Quadratic gravity in first order formalism

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Introduction

General relativity

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We want a quantum formulation of this.

Can General Relativity be formulated as a Quantum Field Theory?

Perturbative expansion of GR

The coupling constant is dimensionful

$$[G] = -2 \text{ in mass units}$$

This means that higher loop corrections will be higher powers of the Riemann tensor

$$R + R^2 + R^3 + ...$$

The theory is non-renormalizable.

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The theory is non-renormalizable.
The "breaking" scale is the Planck mass $M_P \sim 10^{18} GeV$ it can still be regarded as an EFT.

Improving convergence

The problem with renomalizability was the dimensionful coupling parameter. Therefore, the simplest attempt to improve that is to consider quadratic -in curvature- theories.

$$S = \int \sqrt{|g|} d^4x (\alpha_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^2)$$

Indeed, this improves convergence. But it brings a much worse problem.

The propagator falls off as $\frac{1}{p^4}$. This implies that there are ghosts modes that propagates with the wrong sign in the kinetic energy.

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$$\Delta(p) = \int_0^\infty d\mu^2\, \rho(\mu^2)\, \frac{1}{p^2 - \mu^2 + i\epsilon}$$

where $\rho(\mu^2) \geqslant 0$ But

$$\frac{1}{p^4 - m^4} = \frac{1}{2m^2}\left\{\frac{1}{p^2 - m^2} - \frac{1}{p^2 + m^2}\right\}$$

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Quadratic theories are renormalizable but unitarity is lost.

By looking at the Riemann tensor

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}_{\nu\sigma} - \partial_{\sigma}\Gamma^{\mu}_{\nu\rho} + \Gamma^{\mu}_{\lambda\rho}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\lambda}_{\nu\rho}$$

we can see that the structure is that of a gauge field, symbollically.

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$$R \sim \partial\Gamma - \partial\Gamma + [\Gamma, \Gamma]$$

This is clearer when working with a tetrad.

$$R^{ab}_{\mu\nu} = \partial_\mu A^{ab}_\nu - \partial_\nu A^{ab}_\mu + f^{ab}{}_{cd,ef} A^{cd}_\mu A^{ef}_\nu$$

where $[J_{ab}, J_{cd}] = 2if_{ab,cd,ef} J^{ef}$

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where $[J_{ab}, J_{cd}] = 2if_{ab,cd,ef} J^{ef}$ Although it is not fully equivalent

$$R_{\alpha\beta\gamma\delta} \equiv e^a_\alpha e^b_\beta \; R_{ab\gamma\delta} = R_{\gamma\delta\alpha\beta} \equiv e^c_\gamma e^d_\delta R_{cd\alpha\beta}$$

Quadratic gravitational theories are in some sense the closest analogues to Yang-Mills theories.

First Order formalism

There is another formulation of General relativity which is classically equivalent to the usual one.

The first order (Palatini) action reads

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FO and SO are equivalent on shell –even off shell at one loop order–.

The non-metricity is defined as

$$\nabla_\mu g_{\rho\sigma} = -2A_{\mu\rho\sigma}$$

so now $\Gamma^\mu_{\nu\rho} = \left\{ {}^{\mu}_{\nu\rho} \right\} + A^\mu_{\nu\rho}$.

When the non-metricity is non vanishing, the Riemann tensor does not have the usual symmetries

$$R[\Gamma]_{\mu\nu\rho\sigma} \neq R[\Gamma]_{\rho\sigma\mu\nu}$$
$$R[\Gamma]_{(\mu\nu)\rho\sigma} \neq 0$$

It is only antisymmetric in the last two indices by construction.

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What about a quadratic theory of gravity in first order formalism?

Quadratic gravity in first order formalism?

It seems promising,

1. On the one hand, since there are no dimensionful coupling constants (indeed, the theory is Weyl invariant). No new counterterms will appear in perturbation theory. It is power counting renormalizable.

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2. On the other, since now metric and connection are separate fields, the theory is quadratic in derivatives. The propagator behaves like $\frac{1}{p^2}$ so there is no conflict with the Källen-Lehmann's spectral theorem and it seems a good candidate for an unitary theory.

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The price to pay, of course, is that we have now two different fields, and a more complicated tensorial structure.

There are now three different traces of the Riemann tensor.

$$R^{+}[\Gamma]_{\nu\sigma} \equiv g^{\mu\rho}\; R[\Gamma]_{\mu\nu\rho\sigma}$$
$$R^{-}[\Gamma]_{\mu\sigma} \equiv g^{\nu\rho} R[\Gamma]_{\mu\nu\rho\sigma}$$
$$\mathcal{R}_{\rho\sigma} \equiv g^{\mu\nu}\; R[\Gamma]_{\mu\nu\rho\sigma}$$

But only one scalar

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The lagrangian contains all dimension four Weyl invariant operators that can be built out of the metric and the gauge field.

$$S = \int \sqrt{|g|} d^4x \sum_{I=1}^{I=16} g_I O_I$$

$$O_I \equiv R^{\mu}{}_{\nu\rho\sigma}\ \left(D_I\right)_{\mu\mu'}^{\nu\rho\sigma\nu'\rho'\sigma'}\ R^{\mu'}{}_{\nu'\rho'\sigma'}$$

The most important fact is the the theory is Weyl invariant

$$g_{\mu\nu} \rightarrow \Omega^2(x)\, g_{\mu\nu}$$
$$A_{\mu\nu\rho} \rightarrow A_{\mu\nu\rho}$$

First order versus second order

The solution's space is much bigger than that of second order

$$H_{\mu\nu}^{SO} - H_{\mu\nu}^{FO} = -\frac{1}{2}\nabla_\lambda K^\lambda_{(\mu\nu)} + \frac{1}{4} g_{\lambda\mu}\nabla^\rho K^\lambda_{\rho\nu} + \frac{1}{4} g_{\lambda\nu}\nabla^\rho K_{\rho\nu}$$

The first thing to study is when (if at all) those theories describe gravity.

We will have to study the response of the system to external sources:

- Energy momentum sources : $T_{\mu\nu}$
- Gauge field sources: $J_{\mu\nu\lambda}$ which we will assume made out of energy-momentum tensors.

Background field expansion

The general background field expansion reads

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$
$$\Gamma^{\mu}_{\nu\rho} = \bar{\Gamma}^{\mu}_{\nu\rho} + B^{\mu}_{\nu\rho} \equiv \left\{ {\mu \atop \nu\rho} \right\} + A^{\mu}_{\nu\rho}$$

so the action can be written as

$$S = S_0 + S_1 + S_2$$

Of course, since we are expanding around a Levi-Civita background

$$S_0 = \int d^n x \sqrt{|\bar{g}|} \left(\alpha_1 \; \bar{R}_{\mu\nu\rho\sigma} \; \bar{R}^{\mu\nu\rho\sigma} + \alpha_2 \; \bar{R}_{\mu\nu} \; \bar{R}^{\mu\nu} + \alpha_3 \; \bar{R}^2 \right)$$

$$\alpha_1 = g_1 + \frac{1}{2} g_2 - g_3 - \frac{1}{2} g_4 + \frac{1}{2} g_5 + g_6$$
$$\alpha_2 = g_7 + g_8 + g_9 + g_{10} - g_{12} - g_{13}$$
$$\alpha_3 = g_{16}$$

The equations of motion are

$$\left.\frac{\delta S}{\delta g^{\alpha\beta}}\right|_{g_{\mu\nu}=\bar{g}_{\mu\nu}} = \kappa\sqrt{|\bar{g}|}\left\{\frac{1}{2}\bar{g}^{\alpha\beta}\bar{\mathcal{L}} - 2\alpha_1 \bar{R}_{\mu\nu\rho}{}^{\alpha}\bar{R}^{\mu\nu\rho\beta} - 2q_1\bar{R}_{\mu}{}^{\alpha}\bar{R}^{\mu\beta} - \right.$$

$$\left. -2q_2\bar{R}^{\mu\alpha\beta\nu}\bar{R}_{\mu\nu} - 2\alpha_3\bar{R}^{\alpha\beta}\bar{R}\right\}$$

$$\left.\frac{\delta S}{\delta A^{\lambda}_{\alpha\beta}}\right|_{g_{\mu\nu}=\bar{g}_{\mu\nu}} = \sqrt{|\bar{g}|}\left\{4\alpha_1\bar{\nabla}_{\rho}\bar{R}_{\lambda}{}^{\alpha\rho\beta} + 2q_1\left(\bar{\nabla}_{\lambda}\bar{R}^{\alpha\beta} - \delta^{\beta}_{\lambda}\bar{\nabla}_{\mu}\bar{R}^{\alpha\mu}\right) + \right.$$

$$\left. +2q_2\left(\bar{\nabla}^{\alpha}\bar{R}_{\lambda}{}^{\beta} - \bar{g}^{\alpha\beta}\bar{\nabla}_{\mu}\bar{R}_{\lambda}{}^{\mu}\right) + 2\alpha_3\left(\bar{g}^{\alpha\beta}\bar{\nabla}_{\lambda}\bar{R} - \delta^{\beta}_{\lambda}\bar{\nabla}^{\alpha}\bar{R}\right)\right\}$$

$$q_1 = g_7 + g_8 - \frac{1}{2}g_{12} - \frac{1}{2}g_{13}$$

$$q_2 = -g_9 - g_{10} + \frac{1}{2}g_{12} + \frac{1}{2}g_{13}$$

The quadratic term can be written as

$$S^{(2)} = \frac{1}{2}\int \sqrt{|\bar{g}|}d^n x \left\{ h_{\mu\nu} M^{\mu\nu\rho\sigma} h_{\rho\sigma} + h_{\mu\nu} N_{\lambda}^{\mu\nu\rho\sigma} A^{\lambda}_{\rho\sigma} + A^{\alpha}_{\mu\nu}\, K_{\alpha\beta}^{\mu\nu\rho\sigma}\, A^{\beta}_{\rho\sigma} \right\}$$

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$$\begin{aligned}
M^{\alpha\beta\gamma\epsilon} = \kappa^2 \Big\{ & \left(\frac{1}{4} g^{\alpha\beta} g^{\gamma\epsilon} - \frac{1}{2} g^{\alpha\gamma} g^{\beta\epsilon}\right)\left(\alpha_1\ R_{\mu\nu\rho\sigma}\ R^{\mu\nu\rho\sigma} + \alpha_2\ R_{\mu\nu}\ R^{\mu\nu} + \alpha_3\ R^2\right) + \\
& - g^{\gamma\epsilon}\left(2\alpha_1 \bar{R}_{\mu\nu\rho}{}^{\alpha} \bar{R}^{\mu\nu\rho\beta} + 2q_1 \bar{R}_{\mu}{}^{\alpha} \bar{R}^{\mu\beta} + 2q_2 \bar{R}^{\mu\alpha\beta\nu} \bar{R}_{\mu\nu} + 2\alpha_3 \bar{R}^{\alpha\beta} \bar{R}\right) + \\
& + 2\left(2\alpha_1 + g_1 + \frac{1}{2} g_2\right) \bar{R}_{\mu\nu\rho}{}^{\alpha} \bar{R}^{\mu\nu\rho\gamma} \bar{g}^{\beta\epsilon} + 2\left(2q_1 + g_9\right) \bar{R}_{\mu}{}^{\alpha} \bar{R}^{\mu\gamma} \bar{g}^{\beta\epsilon} + \\
& + 4q_2 \bar{R}^{\mu\alpha\gamma\nu} \bar{R}_{\mu\nu} \bar{g}^{\beta\epsilon} + 4\alpha_3 \bar{R}^{\alpha\gamma} \bar{R} \bar{g}^{\beta\epsilon} + 2\left(g_9 + g_{10}\right) \bar{R}_{\mu}{}^{\alpha\beta}{}_{\nu} \bar{R}^{\mu\gamma\epsilon\nu} - \\
& - 2\left(g_3 + \frac{1}{2} g_4\right) \bar{R}_{\mu\nu}{}^{\alpha\gamma} \bar{R}^{\mu\nu\beta\epsilon} + 2g_5 \bar{R}_{\mu}{}^{\alpha}{}_{\nu}{}^{\gamma} \bar{R}^{\nu\beta\mu\epsilon} + 2g_6 \bar{R}_{\mu}{}^{\alpha}{}_{\nu}{}^{\gamma} \bar{R}^{\nu\epsilon\mu\beta} + \\
& + 2\left(g_{12} + g_{13}\right) \bar{R}_{\mu}{}^{\gamma} \bar{R}^{\mu\alpha\beta\epsilon} + 2\left(g_7 + g_8 - g_9\right) \bar{R}^{\alpha\gamma} \bar{R}^{\beta\epsilon} + 2g_{16} \bar{R}^{\alpha\beta} \bar{R}^{\gamma\epsilon} \Big\} + \\
& + \{\alpha \leftrightarrow \beta\} + \{\gamma \leftrightarrow \epsilon\}
\end{aligned}$$

$$
\begin{aligned}
&h^{\gamma\epsilon} N_{\epsilon\lambda}{}^{\alpha\beta}{}_{\gamma} A^{\lambda}_{\alpha\beta} = \\
&= 2h^{\gamma\epsilon} g_1 \kappa \left\{ \bar{R}_{\gamma\nu\rho\sigma} \bar{F}_{\epsilon}{}^{\nu\rho\sigma} - \bar{R}_{\mu\gamma\rho\sigma} \bar{F}^{\mu}{}_{\epsilon}{}^{\rho\sigma} - \bar{R}_{\mu\nu\gamma\sigma} \bar{F}^{\mu\nu}{}_{\epsilon}{}^{\sigma} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\mu\nu\rho}{}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_2 \kappa \left\{ \bar{R}_{\gamma\nu\rho\sigma} \bar{F}_{\epsilon}{}^{\rho\nu\sigma} - \bar{R}_{\mu\gamma\rho\sigma} \bar{F}^{\mu\rho}{}_{\epsilon}{}^{\sigma} - \bar{R}_{\mu\nu\gamma\sigma} \bar{F}^{\mu}{}_{\epsilon}{}^{\nu\sigma} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\mu\rho\nu}{}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_3 \kappa \left\{ -\bar{R}_{\mu\nu\gamma\sigma} \bar{F}^{\nu\mu}{}_{\epsilon}{}^{\sigma} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\nu\mu\rho}{}_{\epsilon} \right\} + \\
&+ h^{\gamma\epsilon} g_4 \kappa \left\{ -\bar{R}_{\mu\gamma\rho\sigma} \bar{F}^{\rho\mu}{}_{\epsilon}{}^{\sigma} - \bar{F}_{\mu\gamma\rho\sigma} \bar{R}^{\rho\mu}{}_{\epsilon}{}^{\sigma} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\rho\mu\nu}{}_{\epsilon} - \bar{F}_{\mu\nu\rho\gamma} \bar{R}^{\rho\mu\nu}{}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_5 \kappa \left\{ -\bar{R}_{\mu\gamma\rho\sigma} \bar{F}^{\rho}{}_{\epsilon}{}^{\mu\sigma} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\rho\nu\mu}{}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_6 \kappa \left\{ -\bar{R}_{\mu\gamma\rho\sigma} \bar{F}^{\rho\sigma\mu}{}_{\epsilon} - \bar{R}_{\mu\nu\rho\gamma} \bar{F}^{\rho}{}_{\epsilon}{}^{\mu\nu} \right\} + \\
&+ 2h^{\gamma\epsilon} g_7 \kappa \left\{ -\bar{R}^{+}_{\gamma\sigma} \bar{F}^{+\sigma}_{\epsilon} - \bar{R}^{+}_{\nu\gamma} \bar{F}^{\nu}_{+\epsilon} \right\} + 2h^{\gamma\epsilon} g_8 \kappa \left\{ -\bar{R}^{+}_{\gamma\sigma} \bar{F}^{\sigma}_{+\epsilon} - \bar{R}^{+}_{\sigma\gamma} \bar{F}^{+\sigma}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_9 \kappa \left\{ \bar{R}^{-}_{\gamma\sigma} \bar{F}^{-\sigma}_{\epsilon} - \bar{R}_{\mu\gamma\epsilon\sigma} \bar{F}^{\mu\sigma}_{-} - \bar{R}^{-}_{\mu\sigma} \bar{F}^{\mu}{}_{\gamma\epsilon}{}^{\sigma} - \bar{R}^{-}_{\mu\gamma} \bar{F}^{\mu}_{-\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_{10} \kappa \left\{ -\bar{R}_{\rho\gamma\epsilon\sigma} \bar{F}^{\sigma\rho}_{-} - \bar{R}^{\mu\sigma}_{-} \bar{F}_{\sigma\gamma\epsilon\mu} \right\} + 2h^{\gamma\epsilon} g_{11} \kappa \left\{ -\mathcal{R}_{\gamma}{}^{\mu} \mathcal{F}_{\epsilon\mu} - \mathcal{R}^{\mu}{}_{\gamma} \mathcal{F}_{\mu\epsilon} \right\} \\
&+ h^{\gamma\epsilon} g_{12} \kappa \left\{ -\bar{R}^{\nu\sigma}_{+} \bar{F}_{\nu\gamma\epsilon\sigma} - \bar{F}^{\nu\sigma}_{+} \bar{R}_{\nu\gamma\epsilon\sigma} - \bar{R}^{+}_{\nu\gamma} \bar{F}^{\nu}_{-\epsilon} - \bar{F}^{+}_{\nu\gamma} \bar{R}^{\nu}_{-\epsilon} \right\} + \\
&+ h^{\gamma\epsilon} g_{13} \kappa \left\{ -\bar{R}^{\nu\sigma}_{+} \bar{F}_{\sigma\gamma\epsilon\nu} - \bar{F}^{\nu\sigma}_{+} \bar{R}_{\sigma\gamma\epsilon\nu} - \bar{R}^{+\sigma}_{\gamma} \bar{F}^{-}_{\sigma\epsilon} - \bar{F}^{+\sigma}_{\gamma} \bar{R}^{-}_{\sigma\epsilon} \right\} + \\
&+ h^{\gamma\epsilon} g_{14} \kappa \left\{ -\bar{R}^{+}_{\gamma\sigma} \mathcal{F}_{\epsilon}{}^{\sigma} - \bar{F}^{+}_{\gamma\sigma} \mathcal{R}_{\epsilon}{}^{\sigma} - \bar{R}^{+}_{\sigma\gamma} \mathcal{F}^{\sigma}{}_{\epsilon} - \bar{F}^{+}_{\sigma\gamma} \mathcal{R}^{\sigma}{}_{\epsilon} \right\} + \\
&+ h^{\gamma\epsilon} g_{15} \kappa \left\{ -\bar{R}_{\mu\gamma\epsilon\sigma} \mathcal{F}^{\mu\sigma} - \bar{F}_{\mu\gamma\epsilon\sigma} \mathcal{R}^{\mu\sigma} - \bar{R}^{-}_{\nu\gamma} \mathcal{F}^{\nu}{}_{\epsilon} - \bar{F}^{-}_{\nu\gamma} \mathcal{R}^{\nu}{}_{\epsilon} \right\} + \\
&+ 2h^{\gamma\epsilon} g_{16} \kappa \left\{ -\bar{R}^{+}_{\gamma\epsilon} \bar{F} - \bar{R} \bar{F}^{+}_{\gamma\epsilon} \right\} + \{\alpha \leftrightarrow \beta\} + \{\gamma \leftrightarrow \epsilon\}
\end{aligned}
$$

$$
\begin{aligned}
K_{\lambda\ \ \ \tau}^{\ \alpha\beta\gamma\epsilon} = \Big[& 8\alpha_1 \delta_\tau^\alpha \bar{R}_\lambda^{\ \ \gamma\beta\epsilon} + 4q_1 \delta_\tau^\alpha \left(\delta_\lambda^\beta \bar{R}^{\gamma\epsilon} - \delta_\lambda^\epsilon \beta R^{\gamma\beta} \right) + \\
& + 4q_2 \delta_\tau^\alpha \left(\bar{g}^{\gamma\beta} \bar{R}_\lambda^{\ \ \epsilon} - \bar{g}^{\gamma\epsilon} \bar{R}_\lambda^{\ \ \beta} \right) + 4\alpha_3 \delta_\tau^\alpha \left(\delta_\lambda^\beta \bar{g}^{\gamma\epsilon} \bar{R} - \delta_\lambda^\epsilon \bar{g}^{\gamma\beta} \bar{R} \right) \Big] + \\
& + 2\bar{\nabla}^\epsilon \bar{\nabla}^\beta \left[\bar{g}_{\lambda\tau} \bar{g}^{\alpha\gamma} (2g_1 + g_2 - g_9) + \delta_\lambda^\gamma \delta_\tau^\alpha (2g_3 + g_4 - g_6 - g_{10}) + \right. \\
& \left. + \delta_\lambda^\alpha \delta_\tau^\gamma (-g_8 + 2g_{11} + g_{14} - g_{16}) \right] + \\
& + 2\bar{\nabla}_\tau \bar{\nabla}^\beta \left[\bar{g}^{\alpha\gamma} \delta_\lambda^\epsilon (2g_5 + 2g_6 - g_{12} - g_{13}) + \delta_\lambda^\alpha \bar{g}^{\gamma\epsilon} (-g_{13} + g_{15} + 2g_{16}) \right] + \\
& + 2\bar{\nabla}_\tau \bar{\nabla}_\lambda \left[-\bar{g}^{\alpha\gamma} \bar{g}^{\beta\epsilon} (g_5 + g_6 + g_7 + g_8) - \bar{g}^{\alpha\beta} \bar{g}^{\gamma\epsilon} (g_{10} + g_{16}) \right] + \\
& + 2\bar{\nabla}_\tau \bar{\nabla}^\epsilon \left[\bar{g}^{\gamma\alpha} \delta_\lambda^\beta (2g_7 + 2g_8 + g_{13} - g_{15}) + \bar{g}^{\alpha\beta} \delta_\lambda^\gamma (2g_{10} + g_{12} + g_{13}) \right] + \\
& + 2\bar{\nabla}^\alpha \bar{\nabla}^\beta \left[2\bar{g}_{\lambda\tau} \bar{g}^{\gamma\epsilon} g_9 + \delta_\tau^\epsilon \delta_\lambda^\gamma (g_{12} + g_{15}) \right] + \\
& + 2\Box \Big[\bar{g}_{\lambda\tau} \bar{g}^{\alpha\gamma} \bar{g}^{\beta\epsilon} (-2g_1 - g_2) - \bar{g}_{\lambda\tau} \bar{g}^{\alpha\beta} \bar{g}^{\gamma\epsilon} g_9 + \\
& + \bar{g}^{\alpha\gamma} \delta_\lambda^\epsilon \delta_\tau^\beta (-2g_3 - g_4 - g_5) + \bar{g}^{\alpha\gamma} \delta_\tau^\epsilon \delta_\lambda^\beta (-g_7 - 2g_{11} - g_{14}) - \\
& - \delta_\tau^\gamma \delta_\lambda^\epsilon \bar{g}^{\alpha\beta} (g_{12} + g_{15}) \Big] + \{\alpha \leftrightarrow \beta\} + \{\gamma \leftrightarrow \epsilon\} + \{\lambda\alpha\beta \leftrightarrow \tau\gamma\epsilon\}
\end{aligned}
$$

In the following, what is important here is the behaviour of these operators.

$$M \sim R^2$$
$$N \sim kR$$
$$K \sim R + k^2$$

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We want to compute the free energy

$$W\left[J_{\alpha\beta\gamma}, T_{\mu\nu}\right] \equiv -\log\ Z\left[J_{\alpha\beta\gamma}, T_{\mu\nu}\right] = \int \sqrt{\bar{g}}\ d^n x \left\{\frac{1}{4} T_{\mu\nu}\left(M^{-1}\right)^{\mu\nu\rho\sigma} T_{\rho\sigma} + \right.$$
$$+ \frac{1}{4}\left(N^{\alpha\beta}_{\mu\nu\lambda}\ \left(M^{-1}\right)^{\rho\sigma}_{\alpha\beta}\ T_{\rho\sigma} + J_{\mu\nu\lambda}\right)\left(K^{\mu\nu\lambda abc} + N^{\mu\nu\lambda}_{uv}\left(M^{-1}\right)^{uvwx} N^{abc}_{wx}\right)^{-1}$$
$$\left.\left(N^{uv}_{abc}\left(M^{-1}\right)_{uvwx} T^{wx} + J_{abc}\right)\right\}$$

Flat background

When $M = 0$, we must consider a theory where all the dynamics come from the gauge field.

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In this case the propagator for the gauge field reads,

$$
\begin{aligned}
(K^{-1})_{\alpha\beta\gamma\epsilon}^{\lambda\ \ \tau} = \frac{1}{k^2}\Big(&\beta_1 g_{\alpha\beta}g_{\gamma\epsilon}g^{\lambda\tau} + \beta_2 g_{\alpha\gamma}g_{\beta\epsilon}g^{\lambda\tau} + \beta_3\delta_\alpha{}^\lambda\delta_\gamma{}^\tau g_{\beta\epsilon} + \beta_4\delta_\alpha{}^\lambda\delta_\beta{}^\tau g_{\gamma\epsilon} + \\
&+ \beta_5\delta_\alpha{}^\tau\delta_\gamma{}^\lambda g_{\beta\epsilon} + \beta_6 g_{\alpha\beta}g_{\gamma\epsilon}\frac{k^\lambda k^\tau}{k^2} + \beta_7 g_{\alpha\gamma}g_{\beta\epsilon}\frac{k^\lambda k^\tau}{k^2} + \beta_8\delta_\beta{}^\tau g_{\gamma\epsilon}\frac{k_\alpha k^\lambda}{k^2} + \\
&+ \beta_9\delta_\gamma{}^\tau g_{\beta\epsilon}\frac{k_\alpha k^\lambda}{k^2} + \beta_{10}\delta_\alpha{}^\tau g_{\beta\epsilon}\frac{k_\gamma k^\lambda}{k^2} + \beta_{11}\delta_\epsilon{}^\tau g_{\alpha\beta}\frac{k_\gamma k^\lambda}{k^2} + \beta_{12} g_{\gamma\epsilon}g^{\lambda\tau}\frac{k_\alpha k_\beta}{k^2} + \\
&+ \beta_{13}\delta_\gamma{}^\lambda\delta_\epsilon{}^\tau\frac{k_\alpha k_\beta}{k^2} + \beta_{14} g_{\beta\epsilon}g^{\lambda\tau}\frac{k_\alpha k_\gamma}{k^2} + \beta_{15}\delta_\beta{}^\lambda\delta_\epsilon{}^\tau\frac{k_\alpha k_\gamma}{k^2} + \beta_{16}\delta_\beta{}^\tau\delta_\epsilon{}^\lambda\frac{k_\alpha k_\gamma}{k^2} + \\
&+ \beta_{17} g_{\gamma\epsilon}\frac{k_\alpha k_\beta k^\lambda k^\tau}{k^4} + \beta_{18} g_{\beta\epsilon}\frac{k_\alpha k_\gamma k^\lambda k^\tau}{k^4} + \beta_{19}\delta_\epsilon{}^\tau\frac{k_\alpha k_\beta k_\gamma k^\lambda}{k^4} + \beta_{20}\delta_\beta{}^\tau\frac{k_\alpha k_\gamma k_\epsilon k^\lambda}{k^4} + \\
&+ \beta_{21} g^{\lambda\tau}\frac{k_\alpha k_\beta k_\gamma k_\epsilon}{k^4} + \beta_{22}\frac{k_\alpha k_\beta k_\gamma k_\epsilon k^\lambda}{k^4}\Big) + \{\alpha\leftrightarrow\beta\} + \{\gamma\leftrightarrow\epsilon\} + \{\lambda\alpha\beta\leftrightarrow\tau\gamma\epsilon\}
\end{aligned}
$$

In this case all the dynamics are driven by the gauge field. In order to understand what sources drive the theory to the gravitational sector, we have to impose that the free energy coincides with the GR prediction, namely

$$J^{a\beta\gamma}(K^{-1})_{\alpha\beta\gamma\mu\nu\rho}J^{\mu\nu\rho} = T^{\mu\nu}\frac{1}{2k^2}\left(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma}\right)T^{\rho\sigma} = \\ = \frac{T_{ab}T^{ab}}{k^2} - \frac{T^a{}_a T^b{}_b}{2k^2}$$

On the other hand, the only constraint on the sources comes from diffeomorphism invariance, which reads

$$\partial_\beta\partial_\gamma \; J^{\alpha\beta\gamma} = 0$$

There are many allowed sources of the form

$$T_{\mu\nu} = t\left(k^2\eta_{\alpha\beta} - k_\alpha k_\beta\right)$$
$$J_{\alpha\beta\gamma} = A\,k_\alpha\eta_{\beta\gamma} + B\,\left(k_\beta\eta_{\alpha\gamma} + k_\gamma\eta_{\alpha\beta}\right)$$

The fate of Weyl Invariance

The theory is, up to now, in the *conformal phase* (Weyl invariant).

This symmetry has to be spontaneously broken.

The simplest possibility to dynamically generate the Einsteing-Hilbert term (thus breaking Weyl invariance) is through interaction with an scalar sector

$$L_s \equiv \sqrt{|g|}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

Quantum corrections will include a term

$$\Delta L = C_\epsilon R\phi^2$$

Einstein-Hilbert

Were the scalar field to get a nonvanishing vacuum expectation value

$$\langle \phi \rangle = v$$

the counterterm implies an Einstein-Hilbert term

$$L_{EH} = M^2 \sqrt{|g|} R$$

The "Planck scale" M is arbitrary, because it comes about through renormalization; it should be pointed out that the only scale present in the problem to begin with is precisely the symmetry breaking one, v.

Conclusions

1. When considering quadratic theories in first order formalism quartic propagators never appear. The theory seems to be both, renormalizable and unitary.
2. The most general lagrangian of this kind -needed to ensure renormalizability- includes sixteen different operators. Most of them probably don't describe gravity.
3. We need to give physical meaning to the connection.
   - When perturbing around flat space it encodes all the physical information.
   - In curved space, both the graviton and the connection drive the dynamics.
4. There remain the mapping of the full parameter space of the theory as well as compute explicitly the quantum corrections to see that everything works according to our expectations.