Quantum Spacetime and Physics Models
Corfu

Generalized Kähler Geometry and current algebras in N=2 superconformal WZW model.

S.E.Parkhomenko

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• Conformal supersymmetric σ-models with extended supersymmetry are important in the construction of superstring compactification.

• $N = 2$ world-sheet supersymmetry $\Rightarrow$ complex Kähler σ -model target space.

• In a more general case the background geometry may include an antisymmetric B-field. The corresponding 2-dimensional supersymmetric σ-model have a second supersymmetry when the target-space has a bi-Hermitian geometry (Gates-Hull-Roček geometry).

• Bi-Hermitian geometry$\Leftrightarrow$generalised Kähler geometry (GKG) .

•*Geometric data of bi-Hermitian geometry:*

$$\begin{aligned} &\{M, g, B, J_l, J_r\} \; J_l^2 = J_r^2 = -1 \\ &g(J_l\cdot, J_l\cdot) = g(J_r\cdot, J_r\cdot) = g(\cdot, \cdot) \end{aligned} \tag{1}$$

• The $N = 2$ supersymmetric WZW models on the compact groups provide a large class of exactly solvable quantum conformal σ-models whose targets supports simultaneously GKG geometry and affine Kac-Moody superalgebra structure causing their exact solution. Therefore it is important to know the exact relation between the GK geometry data and affine Kac-Moody superalgebra conserved currents. In a more general context it would be also important to see if there are GKG targets which allow the W-superalgabras conserved currents. Perhaps Kazama-Suzuki coset models can be related to such targets.

1. Bi-Hermitian data and double Lie group structure in $N = 2$ supersymmetric WZW model on the group G.

• There is a left and right actions of the complex groups $G^{\pm}_{l,r}$ on the group G so the elements of G can be parametrized by the elements from the complex group $G^{\pm}_{l}$ (or $G^{\pm}_{r}$):

$$\{G^{\mathbb{C}}, G_l^+, G_l^-\} \Leftrightarrow \{M = G, g, B, J_l, J_r\} \Leftrightarrow \{G^{\mathbb{C}}, G_r^+, G_r^-\} \qquad (2)$$

•*Thus, the compact group G is endowed with natural complex coordinates. We employ these complex coordinates to rewrite the WZW action on G in the form of supersymmetric σ-model action on the super-world-sheet parametrized by $(\sigma_{0,1}, \theta_{0,1})$:*

$$S = \frac{k}{2}\int d^2\sigma d^2\theta E_{ij}\rho^i_+\rho^j_-, \; E_{ij} = \imath\Omega_{ik}(J_r)^k_j \qquad (3)$$

•Ω_{ij} *is the Semenov-Tian-Shansky simplectic form,* $\rho_{\pm} = D_{\pm}h^+(h^+)^{-1}$, $h \in G_r^+$ *and*

$$g_{ij} = \frac{1}{2}(E_{ij} + E_{ji}), \; B_{ij} = -\frac{1}{2}(E_{ij} - E_{ji}) \qquad (4)$$

2.Canonical variables and Kac-Moody superalgebra currents.

• Having the action, Hamiltonian formalism allows to find out the canonical coordinates and momenta and express the conserved Kac-Moody superalgebra currents by the canonical variables:

$$L^i = -\frac{1}{2}(\Pi(\rho_1^i) + \imath k J_r \rho_1^i), \;\; R^i = -\frac{1}{2}(\Pi(\tilde\rho_1^i) - \imath k J_l \tilde\rho_1^i)$$
$$\rho_1 = D_1 h^+ (h^+)^{-1}, h \in G_r^+, \;\; \tilde\rho_1 = D_1 \tilde h^+ (\tilde h^+)^{-1}, \tilde h \in G_l^+,$$
$$\Pi = \Pi^{ij}\rho_i^* \rho_j^* = \tilde\Pi^{ij}\tilde\rho_i^*\tilde\rho_j^* = \Omega^{-1},$$
$$D_1 = D_+ + D_- \qquad (5)$$

ρ_i^* is a canonically conjugated to ρ_1^i,
$\tilde\rho_i^*$ is a canonically conjugated to $\tilde\rho_1^i$.

$$
\begin{aligned}
[\rho_i^*(\sigma_1,\theta_1),\rho_j^*(\sigma_1',\theta_1')] &= \delta(\sigma_1-\sigma_1')\delta(\theta_1-\theta_1')\phi_{ij}^k\rho_k^*(\sigma_1',\theta_1') \\
[\rho_i^*(\sigma_1,\theta_1),\rho^j(\sigma_1',\theta_1')] &= D_1\delta(\sigma_1-\sigma_1')\delta(\theta_1-\theta_1')\delta_i^j - \\
&\quad \delta(\sigma_1-\sigma_1')\delta(\theta_1-\theta_1')\phi_{ik}^j\rho^k(\sigma_1',\theta_1') \qquad (6)
\end{aligned}
$$

ϕ_{ij}^k are the structure constants of the Lie algebra of the group $G^+ \times G^-$.