Quantum Gravity with Linear Action
and
Gravitational Singularities

George Savvidy

Demokritos National Research Centre

Greece, Athens

Testing Fundamental Physical Principles

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$$(The\ Area\ of\ 4\ D\ universe) = \int_{M_4} R\sqrt{-g}d^4x$$ ~cm^2

R ~ 1/cm^2

and

$\sqrt{-g}d^4x$ ~ cm^4

Gravity with Linear Action
and
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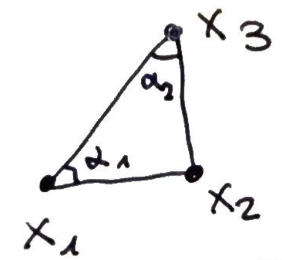
arXiv:1705.01459

Why to consider alternative actions in quantum gravity ?

R.Ambarzumyan
Integral Geometry

the integration is over all triangle shapes

Random triangle R^2



$$d\mu = d^2x_1\, d^2x_2\, d^2x_3$$

$$Z = \int e^{-A} d\mu$$

$$d^2x_2 = \rho_2 d\rho_2 d\varphi_2 \; ; \; d^2x_3 = \rho_3 d\rho_3 d\varphi_3$$

- $$d\mu = d^2x_1\, d\varphi \cdot \rho_2\rho_3 d\rho_2 d\rho_3 d\alpha_1$$

$$\varphi_3 - \varphi_2 = \alpha_1 \; , \; \varphi_2 = \varphi$$

$$\frac{\rho_3}{\sin\alpha_2} = \frac{\rho_2}{\sin\alpha_3} = 2R$$

$$d\mu_S = \underbrace{d^2x_1 \cdot d\varphi} \cdot S\, dS \cdot \frac{d\alpha_1\, d\alpha_2}{\sin\alpha_1 \cdot \sin\alpha_2 \cdot \sin\alpha_3}$$

- $$d\mu_P = \underbrace{d^2x_1 \cdot d\varphi} \cdot P^3 dP \cdot \frac{\sin\alpha_1 \cdot \sin\alpha_2 \cdot \sin\alpha_3\, d\alpha_1 d\alpha_2}{(\sin\alpha_1 + \sin\alpha_2 + \sin\alpha_3)^4}$$

$$Z_S = \int e^{-S} d\mu_S = \infty \; ; \; Z_P = \int e^{-P} d\mu_P < \infty$$

J.Ambjorn,
B.Durhuus,
T.Jonsson

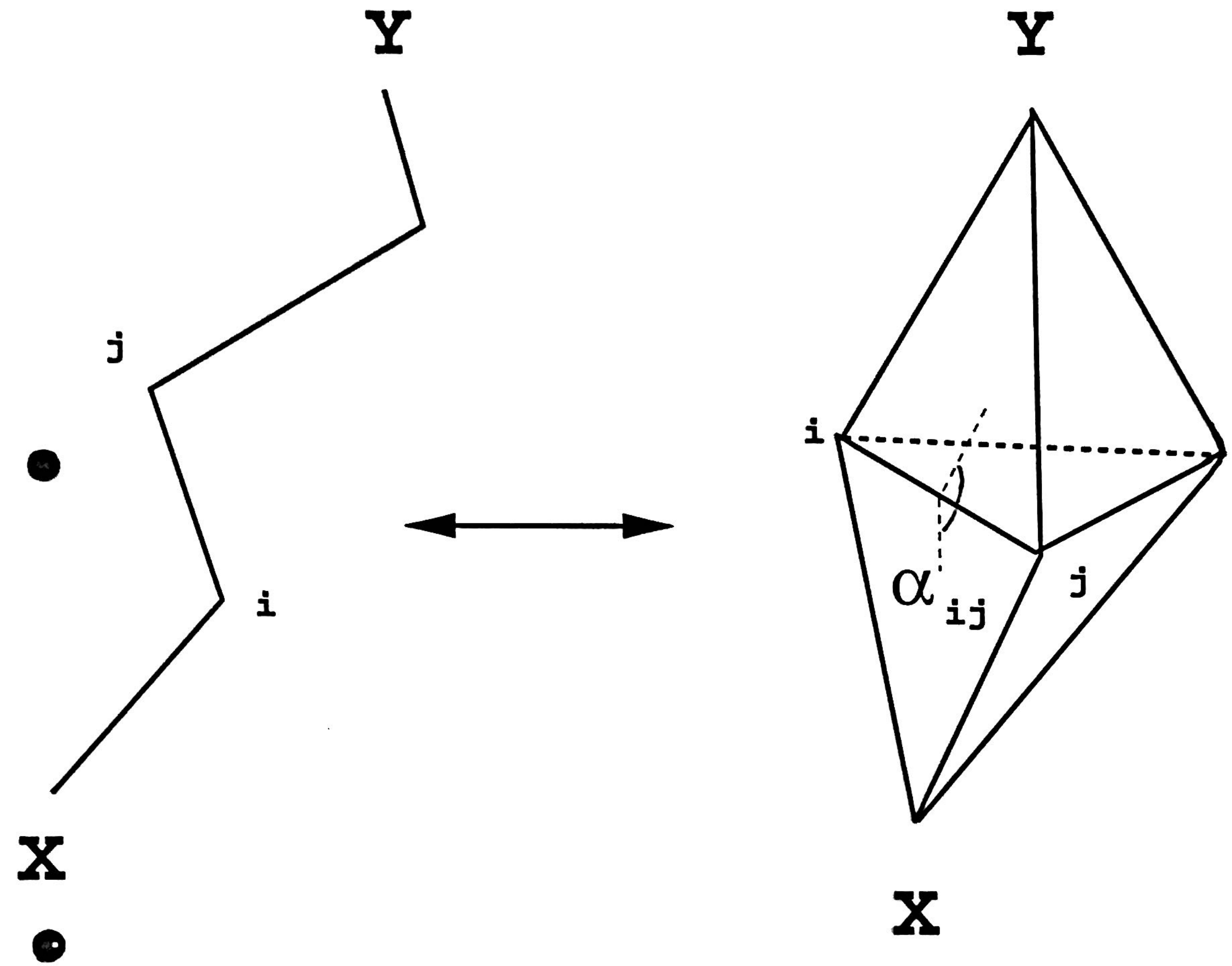
The area action generate spikes

the area measure is singular !

$$\frac{d\alpha}{\alpha}$$

the perimeter measure is not

Extension of Path Integral to Random Surfaces



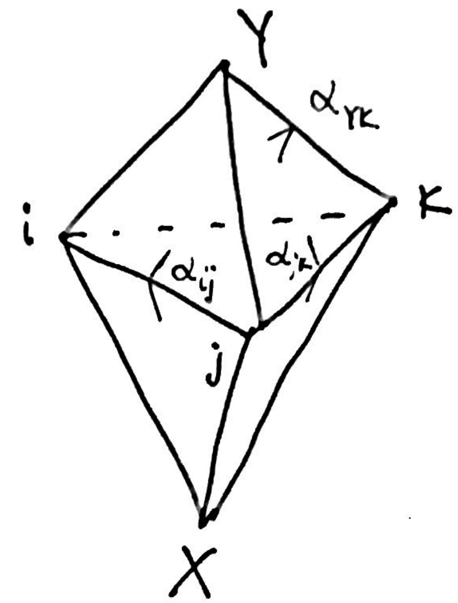
String Action is proportional to the Area of the surface

In this extension the String Action is proportional to the Linear size of the surface

$\mathbf{A}_{xy}$

$$m\sum_{<ij>}\lambda_{ij} \longleftrightarrow m\sum_{<ij>}\lambda_{ij}(\pi-\alpha_{ij})^{\zeta}$$

m - is a mass parameter ζ is a parameter



$\mathcal{M}_2$

- $$A_{XY}(\mathcal{M}_2) = \sum_{\langle ij \rangle} \lambda_{ij} \cdot |\pi - \alpha_{ij}|$$

The linear action suppresses the spikes

λ_{ij} – is the length of the edge $\langle ij \rangle$

- α_{ij} – is the dihedral angle

$$|\pi - \alpha_{ij}| \longrightarrow \Theta(\alpha_{ij})$$

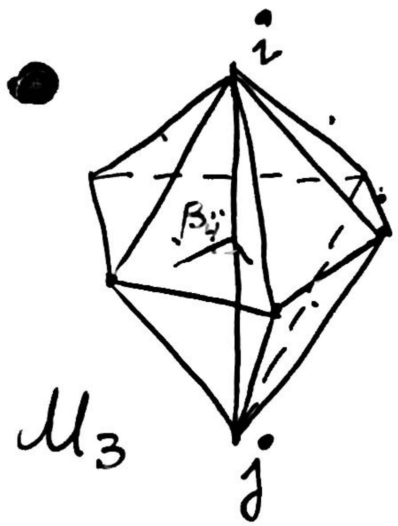
Quantum Gravity

Regge action in <u>3 dimention</u>

1961 $$A(\mathcal{M}_3) = \sum_{\langle ij \rangle} \lambda_{ij} \cdot \left(2\pi - \sum \beta_{ij}\right)$$

cm

The action measures the linear size of the 3D universe



$\mathcal{M}_3$

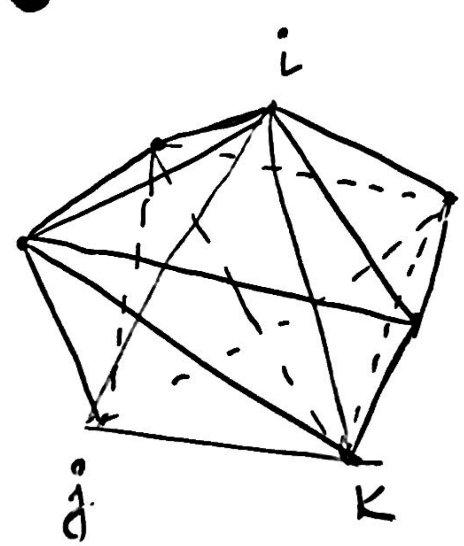
is the discrite version of

$$= \int R^{(3)} d v_3$$

in <u>four-dimention</u> Regge action is:

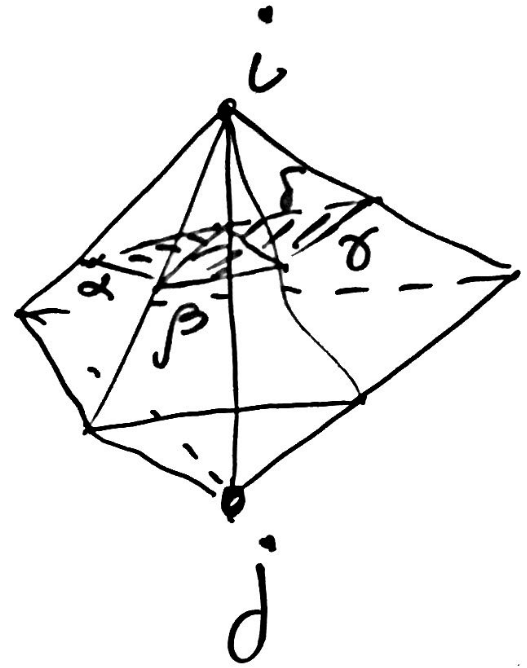
$$S(\mathcal{M}_4) = \sum_{\langle ijk \rangle} \sigma_{ijk} \cdot \left(2\pi - \sum \beta_{ijk}\right)$$

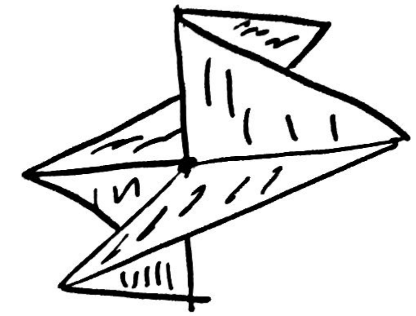
The action measures the Area of the 4D universe



M_4

$$= \int R^{(4)} d v_4 \qquad cm^2$$





2d surface in the section

Regge Gravity in 3D

- $$S_{Gravity} = \sum_{\langle ij \rangle} \lambda_{ij} (2\pi - \alpha_{ij} - \beta_{ij} \cdots)$$

$\dim \mathcal{M}_3 = 3$ [sm]

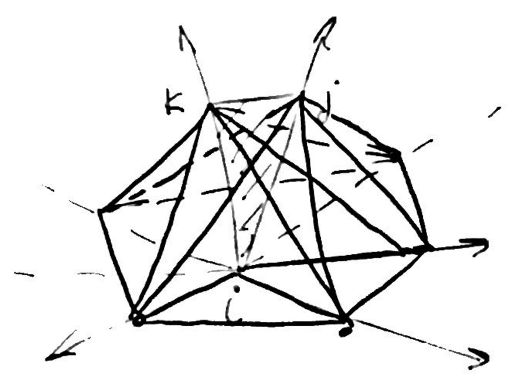
$\dim S_{Grav} = 1$ [sm]

CERN 1995

Four dimensional Gravity with Regge action

$$S(M_4) = \sum_{\langle ijk \rangle} \sigma_{ijk} \cdot (2\pi - \alpha_{ijk} - \beta_{ijk} \cdots)$$

$$= \sum_{\langle ijk \rangle} \sigma_{ijk} \cdot \Theta(\omega_{ijk})$$



σ_{ijk} – the area of the triangle $\langle ijk \rangle$

ω_{ijk} – Euler curvature

$$S(M_4) = \sum_{\{E\}} \chi(M_2^E)$$

summation is over all d-2 dimensional planes $\{E\}$

$$Z_G(\beta) = \sum_{\{M_4\}} \prod_{\{E\}} \exp\{-\beta \cdot \chi(M_2^E)\}$$

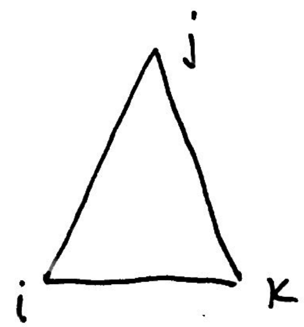
Linear Action

MPLA
11 (1996) 1379
K.S. & G.S.

Extended
Gravity

$$A(\mathcal{M}_4) = \sum_{\langle ijk \rangle} \lambda_{ijk} \cdot \left(2\pi - \sum \beta_{ijk}\right)$$

$[\mathrm{sm}]^1$

- 

$$\lambda_{ijk} = \lambda_{ij} + \lambda_{jk} + \lambda_{ki}$$

$\sqrt{G}$

dual form

$$A(\mathcal{M}_4) = \sum_{\langle ij \rangle} \lambda_{ij} \cdot \sum \left(2\pi - \sum \beta_{ijk}\right)$$

over triangles with common edge $\langle ij \rangle$

- Equation of motion

$$\sum \left(2\pi - \sum \beta_{ijk}\right) = 0$$

over triangles with common edge $\langle ij \rangle$

$$Z(\beta) = \sum_{\{\mathcal{M}_4\}} \exp\left\{-\beta \cdot A(\mathcal{M}_4)\right\}$$

1 *Perimeter-Linear Action*

Thus we shall consider the sum

$$S_L = \sum_{<i,j>} \lambda_{ij} \cdot \sum (2\pi - \sum \beta_{ijk}),$$

where β_{ijk} are the angles on the cone which appear in the normal section of the edge $< ij >$ and triangle $< ijk >$. Combining terms belonging to a given triangle $< ijk >$ we shall get a sum

$$\lambda_{ij} + \lambda_{jk} + \lambda_{ki} = \lambda_{ijk}$$

which is equal to the perimeter of the triangle $< ijk >$:

$$S_L = \sum_{<ijk>} \lambda_{ijk} \cdot \omega^{(2)}_{ijk}$$

where $\omega^{(2)}_{ijk}$ is the deficit angle associated with the triangle $< ijk >$ like in Regge discretisation.

The linear character of the action requires the existence of a new fundamental coupling constant m_P of dimension $1/cm$.

$$S_L = m_P \sum_{<ijk>} \lambda_{ijk} \cdot \omega^{(2)}_{ijk}$$

1 *Regge Area Action* $\Longleftrightarrow$ *Linear Action*

In Regge action the *area* of the triangle σ_{ijk} is multiplied by the deficit angle

$$S_R = \sum_{<ijk>} \sigma_{ijk} \cdot \omega^{(2)}_{ijk}$$

and represents the discretised version of the standard continuous area action in gravity:

$$S_A = -\frac{c^3}{16\pi G} \int R\sqrt{-g} d^4x.$$

The integral $\int R\sqrt{-g}d^4x$ has dimension cm^2 and measures the "area" of the universe.

The linear action can be considered as a "square root" of classical Regge area action.

$$S_R = \sum_{<ijk>} \sigma_{ijk} \cdot \omega^{(2)}_{ijk} \qquad \Longleftrightarrow \qquad S_L = \sum_{<i,j>} \lambda_{ijk} \cdot \omega^{(2)}_{ijk},$$

$$\sigma_{ijk} \quad - \quad \text{the Area} \qquad \Longleftrightarrow \qquad \lambda_{ijk} \quad - \quad \text{the Perimeter},$$

1 *Continuous Linear Action*

It is unknown how to derive a continuous limit of the discretised linear action. One can try to construct a possible linear action for a smooth space-time manifold by using the available geometrical invariants.

These invariants have the following form:

$$I_1 = -\frac{1}{180} R_{\mu\nu\lambda\rho;\sigma} R^{\mu\nu\lambda\rho;\sigma}, \qquad I_2 = +\frac{1}{36} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho} \ ,$$

and we shall consider a linear combination of the above expressions:

$$S_L = -Mc \int \frac{3}{8\pi}(1-\gamma)\sqrt{I_1 + \gamma I_2}\ \sqrt{-g} d^4x,$$

where we introduced the corresponding mass parameter M and the dimensionless parameter γ. The dimension of the invariant $[\sqrt{I_1 + \gamma I_2}]$ is $1/cm^3$, thus the integral S_L has the dimensions of cm and measures the "size" of the universe.

It is similar to the action of the relativistic particle

$$S = -Mc \int ds = -Mc^2 \int \sqrt{1 - \frac{\vec{v}^2}{c^2}}\ dt.$$

Both expressions under the square root are not positive definite . The action develops an imaginary part when $v^2 < c^2$ and quantum mechanical superposition of amplitudes prevents a particle from exceeding the velocity of light. A similar mechanism was implemented in the Born-Infeld modification of electrodynamics with the aim to prevent the appearance of infinite electric fields.

1 *Linear Action - Physical Consequences*

Considering the action

$$S_L = -Mc\int\sqrt{I_1+\gamma I_2}\,\sqrt{-g}d^4x,$$

one can expect that there may appear space-time regions where the expression under the squareroot were negative. These regions became unreachable by the test particles.

If these "locked" space-time regions happen to appear and if that space-time regions include singularities, then one can expect that the gravitational singularities are naturally excluded from the theory due to the fundamental principles of quantum mechanics.

1 *Perturbation Generated by the Linear Action*

The modified action which we shall consider is a sum

$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x - Mc\int \frac{3}{8\pi}(1-\gamma)\sqrt{I_1+\gamma I_2}\,\sqrt{-g}d^4x, \tag{1.1}$$

We shall consider the perturbation of the Schwarzschild solution which is induced by the the additional term in the action and try to understand how it influences the black hole physics and the singularities.

The Schwarzschild solution has the form

$$ds^2 = (1-\frac{r_g}{r})c^2dt^2 - (1-\frac{r_g}{r})^{-1}dr^2 - r^2d\Omega^2\ , \tag{1.2}$$

where $g_{00} = 1-\frac{r_g}{r},\ g_{11} = -(1-\frac{r_g}{r})^{-1},\ g_{22} = -r^2,\ g_{33} = -r^2\sin^2\theta$, and

$$r_g = \frac{2GM}{c^2}, \qquad \sqrt{-g} = r^2\sin\theta.$$

The nontrivial quadratic curvature invariant in this case has the form

$$I_0 = \frac{1}{12}R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} = (\frac{r_g}{r^3})^2$$

and shows that the singularity located at $r=0$ is actually a curvature singularity. The event horizon is located where the metric component g_{rr} diverges, that is, at

$$r_{horizon} = r_g.$$

1 *Perturbation of Schwarzschild solution*

The expressions for the two curvature polynomials of our interest are

$$I_1 = -\frac{1}{180} R_{\mu\nu\lambda\rho;\sigma} R^{\mu\nu\lambda\rho;\sigma}, \qquad I_2 = +\frac{1}{36} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho} \ ,$$

and on the Schwarzschild solution they take the form:

$$I_1 = \frac{r_g^2 (r - r_g)}{r^9}, \qquad I_2 = \frac{r_g^3}{r^9} \ ,$$

The action acquires additional term of the form

$$S_L = -Mc^2 \int \frac{3}{2} \varepsilon \sqrt{1 - \varepsilon \frac{r_g}{r}} \ \frac{r_g}{r^2} dr dt \ ,$$

where $\varepsilon = 1 - \gamma$. As one can see, the expression under the squareroot becomes negative at

$$r < \varepsilon r_g, \qquad 0 < \varepsilon \leq 1 \tag{1.1}$$

and defines the region which is unreachable by the test particles. The size of the region depends on the parameter ε and is smaller than the gravitational radius r_g.

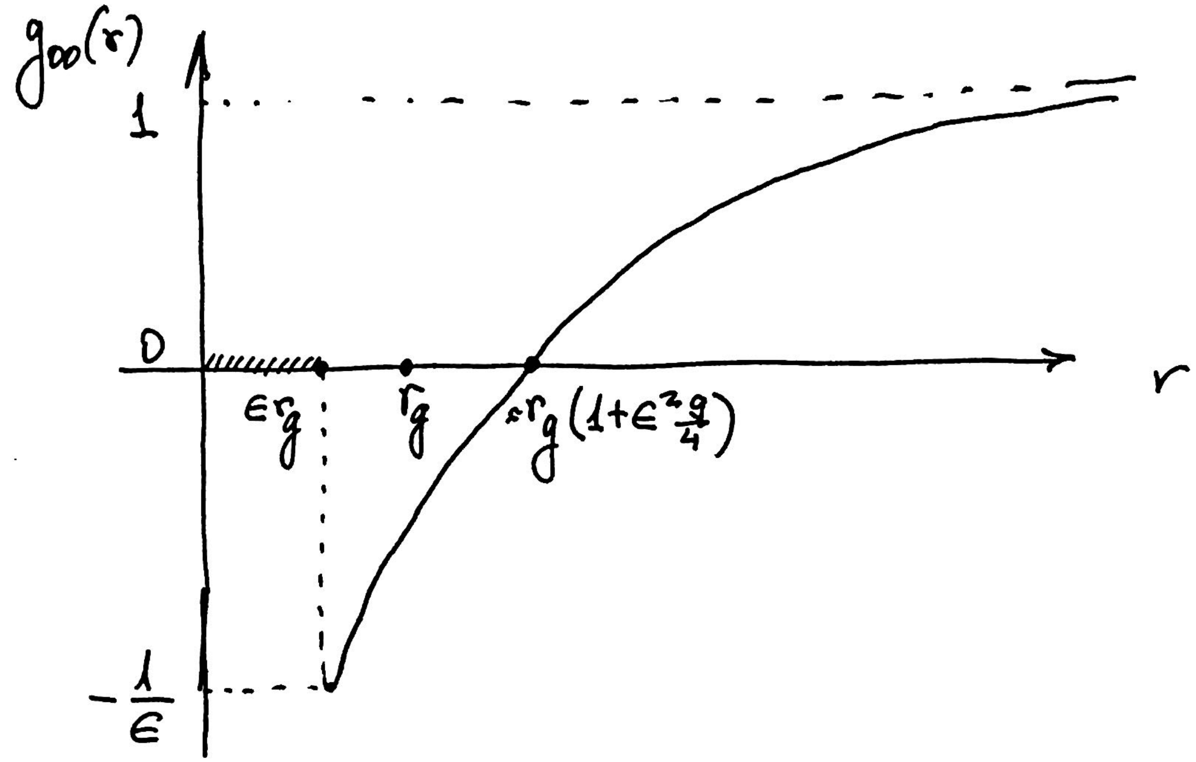


Figure 1:

This result seems to have profound consequences on the gravitational singularity at $r = 0$. In a standard interpretation of the singularities, which appear in spherically symmetric gravitational collapse, the singularity at $r = 0$ is hidden behind an event horizon. In that interpretation the singularities are still present in the theory.

In the suggested scenario it seems possible to eliminate the singularities from the theory based on the fundamental principles of quantum mechanics.

The quantum mechanical amplitude in terms of the path integral has the form

$$\Psi = \int e^{\frac{i}{\hbar}S[g]}\mathcal{D}g_{\mu\nu}(x),$$

where integration is over all diffeomorphism nonequivalent metrics.

For the Schwarzschild massive object the expression for the action is:

$$S = -Mc^2 \int_{\varepsilon r_g}^{\infty} \frac{3}{2}\varepsilon\sqrt{1-\varepsilon\frac{r_g}{r}}\,\frac{r_g}{r^2}drdt = Mc^2 t \tag{0.1}$$

it is proportional to the length t of the space-time trajectory, as it should be for the relativistic particle at rest,

The corresponding amplitude can be written in the form

$$\Psi \approx \exp\,(\frac{i}{\hbar}\sum_n M_n c^2 t),$$

where the summation is over all bodies in the universe.

1 *Perturbation of the Schwarzschild Metric*

The perturbation generates a contribution to the distance invariant ds of the form

$$\delta ds = \frac{3}{2}\int_r^{\infty} \varepsilon\sqrt{1-\varepsilon\frac{r_g}{r}}\,\frac{r_g}{r^2}dr = \left[1-\left(1-\varepsilon\frac{r_g}{r}\right)^{3/2}\right]$$

and the correction to the purely temporal component of the metric tensor is

$$g_{00} = 1-\frac{r_g}{r}-\left[1-\left(1-\varepsilon\frac{r_g}{r}\right)^{3/2}\right]^2.$$

The equation used to determine gravitational time dilation near a massive body is modified in this case and the proper time between events is defined now by the equation

$$d\tau = \sqrt{g_{00}}dt = \sqrt{1-\frac{r_g}{r}-\left[1-\left(1-\varepsilon\frac{r_g}{r}\right)^{3/2}\right]^2}\,dt$$

and therefore $d\tau \leq dt$, as in standard gravity.

It follows that near the gravitational radius $r \approx r_g$ a purely temporal component of the metric tensor has the form $g_{00} \approx 1-\frac{r_g}{r}-\varepsilon^2\frac{9}{4}\left(\frac{r_g}{r}\right)^2+\mathcal{O}(\varepsilon^3)$ and the infinite red shift which appears in the standard case at $r = r_g$ now appears at

$$r \approx r_g(1+\frac{9}{4}\varepsilon^2)+\mathcal{O}(\varepsilon^4).$$

1 *Perturbation of the Trajectories of Test Particles*

The Hamilton-Jacobi equation for geodesics, modified by the perturbation is:

$$g^{\mu\nu}\frac{\partial A}{\partial x^{\mu}}\frac{\partial A}{\partial x^{\nu}} = g^{00}\left(\frac{\partial A}{c\partial t}\right)^2 - \frac{1}{g^{00}}\left(\frac{\partial A}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial A}{\partial \phi}\right)^2 = m^2c^2.$$

The solution has the form $A = -Et + L\phi + A(r)$, where E and L are the energy and angular momentum of the test particle and

$$A(r) = \int \left[\left(g^{00}\frac{E^2}{c^2} - m^2c^2 - \frac{L^2}{r^2}\right)g^{00}\right]^{1/2} dr. \tag{1.1}$$

In the non-relativistic limit $E = E^{'} + mc^2, E^{'} \ll mc^2$, and in terms of a new coordinate $r(r - r_g) = r^{'}$ we shall get

$$A(r) \approx \int \left[(\frac{E^{'2}}{c^2} + 2E^{'}m) + \frac{1}{r^{'}}(4E^{'}mr_g + m^2c^2r_g) - \frac{1}{r^{'2}}\left(L^2 - \frac{3}{2}m^2c^2r_g^2(1+\frac{3}{2}\varepsilon^2)\right)\right]^{1/2} dr^{'} \tag{1.2}$$

and the advance precession of the perihelion $\delta\phi$ expressed in radians per revolution is:

$$\delta\phi = \frac{3\pi m^2c^2r_g^2}{2L^2}(1 + \frac{3}{2}\varepsilon^2) = \frac{6\pi GM}{c^2a(1-e^2)}(1 + \frac{3}{2}\varepsilon^2), \tag{1.3}$$

where a is the semi-major axis and e is the orbital eccentricity. As one can see from the above result, the precession is advanced by the additional factor $1 + \frac{3}{2}\varepsilon^2$.

The upper bound on the value of ε can be extracted from the observational data for the advanced precession of the Mercury perihelion, which is $42,98 \pm 0,04$ seconds of arc per century, thus

$$\varepsilon \leq 0,16 \ .$$

1 *Deflection of Light Ray*

For the light propagation we shall take $m^2 = 0,\;\; E = \omega_0,\; L = \rho\, \omega_0/c$:

$$A(r) = \frac{\omega_0}{c} \int \sqrt{(g^{00} - \frac{\rho^2}{r^2}) g^{00}} dr \approx \frac{\omega_0}{c} \int \sqrt{1 + 2\frac{r_g}{r} - \frac{\rho^2}{r^2}} dr + \mathcal{O}(\varepsilon^2 r_g^2/r^2). \tag{1.1}$$

The trajectory is defined by the equation $\phi + \partial A(r)/\partial\rho = Const$ and in the given approximation the deflection of light ray remains unchanged:

$$\delta\phi = 2\frac{r_g}{\rho},$$

where ρ is the distance from the centre of gravity.

The deflection angle is not influenced by the perturbation, which is of order

$$\mathcal{O}(\varepsilon^2 r_g^2/\rho^2)$$

and does not impose a sensible constraint on ε.

1 *Appendix*

The general form of the linear action has the form:

$$S_L = -m_P c \int \frac{3}{8\pi} \sqrt{\sum_1^3 \eta_i K_i + \sum_1^4 \chi_i J_i + \sum_1^9 \gamma_i I_i} \sqrt{-g} d^4 x \ ,$$

where the curvature invariants have the form

$$\begin{aligned}
&I_0 = \frac{1}{12} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}, \quad I_1 = -\frac{1}{180} R_{\mu\nu\lambda\rho;\sigma} R^{\mu\nu\lambda\rho;\sigma}, \quad I_2 = +\frac{1}{36} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho} \ , \\
&I_3 = -\frac{1}{72} \Box (R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}), \ I_4 = -\frac{1}{90} R_{\mu\nu\lambda\rho;\alpha} R^{\alpha\nu\lambda\rho;\mu}, \ I_5 = -\frac{1}{18} (R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho})_{;\mu;\alpha}, \\
&I_6 = -\frac{1}{18} (R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho})_{;\alpha;\mu} = I_3, \ I_7 = \frac{1}{18} R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho;\alpha;\mu}, \ I_8 = R^{\mu}{}_{\nu\lambda\rho;\mu} R^{\sigma\nu\lambda\rho}{}_{;\sigma} \ , \\
&I_9 = R^{\alpha\nu\lambda\rho} R^{\mu}{}_{\nu\lambda\rho;\mu;\alpha} \ , \quad I_3 = I_5 = I_6 = 5I_1 - I_2, \quad I_4 = I_1, \quad I_7 = I_2 \\
&J_0 = R_{\mu\nu} R^{\mu\nu} \ , \quad J_1 = R_{\mu\nu;\lambda} R^{\mu\nu;\lambda} \ , \quad J_2 = R^{\mu\nu} \Box R_{\mu\nu} \ , \quad J_3 = \Box (R^{\mu\nu} R_{\mu\nu}) \ , \quad J_4 = R_{\mu\sigma}{}^{;\mu} R^{\nu\sigma}{}_{;\nu} \\
&K_0 = R^2 \ , \quad K_1 = R_{;\mu} R^{;\mu} \ , \quad K_2 = R \Box R \ , \quad K_3 = \Box R^2 \ . \qquad (1.1)
\end{aligned}$$

The η_i, χ_i and γ_i are free parameters. Some of the invariants can be expressed through others using Bianchi identities.

Thank You !