsilon^M_{ij} = 0$$

or

$$\mu^M_{ij} = 0 \quad and \quad \epsilon^M_{ij} = 2\epsilon^D_{ij}$$

... depending on relative CP phase of ν_i and ν_j

- **transition moments are suppressed, Glashow-Iliopoulos-Maiani cancellation,**
- **for diagonal moments there is no GIM cancellation**

The first nonzero contribution from neutrino transition moments

$$f_{r_l} \to \cancel{-\frac{3}{2}} + \frac{3}{4}\left(\frac{m_l}{m_W}\right)^2 \quad << 1$$

GIM cancellation

$$\left.\begin{array}{c}\mu_{ij}\\ \epsilon_{ij}\end{array}\right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{l=\ e,\ \mu,\ \tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

$$\mu_B = \frac{e}{2m_e}$$

- $$\left.\begin{array}{c}\mu_{ij}\\ \epsilon_{ij}\end{array}\right\} = 4 \times 10^{-23}\mu_B\left(\frac{m_i \pm m_j}{1\ eV}\right)\sum_{l=\ e,\ \mu,\ \tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

… neutrino radiative decay is very slow

- Dirac ν diagonal (i=j) magnetic moment

$$\epsilon^D_{ii} = 0$$ for CP-invariant interactions

$$\mu^M_{ii} = \epsilon^M_{ii} = 0$$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=\ e,\mu,\tau} r_l \mid U_{li}\mid^2\right) \approx 3.2 \times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

Lee, Shrock, Fujikawa, 1977

- no GIM cancellation
- μ^D_{ii} - to leading order - independent on U_{li} and $m_{\ l=e,\ \mu,\ \tau}$
- $$\mu_e^2 = \sum_{i=1,2,3} \mid U_{ie}\mid^2 \mu^2_{ii}$$ …possibility to measure fundamental μ^D_{ii}

$\mu^D_{ii} = 0$ for massless ν (in the absence of right-handed charged currents)

...the present status...

to have visible $m_\nu \neq 0$

is not an easy task for

theoreticians

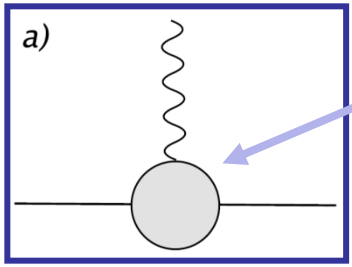
and experimentalists

3.3 Naïve relationship between m_ν and μ_ν

… problem to get large μ_ν *and still acceptable* m_ν

If μ_ν is generated by physics beyond the SM at energy scale Λ ,

P.Vogel e.a., 2006

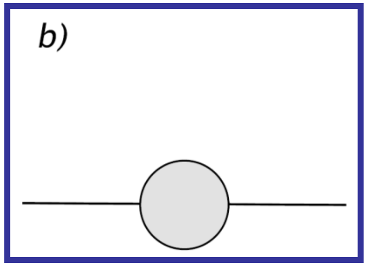


then

$$\boxed{\mu_\nu \sim \frac{eG}{\Lambda},}$$

…combination of constants and loop factors…

contribution to m_ν given by



, then

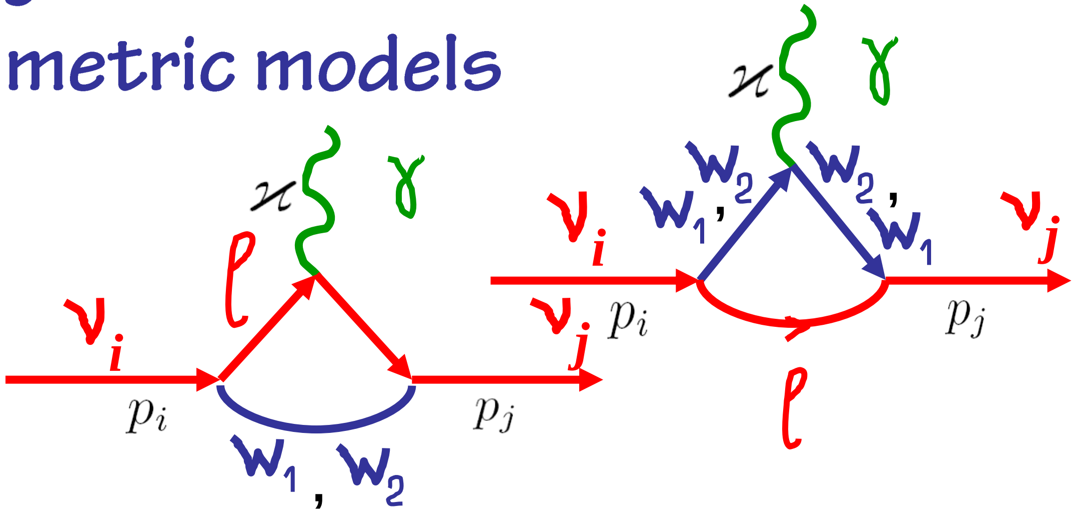
$$\boxed{m_\nu \sim G\Lambda}$$

$$m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18}\mu_B}[\Lambda(\mathrm{TeV})]^2 \ \ \mathrm{eV}$$

Voloshin, 1988;
Barr, Freire,
Zee, 1990

3.6 Neutrino magnetic moment in left-right symmetric models

$$SU_L(2)\times SU_R(2)\times U(1)$$



Gauge bosons mass states

$$W_1 = W_L\cos\xi - W_R\sin\xi$$
$$W_2 = W_L\sin\xi + W_R\cos\xi$$

with mixing angle ξ **of gauge bosons** $W_{L,R}$ **with pure** $(V\pm A)$ **couplings**

Kim, 1976; Marciano, Sanda, 1977;
Beg, Marciano, Ruderman, 1978

$$\boxed{\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[\underline{m_l}\left(1-\frac{m^2_{W_1}}{m^2_{W_2}}\right)\sin 2\xi + \frac{3}{4}\underline{m_{\nu_l}}\left(1+\frac{m^2_{W_1}}{m^2_{W_2}}\right)\right]}$$

… charged lepton mass …

… neutrino mass …

Large magnetic moment

$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}, m_{e^-})$

- In the L-R symmetric models $(SU(2)_L \times SU(2)_R \times U(1))$

Kim, 1976
Beg, Marciano, Ruderman 1978

- **Voloshin, 1988**
  "On compatibility of small m_ν with large μ_ν of neutrino",
  Sov.J.Nucl.Phys. 48 (1988) 512

  … there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

Z.Z.Xing, Y.L.Zhou,
"Enhanced electromagnetic transition dipole moments and radiative decays of massive neutrinos due to the seesaw-induced non-unitary effects"
Phys.Lett.B 715 (2012) 178

- ***Bar, Freire, Zee, 1990***
- **supersymmetry**
- **extra dimensions**

considerable enhancement of μ_ν to experimentally relevant range

- **model-independent constraint μ_ν**

$$\mu_\nu^D \leq 10^{-15} \mu_B$$

$$\mu_\nu^M \leq 10^{-14} \mu_B$$

for BSM ($\Lambda \sim 1\ \text{TeV}$) without fine tuning and under the assumption that $\delta m_\nu \leq 1\ \text{eV}$

Bell, Cirigliano, Ramsey-Musolf, Vogel, Wise, 2005

✓ magnetic moment in experiments

(most easily understood
and accessible for experimental
studies are dipole moments)

Studies of ν-e scattering
- most sensitive method for experimental investigation of μ_ν

Cross-section:

- $$\boxed{\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}}$$

where the Standard Model contribution

- $$\boxed{\left(\frac{d\sigma}{dT}\right)_{\rm SM} = \frac{G_{\rm F}^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right],}$$

T is the electron recoil energy and

- $$\boxed{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1 - T/E_\nu}{T}\right]\mu_\nu^2}$$

$$\boxed{\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left|\sum_i U_{li} e^{-iE_iL}\mu_{ji}\right|^2}$$

$$\mu_{ij} \to |\mu_{ij} - \epsilon_{ij}|$$

$$g_V = \begin{cases} 2\sin^2\theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2\sin^2\theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases}$$

for anti-neutrinos $g_A \to -g_A$

- to incorporate charge radius: $g_V \to g_V + \boxed{\frac{2}{3}M_W^2\langle r^2\rangle\sin^2\theta_W}$???

3.11 ν charge radius and anapole moment

$$\boxed{\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5+f_A(q^2)(q^2\gamma_\mu-q_\mu\not{q})\gamma_5}$$

1. electric

2. magnetic

dipole

3. electric

4. anapole

Although it is usually assumed that ν are electrically neutral (charge quantization implies $Q\sim\frac{1}{3}e$), ν can dissociates into charged particles so that $f_Q(q^2)\neq 0\ for\ q^2\neq 0$:

$$\boxed{f_Q(q^2)=f_Q(0)+q^2\frac{df_Q}{dq^2}(0)+\cdots,}$$

where the massive ν charge radius

$$\boxed{\langle r_\nu^2\rangle=-6\frac{df_Q}{dq^2}(0)}$$

For massless ν

anapole moment

$$\boxed{a_\nu=f_A(q^2)=\frac{1}{6}\langle r_\nu^2\rangle}$$

Interpretation of charge radius as an observable is rather delicate issue: $\langle r_\nu^2\rangle$ represents a correction to tree-level electroweak scattering amplitude between ν and charged particles, which receives radiative corrections from several diagrams (including γ exchange) to be considered simultaneously ⟹ calculated CR is infinite and gauge dependent quantity. For massless ν, a_ν and $\langle r_\nu^2\rangle$ can be defined (finite and gauge independent) from scattering cross section. ⟹

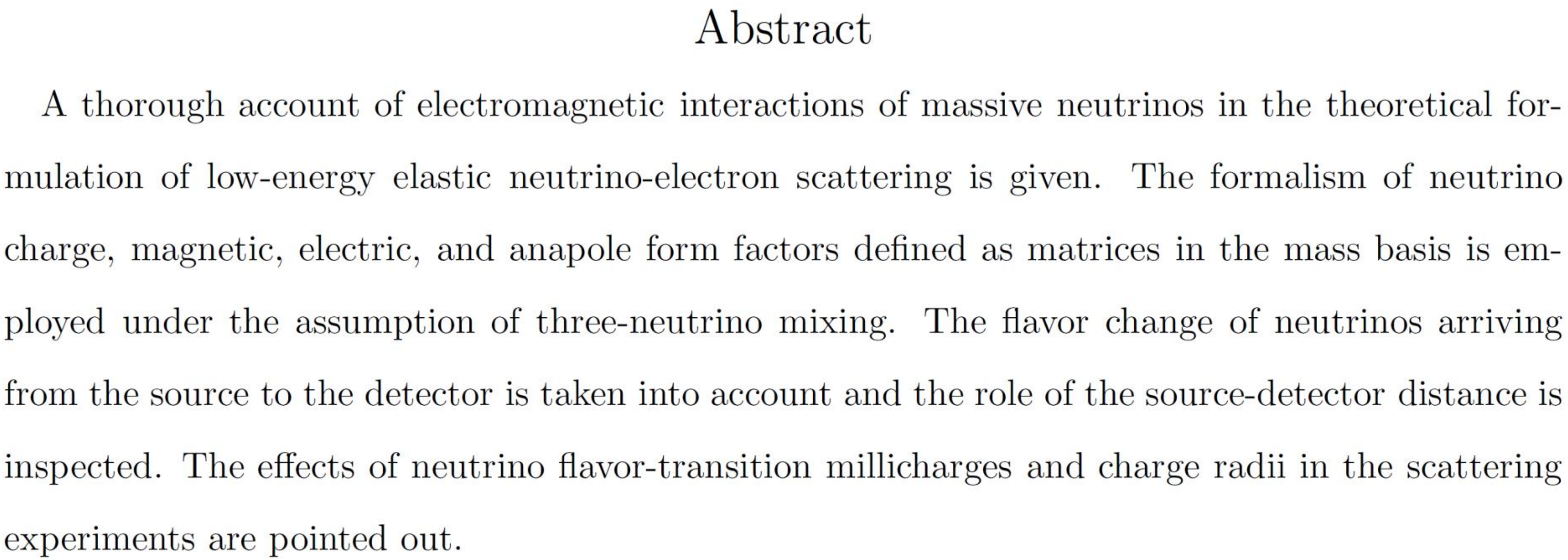
? ? ? For massive ν ? ? ?

Bernabeu, Papavassiliou, Vidal,
Nucl.Phys. B 680 (2004) 450

K. Kouzakov, A. Studenikin,

Phys. Rev. D 95 (2017) 055013

"Electromagnetic properties of massive neutrinos in low-energy elastic neutrino-electron scattering"

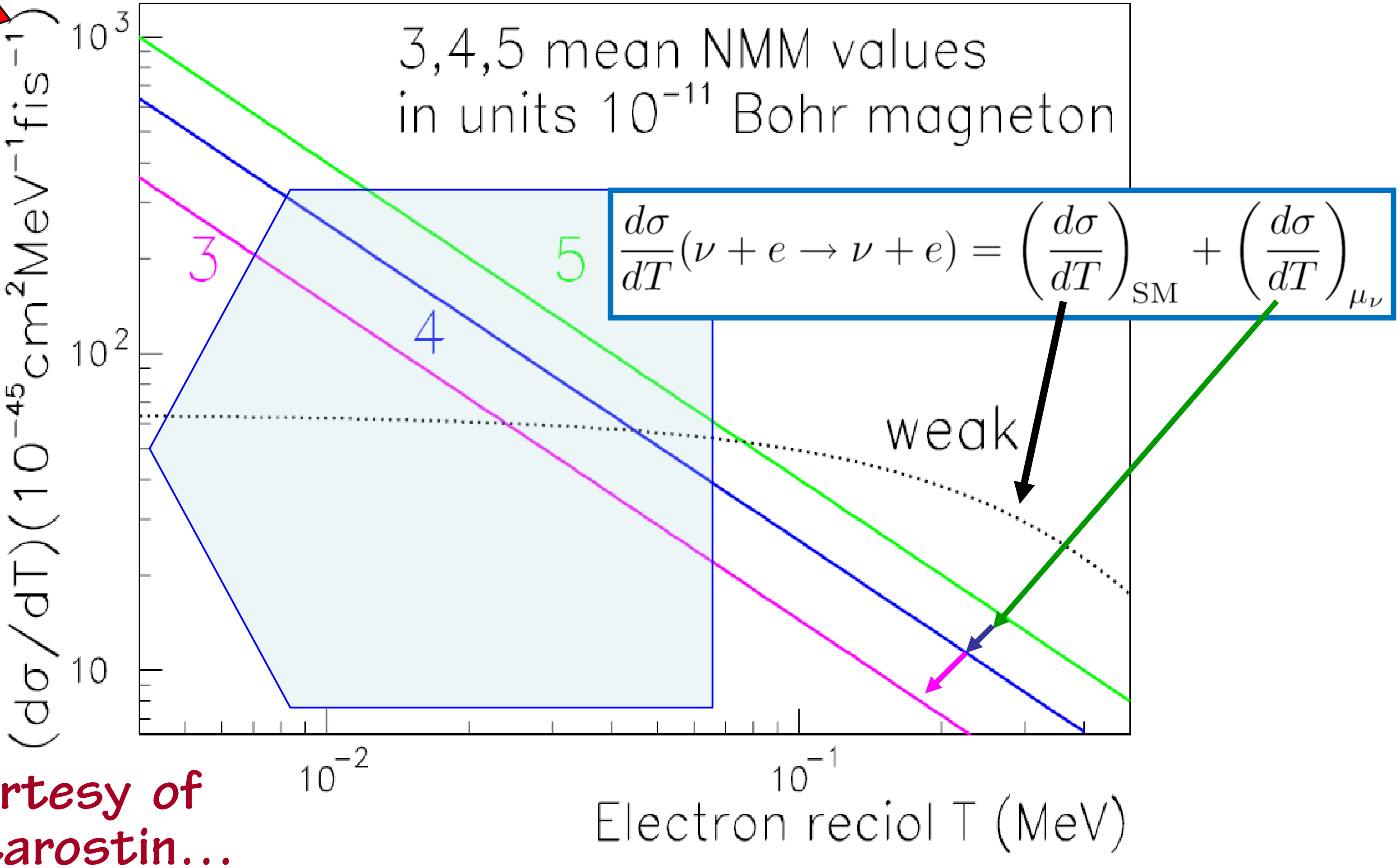


Abstract

A thorough account of electromagnetic interactions of massive neutrinos in the theoretical formulation of low-energy elastic neutrino-electron scattering is given. The formalism of neutrino charge, magnetic, electric, and anapole form factors defined as matrices in the mass basis is employed under the assumption of three-neutrino mixing. The flavor change of neutrinos arriving from the source to the detector is taken into account and the role of the source-detector distance is inspected. The effects of neutrino flavor-transition millicharges and charge radii in the scattering experiments are pointed out.

Magnetic moment contribution dominates at low electron recoil energies when $\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}$ **and** $\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4}\mu_\nu^2$

… the lower the smallest measurable electron recoil energy is, smaller values of μ_ν^2 can be probed in scattering experiments …



$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$ ●

… courtesy of
A.Starostin…

MUNU experiment at Bugey reactor (2005)

$$\boxed{\mu_\nu \leq 9 \times 10^{-11} \mu_B}$$

TEXONO collaboration at Kuo-Sheng power plant (2006)

$$\boxed{\mu_\nu \leq 7 \times 10^{-11} \mu_B}$$

GEMMA (2007)

$$\boxed{\mu_\nu \leq \; 5.8 \times 10^{-11} \mu_B}$$

GEMMA I 2005 - 2007

BOREXINO (2008)

$$\boxed{\mu_\nu \leq \; 5.4 \times 10^{-11} \mu_B}$$

...was considered as the world best constraint...

$$\boxed{\mu_\nu \leq \; 8.5 \times 10^{-11} \mu_B \quad (\nu_\tau, \; \nu_\mu)}$$

based on first release of
BOREXINO data

Montanino,
Picariello,
Pulido,
PRD 2008

...attempts to
improve bounds

GEMMA (2005-2012)

Germanium Experiment for Measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at Kalinin Nuclear Power Plant

World best experimental limit

- $$\boxed{\mu_\nu < 2.9 \times 10^{-11} \mu_B}$$

June 2012

A. Beda et al, in: Special Issue on "Neutrino Physics", Advances in High Energy Physics (2012) 2012,
editors: J. Bernabeu, G. Fogli, A. McDonald, K. Nishikawa

… quite realistic prospects of the near future … 2018

- $$\boxed{\mu_\nu^a \sim \; 0.7 \times 10^{-12} \; \mu_B}$$

unprecedentedly low threshold

$$\boxed{T \sim 200 \;\; eV}$$

… quite recent claim that ν-e cross section should be increased by Atomic Ionization Effect:

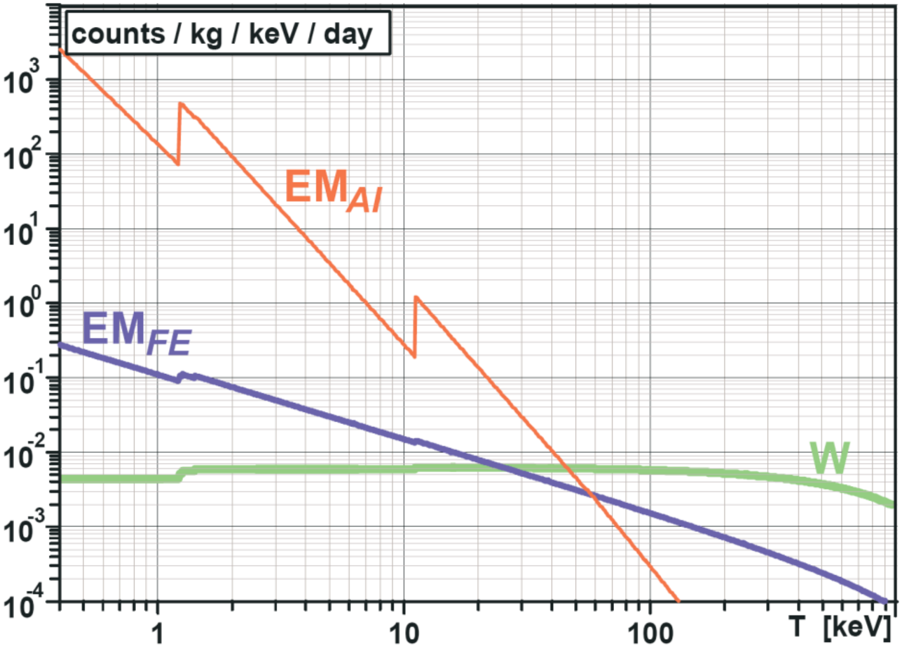
?

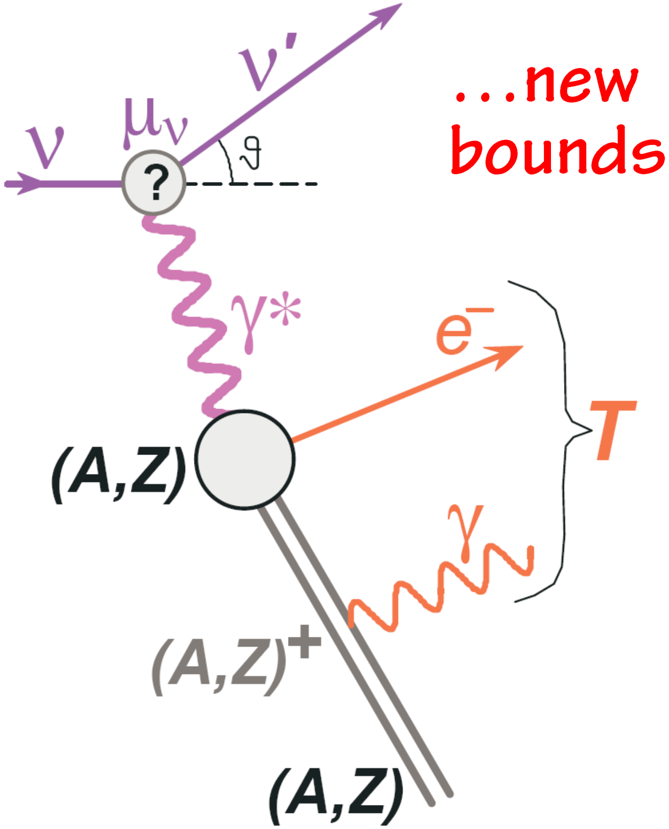
H.Wong et al. (TEXONO Coll.), PRL 105 (2010) 061801

(ν scattering on bound e)

$$\begin{array}{r}\nu + (A,Z) \longrightarrow \nu\,' + (A,Z)^+ + e^- \\ \downarrow \text{recombination} \\ (A,Z) \quad + \gamma\end{array}$$

… an interesting hypothetical possibility to improve bounds…





…new bounds …

… better limits on ν effective magnetic moment …

$$\boxed{\mu_\nu < 1.3 \times 10^{-11} \mu_B}$$?

H.Wong et al.,
(TEXONO Coll.),
PRL 105 (2010)
061801

… atomic ionization effect accounted for …

… however …

$$\boxed{\mu_\nu < 5.0 \times 10^{-12} \mu_B}$$?

… atomic ionization effect accounted for …

A.Beda et al.
(GEMMA Coll.),
arXiv: 1005.2736,
16 May 2010

$$\boxed{\mu_\nu < 3.2 \times 10^{-11} \mu_B}$$

… ν-e scattering on free electrons …
(without atomic ionization)

K.Kouzakov, A.Studenikin,

- “Magnetic neutrino scattering on atomic electrons revisited” **Phys.Lett. B** 105 (2011) 061801,
- “Electromagnetic neutrino-atom collisions: The role of electron binding” **Nucl.Phys.** (Proc.Suppl.) 217 (2011) 353

K.Kouzakov, A.Studenikin, M.Voloshin,

- “Neutrino electromagnetic properties and new bounds on neutrino magnetic moments” **J.Phys.: Conf.Ser**. 375 (2012) 042045
- “Neutrino-impact ionization of atoms in search for neutrino magnetic moment”, **Phys.Rev.D** 83 (2011) 113001
- “On neutrino-atom scattering in searches for neutrino magnetic moments” **Nucl.Phys.B** (Proc.Supp.) 2011 (Proc. of Neutrino 2010 Conf.)
- “Testing neutrino magnetic moment in ionization of atoms by neutrino impact”, **JETP Lett.** 93 (2011) 699

M.Voloshin,

- “Neutrino scattering on atomic electrons in search for neutrino magnetic moment” **Phys.Rev.Lett.** 105 (2010) 201801

Effective ν_e magnetic moment measured in ν-e scattering experiments ? $\boxed{\mu_e^2}$

Two steps:

1) consider ν_e as superposition of mass eigenstates ($i=1,2,3$) at some distance L from the source, and then sum up magnetic moment contributions to ν-e scattering amplitude (of each of mass components) induced by their magnetic moments

$$\boxed{A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}}$$

J.Beacom, P.Vogel, 1999

2) amplitudes combine incoherently in total cross section

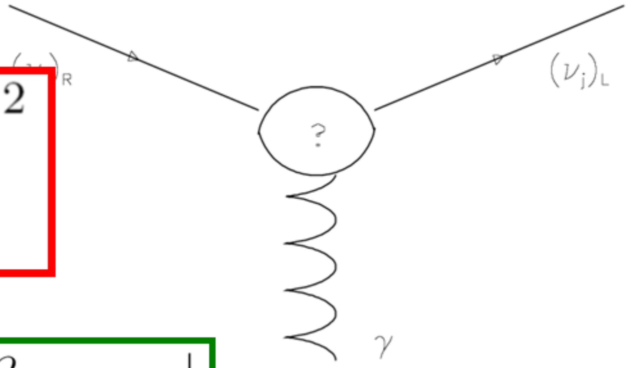
$$\boxed{\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2}$$

C.Giunti, A.Studenikin, 2009

NB! Summation over $j=1,2,3$ is outside the square because of **incoherence** of different final mass states contributions to cross section.

Effective ν magnetic moment in experiments

(for neutrino produced as ν_l with energy $\boldsymbol{E_\nu}$
and after traveling a distance $\boldsymbol{L}$)



$$\boxed{\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_iL} \mu_{ji} \right|^2}$$

where **neutrino mixing matrix**

$$\mu_{ij} \equiv |\beta_{ij} - \varepsilon_{ij}|$$

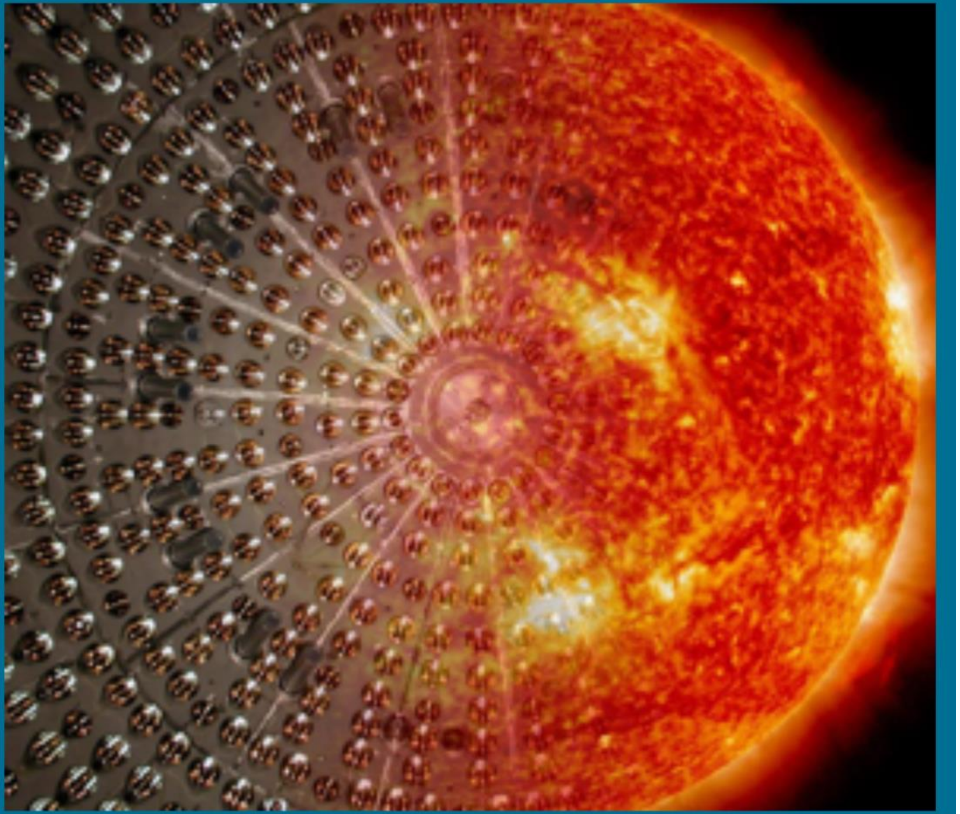
magnetic and **electric** moments

- Observable μ_ν is an effective parameter that depends on neutrino flavour composition at the detector.

- Implications of μ_ν limits from different experiments (reactor , solar ^{8}B and ^{7}Be) are different.

TAUP Topics in Astroparticle and Underground Physics

Limiting the effective magnetic moment of solar neutrinos with the Borexino detector



Livia Ludhova
on behalf of
the Borexino collaboration

IKP-2 FZ Jülich,
RWTH Aachen,
and JARA Institute, Germany

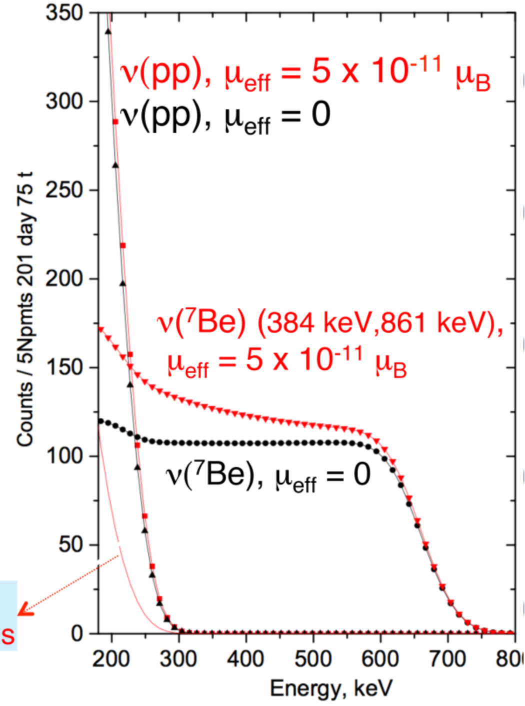
$\sigma(e^- - \nu$ scattering) with NMM > 0

- In addition to the weak-interaction term σ_{WI}, there appears an **additional electro-magnetic term σ_{EM}, proportional to NMM:**

$$\frac{d\sigma_{EM}}{dT_e}(T_e, E_\nu) = \pi\, r_0^2\, \mu_{eff}^2 \left(\frac{1}{T_e} - \frac{1}{E_\nu}\right)$$

$r_0 = 1.818 \times 10^{-13}$ cm (electron radius)

- μ_{eff}: for a mixture of mass eigenstates
- 1-photon exchange + ν flips helicity (WI and EM terms do not interfere)
- For $T_e << E_\nu$: $\sigma_{TOTAL} \sim 1/T_e$, the spectrum of the scattered electron is influenced mostly at low energies.

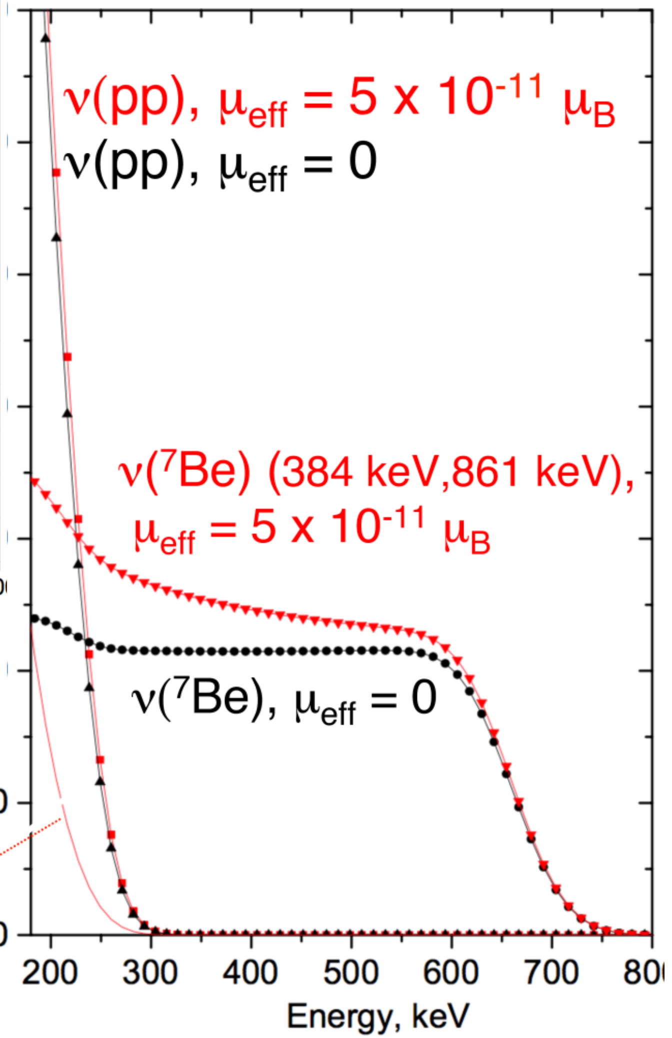


^{7}Be-ν: strong change of the shape
MAJOR SENSITIVTY TO NMM

Difference for pp shapes

pp-ν: the change of the shape is almost equivalent to the change of only normalization
CONSTRAINING PP FLUX HELPS!

MM > 0



^{7}Be-ν: strong change of the shape
MAJOR SENSITIVTY TO NMM

Difference for pp shapes

pp-ν: the change of the shape is almost equivalent to the change of only normalization
CONSTRAINING PP FLUX HELPS!

NEW

NMM results from Phase 2

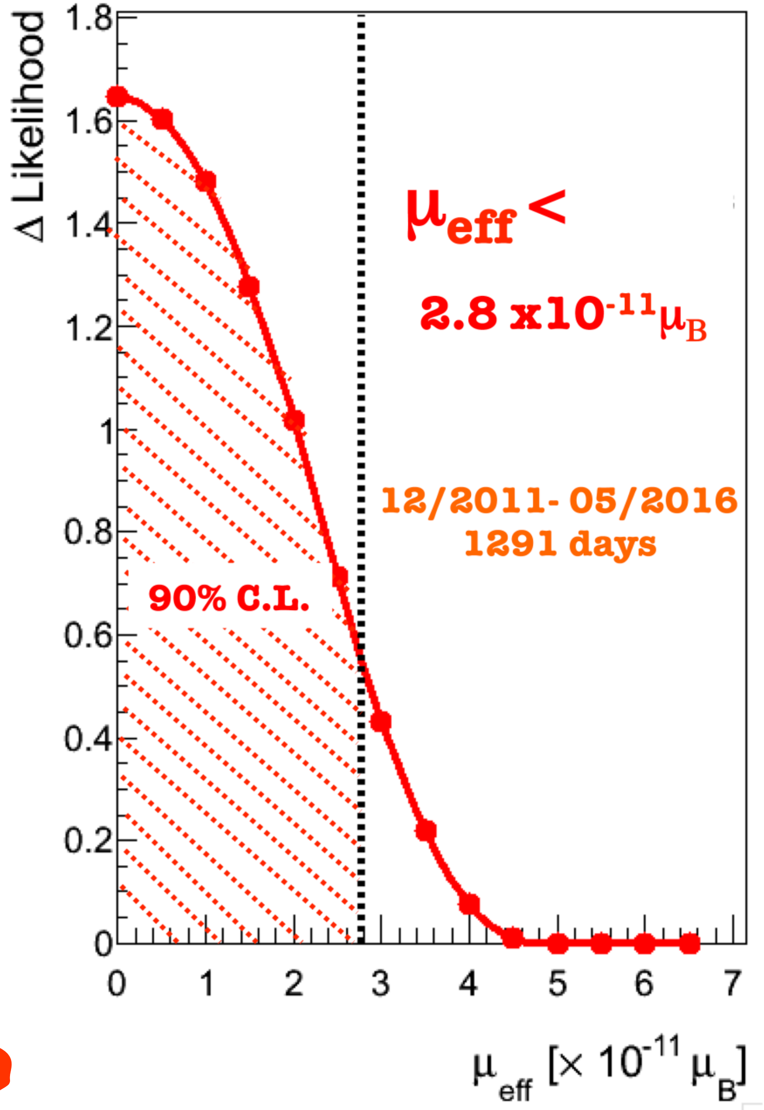
JÜLICH FORSCHUNGSZENTRUM

Data selection:
Fiducial volume: R < 3.021 m, |z| < 1.67 m
Muon, ^{214}Bi-^{214}Po, and noise suppression
Free fit parameters: solar-ν (pp, ^{7}Be) and **backgrounds** (^{85}Kr,^{210}Po, ^{210}Bi, ^{11}C, external bgr.), **response parameters** (light yield, ^{210}Po position and width, ^{11}C edge (2 x 511 keV), 2 energy resolution parameters)
Constrained parameters: ^{14}C, pile up
Fixed parameters: pep-, CNO-, ^{8}B-ν rates
Systematics: treatment of pile-up, energy estimators, pep and CNO constraints with LZ and HZ SSM

Without radiochemical constraint
$\mu_{eff} < 4.0 \times 10^{-11} \mu_B$ (90% C.L.)

With radiochemical constraint
$\mu_{eff} < 2.6 \times 10^{-11} \mu_B$ (90% C.L.)
adding systematics
$\mu_{eff} < 2.8 \times 10^{-11} \mu_B$ **(90% C.L.)**

Profiling μ_{eff} with σ_{EM} for pp &^{7}Be



$\mu_{eff} <$
$2.8 \times 10^{-11} \mu_B$

12/2011- 05/2016
1291 days

②

Experimental limits for different effective μ_ν

Method		Experiment	Limit	CL	Reference
		Krasnoyarsk	$\mu_{\nu_e} < 2.4 \times 10^{-10}\ \mu_B$	90%	Vidyakin *et al.* (1992)
		Rovno	$\mu_{\nu_e} < 1.9 \times 10^{-10}\ \mu_B$	95%	Derbin *et al.* (1993)
Reactor $\bar{\nu}_e$-e^-	●	MUNU	$\mu_{\nu_e} < 0.9 \times 10^{-10}\ \mu_B$	90%	Daraktchieva *et al.* (2005)
	●	TEXONO	$\mu_{\nu_e} < 7.4 \times 10^{-11}\ \mu_B$	90%	Wong *et al.* (2007)
	●	GEMMA	$\mu_{\nu_e} < 2.9 \times 10^{-11}\ \mu_B$	90%	Beda *et al.* (2012)
Accelerator ν_e-e^-		LAMPF	$\mu_{\nu_e} < 10.8 \times 10^{-10}\ \mu_B$	90%	Allen *et al.* (1993)
Accelerator $(\nu_\mu, \bar{\nu}_\mu)$-e^-		BNL-E734	$\mu_{\nu_\mu} < 8.5 \times 10^{-10}\ \mu_B$	90%	Ahrens *et al.* (1990)
		LAMPF	$\mu_{\nu_\mu} < 7.4 \times 10^{-10}\ \mu_B$	90%	Allen *et al.* (1993)
		LSND	$\mu_{\nu_\mu} < 6.8 \times 10^{-10}\ \mu_B$	90%	Auerbach *et al.* (2001)
Accelerator $(\nu_\tau, \bar{\nu}_\tau)$-$e^-$		DONUT	$\mu_{\nu_\tau} < 3.9 \times 10^{-7}\ \mu_B$	90%	Schwienhorst *et al.* (2001)
Solar ν_e-e^-		Super-Kamiokande	$\mu_S(E_\nu \gtrsim 5\,\text{MeV}) < 1.1 \times 10^{-10}\ \mu_B$	90%	Liu *et al.* (2004)
	●	Borexino	$\mu_S(E_\nu \lesssim 1\,\text{MeV}) < 5.4 \times 10^{-11}\ \mu_B$	90%	Arpesella *et al.* (2008)

… next talk by Livia Ludhova …

C. Giunti, A. Studenikin, "Electromagnetic interactions of neutrinos: a window to new physics", Rev. Mod. Phys. 87 (2015) 531

… A remark on electric charge of ν … Beyond Standard Model…

ν neutrality *Q=0* is attributed to

gauge invariance
+
anomaly cancellation constraints

imposed in SM of electroweak interactions

Foot, Joshi, Lew, Volkas, 1990;
Foot, Lew, Volkas, 1993;
Babu, Mohapatra, 1989, 1990
Foot , He (1991)

- *…General proof:*
- *In SM :*

$SU(2)_L \times U(1)_Y$

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

- *In SM (without ν_R) triangle anomalies cancellation constraints* → *certain relations among particle hypercharges Y, that is enough to fix all Y so that they, and consequently Q, are quantized*

Q=0

- *Q=0* *is proven also by direct calculation in SM within different gauges and methods*

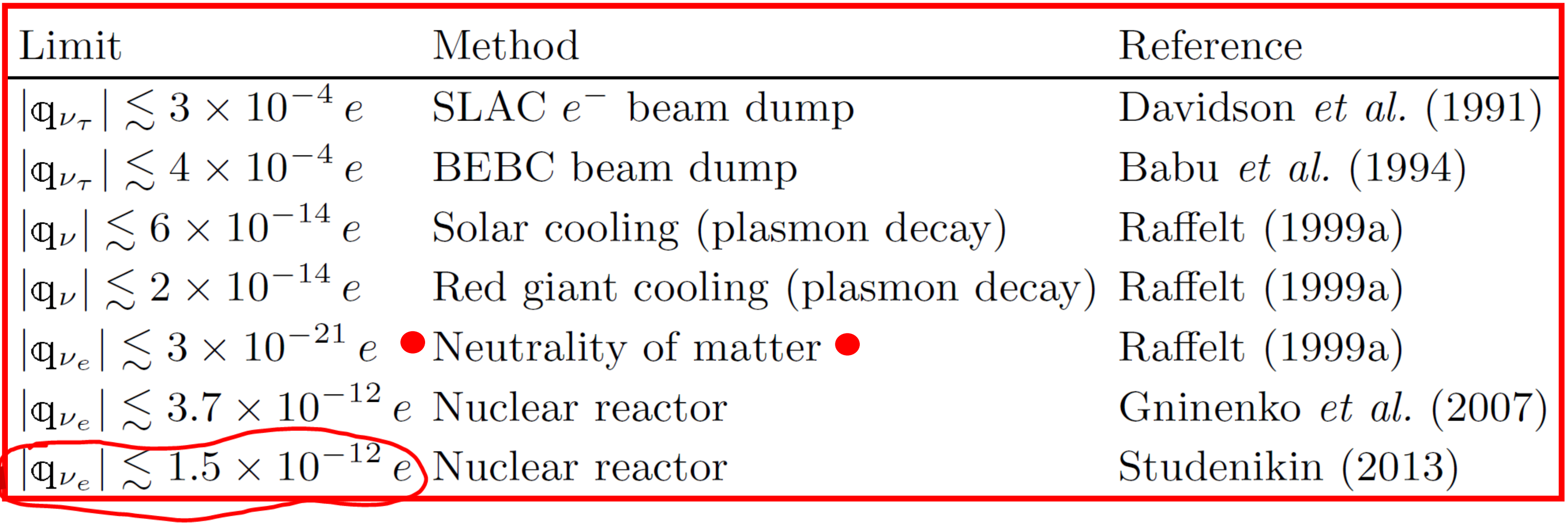
Bardeen, Gastmans, Lautrup, 1972;
Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000;
Beg, Marciano, Ruderman, 1978;
Marciano, Sirlin, 1980; Sakakibara, 1981;
- *M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)*

- *… However, strict requirements for* ***Q*** *quantization may disappear in extensions of standard* $SU(2)_L \times U(1)_Y$ *EW model if* ν_R *with* $\boldsymbol{Y \neq 0}$ *are included : in the absence of* ***Y*** *quantization electric charges* ***Q*** *gets dequantized* → **millicharged ν**

②

Experimental limits for different effective q_ν

C. Giunti, A. Studenikin, "Electromagnetic interactions of neutrinos: a window to new physics", Rev. Mod. Phys. 87 (2015) 531



Limit	Method	Reference
$\lvert q_{\nu_\tau}\rvert \lesssim 3 \times 10^{-4}\,e$	SLAC e^- beam dump	Davidson *et al.* (1991)
$\lvert q_{\nu_\tau}\rvert \lesssim 4 \times 10^{-4}\,e$	BEBC beam dump	Babu *et al.* (1994)
$\lvert q_{\nu}\rvert \lesssim 6 \times 10^{-14}\,e$	Solar cooling (plasmon decay)	Raffelt (1999a)
$\lvert q_{\nu}\rvert \lesssim 2 \times 10^{-14}\,e$	Red giant cooling (plasmon decay)	Raffelt (1999a)
$\lvert q_{\nu_e}\rvert \lesssim 3 \times 10^{-21}\,e$	● Neutrality of matter ●	Raffelt (1999a)
$\lvert q_{\nu_e}\rvert \lesssim 3.7 \times 10^{-12}\,e$	Nuclear reactor	Gninenko *et al.* (2007)
$\lvert q_{\nu_e}\rvert \lesssim 1.5 \times 10^{-12}\,e$	Nuclear reactor	Studenikin (2013)

A. Studenikin: "New bounds on neutrino electric millicharge from limits on neutrino magnetic moment", Eur.Phys.Lett. 107 (2014) 2100

C.Patrignani et al (Particle Data Group), "The Review of Particle Physics 2016" Chinese Physics C 40 (2016) 100001

Bounds on millicharge q_ν from μ_ν (GEMMA Coll. data)

ν-e cross-section

$$\boxed{\left(\frac{d\sigma}{dT}\right)_{\nu-e} = \left(\frac{d\sigma}{dT}\right)_{SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu} + \left(\frac{d\sigma}{dT}\right)_{q_\nu}}$$

two not seen contributions:

$$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu^a} \approx \pi\alpha^2 \frac{1}{m_e^2 T}\left(\frac{\mu_\nu^a}{\mu_B}\right)^2$$

$$\left(\frac{d\sigma}{dT}\right)_{q_\nu} \approx 2\pi\alpha \frac{1}{m_e T^2} q_\nu^2$$

Bounds on q_ν from

… unobservable effects of New Physics

$$R = \frac{\left(\frac{d\sigma}{dT}\right)_{q_\nu}}{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu^a}} = \frac{2m_e}{T} \frac{\left(\frac{q_\nu}{e_0}\right)^2}{\left(\frac{\mu_\nu^a}{\mu_B}\right)^2} < 1$$

Studenikin, Eurphys. Lett. 107 (2014) 21001

● Particle Data Group, 2016 ●

Expected new constraints from GEMMA:

now $\mu_\nu^a < 2.9 \times 10^{-11} \mu_B$ ($T \sim 2.8\ keV$)

Constraints on q_ν

$$\boxed{|\, q_\nu \,| < 1.5 \times 10^{-12} e_0}$$ ●

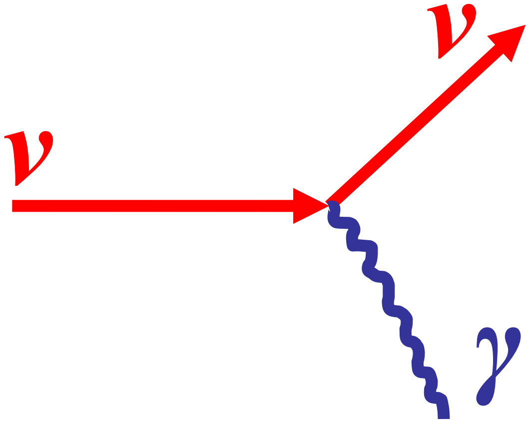
2018 (expected)

… unprecedentedly low threshold …

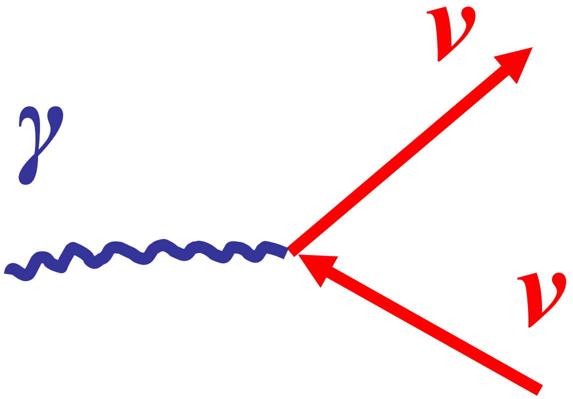
$$\boxed{\mu_\nu^a \sim\ 0.7 \times 10^{-12}\ \mu_B}\ \left(\boxed{T \sim 200\ \ eV}\right)$$

$$\boxed{|q_\nu| < 1.1 \times 10^{-13} e_0}$$

③ ν electromagnetic interactions

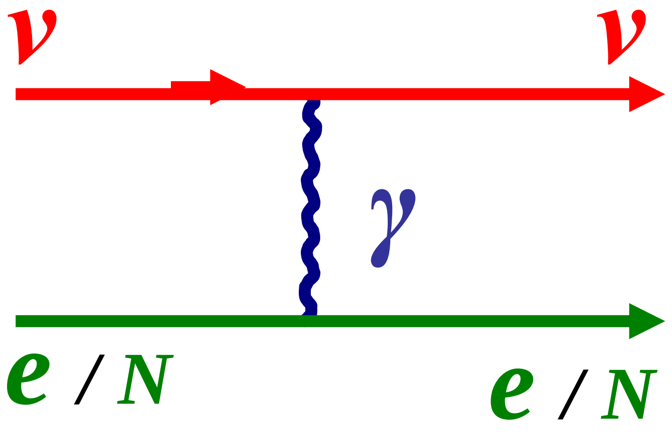


ν decay, Cherenkov radiation

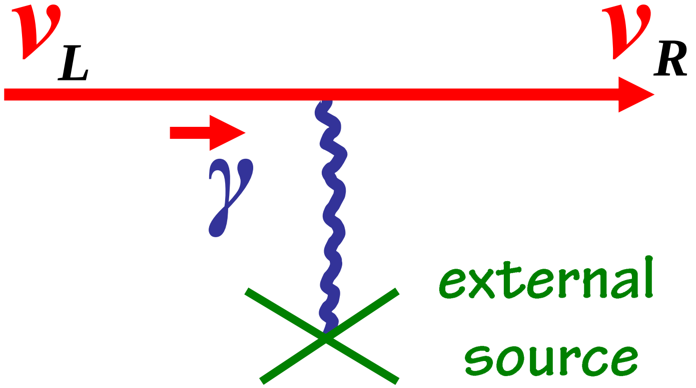


γ decay in plasma

!!!



Scattering

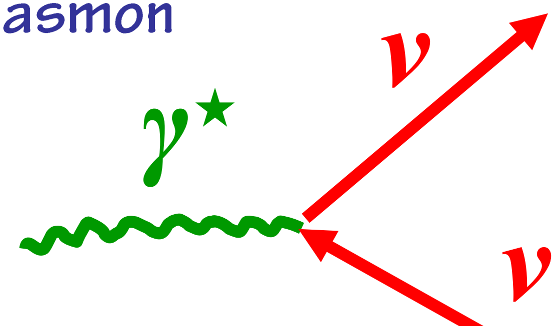


Spin precession

② Astrophysical bound on μ_ν

G.Raffelt, PRL 1990

comes from cooling of red gaint stars by plasmon decay $\gamma^\star \longrightarrow \nu\nu$



$$\boxed{L_{int} = \frac{1}{2}\sum_{a,b}\left(\mu_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\psi_b + \epsilon_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\gamma_5\psi_b\right)}$$

neutrino flavour states

$$\epsilon_\alpha k^\alpha = 0$$

Matrix element

$$|M|^2 = M_{\alpha\beta}p^\alpha p^\beta, \quad M_{\alpha\beta} = 4\mu^2(2k_\alpha k_\beta - 2k^2\epsilon^*_\alpha\epsilon_\beta - k^2 g_{\alpha,\beta}),$$

Decay rate

$$\boxed{\Gamma_{\gamma\rightarrow\nu\bar{\nu}} = \frac{\mu^2}{24\pi}\frac{(\omega^2 - k^2)^2}{\omega}}$$

= 0 in vacuum $\omega = k$

In the classical limit $\gamma^\star$ - like a massive particle with $\omega^2 - k^2 = \omega_{pl}^2$

Energy-loss rate per unit volume

$$\boxed{Q_\mu = g\int\frac{d^3k}{(2\pi)^3}\omega f_{BE}\Gamma_{\gamma\rightarrow\nu\bar{\nu}}}$$

$$\mu^2 \rightarrow \sum_{a,b}\left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2\right)$$

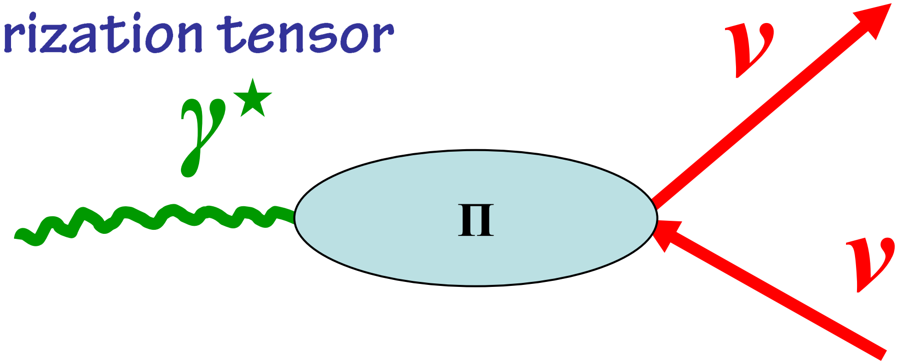
distribution function of plasmons

Astrophysical bound on μ_ν

$$\boxed{Q_\mu = g \int \frac{d^3k}{(2\pi)^3}\omega f_{BE}\Gamma_{\gamma\to\nu\bar{\nu}}}$$

Energy-loss rate per unit volume

Magnetic moment plasmon decay enhances the Standard Model photo-neutrino cooling by photon polarization tensor



more fast star cooling

In order not to delay helium ignition (≤5% in Q)

… best astrophysical limit on ν magnetic moment…

$$\boxed{\mu \leq 3\times 10^{-12}\mu_B}$$

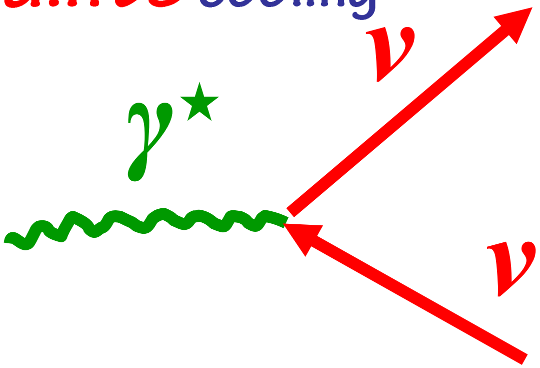
G.Raffelt, PRL 1990

$$\mu^2 \to \sum_{a,b}\left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2\right)$$

Dobroliubov, Ignatiev (1990); Babu, Volkas (1992);
Mohapatra, Nussinov (1992) …

- Constraints on neutrino **millicharge** from **red gaints** cooling

$$\gamma^\star \longrightarrow \nu\nu$$



Interaction Lagrangian

$$\boxed{L_{int} = -iq_\nu \bar{\psi}_\nu \gamma^\mu \psi_\nu A^\mu}$$

millicharge

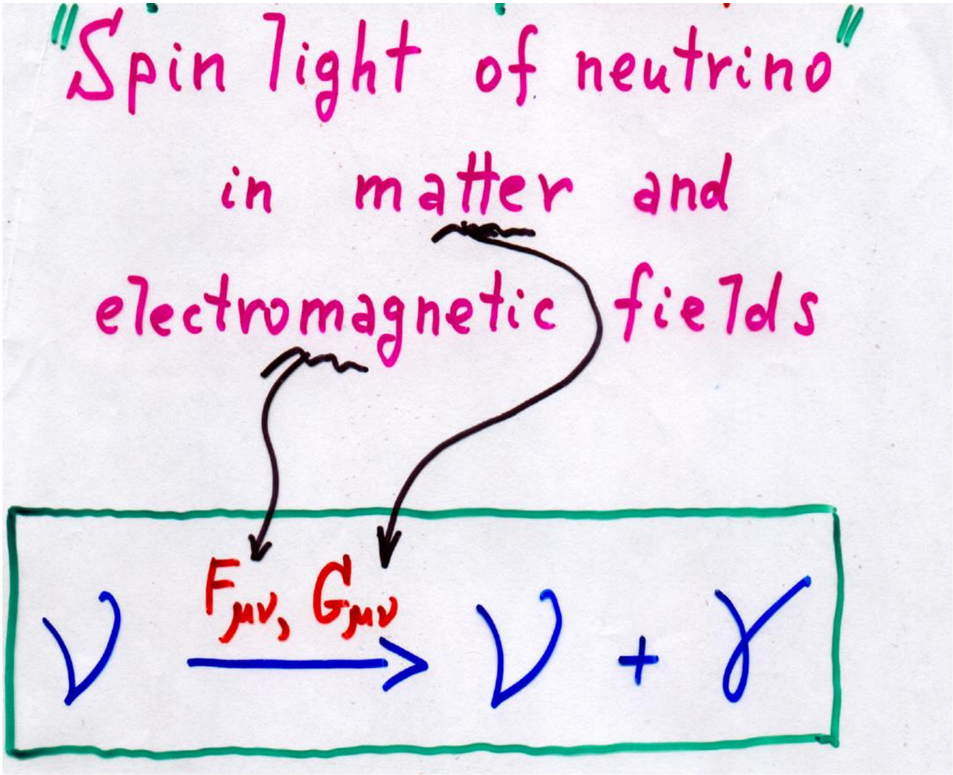
Decay rate

$$\boxed{\Gamma_{q_\nu} = \frac{{q_\nu}^2}{12\pi}\omega_{pl}\left(\frac{\omega_{pl}}{\omega}\right)}$$

- $\boxed{q_\nu \leq 2 \times 10^{-14} e}$ …to avoid helium ignition in low-mass red gaints **Halt, Raffelt, Weiss, PRL1994**
- $\boxed{q_\nu \leq 3 \times 10^{-17} e}$ … absence of anomalous energy-dependent dispersion of SN1987A ν signal, most model independent
- … from “charge neutrality” of neutron… $\boxed{q_\nu \leq 3 \times 10^{-21} e}$

• New mechanism of electromagnetic radiation

$SL\nu$



"Spin light of neutrino" in matter and electromagnetic fields

$$\nu \xrightarrow{F_{\mu\nu},\, G_{\mu\nu}} \nu + \gamma$$

A. Egorov, A. Lobanov, A. Studenikin,
Phys.Lett. B 491 (2000) 137

Lobanov, Studenikin,
Phys.Lett. B 515 (2001) 94
Phys.Lett. B 564 (2003) 27
Phys.Lett. B 601 (2004) 171

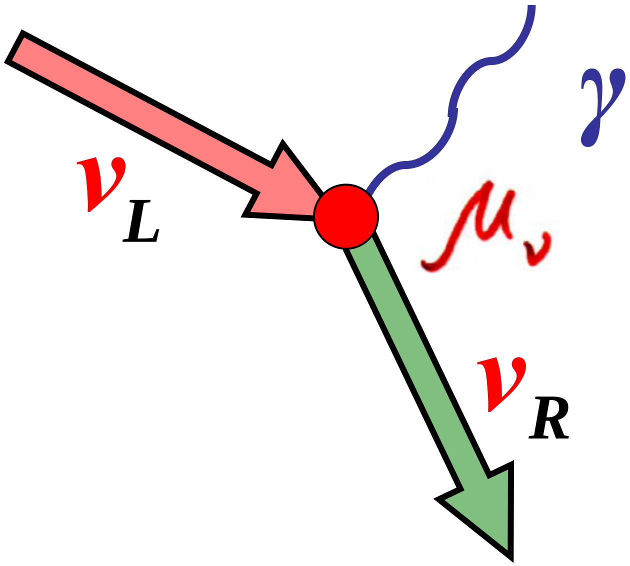
Studenikin, A.Ternov,
Phys.Lett. B 608 (2005) 107

A. Grigoriev, Studenikin, Ternov,
Phys.Lett. B 622 (2005) 199

Studenikin,
J.Phys.A: Math.Gen. 39 (2006) 6769
J.Phys.A: Math.Theor. 41 (2008) 16402

Grigoriev, A. Lokhov, Studenikin, Ternov,
Nuovo Cim. 35 C (2012) 57
Phys.Lett.B 718 (2012) 512

Neutrino – photon coupling



broad neutrino lines
account for interaction
with environment

"Spin light of neutrino in matter"

$SL\nu$

- … within the quantum treatment based on method of exact solutions …

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L^{CC}_{eff} + \Delta L^{NC}_{eff} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^2\theta_W)j^{\mu} - \lambda^{\mu}\right)$$

matter polarization

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\right\}\Psi(x) = 0}$$

A.Studenikin, A.Ternov, hep-ph/0410297;
***Phys.Lett.B* 608 (2005) 107**

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02.

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Quantum theory of spin light of neutrino

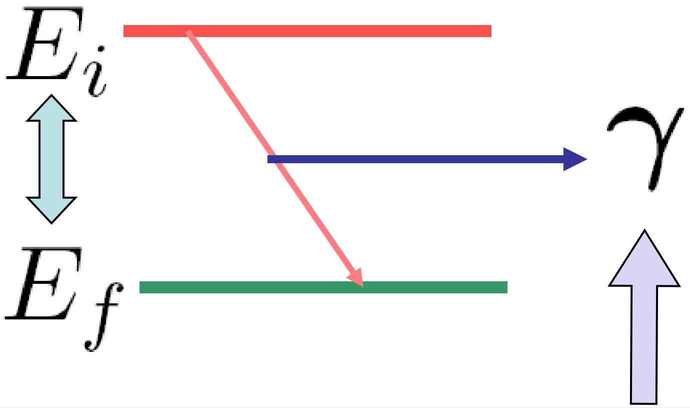
$SL\nu$

Quantum treatment of *spin light of neutrino* in matter
showns that this process originates from the **two subdivided phenomena:**

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,

$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$



the radiation of the photon in the process of the neutrino transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

A.Grigoriev, A.Lokhov,
A.Ternov, A.Studenikin

The effect of plasmon mass on Spin Light of Neutrino in dense matter

Phys.Lett. B 718 (2012) 512

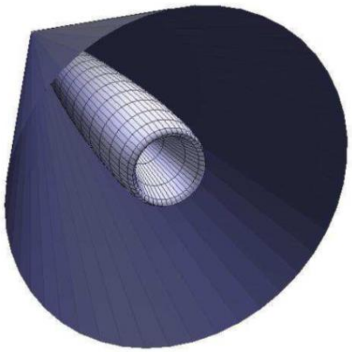


Figure 1: 3D representation of the radiation power distribution.

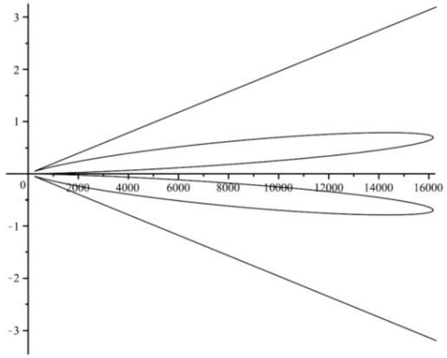


Figure 2: The two-dimensional cut along the symmetry axis. Relative units are used.

4. Conclusions

We developed a detailed evaluation of the spin light of neutrino in matter accounting for effects of the emitted plasmon mass. On the base of the exact solution of the modified Dirac equation for the neutrino wave function in the presence of the background matter the appearance of the threshold for the considered process is confirmed. The obtained exact and explicit threshold condition relation exhibit a rather complicated dependance on the matter density and neutrino mass. The dependance of the rate and power on the neutrino energy, matter density and the angular distribution of the $SL\nu$ is investigated in details. It is shown how the rate and power wash out when the threshold parameter $a = m_\gamma^2/4\tilde{n}p$ approaching unity. From the performed detailed analysis it is shown that the $SL\nu$ mechanism is practically insensitive to the emitted plasmon mass for very high densities of matter (even up to $n = 10^{41} cm^{-3}$) for ultra-high energy neutrinos for a wide range of energies starting from $E = 1$ TeV. This conclusion is of interest for astrophysical applications of $SL\nu$ radiation mechanism in light of the recently reported hints of $1 \div 10$ PeV neutrinos observed by IceCube [17].

A. Grigoriev, A. Lokhov, A. Ternov, A. Studenikin

Spin light of neutrino in astrophysical environments

arXiv: 1705.07481

ν manifests its electromagnetic properties most clearly under influence of astrophysical extreme external conditions:

- strong external electromagnetic fields

and

- dense background matter

—

ν in extreme external conditions (strong fields and dense matter)

A. Studenikin,

- "Quantum treatment of neutrino in background matter",
  J. Phys. A: Math. Gen. 39 (2006) 6769-6776
- "Neutrinos and electrons in background matter: a new approach",
  Ann.Fond. de Broglie 31 (2006) 289-316
- "Method of wave equations exact solutions in studies of neutrinos and electron interactions in dense matter",
  J.Phys.A: Math.Theor. 41 (2008) 164047

...«method of exact solutions»

• … astrophysical bound on millicharge q_ν from

②

ν energy quantization in rotating magnetized media

Grigoriev, Savochkin, Studenikin, Russ. Phys. J. 50 (2007) 845

Studenikin, J. Phys. A: Math. Theor. 41 (2008) 164047

Balantsev, Popov, Studenikin,
J. Phys. A: Math. Theor. 44 (2011) 255301

Balantsev, Studenikin, Tokarev,
Phys. Part. Nucl. 43 (2012) 727
Phys. Atom. Nucl. 76 (2013) 489

Studenikin, Tokarev, Nucl. Phys. B 884 (2014) 396

Millicharged ν in rotating magnetized matter

Balatsev, Tokarev, Studenikin,
Phys.Part.Nucl., 2012,
Phys.Atom.Nucl., Nucl.Phys. B, 2013,
Studenikin, Tokarev, Nucl.Phys.B (2014) •

Modified Dirac equation for ν wave function

$$\boxed{\Big(\gamma_\mu(p^\mu + q_0 A^\mu) - \frac{1}{2}\gamma_\mu(c_l+\gamma_5)f^\mu - \frac{i}{2}\mu\sigma_{\mu\nu}F^{\mu\nu} - m\Big)\,\Psi(x) = 0}$$

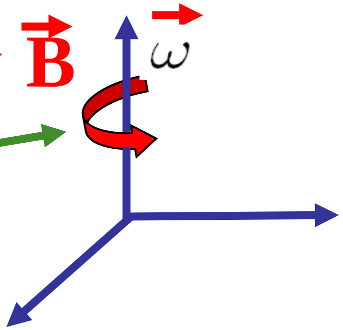
external magnetic field

$$\boxed{V_m = \tfrac{1}{2}\gamma_\mu(c_l+\gamma_5)f^\mu} \qquad c_l = 1$$

matter potential

rotating matter

$$\boxed{f^\mu = -Gn_n(1, -\epsilon y\omega, \epsilon x\omega, 0)}$$



rotation angular frequency

ν energy is quantized in rotating matter

A.Studenikin, I.Tokarev,
Nucl.Phys.B (2014)

$$G = \frac{G_F}{\sqrt{2}}$$

$$\boxed{p_0 = \sqrt{p_3^2 + 2N|2Gn_n\omega - \epsilon q_\nu B| + m^2} - Gn_n - q\phi}$$

$$\boxed{N = 0, 1, 2, \ldots}$$

integer number

matter rotation
frequency

scalar potential
of electric field

ν energy is quantized in rotating matter
like electron energy in magnetic field
(Landau energy levels):

$$\boxed{p_0^{(e)} = \sqrt{m_e^2 + p_3^2 + 2\gamma N}, \quad \gamma = eB, \quad N = 0, 1, 2, \ldots}$$

In quasi-classical approach

- ✓ quantum states in rotating matter
- ✓ motion in circular orbits

$$\boxed{R = \int_0^\infty \Psi_L^\dagger \, \mathrm{r} \, \Psi_L \, d\mathbf{r} = \sqrt{\frac{2N}{|2Gn_n\omega - \epsilon q_0 B|}}}$$

due to effective Lorentz force

$$\boxed{\mathbf{F}_{eff} = q_{eff}\mathbf{E}_{eff} + q_{eff}\,[\boldsymbol{\beta} \times \mathbf{B}_{eff}]}$$

A. Studenikin,
J.Phys.A: Math.Theor.
41(2008) 164047

$$q_{eff}\mathbf{E}_{eff} = q_m\mathbf{E}_m + q_0\mathbf{E} \qquad q_{eff}\mathbf{B}_{eff} = |q_m B_m + q_0 B|\mathbf{e}_z$$

where

$$\boxed{q_m = -G, \quad \mathbf{E}_m = -\boldsymbol{\nabla} n_n, \quad \mathbf{B}_m = 2n_n\boldsymbol{\omega}}$$

matter induced “charge”, “electric” and “magnetic” fields

… we predict :

A.Studenikin, I.Tokarev,
Nucl.Phys.B (2014)

$E \sim 1\ eV$

1) low-energy ν are trapped in circular orbits inside rotating neutron stars

$$R = \sqrt{\frac{2N}{Gn\omega}} < R_{NS} = 10\ km$$

$$R_{NS} = 10\ km$$
$$n = 10^{37} cm^{-3}$$
$$\omega = 2\pi \times 10^{3}\ s^{-1}$$

2) rotating neutron stars as filters for low-energy relic ν ?

$$T_\nu \sim 10^{-4}\ \mathrm{eV}$$

… we predict :

3) high-energy ν are deflected inside a rotating astrophysical transient sources (GRBs, SNe, AGNs)

absence of light in correlation with ν signal reported by ANTARES Coll.

M.Ageron et al,
Nucl.Instrum.Meth. A692 (2012) 184

● Millicharged ν as star rotation engine

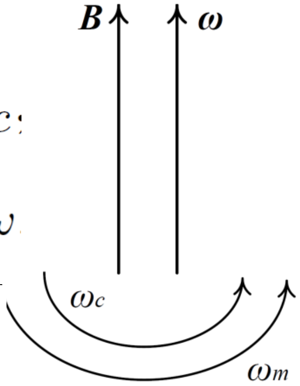
- Single ν generates feedback force with projection on rotation plane

- $F = (q_0 B + 2Gn_n\omega)\sin\theta$

$$\Omega = \omega_m + \omega_c;$$

$$\omega_m = \frac{2Gn_n}{p_0 + Gn_n}\omega,$$

$$\omega_c = \frac{q_0 B}{p_0 + Gn_n}$$

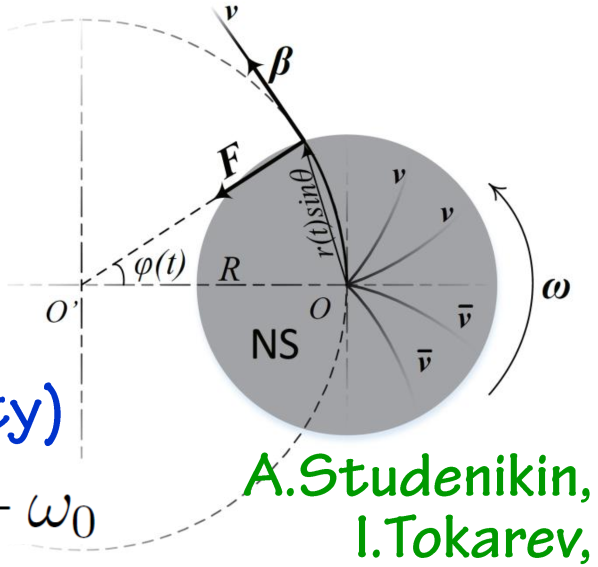


single ν torque

- $M_0(t) = \sqrt{1 - \frac{r^2(t)\Omega^2\sin^2\theta}{4}}\,Fr(t)\sin\theta$

total N_ν torque

$$M(t) = \frac{N_\nu}{4\pi}\int M_0(t)\sin\theta d\theta d\varphi$$



- Should effect initial star rotation (shift of star angular velocity)

$$\boxed{|\Delta\omega| = \frac{5N_\nu}{6M_S}(q_0 B + 2Gn_n\omega_0)}$$

$$\Delta\omega = \omega - \omega_0$$

A.Studenikin,
I.Tokarev,
Nucl.Phys.B (2014)

• ν Star Turning mechanism (νST)

A. Studenikin, I. Tokarev, Nucl. Phys. B 884 (2014) 396

Escaping millicharged νs move on curved orbits inside magnetized rotating star and feedback of effective Lorentz force should effect initial star rotation

- New astrophysical constraint on ν millicharge

$$\boxed{\frac{|\triangle\omega|}{\omega_0} = 7.6\varepsilon \times 10^{18} \left(\frac{P_0}{10 \text{ s}}\right) \left(\frac{N_\nu}{10^{58}}\right) \left(\frac{1.4M_\odot}{M_S}\right) \left(\frac{B}{10^{14}G}\right)}$$

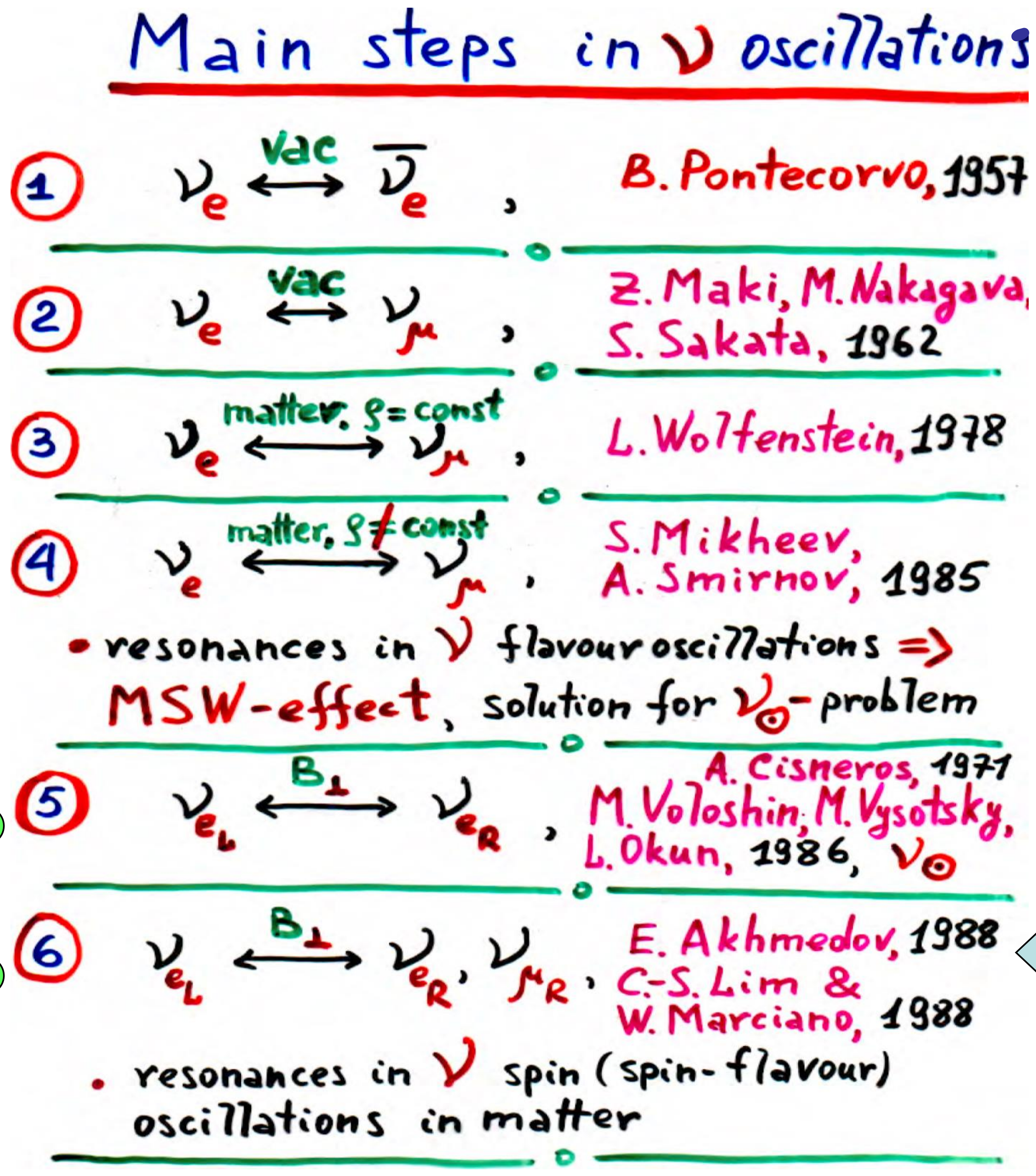
- $|\triangle\omega| < \omega_0$! ...to avoid contradiction of νST impact with observational data on pulsars ...

$$\boxed{q_0 < 1.3 \times 10^{-19} e_0}$$

● ... best astrophysical bound ...

- ν spin and spin-flavour oscillations in transversal matter currents

Studenikin (2004)



Main steps in ν oscillations

① $\nu_e \overset{\text{vac}}{\longleftrightarrow} \bar{\nu}_e$, B. Pontecorvo, 1957

② $\nu_e \overset{\text{vac}}{\longleftrightarrow} \nu_\mu$, Z. Maki, M. Nakagava, S. Sakata, 1962

③ $\nu_e \overset{\text{matter, }\rho=\text{const}}{\longleftrightarrow} \nu_\mu$, L. Wolfenstein, 1978

④ $\nu_e \overset{\text{matter, }\rho\neq\text{const}}{\longleftrightarrow} \nu_\mu$, S. Mikheev, A. Smirnov, 1985

- resonances in ν flavour oscillations ⇒ MSW-effect, solution for $\nu_\odot$-problem

⑤ $\nu_{e_L} \overset{B_\perp}{\longleftrightarrow} \nu_{e_R}$, A. Cisneros, 1971; M. Voloshin, M. Vysotsky, L. Okun, 1986, $\nu_\odot$

⑥ $\nu_{e_L} \overset{B_\perp}{\longleftrightarrow} \nu_{e_R}, \nu_{\mu_R}$, E. Akhmedov, 1988; C.-S. Lim & W. Marciano, 1988

- resonances in ν spin (spin-flavour) oscillations in matter

60 years !

Bruno Pontecorvo
1913–1993

only in $B_\perp$ and matter at rest

④ ν spin and spin-flavour oscillations in $B_\perp$

Consider **two different neutrinos**: $\nu_{e_L}, \quad \nu_{\mu_R}, \quad m_L \neq m_R$

with **magnetic moment interaction**

$$\boxed{L \sim \bar{\nu}\sigma_{\lambda\rho}F^{\lambda\rho}\nu\ ' = \bar{\nu}_L\sigma_{\lambda\rho}F^{\lambda\rho}\nu_R\ ' + \bar{\nu}_R\sigma_{\lambda\rho}F^{\lambda\rho}\nu_L\ '}\ .$$

Twisting magnetic field $B = |\mathbf{B}_\perp|e^{i\phi(t)}$ or solar ν etc ...

ν *evolution equation*

$$\boxed{i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = H\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}}$$

$$\boxed{H = \begin{pmatrix} E_L & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & E_R \end{pmatrix} = \ldots \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tilde{H}}$$

$$\boxed{\tilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{V_{\nu_e}}{2} & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & \frac{\Delta m^2}{4E} - \frac{V_{\nu_e}}{2} \end{pmatrix}}$$

Probability of $\nu_{e_L} \Longleftrightarrow \nu_{\mu_R}$ oscillations in $\boxed{B = |\mathbf{B}_\perp| e^{i\phi(t)}}$

- $$\boxed{P_{\nu_L \nu_R} = \sin^2\beta \, \sin^2\Omega z, \quad \sin^2\beta = \frac{(\mu_{e\mu}B)^2}{(\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}}$$

$$\boxed{\Delta_{LR} = \frac{\Delta m^2}{2}(\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}}$$

$$\boxed{\Omega^2 = (\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$$

- Resonance amplification of oscillations in matter:

$$\boxed{\Delta_{LR} \to 0} \Longrightarrow \boxed{\sin^2\beta \to 1}$$

Akhmedov, 1988
Lim, Marciano

… similar to
MSW effect

In magnetic field

$$\nu_{e_L} \ \ \nu_{\mu_R}$$

$$i\frac{d}{dz}\nu_{e_L} = -\frac{\Delta_{LR}}{4E}\nu_{e_L} + \mu_{e\mu}B\nu_{\mu_R}$$

$$i\frac{d}{dz}\nu_{\mu_L} = \frac{\Delta_{LR}}{4E}\nu_{\mu_L} + \mu_{e\mu}B\nu_{e_R}$$

Neutrino conversions and oscillations in magnetic field

● ⊛ $\nu_\odot$ problem

$\nu_L \overset{B}{\longleftrightarrow} \nu_R$

Cisneros, 1971

* { Voloshin, Vysotsky, Okun, 1986
  Barbieri, Fiorentini, 1988

⊙ twisting B { Smirnov, 1991
  Akhmedov, Petcov, Smirnov, 1993

…for recent analysis see

J.Pulido, 2006, TAUP-09; ●
A.Balantekin, C.Volpe, 2005

…subdominant contribution to LMA – MSW solution…

$\nu_\odot$

● ⊛ Supernova $\nu_L \overset{B}{\longleftrightarrow} \nu_R$

● Dar, 1987

Fujikawa, Shrock, 1988

Voloshin, 1988

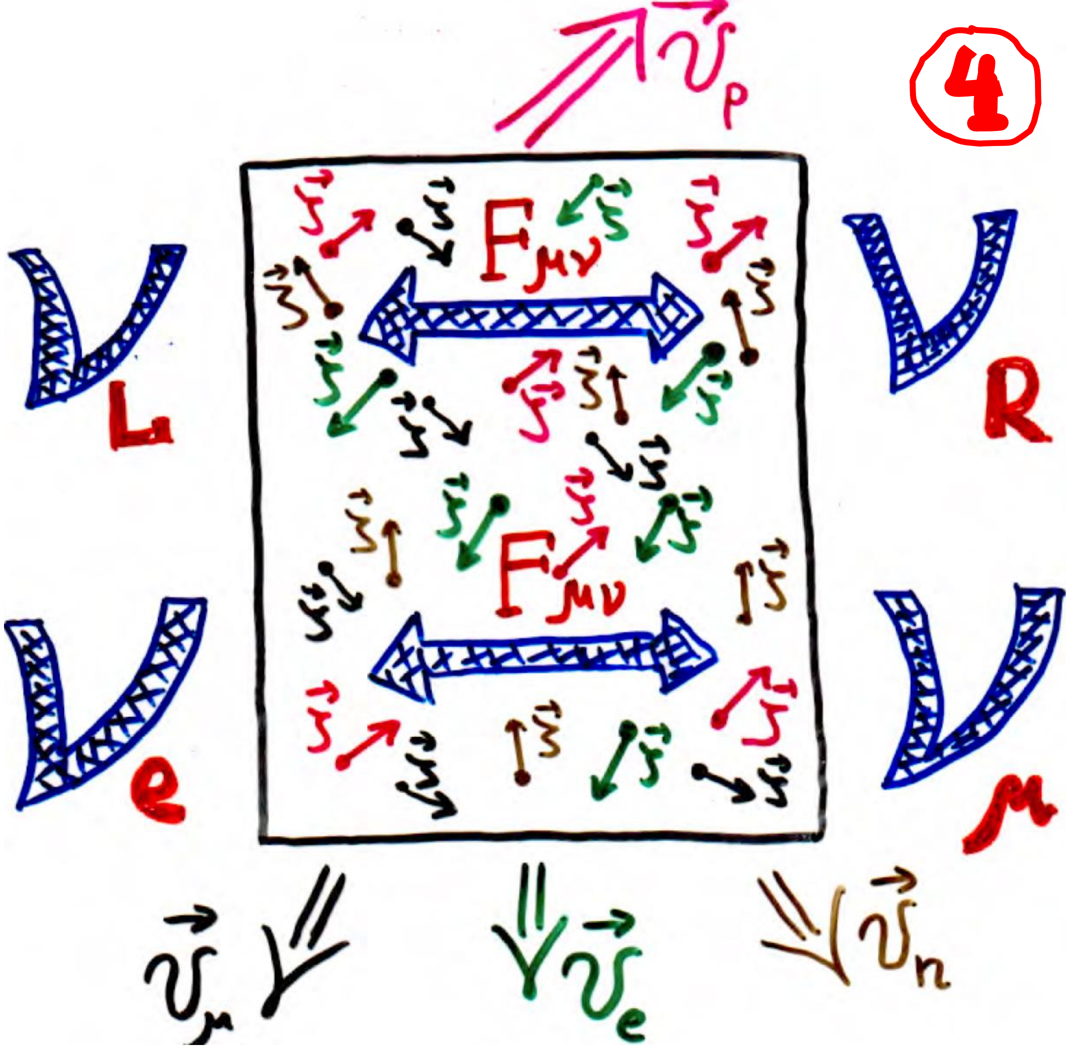
⊛ Spin-flavour oscillations in early universe – strong $B_\perp$ ⟹ population of ν wrong-helicity states (r.h.) would accelerate expansion of universe (???)

neutrino spin and flavor oscillations in moving matter

A.Egorov, A.Lobanov,
A.Studenikin,
Phys.Lett.B 491
(2000) 137

A.Lobanov,
A.Studenikin,
Phys.Lett.B 515
(2001) 94

A.Lobanov, A.Grigoriev,
A.Studenikin,
Phys.Lett.B 535
(2002) 187



moving matter components

$f = e, n, p, \mu,$ etc

with polarizations

$\vec{\zeta}, \vec{\zeta}, \vec{\zeta}, \vec{\zeta},$ etc

ν spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,
JHEP 09 (2002) 016

General types non-derivative interaction with external fields

$$\boxed{\begin{gathered}-\mathcal{L} = g_s s(x)\bar\nu\nu + g_p\pi(x)\bar\nu\gamma^5\nu + g_v V^\mu(x)\bar\nu\gamma_\mu\nu + g_a A^\mu(x)\bar\nu\gamma_\mu\gamma^5\nu + \\ + \frac{g_t}{2}T^{\mu\nu}\bar\nu\sigma_{\mu\nu}\nu + \frac{g'_t}{2}\Pi^{\mu\nu}\bar\nu\sigma_{\mu\nu}\gamma_5\nu,\end{gathered}}$$

scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields:

$$s, \pi, V^\mu = (V^0, \vec V), A^\mu = (A^0, \vec A),$$

$$T_{\mu\nu} = (\vec a, \vec b), \Pi_{\mu\nu} = (\vec c, \vec d)$$

Relativistic equation (quasiclassical) for ν spin vector:

$$\begin{aligned}\dot{\vec\zeta}_\nu = 2g_a&\left\{A^0[\vec\zeta_\nu\times\vec\beta] - \frac{m_\nu}{E_\nu}[\vec\zeta_\nu\times\vec A] - \frac{E_\nu}{E_\nu+m_\nu}(\vec A\vec\beta)[\vec\zeta_\nu\times\vec\beta]\right\} \\ &+2g_t\left\{[\vec\zeta_\nu\times\vec b] - \frac{E_\nu}{E_\nu+m_\nu}(\vec\beta\vec b)[\vec\zeta_\nu\times\vec\beta] + [\vec\zeta_\nu\times[\vec a\times\vec\beta]]\right\} + \\ &+2ig'_t\left\{[\vec\zeta_\nu\times\vec c] - \frac{E_\nu}{E_\nu+m_\nu}(\vec\beta\vec c)[\vec\zeta_\nu\times\vec\beta] - [\vec\zeta_\nu\times[\vec d\times\vec\beta]]\right\}.\end{aligned}$$

● *Neither* ***s*** *nor* $\boldsymbol{\pi}$ *nor* ***V*** *contributes to spin evolution*

- **Electromagnetic interaction**

$$T_{\mu\nu} = F_{\mu\nu} = (\vec E, \vec B)$$

- **SM weak interaction**

$$G_{\mu\nu} = (-\vec P, \vec M)$$

$$\vec M = \gamma(A^0\vec\beta - \vec A)$$
$$\vec P = -\gamma[\vec\beta\times\vec A],$$

Neutrino spin evolution in arbitrary electromagnetic field $F_{\mu\nu}$ and moving and polarized matter

Bargmann-Michel-Telegdi equation for spin vector S_μ of neutral particle:

$$\frac{dS^\mu}{d\tau} = 2\mu\left[F^{\mu\nu}S_\nu - u^\mu\left(u_\nu F^{\nu\lambda}S_\lambda\right)\right] + \underbrace{2\epsilon\left[\tilde{F}^{\mu\nu}S_\nu - u^\mu\left(u_\nu \tilde{F}^{\nu\lambda}S_\lambda\right)\right]}_{\cancel{\text{T-invariance}}}$$

magnetic

dipole moments

electric

- ~~direct interaction~~ of ~~ν~~ spin with $F_{\mu\nu}$
- $\mathcal{P}$ invariant theory

arbitrary e.m. field

Lorentz invariant generalization of BMT eq. :

$$F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}$$

ν interactions with moving and polarized matter

Substitution $\boxed{F_{\mu\nu} \longrightarrow F_{\mu\nu} + G_{\mu\nu}}$

implies:

✓ $$\boxed{\begin{aligned} \vec{B} &\longrightarrow \vec{B} + \vec{M} \\ \vec{E} &\longrightarrow \vec{E} - \vec{P} \end{aligned}}$$

effects of ν interaction with moving and polarized matter

… once more…

For SM+SU(2)-singlet ν_R and matter $f=e$

Bargmann-Michel-Telegdi eq

$$\boxed{\frac{d\vec S_\nu}{dt} = \frac{2\mu_\nu}{\gamma_\nu}\left[\vec S_\nu \times \left(\vec B_0 + \boxed{\vec M_0}\right)\right] \quad ,}$$

in rest frame of neutrino

interaction of neutrino with an electromagnetic field

$$\vec B_0 = \gamma_\nu \left(\underline{\vec B_\perp} + \frac{1}{\gamma_\nu}\underline{\vec B_\parallel} + \sqrt{1-\frac{1}{\gamma_\nu^2}}\,[\underline{\vec E_\perp} \times \vec n] \right),$$

interaction of neutrino with matter

$$\vec M_0 = \gamma_\nu \rho n_e \left(\underline{\vec\beta_\nu} (1 - \underline{\vec\beta_\nu}\,\underline{\vec v_e}) - \frac{1}{\gamma_\nu}\underline{\vec v_{e\perp}} \right),$$

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1 + 4\sin^8\theta_W),$$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

matter density

$\boxed{\parallel}$ $\boxed{\perp}$

→ ν spin precession in moving matter !!!

without any magnetic field !!!

Physics of Atomic Nuclei, Vol. 67, No. 5, 2004, pp. 993–1002. Translated from Yadernaya Fizika, Vol. 67, No. 5, 2004, pp. 1014–1024.
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ELEMENTARY PARTICLES AND FIELDS

Theory

Phys.Atom.Nucl. 67 (2004) 993-1002

Neutrino in Electromagnetic Fields and Moving Media

A. I. Studenikin*

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Received March 26, 2003; in final form, August 12, 2003

Abstract—The history of the development of the theory of neutrino-flavor and neutrino-spin oscillations in electromagnetic fields and in a medium is briefly surveyed. A new Lorentz-invariant approach to describing neutrino oscillations in a medium is formulated in such a way that it makes it possible to consider the motion of a medium at an arbitrary velocity, including relativistic ones. This approach permits studying neutrino-spin oscillations under the effect of an arbitrary external electromagnetic field. In particular, it is predicted that, in the field of an electromagnetic wave, new resonances may exist in neutrino oscillations. In the case of spin oscillations in various electromagnetic fields, the concept of a critical magnetic-field-component strength is introduced above which the oscillations become sizable. The use of the Lorentz-invariant formalism in considering neutrino oscillations in moving matter leads to the conclusion that the relativistic motion of matter significantly affects the character of neutrino oscillations and can radically change the conditions under which the oscillations are resonantly enhanced. Possible new effects in neutrino oscillations are discussed for the case of neutrino propagation in relativistic fluxes of matter.

STUDENIKIN PHYSICS OF ATOMIC NUCLEI Vol. 67 No. 5 2004

Consider

$$\nu_{e_L} \to \nu_{e_R}, \quad \nu_{e_L} \to \nu_{\mu_R}$$

$$P(\nu_i \to \nu_j) = \sin^2(2\theta_{\text{eff}}) \sin^2 \frac{\pi x}{L_{\text{eff}}}, \quad i \neq j$$

$$L_{\text{eff}} = \frac{2\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}}$$

$$\sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad \Delta_{\text{eff}}^2 = \frac{\mu}{\gamma_\nu}\left|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}\right|. \qquad E_{\text{eff}} = \mu\left|\mathbf{B}_\perp + \frac{1}{\gamma_\nu}\mathbf{M}_{0\perp}\right|$$

- A.Studenikin, "Status and perspectives of neutrino magnetic moments"
J.Phys.Conf.Ser. 718 (2016) 062076

$$\vec{M}_0 = \gamma_\nu \rho n_e \left(\underline{\underline{\vec{\beta}_\nu}} (1 - \underline{\underline{\vec{\beta}_\nu}} \underline{\underline{\vec{v}_e}}) - \frac{1}{\gamma_\nu} \underline{\underline{\vec{v}_{e_\perp}}}\right),$$

transversal matter current

matter density

$\parallel$

$\perp$

$$\gamma_\nu = \frac{E_\nu}{m_\nu},$$

where

$$\rho = \frac{G_F}{2\mu_\nu\sqrt{2}}(1 + 4\sin^2\theta_W)$$

Physics of Atomic Nuclei, Vol. 67, No. 5, 2004, pp. 993–1002. Translated from Yadernaya Fizika, Vol. 67, No. 5, 2004, pp. 1014–1024.

ELEMENTARY PARTICLES AND FIELDS

Theory

Phys.Atom.Nucl. 67 (2004) 993-1002, hep-ph/04070100

Neutrino in Electromagnetic Fields and Moving Media

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Received March 26, 2003; in final form, August 12, 2003

The possible emergence of neutrino-spin oscillations (for example, $\nu_{eL} \leftrightarrow \nu_{eR}$) owing to neutrino interaction with matter under the condition that there exists a nonzero transverse current component or matter polarization (that is, $\mathbf{M}_{0\perp} \neq 0$) is the most important new effect that follows from the investigation of neutrino-spin oscillations in Section 4. So far, it has been assumed that neutrino-spin oscillations may arise only in the case where there exists a nonzero transverse magnetic field in the neutrino rest frame.

$\nu_{e_L} \to \nu_{e_R}, \quad \nu_{e_L} \to \nu_{\mu_R}$

… the effect of ν helicity conversions and oscillations induced by transversal matter currents has been recently confirmed:

- J. Serreau and C. Volpe, "Neutrino-antineutrino correlations in dense anisotropic media", Phys .Rev. D90 (2014) 125040
- V. Ciriglianoa, G. M. Fuller, and A. Vlasenko, "A new spin on neutrino quantum kinetics" Phys. Lett. B747 (2015) 27
- A. Kartavtsev, G. Raffelt, and H. Vogel, "Neutrino propagation in media: flavor-, helicity-, and pair correlations", Phys. Rev. D91 (2015) 125020
- A. Dobrynina, A. Kartavtsev, and G. Raffelt, "Helicity oscillations of Dirac and Majorana neutrinos", Phys. Rev. D93 (2016) 125030

Neutrino spin (spin-flavour) oscillations in transversal matter currents

… quantum treatment …

- Studenikin

PoS (2017) NOW2016_070

Two flavour ν states

$$\boxed{\nu_e^{\pm} = \nu_1^{\pm}\cos\theta + \nu_2^{\pm}\sin\theta, \quad \nu_\mu^{\pm} = -\nu_1^{\pm}\sin\theta + \nu_2^{\pm}\cos\theta}$$

- Popov, Pustoshny, Studenikin,

Poster # 129

two ν mass

$$\boxed{\nu_\alpha^{\pm} = C_\alpha\sqrt{\frac{E_\alpha + m_\alpha}{2E_\alpha}}\begin{pmatrix} u^{\pm} \\ \frac{\sigma \mathbf{p}_\alpha}{E_\alpha + m_\alpha}u^{\pm}\end{pmatrix} e^{i\mathbf{p}_\alpha \mathbf{x}}, \alpha = 1, 2}$$

two helicities

$$\boxed{u^{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u^{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

ν interaction with moving matter composed of neutrons :

$$\boxed{L_{eff} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma_5}{2}\nu\right)}$$

transversal and longitudinal currents

$$\vec{j}_{\perp} + \vec{j}_{\parallel}$$

$$\boxed{f^{\mu} = -\frac{G_F}{2\sqrt{2}}j_n^{\mu}} \qquad \boxed{j_n^{\mu} = n(1, \mathbf{v})} \qquad n = \frac{n_0}{\sqrt{1-v^2}}$$

Two flavour ν with two helicities in moving matter

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^+\\ \nu_e^-\\ \nu_\mu^+\\ \nu_\mu^-\end{pmatrix}=\left\{H_{vac}^{eff}+\Delta H^{eff}\right\}\begin{pmatrix}\nu_e^+\\ \nu_e^-\\ \nu_\mu^+\\ \nu_\mu^-\end{pmatrix}$$

$$\Delta H^{eff}=\Delta H^{eff}_{v=0}+\Delta H^{eff}_{\vec{j}_\parallel+\vec{j}_\perp}$$

$$\vec{j}_\perp + \vec{j}_\parallel$$

Contribution of matter currents

$$\Delta H^{eff}=\begin{pmatrix}\Delta_{ee}^{++} & \Delta_{ee}^{+-} & \Delta_{e\mu}^{++} & \Delta_{e\mu}^{+-}\\ \Delta_{ee}^{-+} & \Delta_{ee}^{--} & \Delta_{e\mu}^{-+} & \Delta_{e\mu}^{--}\\ \Delta_{\mu e}^{++} & \Delta_{\mu e}^{+-} & \Delta_{\mu\mu}^{++} & \Delta_{\mu\mu}^{+-}\\ \Delta_{\mu e}^{-+} & \Delta_{\mu e}^{--} & \Delta_{\mu\mu}^{-+} & \Delta_{\mu\mu}^{--}\end{pmatrix}$$

$$\Delta_{kl}^{ss'}=\langle\nu_k^s|\Delta H^{SM}|\nu_l^{s'}\rangle \quad k,l=e,\mu \qquad s,s'=\pm$$

$$\Delta H^{SM}=-\frac{G_F}{2\sqrt{2}}\frac{n}{\sqrt{1-v^2}}\left(1-\gamma_0\boldsymbol{\gamma}\mathbf{v}\right)\left(1+\gamma_5\right)$$

$$\nu_e^\pm=\nu_1^\pm\cos\theta+\nu_2^\pm\sin\theta, \qquad \nu_\mu^\pm=-\nu_1^\pm\sin\theta+\nu_2^\pm\cos\theta$$

$$\gamma_{\alpha\alpha'}{}^{-1}=\frac{1}{2}\left(\gamma_\alpha^{-1}+\gamma_{\alpha'}^{-1}\right) \qquad \widetilde{\gamma}_{\alpha\alpha'}^{-1}=\frac{1}{2}\left(\gamma_\alpha^{-1}-\gamma_{\alpha'}^{-1}\right)$$

$$\Delta_{\alpha\alpha'}^{ss'}=\frac{G_F}{2\sqrt{2}}\frac{n_0}{\sqrt{1-v^2}}\left\{u_\alpha^{s\,T}\left[\left(1-\sigma_3\right)\left(v_\parallel-1\right)+\left(\gamma_{\alpha\alpha'}{}^{-1}\sigma_1+i\widetilde{\gamma}_{\alpha\alpha'}^{-1}\sigma_2\right)v_\perp\right]u_{\alpha'}^{s'}\right\}\alpha=1,2$$

$$\Delta_{\alpha\alpha'}^{ss'}=\frac{G_F}{2\sqrt{2}}\frac{n_0}{\sqrt{1-v^2}}\left\{u_\alpha^{s\,T}\left[\begin{pmatrix}0 & 0\\ 0 & 2\end{pmatrix}\left(v_\parallel-1\right)+\begin{pmatrix}0 & \gamma_\alpha^{-1}\\ \gamma_{\alpha'}^{-1} & 0\end{pmatrix}v_\perp\right]u_{\alpha'}^{s'}\right\}$$

$$u^+=\begin{pmatrix}1\\0\end{pmatrix}, \qquad u^-=\begin{pmatrix}0\\1\end{pmatrix}$$

two helicity states

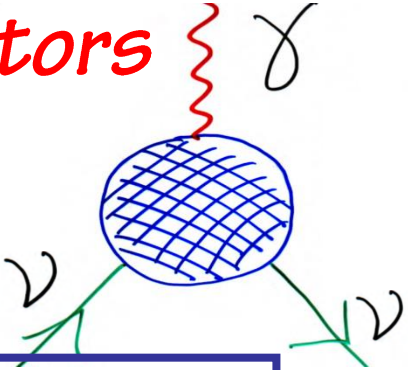
$$\gamma_\alpha^{-1}=\frac{m_\alpha}{E_\alpha}$$

- longitudinal current $j_\parallel$ does not change ν helicity
- transversal current $j_\perp$ do change ν helicity

Studenikin, 2004, 2017; Popov, Pustoshny, Studenikin, Poster # 129

Conclusions

ν e.m. vertex function → 4 form factors



charge dipole magnetic and electric

- $$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
  $$f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$ anapole

- EM properties → a way to distinguish Dirac and Majorana ν

- Standard Model with ν_R $(m_\nu \neq 0)$: $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3 \cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1eV}\right)$

- In extensions of SM → enhancement of ν magnetic moment , even electrically millicharged ν

- Limits from reactor ν-e scattering experiments (2012):

$\mu_\nu < 2.9 \times 10^{-11} \mu_B$ A.Beda et al. (GEMMA Coll.)

$| q_\nu | < 1.5 \times 10^{-12} e_0$ Studenikin 2013

$\mu_\nu < 2.8 \times 10^{-11} \mu_B$ Borexino 2017

- Limits from astrophysics, star cooling (1990):

$\mu_\nu < 3 \times 10^{-12} \mu_B$ Raffelt

$q_0 < 1.3 \times 10^{-19} e_0$ νST, 2014 mechanism

μ_ν is "presently known" to be in the range

$$\boxed{10^{-20}\mu_B \leq \ \mu_\nu \ \leq 10^{-\mathbf{11}}\mu_B}$$

μ_ν provides a tool for exploration possible physics beyond the Standard Model

- Due to smallness of neutrino-mass-induced magnetic moments,

$$\boxed{\mu_{ii} \approx 3.2 \times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B}$$

any indication for non-trivial electromagnetic properties of ν, that could be obtained within reasonable time in the future, would give evidence for BESM physics

Beyond Extended Standard Model

μ_ν interactions could have important effects in astrophysical and cosmological environments

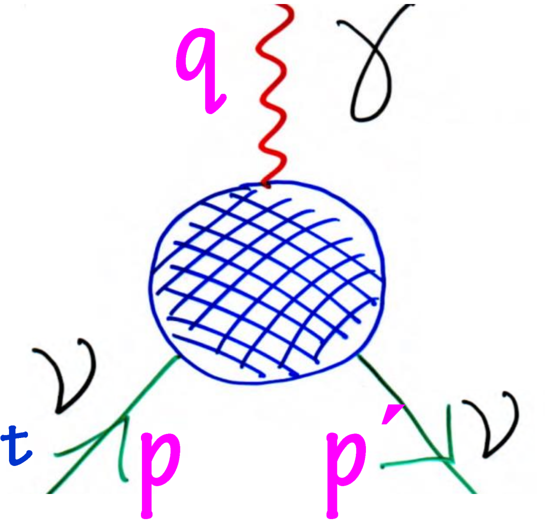
future high-precision observations of supernova ν fluxes (for instance, in JUNO experiment) may reveal effect of collective spin-flavour oscillations due to Majorana

$$\mu_\nu \sim 10^{-21} \mu_B$$

- A. de Gouvea, S. Shalgar,
  Cosmol. Astropart. Phys. 04 (2013) 018

back up slides

ν electromagnetic vertex function



$$< \psi(p')|J_\mu^{EM}|\psi(p) > = \bar{u}(p')\Lambda_\mu(q,l)u(p)$$

Matrix element of electromagnetic current is a Lorentz vector

$\Lambda_\mu(q,l)$ should be constructed using ●

matrices $\hat{1}, \quad \gamma_5, \quad \gamma_\mu, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu},$

tensors $g_{\mu\nu}, \quad \epsilon_{\mu\nu\sigma\gamma}$

vectors q_μ and l_μ

$$\boxed{q_\mu = p'_\mu - p_\mu, \; l_\mu = p'_\mu + p_\mu}$$

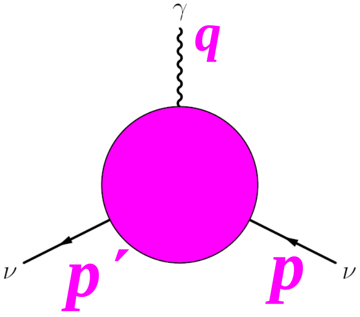
● Lorentz covariance (1) and electromagnetic gauge invariance (2) $\Longrightarrow$

Vertex function $\Lambda_\mu(q,l)$ ⟹ there are three sets of operators:

- $\hat{\mathbf{1}}q_\mu,\quad \hat{\mathbf{1}}l_\mu,\quad \gamma_5 q_\mu,\quad \gamma_5 l_\mu$

  $\not{q}q_\mu,\quad \not{l}q_\mu,\quad \gamma_5 q_\mu,\quad \gamma_5\not{q}q_\mu,\quad \gamma_5\not{l}q_\mu,\quad \sigma_{\alpha\beta}q^\alpha l^\beta q_\mu,\quad \{q_\mu \leftrightarrow l_\mu\}$
- $\gamma_\mu,\quad \gamma_5\gamma_\mu,\quad \sigma_{\mu\nu}q^\nu,\quad \sigma_{\mu\nu}l^\nu.$
- $\epsilon_{\mu\nu\sigma\gamma}\sigma^{\alpha\beta}q^\nu,\quad \epsilon_{\mu\nu\sigma\gamma}\sigma^{\alpha\beta}l^\nu,\quad \epsilon_{\mu\nu\sigma\gamma}\sigma^{\nu\beta}q_\beta q^\sigma l^\gamma,$

  $\epsilon_{\mu\nu\sigma\gamma}\sigma^{\nu\beta}l_\beta q^\sigma l^\gamma,\quad \epsilon_{\mu\nu\sigma\gamma}\gamma^\nu q^\sigma l^\gamma\hat{\mathbf{1}},\quad \epsilon_{\mu\nu\sigma\gamma}\gamma^\nu q^\sigma l^\gamma\gamma_5$



✓ **vertex function** (using Gordon-like identities)

$$\boxed{\begin{aligned}\Lambda_\mu(q,l) = {} & f_1(q^2)q_\mu + f_2(q^2)q_\mu\gamma_5 + f_3(q^2)\gamma_\mu + \\ & f_4(q^2)\gamma_\mu\gamma_5 + f_5(q^2)\sigma_{\mu\nu}q^\nu + f_6(q^2)\epsilon_{\mu\nu\rho\gamma}\sigma^{\rho\gamma}q^\nu,\end{aligned}}$$

the only dependence on q^2 emains because $p^2 = p'^2 = m^2$, $l^2 = 4m^2 - q^2$

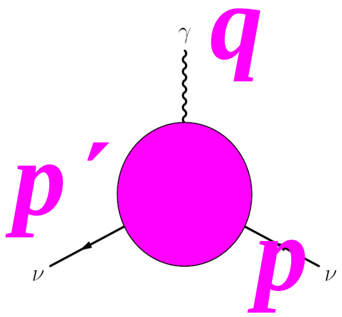
Gordon-like identities

$$
\begin{aligned}
\bar{u}(\mathbf{p}_1)\gamma^\mu u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[l^\mu + i\sigma^{\mu\nu}q_\nu]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)\gamma^\mu\gamma_5 u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[\gamma_5 q^\mu + i\gamma_5\sigma^{\mu\nu}l_\nu]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}l_\nu u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)q^\nu u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}q_\nu u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)[2m\gamma^\mu l^\mu]u(\mathbf{p}_2)
\end{aligned}
$$

$$
\begin{aligned}
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}\gamma_5 q_\nu u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)l^\mu\gamma_5 u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{-i[q^\alpha \ \not{l} - l^\alpha \ \not{q}] + i(q^2 - 4m^2)\gamma^\alpha + \\
& 2im(l^\alpha + q^\alpha)\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{i[q^\alpha \ \not{l} - l^\alpha \ \not{q}]\gamma_5 + iq^2\gamma_5\gamma^\alpha - \\
& 2im(l^\alpha + q^\alpha)\gamma_5\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\gamma_\nu\gamma_5]u(\mathbf{p}_2) &= \frac{i}{2m}\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}q^\rho]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}l^\rho]u(\mathbf{p}_2) &= 0
\end{aligned}
$$

Electromagnetic gauge invariance (2)

(requirement of current conservation)



$$\boxed{\partial_\mu j^\mu = 0}$$

$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0,$$

$$f_1(q^2) = 0, \qquad f_2(q^2)q^2 + 2mf_4(q^2) = 0$$

vertex function

$$\Lambda_\mu(q) \;=\; f_Q(q^2)\gamma_\mu \;+\; f_M(q^2)i\sigma_{\mu\nu}q^\nu \;+$$
$$f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 \;+\; f_A(q^2)(q^2\gamma_\mu \;-\; q_\mu \not{q})\gamma_5$$

charge

dipole electric and magnetic

anapole

4 Form Factors

... consistent with

Lorentz-covariance (1)

+

electromagnetic gauge invariance (2)

Matrix element of electromagnetic current between neutrino states

$$\langle\nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p),$$

where vertex function generally contains 4 form factors

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

1. electric

dipole

2. magnetic

3. electric

4. anapole

- Hermiticity and discrete symmetries of EM current J_μ^{EM} put constraints on form factors

Dirac ν

1) CP invariance + Hermiticity ⟹ $f_E = 0$,

2) at zero momentum transfer only electric Charge $f_Q(0)$ and magnetic moment $f_M(0)$ contribute to $H_{int} \sim J_\mu^{EM} A^\mu$

3) Hermiticity itself ⟹ three form factors are real: $Im f_Q = Im f_M = Im f_A = \mathbf{0}$

Majorana ν

1) from CPT invariance (regardless CP or ~~CP~~).

$$f_Q = f_M = f_E = 0$$

...as early as 1939, W.Pauli...

- EM properties ⟹ a way to distinguish Dirac and Majorana ν

… A remark on electric charge of ν …

Beyond Standard Model…

ν **neutrality *Q=0* is attributed to**

gauge invariance
+
anomaly cancellation constraints

imposed in SM of electroweak interactions

- ***…General proof:***

Foot, Joshi, Lew, Volkas, 1990;
Foot, Lew, Volkas, 1993;
Babu, Mohapatra, 1989, 1990
Foot , He (1991)

- ***In SM :***

$SU(2)_L \times U(1)_Y$

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

- ***In SM (without ν_R) triangle anomalies cancellation constraints → certain relations among particle hypercharges Y, that is enough to fix all Y so that they, and consequently Q, are quantized***

$\boxed{Q=0}$

- $\boxed{Q=0}$ ***is proven also by direct calculation in SM within different gauges and methods***

Bardeen, Gastmans, Lautrup, 1972;
Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000;
Beg, Marciano, Ruderman, 1978;
Marciano, Sirlin, 1980; Sakakibara, 1981;
M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)

- ***… However, strict requirements for Q quantization may disappear in extensions of standard $SU(2)_L \times U(1)_Y$ EW model if ν_R with $Y \neq 0$ are included : in the absence of Y quantization electric charges Q gets dequantized***

→ **millicharged ν**

Astrophysics bounds on μ_ν

$$\boxed{\mu_\nu(\text{astro}) < 10^{-10}\text{–}10^{-12}\ \mu_{\rm B}}$$

Mostly derived from consequences of helicity-state change in astrophysical medium:

- **available degrees of freedom in BBN,**
- **stellar cooling via plasmon decay,**
- **cooling of SN1987a**

Red Giant lumin.
$\mu_\nu < 3\cdot 10^{-12}\mu_B$
G. Raffelt, D. Dearborn,
J. Silk, 1989.

Bounds depend on

- **modeling of astrophysical systems,**
- **on assumptions on the neutrino properties.**

Generic assumption:

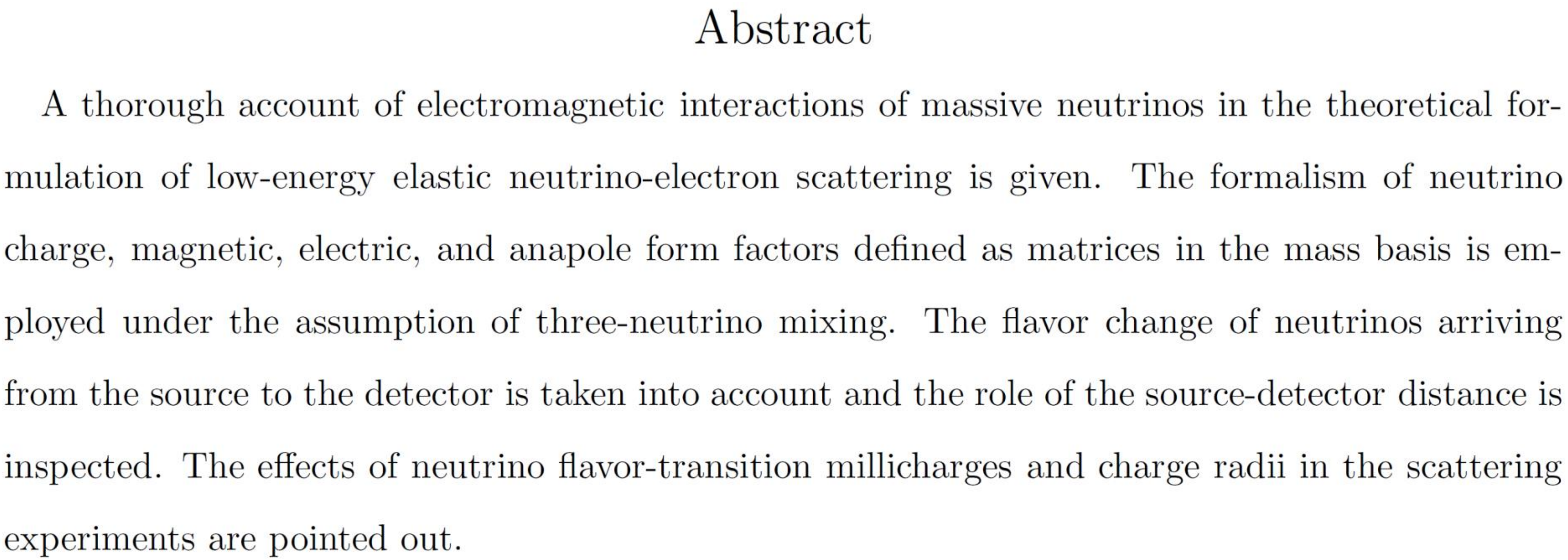
- **absence of other nonstandard interactions except for** μ_ν

A global treatment would be desirable, incorporating oscillation and matter effects as well as the complications due to interference and competitions among various channels

K. Kouzakov, A. Studenikin,

Phys. Rev. D 95 (2017) 055013

“Electromagnetic properties of massive neutrinos in low-energy elastic neutrino-electron scattering”

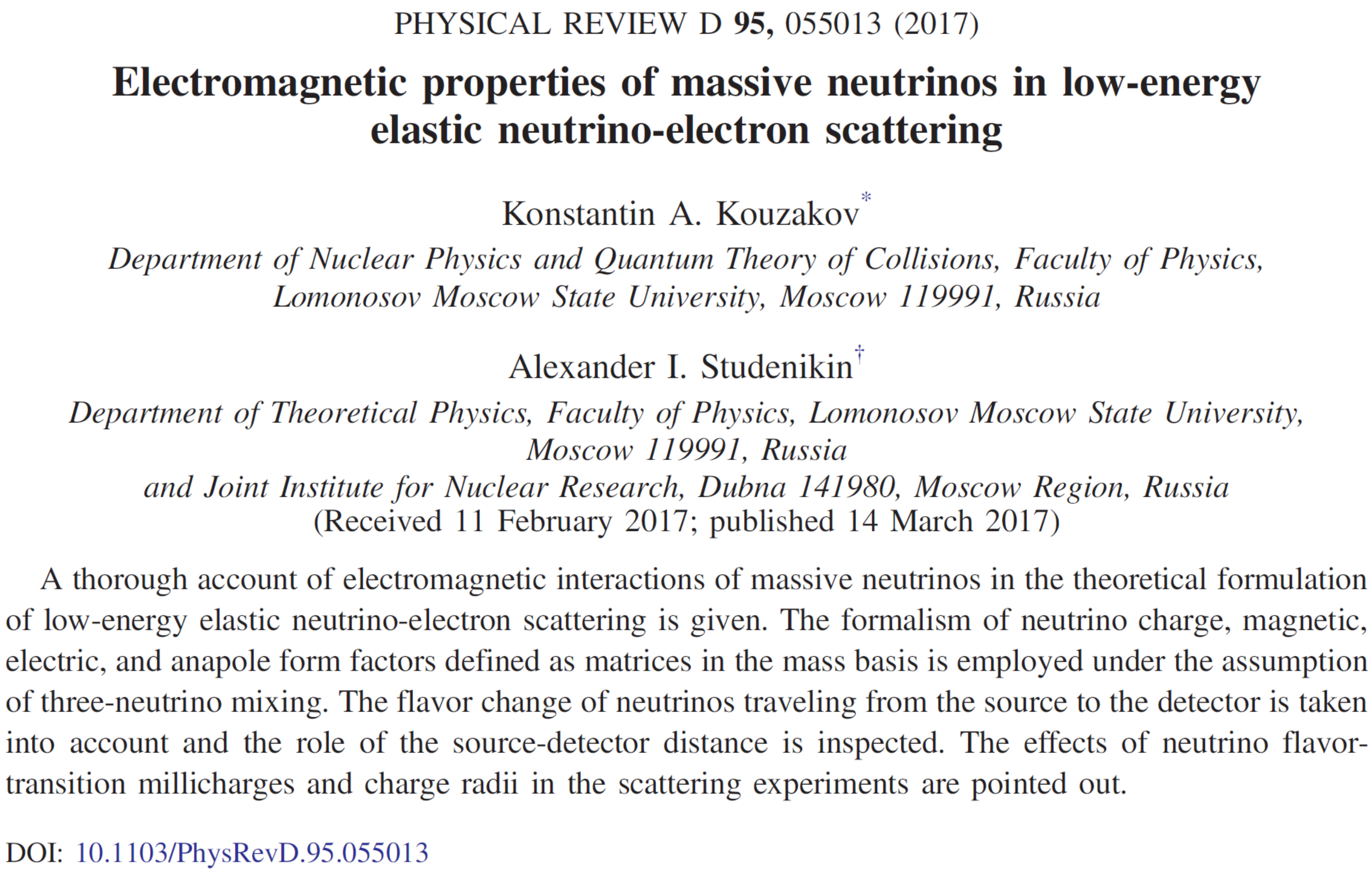


Abstract

A thorough account of electromagnetic interactions of massive neutrinos in the theoretical formulation of low-energy elastic neutrino-electron scattering is given. The formalism of neutrino charge, magnetic, electric, and anapole form factors defined as matrices in the mass basis is employed under the assumption of three-neutrino mixing. The flavor change of neutrinos arriving from the source to the detector is taken into account and the role of the source-detector distance is inspected. The effects of neutrino flavor-transition millicharges and charge radii in the scattering experiments are pointed out.

arXiv: 1703.0040 Mar 2017

… comprehensive analysis of ν-e scattering …



PHYSICAL REVIEW D **95,** 055013 (2017)

Electromagnetic properties of massive neutrinos in low-energy elastic neutrino-electron scattering

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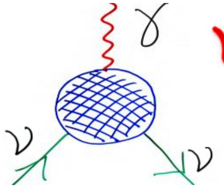
A thorough account of electromagnetic interactions of massive neutrinos in the theoretical formulation of low-energy elastic neutrino-electron scattering is given. The formalism of neutrino charge, magnetic, electric, and anapole form factors defined as matrices in the mass basis is employed under the assumption of three-neutrino mixing. The flavor change of neutrinos traveling from the source to the detector is taken into account and the role of the source-detector distance is inspected. The effects of neutrino flavor-transition millicharges and charge radii in the scattering experiments are pointed out.

DOI: 10.1103/PhysRevD.95.055013



ν electromagnetic interactions

mass states $\nu_j\ ,\ m_j\ (j=1,2,3)$

$q = p_j - p_k$

$$\boxed{\mathcal{H}^{(\nu)}_{\rm em} = j^{(\nu)}_\lambda A^\lambda = \sum_{j,k=1}^{3} \overline{\nu}_j \Lambda^{jk}_\lambda \nu_k A^\lambda}$$

$$\boxed{\Lambda_\lambda(q) = \left(\gamma_\lambda - \frac{q_\lambda \not{q}}{q^2}\right)\left[f_Q(q^2) + f_A(q^2)q^2\gamma^5\right] - i\sigma_{\lambda\rho}q^\rho\left[f_M(q^2) + if_E(q^2)\gamma^5\right]}$$

Elastic neutrino-electron scattering

at energy-momentum transfer $q = (T, \mathbf{q})$

$$\boxed{\nu_l(L) + e^- \rightarrow \nu_j + e^-}$$

flavour state $|\nu_\ell\rangle$ in the source arrives to the detector as

$$\boxed{|\nu_\ell(L)\rangle = \sum_{k=1}^{3} U^*_{\ell k} e^{-i\frac{m_k^2}{2E_\nu}L} |\nu_k\rangle}$$

Matrix element of weak interactions

$$\boxed{\mathcal{M}^{(w)}_j = \frac{G_F}{\sqrt{2}} \sum_{k=1}^{3} U^*_{\ell k} e^{-i\frac{m_k^2}{2E_\nu}L} \left[(g'_V)_{jk}\bar{u}_j\gamma_\lambda(1-\gamma^5)u_k J^\lambda_V(q) - (g'_A)_{jk}\bar{u}_j\gamma_\lambda(1-\gamma^5)u_k J^\lambda_A(q)\right]}$$

$$(g'_V)_{jk} = \delta_{jk}g_V + U^*_{ej}U_{ek} \quad (g'_A)_{jk} = \delta_{jk}g_A + U^*_{ej}U_{ek} \quad g_V = 2\sin^2\theta_W - 1/2,\ g_A = -1/2$$

Electron transition V and A currents in detector

$\boxed{|i\rangle \text{ and } |f\rangle}$

states of detector

$$\boxed{J^\lambda_V(q) = \langle f| \sum_d e^{i\mathbf{q}\cdot\mathbf{r}_d} \gamma^0_d \gamma^\lambda_d |i\rangle} \qquad \boxed{J^\lambda_A(q) = \langle f| \sum_d e^{i\mathbf{q}\cdot\mathbf{r}_d} \gamma^0_d \gamma^\lambda_d \gamma^5_d |i\rangle}$$

over all electrons of detector

$$\boxed{\mathcal{E}_f - \mathcal{E}_i = T}$$

energy transfer

Matrix element of electromagnetic interactions

$$\boxed{\mathcal{M}_j^{(\gamma)} = \mathcal{M}_j^{(Q)} + \mathcal{M}_j^{(\mu)}}$$

- $$\boxed{\mathcal{M}_j^{(Q)} = \frac{4\pi\alpha}{q^2}\sum_{k=1}^{3} U_{\ell k}^{*} e^{-i\frac{m_k^2}{2E_\nu}L} \bar{u}_j \left(\gamma_\lambda - \frac{q_\lambda \not{q}}{q^2}\right)\left[(e_\nu)_{jk} + \frac{q^2}{6}\langle r_\nu^2\rangle_{jk}\right] u_k J_V^\lambda(q)}$$

millicharge $\boxed{(e_\nu)_{jk} = e_{jk}}$ charge radius and anapole moment $\boxed{\langle r_\nu^2\rangle_{jk} = \langle r^2\rangle_{jk} + 6\gamma^5 a_{jk}}$

- $$\boxed{\mathcal{M}_j^{(\mu)} = -i\frac{2\pi\alpha}{m_e q^2}\sum_{k=1}^{3} U_{\ell k}^{*} e^{-i\frac{m_k^2}{2E_\nu}L} \bar{u}_j \sigma_{\lambda\rho} q^\rho (\mu_\nu)_{jk} u_k J_V^\lambda(q)}$$

$\boxed{(\mu_\nu)_{jk} = \mu_{jk} + i\gamma^5 \varepsilon_{jk}}$

magnetic & electric dipole moments

nonmoving matter !!!

Helicity-conserving amplitudes

$$\mathcal{M}_j^{(w,Q)} = \mathcal{M}_j^{(w)} + \mathcal{M}_j^{(Q)}$$

$$= \frac{G_F}{\sqrt{2}}\sum_{k=1}^{3} U_{\ell k}^{*} e^{-i\frac{m_k^2}{2E_\nu}L} \left\{ \left[(g_V')_{jk} + \tilde{Q}_{jk}\right] \bar{u}_j \gamma_\lambda (1-\gamma^5) u_k J_V^\lambda(q) \right.$$

$$\left. -(g_A')_{jk} \bar{u}_j \gamma_\lambda (1-\gamma^5) u_k J_A^\lambda(q) \right\}$$

$$\tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{jk}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{jk}\right]$$

Differential cross section measured in scattering experiment

the final massive state is not resolved in experiment

$$\boxed{\frac{d\sigma}{dT}=\frac{1}{32\pi^2}\int\limits_{T^2}^{(2E_\nu-T)^2}\frac{d\mathbf{q}^2}{E_\nu^2}\int\limits_0^{2\pi}d\varphi_{\mathbf{q}}\,|\mathcal{M}_{fi}|^2\,\delta(T-\mathcal{E}_f+\mathcal{E}_i)}$$

1) ν masses are neglected

2) $p_j = p' \qquad p_k = p$

3) averaging (summation) over initial (final) spin polariz.

4) $\varphi_{\mathbf{q}}$ is azimutal angle

$$\left|\mathcal{M}_{fi}^{(w,Q)}\right|^2=\sum_{j=1}^{3}\left|\tilde{\mathcal{M}}_j^{(w,Q)}\right|^2 \qquad \boxed{|\mathcal{M}_{fi}|^2=\sum_{j=1}^{3}\left\{\left|\mathcal{M}_j^{(w,Q)}\right|^2+\left|\mathcal{M}_j^{(\mu)}\right|^2\right\}}$$

$$\begin{aligned}
= 4G_F^2\Big\{ & C_1\left[2|p\cdot J_V(q)|^2-(p\cdot p')J_V(q)\cdot J_V^*(q)-i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_V^{\lambda}(q)J_V^{\lambda'*}(q)\right] \\
& +C_2\Big[(p\cdot J_A(q))\,(p'\cdot J_A^*(q))+(p'\cdot J_A(q))\,(p\cdot J_A^*(q))-(p\cdot p')J_A(q)\cdot J_A^*(q) \\
& -i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_A^{\lambda}(q)J_A^{\lambda'*}(q)\Big]+2\mathrm{Re}\Big\{C_3\Big[(p\cdot J_V(q))\,(p'\cdot J_A^*(q)) \\
& +(p'\cdot J_A(q))\,(p\cdot J_V^*(q))-(p\cdot p')J_V(q)\cdot J_A^*(q)-i\varepsilon_{\lambda\rho\lambda'\rho'}p'^{\rho}p^{\rho'}J_V^{\lambda}(q)J_A^{\lambda'*}(q)\Big]\Big\}\Big\}
\end{aligned}$$

… complicated intersection of weak and electromagnetic interactions with effects of mixing …

$$C_1=\sum_{j,k,k'=1}^{3}U_{\ell k}^*U_{\ell k'}e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}\left[(g_V')_{jk}+\tilde{Q}_{jk}\right]\left[(g_V')_{jk'}^*+\tilde{Q}_{jk'}^*\right]$$

$$C_2=\sum_{j,k,k'=1}^{3}U_{\ell k}^*U_{\ell k'}e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}(g_A')_{jk}(g_A')_{jk'}^*$$

$$C_3=\sum_{j,k,k'=1}^{3}U_{\ell k}^*U_{\ell k'}e^{-i\frac{\delta m_{kk'}^2}{2E_\nu}L}\left[(g_V')_{jk}+\tilde{Q}_{jk}\right](g_A')_{jk'}^*$$

$$\boxed{\tilde{Q}_{jk}=\frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{jk}}{q^2}+\frac{1}{6}\langle r_\nu^2\rangle_{jk}\right]}$$

$$\boxed{(g_V')_{jk}=\delta_{jk}g_V+U_{ej}^*U_{ek}}$$

$$\boxed{(g_A')_{jk}=\delta_{jk}g_A+U_{ej}^*U_{ek}}$$

$$\delta m_{kk'}^2=m_k^2-m_{k'}^2$$

… it is usually claimed …

$$g_V\rightarrow g_V+\boxed{\tfrac{2}{3}M_W^2\langle r^2\rangle\sin^2\theta_W}$$

Magnetic moment part of cross section

$$\frac{d\sigma}{dT} = \frac{1}{32\pi^2}\int\limits_{T^2}^{(2E_\nu - T)^2}\frac{d\mathbf{q}^2}{E_\nu^2}\int\limits_0^{2\pi} d\varphi_{\mathbf{q}}\,|\mathcal{M}_{fi}|^2\,\delta(T-\mathcal{E}_f+\mathcal{E}_i)$$

$$|\mathcal{M}_{fi}|^2 = \sum_{j=1}^{3}\left\{\left|\mathcal{M}_j^{(w,Q)}\right|^2 + \left|\mathcal{M}_j^{(\mu)}\right|^2\right\}$$

$$\left|\mathcal{M}_{fi}^{(\mu)}\right|^2 = \sum_{j=1}^{3}\left|\mathcal{M}_j^{(\mu)}\right|^2 = \frac{32\pi^2\alpha^2}{m_e^2|q^2|}|\mu_\nu(L,E_\nu)|^2|p\cdot J_V(q)|^2$$

$$|\mu_\nu(L,E_\nu)|^2 = \sum_{j=1}^{3}\left|\sum_{k=1}^{3} U_{\ell k}^{*}\, e^{-i\frac{m_k^2}{2E_\nu}L}(\mu_\nu)_{jk}\right|^2$$

Giunti, Studenikin,
Rev. Mod. Phys. 2015

For Dirac antineutrinos

$$(e_\nu)_{jk} \to (e_{\bar\nu})_{jk} = -e_{kj} \qquad (\mu_\nu)_{jk} \to (\mu_{\bar\nu})_{jk} = -\mu_{kj} - i\gamma^5\varepsilon_{kj} \qquad \langle r_\nu^2\rangle_{jk} \to \langle r_{\bar\nu}^2\rangle_{jk} = -\langle r^2\rangle_{kj} + 6\gamma^5 a_{kj}$$

$$(g'_V)_{jk} \;\to\; -(g'_V)^*_{jk} \qquad (g'_A)_{jk} \;\to\; -(g'_A)^*_{jk} \qquad \varepsilon_{\lambda\rho\lambda'\rho'} \;\to\; -\varepsilon_{\lambda\rho\lambda'\rho'} \qquad U_{\ell k} \;\to\; U^*_{\ell k}$$

Free-electron approximation $T \gg E_b$

electrons are free and at rest

energy transfer

electron binding energy in detector

ν-e scattering cross section (free e)

$$\boxed{\frac{d\sigma}{dT} = \frac{1}{32\pi^2}\int\limits_{T^2}^{(2E_\nu - T)^2} \frac{d\mathbf{q}^2}{E_\nu^2}\int\limits_0^{2\pi} d\varphi_{\mathbf{q}} \left|\mathcal{M}_{fi}\right|^2 \delta(T - \sqrt{\mathbf{q}^2 + m_e^2} + m_e)}$$

$$J_A^\lambda(q) = \frac{1}{2\sqrt{E_e' m_e}}\bar{u}_e'\gamma^\lambda\gamma^5 u_e$$

$$J_V^\lambda(q) = \frac{1}{2\sqrt{E_e' m_e}}\bar{u}_e'\gamma^\lambda u_e$$

$$E_e' = m_e + T$$

final electron energy

Finally cross section (free e)

$$\boxed{\frac{d\sigma^{\mathrm{FE}}}{dT} = \frac{d\sigma^{\mathrm{FE}}_{(w,Q)}}{dT} + \frac{d\sigma^{\mathrm{FE}}_{(\mu)}}{dT}}$$

where

$$\frac{d\sigma^{\mathrm{FE}}_{(\mu)}}{dT} = \frac{\pi\alpha^2}{m_e^2}\left|\mu_\nu(L, E_\nu)\right|^2 \left(\frac{1}{T} - \frac{1}{E_\nu}\right)$$

and

$$\frac{d\sigma^{\mathrm{FE}}_{(w,Q)}}{dT} = \frac{G_F^2 m_e}{2\pi}\left[C_1 + C_2 - 2\mathrm{Re}\left\{C_3\right\} + (C_1 + C_2 + 2\mathrm{Re}\left\{C_3\right\})\left(1 - \frac{T}{E_\nu}\right) + (C_2 - C_1)\frac{T m_e}{E_\nu^2}\right]$$

The role of ν flavor oscillations

- Manifestation of ν electromagnetic properties depends on ν state $\nu_\ell(L)$ in the detector
- The obtained **cross section** depends on flavor transition **amplitude** and **probability**

$$\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu) = \langle\nu_{\ell'}|\nu_\ell(L)\rangle = \sum_{k=1}^{3} U^*_{\ell k} U_{\ell' k} e^{-i\frac{m_k^2}{2E_\nu}L}$$

and

$$P_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu) = |\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)|^2$$

$$\frac{d\sigma^{\mathrm{FE}}_{(w,Q)}}{dT} = \frac{G_F^2 m_e}{2\pi}\left[C_1 + C_2 - 2\mathrm{Re}\left\{C_3\right\} + (C_1 + C_2 + 2\mathrm{Re}\left\{C_3\right\})\left(1 - \frac{T}{E_\nu}\right) + (C_2 - C_1)\frac{Tm_e}{E_\nu^2}\right]$$

$$C_1 = g_V^2 + 2g_V P_{\nu_\ell\to\nu_e}(L,E_\nu) + P_{\nu_\ell\to\nu_e}(L,E_\nu) + 2g_V \sum_{\ell',\ell''=e,\mu,\tau} \mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)\mathcal{A}^*_{\nu_\ell\to\nu_{\ell''}}(L,E_\nu)\tilde{Q}_{\ell''\ell'}$$
$$+2\mathrm{Re}\left\{\mathcal{A}^*_{\nu_\ell\to\nu_e}(L,E_\nu)\sum_{\ell'=e,\mu,\tau}\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)\tilde{Q}_{e\ell'}\right\} + \sum_{\ell',\ell'',\ell'''=e,\mu,\tau}\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)\mathcal{A}^*_{\nu_\ell\to\nu_{\ell''}}(L,E_\nu)\tilde{Q}_{\ell''\ell'''}\tilde{Q}_{\ell'''\ell'}$$

$$C_2 = g_A^2 + 2g_A P_{\nu_\ell\to\nu_e}(L,E_\nu) + P_{\nu_\ell\to\nu_e}(L,E_\nu)$$

$$C_3 = g_V g_A + (g_V + g_A + 1)P_{\nu_\ell\to\nu_e}(L,E_\nu) + g_A \sum_{\ell',\ell''=e,\mu,\tau}\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)\mathcal{A}^*_{\nu_\ell\to\nu_{\ell''}}(L,E_\nu)\tilde{Q}_{\ell''\ell'}$$
$$+\mathcal{A}^*_{\nu_\ell\to\nu_e}(L,E_\nu)\sum_{\ell'=e,\mu,\tau}\mathcal{A}_{\nu_\ell\to\nu_{\ell'}}(L,E_\nu)\tilde{Q}_{e\ell'}$$

Generalized ν charge

Up to now we have used $\tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{jk}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{jk}\right]$ in mass basis

Finally we have in flavour basis

$$\boxed{\tilde{Q}_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} \tilde{Q}_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F}\left[\frac{(e_\nu)_{\ell'\ell}}{q^2} + \frac{1}{6}\langle r_\nu^2\rangle_{\ell'\ell}\right]}$$

where

$$\boxed{(e_\nu)_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} (e_\nu)_{jk}}$$

$$\boxed{\langle r_\nu^2\rangle_{\ell'\ell} = \sum_{j,k=1}^{3} U_{\ell' j} U_{\ell k}^{*} \langle r_\nu^2\rangle_{jk}}$$

millicharge

in ν flavour basis

charge radius

- **Short-baselin case** $\boxed{L \ll L_{kk'} = 2E_\nu/|\delta m^2_{kk'}|} \longrightarrow \boxed{e^{-i(\delta m^2_{kk'}/2E_\nu)L} = 1}$

- $P_{\nu_\ell \to \nu_e}(L, E_\nu) = \delta_{\ell e}$ $\qquad$ $\mathcal{A}_{\nu_\ell \to \nu_{\ell'}}(L, E_\nu)\mathcal{A}^*_{\nu_\ell \to \nu_{\ell''}}(L, E_\nu) = \delta_{\ell\ell'}\delta_{\ell\ell''}$

effect of ν flavor change is insignificant
($\nu_\ell(L)$ is as in the source)

$$C_1 = (g_V + \delta_{\ell e} + \tilde{Q}_{\ell\ell})^2 + \sum_{\ell'=e,\mu,\tau} (1 - \delta_{\ell'\ell}) \left|\tilde{Q}_{\ell'\ell}\right|^2 \qquad C_2 = (g_A + \delta_{\ell e})^2$$

$$C_3 = (g_V + \delta_{\ell e})(g_A + \delta_{\ell e}) + (g_A + \delta_{\ell e})\tilde{Q}_{\ell\ell}$$

weak-electromagnetic interference term contains only flavour-diagonal millicharges and charge radii

- Effective magnetic moment

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j=1}^{3} \sum_{k,k'=1}^{3} U^*_{\ell k} U_{\ell k'} (\mu_\nu)_{jk} (\mu_\nu)^*_{jk'} = \sum_{\ell'=e,\mu,\tau} |(\mu_\nu)_{\ell'\ell}|^2$$ where

$$(\mu_\nu)_{\ell'\ell} = \sum_{j,k=1}^{3} U^*_{\ell k} U_{\ell' j} (\mu_\nu)_{jk}$$ is the effective magnetic moment in flavor basis

- Long-baselin case $\boxed{L \gg L_{kj} = 2E_\nu/|\delta m^2_{kk'}|}$ $\boxed{\exp(-i\delta m^2_{kk'}/2E_\nu) \;=\; \delta_{kk'}}$

effect of decoherence

$$C_1 = g_V^2 + 2g_V P_{\nu_\ell\to\nu_e} + P_{\nu_\ell\to\nu_e} + \sum_{j,k=1}^{3} |U_{\ell k}|^2 \left|\tilde{Q}_{jk}\right|^2 + 2g_V \sum_{j=1}^{3} |U_{\ell j}|^2 \tilde{Q}_{jj} + 2 \sum_{j,k=1}^{3} |U_{\ell k}|^2 \mathrm{Re}\left\{U_{ej} U_{ek}^* \tilde{Q}_{jk}\right\}$$

$$C_2 = g_A^2 + 2g_A P_{\nu_\ell\to\nu_e} + P_{\nu_\ell\to\nu_e}$$

$$C_3 = g_V g_A + (g_V + g_A + 1) P_{\nu_\ell\to\nu_e} + g_A \sum_{j=1}^{3} |U_{\ell j}|^2 \tilde{Q}_{jj} + 2 \sum_{j,k=1}^{3} |U_{\ell k}|^2 U_{ej} U_{ek}^* \tilde{Q}_{jk}$$

where the flavour transition probability $P_{\nu_\ell\to\nu_e} = \sum_{k=1}^{3} |U_{\ell k}|^2 |U_{ek}|^2$ does not depend on source-detector distance and ν energy

- Effective magnetic moment $|\mu_\nu(L, E_\nu)|^2 = \sum_{j,k=1}^{3} |U_{\ell k}|^2 \left|(\mu_\nu)_{jk}\right|^2$ is independent of L and E

Concluding remarks

Kouzakov, Studenikin
Phys. Rev. D 95 (2017)
055013

- cross section of ν-e is of ν electromagnetic form factors
- in short-baseline experiments one studies form factors in flavour basis
- long-baseline experiments more convenient to interpret in terms of fundamental form factors in mass basis
- ν millicharge when it is constrained in reactor short-baseline experiments (GEMMA, for instance) should be interpreted as

$$\boxed{|e_{\nu_e}| = \sqrt{|(e_\nu)_{ee}|^2 + |(e_\nu)_{\mu e}|^2 + |(e_\nu)_{\tau e}|^2}}$$

- ν charge radius in ν-e elastic scattering can't be considered as a shift $g_V \to g_V + \boxed{\frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2 \theta_W}$, there are also contributions from flavor-transition charge radii

GEMMA

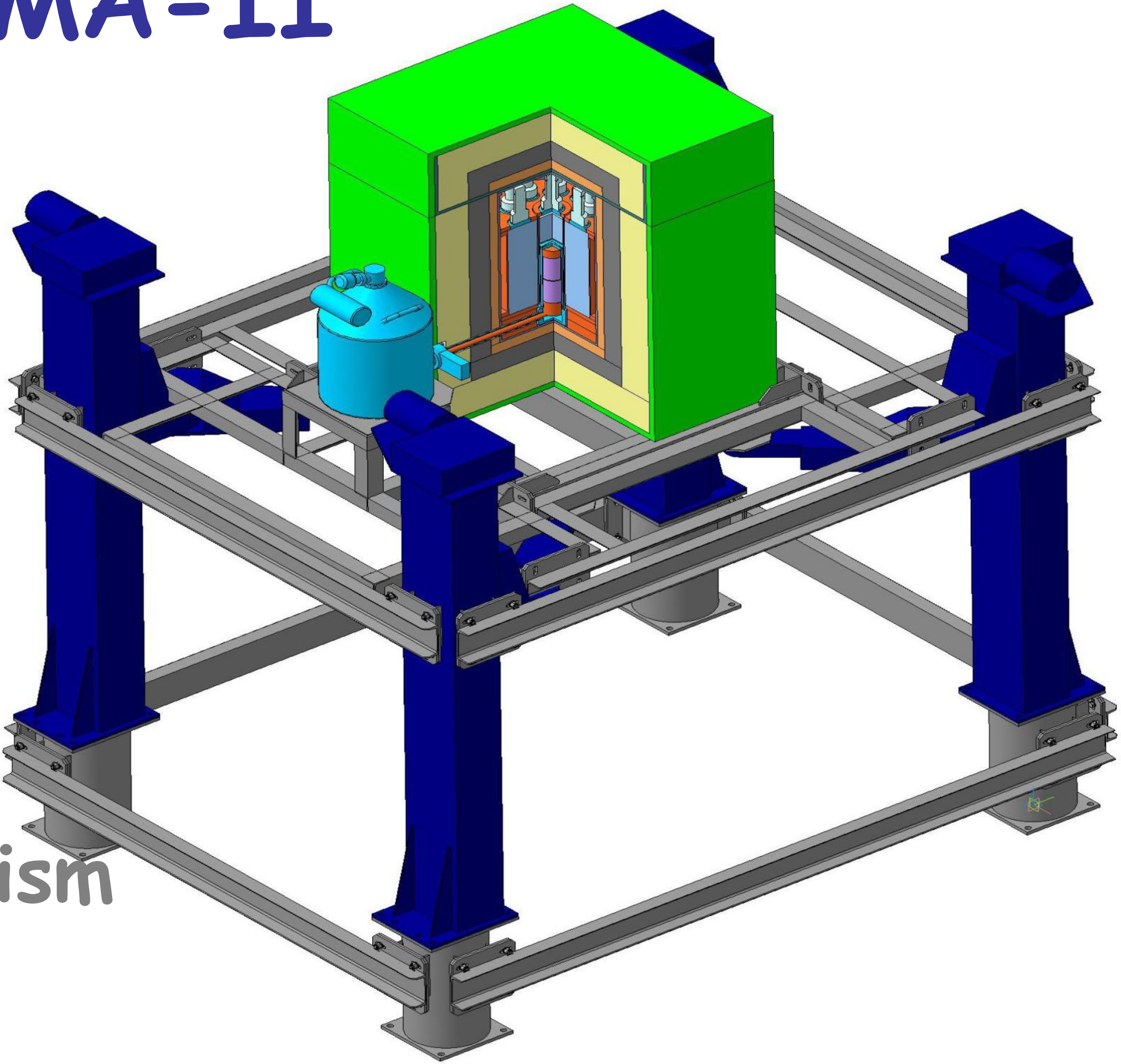
#2: 14 m

#3: 10 m

KNPP

Udomlya
Russia

GEMMA-II



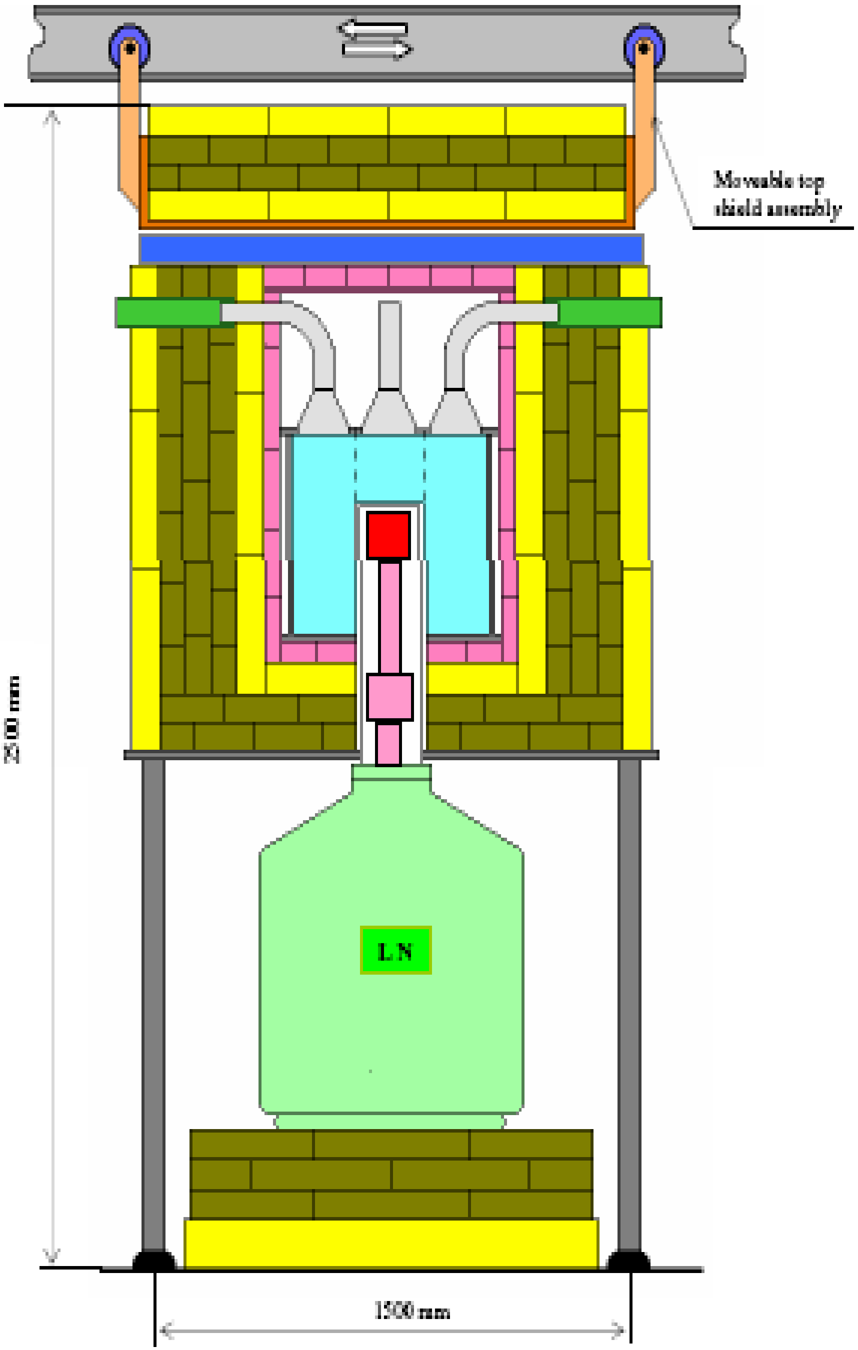
Lifting mechanism

Experiment GEMMA

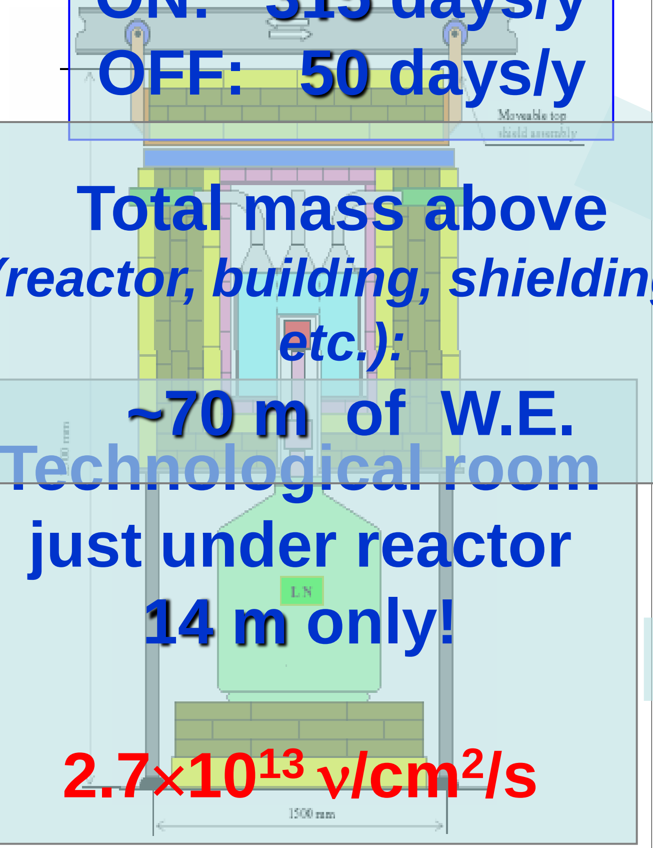
(Germanium Experiment for measurement of Magnetic Moment of Antineutrino)

[Phys. of At. Nucl.,67(2004)1948]

- Spectrometer includes a HPGe detector of 1.5 kg installed within NaI active shielding.
- HPGe + NaI are surrounded with multi-layer passive shielding : electrolytic copper, borated polyethylene and lead.



Reactor unit # 2 of the "Kalinin" Nuclear Power Plant (400 km North from Moscow)

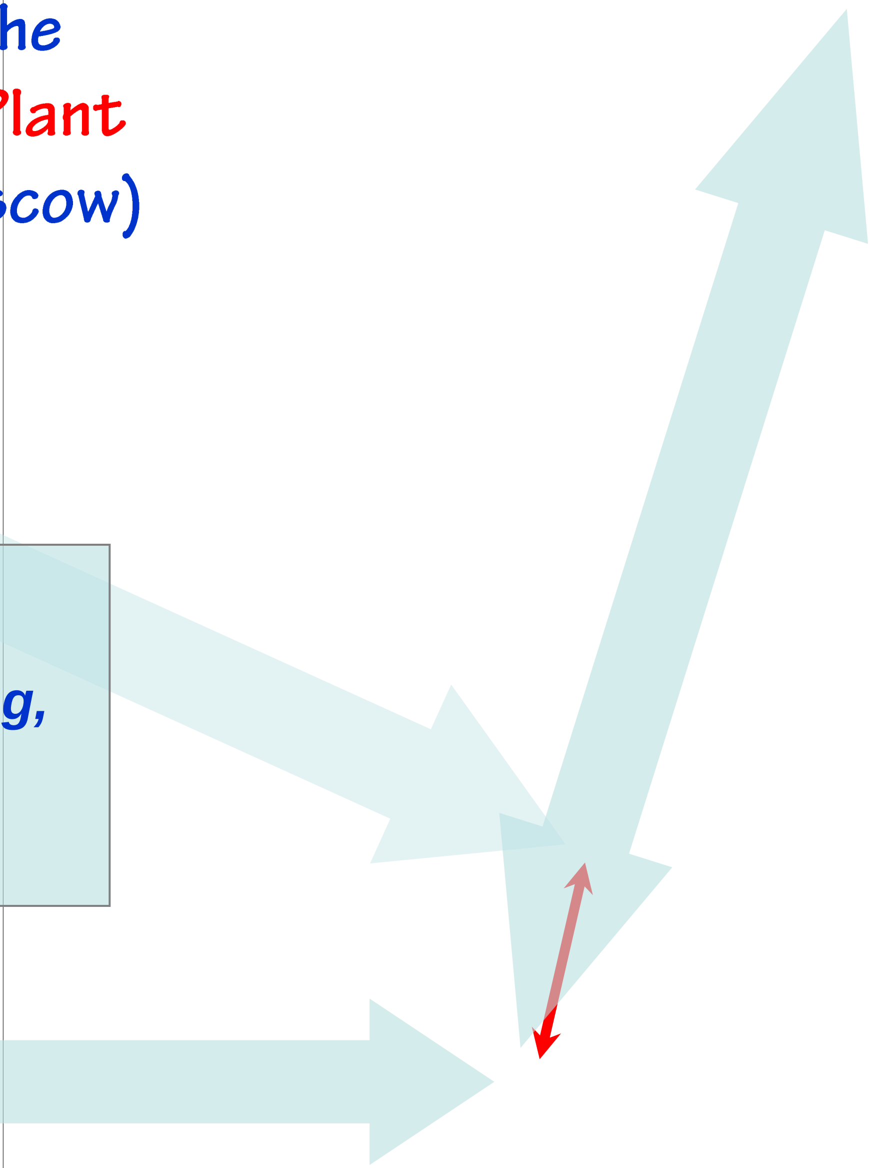


Power: 3 GW
ON: 315 days/y
OFF: 50 days/y

Total mass above
(reactor, building, shielding, etc.):
~70 m of W.E.

Technological room just under reactor
14 m only!

2.7×10^{13} ν/cm^2/s



... courtesy of D.Medvedev...

GEMMA background conditions

- γ-rays were measured with Ge detector. The main sources are: ^{137}Cs, ^{60}Co, ^{134}Cs.
- Neutron background was measured with ^{3}He counters, i.e., thermal neutrons were counted. Their flux at the facility site turned out to be 30 times lower than in the outside laboratory room.
- Charged component of the cosmic radiation (muons) was measured to be 5 times lower than outside.

Experimental sensitivity

$$\mu_{\nu} \propto \frac{1}{\sqrt{N_{\nu}}}\left(\frac{B}{mt}\right)^{\frac{1}{4}}$$

N_ν : number of signal events expected

B : background level in the ROI

m : target (=detector) mass

t : measurement time

$$N_\nu \sim \varphi_\nu \left(\sim Power / r^2\right)$$

$$\sim \left(T_{max} - T_{min} / T_{max} * T_{min} \right)^{1/2}$$

GEMMA I

φ_ν ~ 2.7×10^{13} ν / cm^2 / s

t ~ 4 years

B ~ 2.5 keV^{-1} kg^{-1} day^{-1}

m ~ 1.5 kg

T_{th} ~ 2.8 keV

$$\mu_\nu \leq 2.9 \times 10^{-11} \mu_B$$

… courtesy of D.Medvedev…

Data Set

- I phase – 5184 h ON, 1853 h OFF

$$\boxed{\mu_\nu < 5.8*10^{-11}\mu_B}$$

- II phase – 6798 h ON, 1021 h OFF
- I+II – 11982 h ON, 2874 h OFF

$$\boxed{\mu_\nu < 3.2*10^{-11}\mu_B}$$

- III phase – 6152 h ON, 1613 h OFF
- I+II+III – 18134 h ON, 4487 h OFF

$$\boxed{\mu_\nu < 2.9*10^{-11}\mu_B}$$

Beda A.G. et al. **// Advances in High Energy Physics. 2012. V. 2012, Article ID 350150.**
Beda A.G. et al. **// Physics of Particles and Nuclei Letters, 2013, V. 10, №2, pp. 139–143.**

Sensitivity of future experiments

B = 0.2 1/keV/kg/day (background level in ROI)

Mass, kg	Threshold, keV	Sensitivity, $10^{-12}\mu_B$
4.5	0.4	5.8
10	0.4	4.7
20	0.4	4.0
4.5	0.3	5.6
10	0.3	4.6
20	0.3	3.9

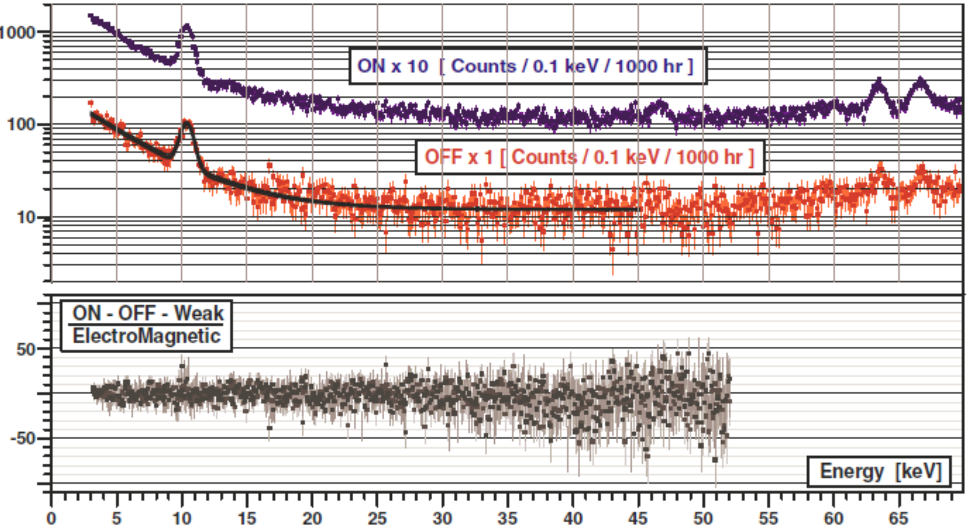
… courtesy of D.Medvedev…

… the obtained constraint on neutrino millicharge q_ν

- rough order-of-magnitude estimation,
- exact values should be evaluated using the
- corresponding statistical procedures

this is because limits on neutrino μ_ν are derived from GEMMA experiment data taken over an extended energy range 2.8 keV --- 55 keV, rather than at a single electron energy-bin at threshold

A.Studenikin : "New bounds on neutrino electric millicharge from limits on neutrino magnetic moment",
Eur.Phys.Lett. 107 (2014) 2100, arXiv:1302.1168



Difference between reactor on and off electron recoil energy spectra (with account for weak interaction contribution) normalized by theoretical electromagnetic spectra

A. Beda et al, Adv. High Energy Phys. 2012(2012) 350150

- Limit evaluated using statistical procedures is of the same order as previously discussed

- $$\boxed{| q_\nu |< 2.7 \times 10^{-12} e_0 \;\; (90\% \text{ C.L.})}$$

A.Studenikin : “New bounds on neutrino electric millicharge from limits on neutrino magnetic moment”, Eur.Phys.Lett. 107 (2014) 2100, arXiv:1302.1168

- V.Brudanin, D.Medvedev, A.Starostin, A.Studenikin : “New bounds on neutrino electric millicharge from GEMMA experiment on neutrino magnetic moment”, arXiv: 1411.2279

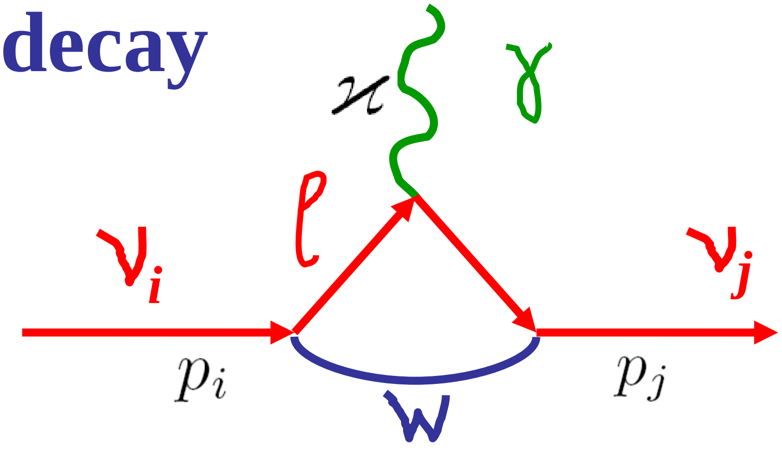
Radiative decay

3.7 Neutrino radiative decay

$$\nu_i \longrightarrow \nu_j + \gamma$$

$$m_i > m_j$$

$$\boxed{L_{int} = \frac{1}{2}\bar{\psi}_i\sigma_{\alpha\beta}(\sigma_{ij} + \epsilon_{ij}\gamma_5)\psi_j F^{\alpha\beta} + h.c.}$$



Radiative decay rate

Petkov 1977; Zatsepin, Smirnov 1978;
Bilenky, Petkov 1987; Pal, Wolfenstein 1982

$$\boxed{\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3} \approx 5\left(\frac{\mu_{eff}}{\mu_B}\right)^2\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3\left(\frac{m_i}{1\ eV}\right)^3\ s^{-1}$$

$$\boxed{\mu_{eff}^2 = \mid \mu_{ij} \mid^2 + \mid \epsilon_{ij} \mid^2}$$

- Radiative decay has been constrained from absence of decay photons:
  1) reactor $\bar{\nu}_e$ and solar ν_e fluxes,
  2) SN 1987A ν burst (all flavours),
  3) spectral distortion of CMBR

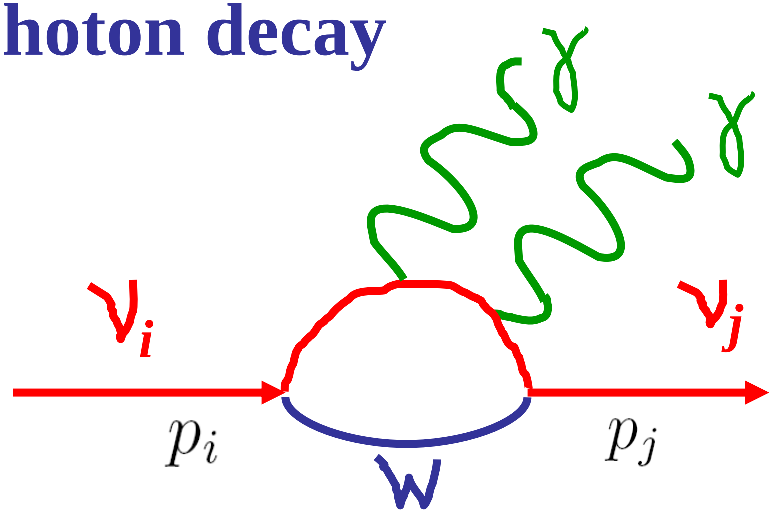
Raffelt 1999
Kolb, Turner 1990;
Ressell, Turner 1990

3.8

Neutrino radiative two-photon decay

$$\nu_i \longrightarrow \nu_j + \gamma + \gamma$$

$$m_i > m_j$$



fine structure constant

$$\boxed{\Gamma_{\nu_i \to \nu_j + \gamma + \gamma} \sim \frac{\alpha_{QED}}{4\pi} \Gamma_{\nu_i \to \nu_j + \gamma}}$$

$$\boxed{f(r_l) \approx \frac{3}{2}\Big(\cancel{1} - \frac{1}{2}\big(\frac{m_l}{m_W}\big)^2\Big) \to (m_i/m_l)^2}$$

Nieves, 1983; Ghosh, 1984

… there is no GIM cancellation…

… can be of interest for certain range of ν masses…

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right)$$

matter current

where $$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^{2}\theta_W)j^{\mu} - \lambda^{\mu}\Big)$$

matter polarization

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m\right\}\Psi(x) = 0}$$

A.Studenikin, A.Ternov, hep-ph/0410297;
***Phys.Lett.B* 608 (2005) 107**

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,’88; J.Panteleone,’91; K.Kiers, N.Weiss, M.Tytgat,’97-’98; P.Manheim,’88; D.Nötzold, G.Raffelt,’88; J.Nieves,’89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang’91; M.Kachelriess,’98; A.Kusenko, M.Postma,’02.

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Neutrino wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix} \sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}} \\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;\,e^{i\delta} \\ s\varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}} \\ \varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;\,e^{i\delta} \end{pmatrix}}$$

A.Studenikin, A.Ternov,
***Phys.Lett.B* 608 (2005) 107**

$$\eta = \mathrm{sign}\left(1-s\alpha\frac{m}{p}\right),\ \delta = \arctan\left(p_2/p_1\right)$$

A.Grigoriev, A.Studenikin, A.Ternov,
***Phys.Lett.B* 622 (2005) 199**

$$\boxed{E_\varepsilon-\alpha m=\varepsilon\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}}$$

The quantity $\boxed{\varepsilon=\pm 1}$ splits the solutions into the two branches that in the limit of vanishing matter density, $\boxed{\alpha\rightarrow 0,}$ reproduce the positive and negative-frequency solutions, respectively.

Quantum theory of spin light of neutrino (I)

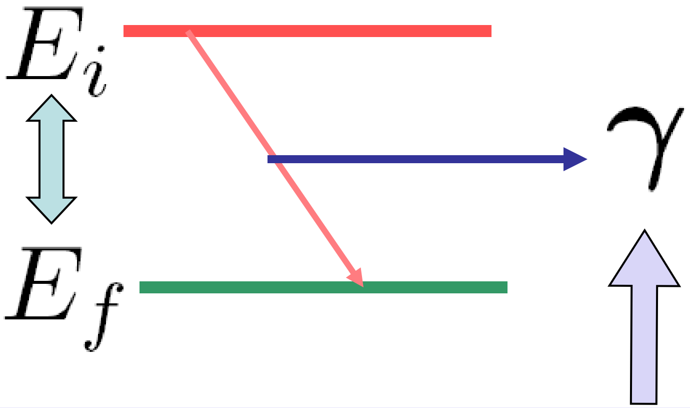
Quantum treatment of *spin light of neutrino* in matter
shown*s* that this process originates from the **two subdivided phenomena:**

$\boxed{SL\nu}$

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,

$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2} + m^2 + \alpha m$$

$$s = \pm 1$$



the radiation of the photon in the process of the neutrino transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

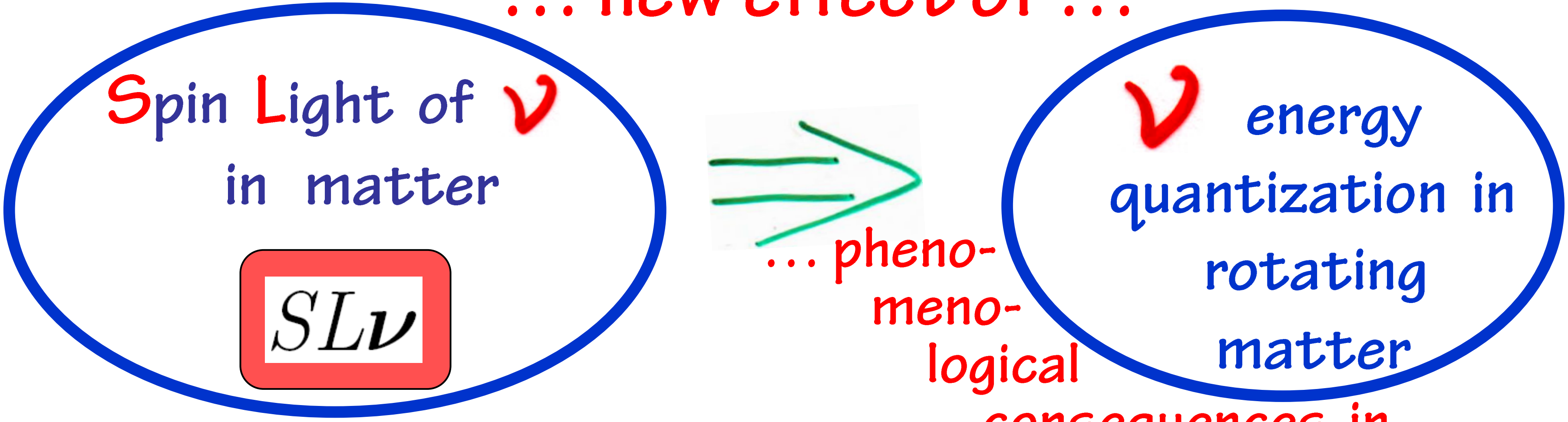
A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

- ν quantum states in dense magnetized matter

… new effect of …



Spin Light of ν in matter

$SL\nu$

ν energy quantization in rotating matter

… phenomenological consequences in astrophysics (pulsars)

ν in matter treated within «method of exact solutions» (Dirac equation with matter potential for ν)

5

New mechanism of electromagnetic radiation

SLe_{ν}

I. Balantsev, A. Studenikin,

"Spin Light of Electron in dense Neutrino fluxes", arXiv: 1405.6598,

"Spin light of relativistic electrons in neutrino fluxes", arXiv: 1502.05346,

"From electromagnetic neutrinos to new electromagnetic radiation mechanism in neutrino fluxes", Int.J.Mod.Phys. A30 (2015) 17, 1530044

2015
the YEAR of LIGHT . . .
(United Nations)

③ I. Balantsev, A. Studenikin

"From electromagnetic neutrinos to new electromagnetic radiation mechanism in neutrino fluxes" Int. J. Mod. Phys. A 30 (2015) 1530044

$\boxed{SL\nu} \Rightarrow SLe_{\nu}$

$SL\nu$

Spin light of electron in dense neutrino fluxes SLe_{ν}

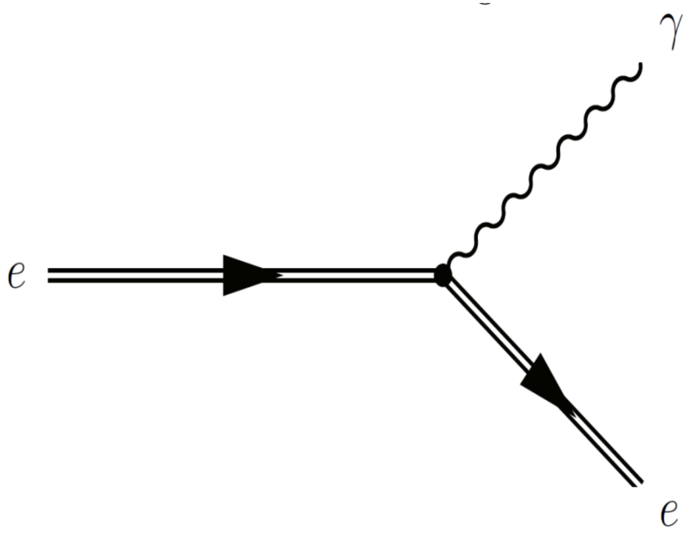
I.Balantsev, A.Studenikin, I

Int.J.Mod.Phys. A 30 (2015) 17, 1530044,
arXiv: 1405.6598, arXiv: 1502.05346

- Electrons in background matter potential (ultra-relativistic ν flux)

$$f^{\mu} = G(n,0,0,n)$$

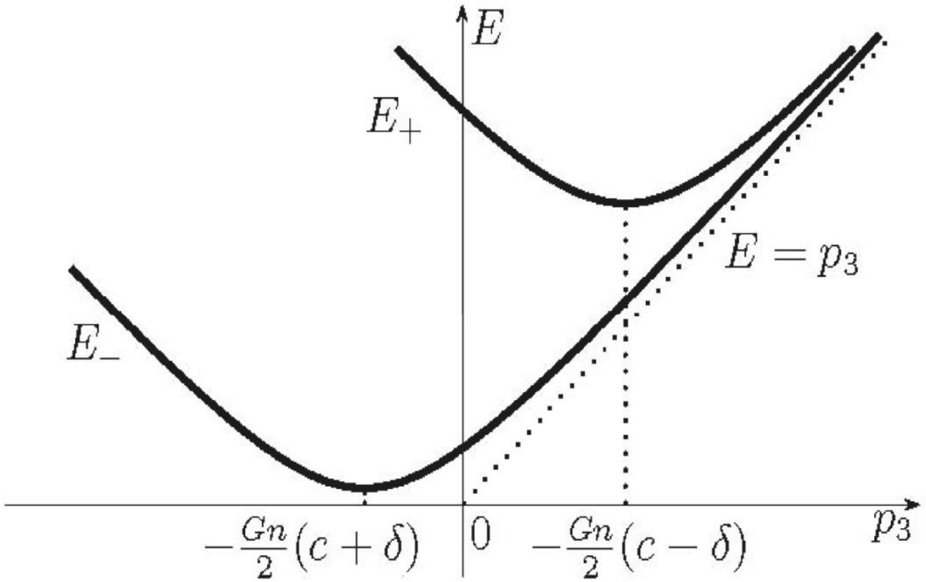
$$n = \frac{n_e + n_\mu + n_\tau}{3}$$



$$\boxed{\left(\gamma_\mu p^\mu + \gamma_\mu \frac{c + \delta_e \gamma^5}{2} f^\mu - m\right)\Psi(x) = 0}$$

$$c = \delta_e - 12\sin^2\theta_W$$

$$\delta_e = \frac{n_\mu + n_\tau - n_e}{n}$$



Energy spectrum of electrons in relativistic ν flux

Fig. 1. The dependence of the electron energies in two different spin states, $E_+(\boldsymbol{p})$ and $E_-(\boldsymbol{p})$, on the momentum component p_3.

$$\boldsymbol{p} = (\boldsymbol{p}_\perp, p_3)$$

$$\boxed{E_s^\varepsilon(\boldsymbol{p}) = \varepsilon\sqrt{m^2 + \boldsymbol{p}_\perp^2 + \Big(p_3 + A\Big)^2} - A} \quad A = \frac{Gn}{2}(c - s\delta), \ \delta = |\delta_e|$$

Wave function of electrons

$$\psi_i(\boldsymbol{r}, t) = \mathrm{e}^{i(-E_+ t + \boldsymbol{p}\boldsymbol{r})}\tilde{\psi}_i, \qquad \psi_f(\boldsymbol{r}, t) = \mathrm{e}^{i(-E_- t + \boldsymbol{p}\boldsymbol{r})}\tilde{\psi}_f$$

$$\tilde{\psi}_i = \frac{1}{L^{\frac{3}{2}} C_+}\begin{pmatrix} 0 \\ m \\ p_\perp \mathrm{e}^{-i\phi} \\ E_+ - p_3 \end{pmatrix}, \quad \tilde{\psi}_f = \frac{1}{L^{\frac{3}{2}} C_-}\begin{pmatrix} E_- - p_3 \\ -p_\perp e^{i\phi} \\ m \\ 0 \end{pmatrix} \quad C_\pm = \sqrt{m^2 + p_\perp^2 + (E_\pm - p_3)^2}$$

SLe_{ν} in case of relativistic electrons in dense ν fluxes at supernovae environment

C. Frohlich, P. Hauser, M. Liebendorfer, G. Martinez-Pinedo, F.-K. Thielemann *et al.*, Composition of the innermost supernova ejecta, *Astrophys.J.* **637**, 415 (2006).
H.-T. Janka, K. Langanke, A. Marek, G. Martinez-Pinedo and B. Mueller, Theory of core-collapse supernovae, *Phys.Rept.* **442**, 38 (2007).

each second a reasonable part of ν flux energy can be transformed to gamma-rays

I.Balantsev, A.Studenikin,
Int.J.Mod.Phys. A 30 (2015) 17, 1530044

- new mechanism of electromagnetic radiation in the Year of Light

Spin Light

(end)

ν spin and flavor oscillations in arbitrary magnetic fields $\vec{B} = \vec{B}_{\perp} + \vec{B}_{\parallel}$

- A. Studenikin, "Neutrino electromagnetic properties: three new effects in neutrino spin oscillations"
  EPJ Web Conf. 125 (2016) 04018,
  arXiv:1705.05944

- A. Grigoriev
  R. Fabbricatore
  A. Studenikin
  "Neutrino spin-flavour oscillations derived from the mass basis"
  J. Phys.: Conf. Ser. 718 (2016) 062058 TAUP 2015 (2016)
  arXiv:1604.01245

- A. Dmitriev
  R. Fabbricatore
  A. Studenikin
  "Neutrino electromagnetic properties: new approach to oscillations in arbitrary magnetic field"
  arxXiv: 1506.05311

Two ν mass states with two helicities in $\vec{B} = \vec{B}_{\perp} + \vec{B}_{\parallel}$

Electromagnetic interaction of ν with μ_ν $(\alpha, \alpha' = 1,2)$

$$H_{EM} = \frac{1}{2}\mu_{\alpha\beta}\bar{\nu}_\beta\sigma_{\mu\nu}\nu_\alpha F^{\mu\nu} + h.c.$$

with constant $\vec{B} = \vec{B}_\perp + \vec{B}_{\|}$

$$\boxed{H_{EM} = -\mu_{\alpha\alpha'}\bar{\nu}_{\alpha'}\mathbf{\Sigma B}\nu_\alpha + h.c.,} \qquad \Sigma_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$$

- Consider two ν mass states $(\alpha, \alpha' = 1,2)$
  with two helicities $(s = \pm 1)$ arXiv:1604.01245

Evolution equation $\boxed{i\frac{d}{dt}\nu_m(t) = H_{eff}\nu_m(t)}$,

where effective oscillation Hamiltonian $\boxed{H_{eff} = H_{vac} + H_B}$

and $\boxed{H_B = \langle \nu_{\alpha,s}|H_{EM}|\nu_{\alpha',s'}\rangle}$

free ν helicity states $\nu_{\alpha,s} = C_\alpha\sqrt{\frac{E_\alpha + m_\alpha}{2E_\alpha}}\begin{pmatrix} u_s \\ \frac{\mathbf{\Sigma p}_\alpha}{E_\alpha + m_\alpha}u_s \end{pmatrix} e^{i\mathbf{p}_\alpha x},$ $\quad u_{s=1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\quad u_{s=-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

For two ν mass states $(\alpha, \alpha' = 1,2)$ with two helicities $(s = \pm 1)$ in $\vec{B} = \vec{B}_{\perp} + \vec{B}_{||}$

- Evolution equation (ν mass states)

arXiv:1705.05944

arXiv:1604.01245

$$
i\frac{d}{dt}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} E_1 + \mu_{11}\frac{B_{||}}{\gamma_{11}} & \mu_{11}B_{\perp} & \mu_{12}\frac{B_{||}}{\gamma_{12}} & \mu_{12}B_{\perp} \\ \mu_{11}B_{\perp} & E_1 - \mu_{11}\frac{B_{||}}{\gamma_{11}} & \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{||}}{\gamma_{12}} \\ \mu_{12}\frac{B_{||}}{\gamma_{12}} & \boxed{\mu_{12}B_{\perp}} & E_2 + \boxed{\mu_{22}\frac{B_{||}}{\gamma_{22}}} & \mu_{22}B_{\perp} \\ \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{||}}{\gamma_{12}} & \mu_{22'}B_{\perp} & E_2 - \mu_{22}\frac{B_{||}}{\gamma_{22}} \end{pmatrix}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix}
$$

$$
E_{\alpha} = \sqrt{\mathbf{p}^2 + m_{\alpha}^2} \approx |\mathbf{p}| + \frac{m_{\alpha}^2}{2|\mathbf{p}|}, \quad \alpha = 1, 2. \qquad \gamma_{\alpha\alpha'}^{-1} = \frac{1}{2}\left(\frac{m_{\alpha}}{E_{\alpha}} + \frac{m_{\alpha'}}{E_{\alpha'}}\right)
$$

- mixings between two different helicity states are due to $B_{\perp}$
  - couplings with $B_{||}$ shift ν energies
    - mixing between different mass states is due to transition magnetic moment interactions with $B_{||}$

Two ν mass states $(\alpha, \alpha' = 1,2)$ with two helicities $(s=\pm 1)$

in $\vec{B} = \vec{B}_{\perp} + \vec{B}_{||}$

Effective oscillation Hamiltonian $\boxed{H_{em} = H_{vac} + H_{\mu_\nu}}$

$$\boxed{H_{\mu_\nu} = -\frac{1}{2}\mu_{\alpha\alpha'}\overline{\nu}_{\alpha'}\sigma_{\mu\nu}\nu_{\alpha}F^{\mu\nu} + h.c. = -\frac{1}{2}\mu_{\alpha\alpha'}\overline{\nu}_{\alpha'}\boldsymbol{\Sigma B}\nu_{\alpha} + h.c.}$$

$$\gamma^{-1}_{\alpha\alpha'} = \frac{1}{2}\left(\frac{m_\alpha}{E_\alpha} + \frac{m_{\alpha'}}{E_{\alpha'}}\right)$$

- Evolution equation (mass states) $E_\alpha = \sqrt{\mathbf{p}^2 + m_\alpha^2} \approx |\mathbf{p}| + \frac{m_\alpha^2}{2|\mathbf{p}|}, \quad \alpha = 1, 2.$

$$\boxed{i\frac{d}{dt}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} E_1 + \mu_{11}\frac{B_{||}}{\gamma_{11}} & \mu_{11}B_{\perp} & \mu_{12}\frac{B_{||}}{\gamma_{12}} & \mu_{12}B_{\perp} \\ \mu_{11}B_{\perp} & E_1 - \mu_{11}\frac{B_{||}}{\gamma_{11}} & \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{||}}{\gamma_{12}} \\ \mu_{12}\frac{B_{||}}{\gamma_{12}} & \mu_{12}B_{\perp} & E_2 + \mu_{22}\frac{B_{||}}{\gamma_{22}} & \mu_{22}B_{\perp} \\ \mu_{12}B_{\perp} & -\mu_{12}\frac{B_{||}}{\gamma_{12}} & \mu_{22'}B_{\perp} & E_2 - \mu_{22}\frac{B_{||}}{\gamma_{22}} \end{pmatrix}\begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix}}$$

- mixings between two different helicity states are due to $B_{\perp}$
  - couplings with $B_{||}$ shift ν energies **NEW**
    - mixing between different mass states is due to transition magnetic moment interactions with $B_{||}$

arXiv:1604.01245

- **Evolution equation (flavour states)** $\nu_f = U\nu_m$

For relativistic $\nu_f = (\nu_e^R, \nu_e^L, \nu_\mu^R, \nu_\mu^L)^\tau$ (flavour chiral states)

$$\nu_e^{R,L} = \nu_{1,s=\pm1}\cos\theta + \nu_{2,s=\pm1}\sin\theta, \quad \nu_\mu^{R,L} = -\nu_{1,s=\pm1}\sin\theta + \nu_{2,s=\pm1}\cos\theta$$

- **Magnetic moment interaction Hamiltonian for flavour ν**

$$\tilde{H}_B^f = \begin{pmatrix} (\frac{\mu}{\gamma})_{ee}B_{||} & \mu_{ee}B_\perp & (\frac{\mu}{\gamma})_{e\mu}B_{||} & \mu_{e\mu}B_\perp \\ \mu_{ee}B_\perp & -(\frac{\mu}{\gamma})_{ee}B_{||} & \mu_{e\mu}B_\perp & -(\frac{\mu}{\gamma})_{e\mu}B_{||} \\ (\frac{\mu}{\gamma})_{e\mu}B_{||} & \mu_{e\mu}B_\perp & (\frac{\mu}{\gamma})_{\mu\mu}B_{||} & \mu_{\mu\mu}B_\perp \\ \mu_{e\mu}B_\perp & -(\frac{\mu}{\gamma})_{e\mu}B_{||} & \mu_{\mu\mu}B_\perp & -(\frac{\mu}{\gamma})_{\mu\mu}B_{||} \end{pmatrix}$$

$$\begin{aligned} \mu_{ee} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta \\ \mu_{e\mu} &= \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22}-\mu_{11})\sin 2\theta \\ \mu_{\mu\mu} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta - \mu_{12}\sin 2\theta \end{aligned}$$

$$\begin{aligned} \left(\frac{\mu}{\gamma}\right)_{ee} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta \\ \left(\frac{\mu}{\gamma}\right)_{e\mu} &= \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta \\ \left(\frac{\mu}{\gamma}\right)_{\mu\mu} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta \end{aligned}$$

arXiv:1604.01245

effective magnetic moments in flavour basis ●

Flavour mixing and oscillations in $B_{||}$: $\nu_e^L \leftrightarrow \nu_\mu^L$

(in case $B = B_{||}$ $\nu^L_{e,\mu}$ decouple from $\nu^R_{e,\mu}$)

neutrino flavor evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L\\ \nu_\mu^L\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta - (\frac{\mu}{\gamma})_{ee}B_{||} & \frac{\Delta m^2}{4E}\sin 2\theta - (\frac{\mu}{\gamma})_{e\mu}B_{||} \\ \frac{\Delta m^2}{4E}\sin 2\theta - (\frac{\mu}{\gamma})_{e\mu}B_{||} & \frac{\Delta m^2}{4E}\cos 2\theta - (\frac{\mu}{\gamma})_{\mu\mu}B_{||}\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_\mu^L\end{pmatrix}$$

- **Probability of neutrino flavour oscillations:**

$$P_{\nu_e^L\to\nu_\mu^L} = \frac{\left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{||}\right)^2}{\left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{||}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + 2\frac{\mu_{12}}{\gamma_{12}}B_{||}\sin 2\theta\right)^2}\sin^2\left(\frac{1}{2}\sqrt{D}x\right)$$

$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta$$

$$D = \left(\frac{\Delta m^2}{2E}\sin 2\theta - 2(\frac{\mu}{\gamma})_{e\mu}B_{||}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + 2\frac{\mu_{12}}{\gamma_{12}}B_{||}\sin 2\theta\right)^2$$

- effective magnetic moment in flavour basis

arXiv:1604.01245
arXiv:1705.05944

$B_{||}$ generates flavour mixing and also can produce resonance amplification of oscillations