

Reduction of Couplings in Finite Theories and in the MSSM

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- ▶ What happens as we approach the Planck scale? or just as we go up in energy...
- ▶ What happened in the early Universe?
- ▶ How do we go from a fundamental theory to eW field theory as we know it?
- ▶ How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- ▶ How do particles get their very different masses?
- ▶ What is the nature of the Higgs?
- ▶ Is there one or many? How this affects all the above?
- ▶ Where is the new physics??

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale \rightarrow Planck scale

\Rightarrow **reduction of couplings**

resulting theory: less free parameters \therefore more predictive

Zimmermann 1985

Gauge Yukawa Unification – GYU

Remarkable: reduction of couplings provides a way to relate two previously unrelated sectors

gauge and Yukawa couplings

Reduction of couplings in third generation provides predictions for quark masses (top and bottom)

Including soft breaking terms gives Higgs masses and SUSY spectrum

Kapetanakis, M.M., Zoupanos (1993), Kubo, M.M., Olechowski, Tracas, Zoupanos (1995,1996,1997); Oehme (1995); Kobayashi, Kubo, Raby, Zhang (2005); Gogoladze, Mimura, Nandi (2003,2004); Gogoladze, Li, Senoguz, Shafi, Khalid, Raza (2006,2011); M.M., Tracas, Zoupanos (2014)

Gauge Yukawa Unification in Finite Theories

Dimensionless sector of all-loop finite $SU(5)$ model

$$M_{top} \sim 178 \text{ GeV}$$

large $\tan \beta$, heavy SUSY spectrum

Kapetanakis, M.M., Zoupanos, Z.f.Physik (1993)

$$M_{top}^{exp} 176 \pm 18 \text{ GeV}$$

found in 1995

$$M_{top}^{th} \sim 172.5$$

2007

$$M_{top}^{exp} 173.1 \pm .09 \text{ GeV}$$

2013

Very promising, a more detailed analysis was clearly needed

$$\text{Higgs mass} \sim 121 - 126 \text{ GeV}$$

Heinemeyer M.M., Zoupanos, JHEP, 2007; Phys.Lett.B (2013)

$$M_H^{exp} 126 \pm 1 \text{ GeV}$$

2013

Gauge Yukawa Unification in the MSSM

- ▶ Possible to have a reduced system in the third generation compatible with quark masses

large $\tan \beta$, heavy SUSY spectrum

- ▶ Higgs mass $\sim 123 - 126$ GeV

M.M., Tracas, Zoupanos, Phys.Lett.B 2014

- ▶ Recently reduced also the soft breaking terms

S. Heinemeyer, M.M., Tracas, Zoupanos, work in progress

new predictions for Higgs mass and susy spectrum

Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \dots, g_N) = 0$ satisfies

$$\mu d\Phi/d\mu = \sum_{i=1}^N \beta_i \partial\Phi/\partial g_i = 0.$$

$g_i = \text{coupling}$, β_i its β function

Finding the $(N - 1)$ independent Φ 's is equivalent to solve the
reduction equations (RE)

$$\beta_g (dg_i/dg) = \beta_i ,$$

$i = 1, \dots, N$

- ▶ Reduced theory: only one independent coupling and its β function
- ▶ complete reduction: power series solution of RE

$$g_a = \sum_{n=0} \rho_a^{(n)} g^{2n+1}$$

- ▶ uniqueness of the solution can be investigated at one-loop
valid at all loops

Zimmermann, Oehme, Sibold (1984,1985)

- ▶ The complete reduction might be too restrictive, one may use fewer Φ 's as RGI constraints
- ▶ Reduction of couplings is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \beta^N = 0$$

- ▶ SUSY no-renormalization theorems
 - ▶ \Rightarrow **only study one and two-loops**
 - ▶ guarantee that is gauge and reparameterization invariant to **all loops**

Reduction of couplings: the Standard Model

It is possible to make a reduced system in the Standard Model in the matter sector:

solve the REs, reduce the Yukawa and Higgs in favour of α_S gives

$$\alpha_t/\alpha_S = \frac{2}{9} ; \quad \alpha_\lambda/\alpha_S = \frac{\sqrt{689} - 25}{18} \simeq 0.0694$$

border line in RG surface, Pendleton-Ross infrared fixed line
But including the corrections due to non-vanishing gauge couplings up to two-loops, changes these relations and gives

$$M_t = 98.6 \pm 9.2 \text{ GeV}$$

and

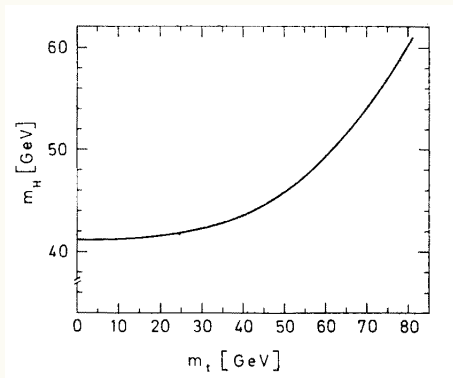
$$M_h = 64.5 \pm 1.5 \text{ GeV}$$

Both out of the experimental range, but pretty impressive

General asymptotic reduction in SM

More parameters (couplings):

if top and Higgs heavy \Rightarrow new physics heavy



Kubo, Sibold and Zimmermann, 1985, 1989; Sibold and Zimmermann, 1987; Sibold 1987; Kubo 1991

Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2} m^{ij} \phi_i \phi_j + \frac{1}{6} C^{ijk} \phi_i \phi_j \phi_k ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3C_2(G), \quad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i) .$$

$C_2(G)$ quadratic Casimir invariant, $T(R_i)$ Dynkin index of R_i , C_{ijk} Yukawa coup., g gauge coup.

- ▶ **restricts the particle content of the models**
- ▶ **relates the gauge and Yukawa sectors**

- ▶ One-loop finiteness \Rightarrow two-loop finiteness

Jones, Mezincescu and Yao (1984,1985)

- ▶ One-loop finiteness restricts the choice of irreps R_i , as well as the Yukawa couplings
- ▶ Cannot be applied to the susy Standard Model (SSM):
 $C_2[U(1)] = 0$
- ▶ The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g , which is solution (**isolated and non-degenerate**) to the reduction equations

Theorem

Lucchesi, Piguet, Sibold 1988

Consider an $N = 1$ supersymmetric Yang-Mills theory, with simple gauge group. If the following conditions are satisfied

1. There is no gauge anomaly.
2. The gauge β -function vanishes at one-loop

$$\beta_g^{(1)} = 0 = \sum_i I(R_i) - 3 C_2(G).$$

3. There exist solutions of the form

$$C_{ijk} = \rho_{ijk} g, \quad \rho_{ijk} \in \mathfrak{G}$$

to the conditions of vanishing one-loop matter fields anomalous dimensions

$$\gamma_j^{i(1)} = 0 = \frac{1}{32\pi^2} [C^{ikl} C_{jkl} - 2g^2 C_2(R_i)\delta_{ij}].$$

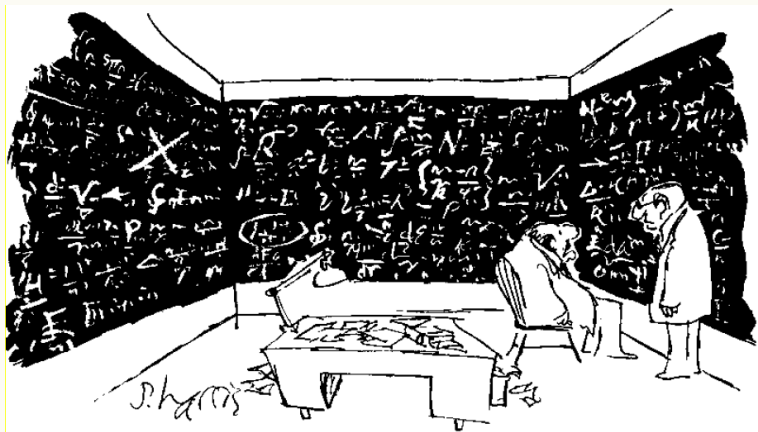
4. these solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β -functions:

$$\beta_{ijk} = 0.$$

Then, each of the solutions can be uniquely extended to a formal power series in g , and the associated super Yang-Mills models depend on the single coupling constant g with a β function which vanishes at all-orders.

SUSY breaking soft terms

Introduce over 100 new free parameters



"Whatever happened to *elegant* solutions?"

RGI in the Soft Supersymmetry Breaking Sector

Supersymmetry is essential. It has to be broken, though. . .

$$-\mathcal{L}_{\text{SB}} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^j \phi^{*i} \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.}$$

h trilinear couplings (A), b^{ij} bilinear couplings, m^2 squared scalar masses, M unified gaugino mass

The RGI method has been extended to the SSB of these theories.

- ▶ One- and two-loop finiteness conditions for SSB have been known for some time

Jack, Jones, et al.

- ▶ It is also possible to have all-loop RGI relations in the finite and non-finite cases

Kazakov; Jack, Jones, Pickering

SSB terms depend only on g and the unified gaugino mass M
universality conditions

$$h = -MC, \quad m^2 \propto M^2, \quad b \propto M\mu$$

Very appealing! But too restrictive

it leads to phenomenological problems:

- ▶ Charge and colour breaking vacua
- ▶ Incompatible with radiative electroweak breaking
- ▶ The lightest susy particle (LSP) is charged

Possible to relax the universality condition to a sum-rule for the soft scalar masses

⇒ better phenomenology.

Kobayashi, Kubo, M.M., Zoupanos

All-loop RGI relations in the soft sector

From reduction equations

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}$$

we assume the existence of a RGI surface on which

$$h^{ijk} = -M \frac{dC(g)^{ijl}}{d \ln g}$$

holds too in all-orders. Then one can prove, that the following relations are RGI to all-loops

$$M = M_0 \frac{\beta_g}{g},$$

$$h^{ijk} = -M_0 \beta_C^{ijk},$$

$$b^{jj} = -M_0 \beta_\mu^{jj},$$

$$(m^2)^i_j = \frac{1}{2} |M_0|^2 \mu \frac{d\gamma^i_j}{d\mu}, \quad (1)$$

where M_0 is an arbitrary reference mass scale. If $M_0 = m_{3/2}$ we get exactly the **anomaly mediated** breaking terms

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n} \Rightarrow h^{ijk} = -MC^{ijk} + \dots = -M\rho_{(0)}^{ijk} g + O(g^5)$$

If lowest order coefficients $\rho_{(0)}^{ijk}$ and $(m^2)_j^i$ satisfy diagonality relations

$$\rho_{ipq(0)} \rho_{(0)}^{jpq} \propto \delta_j^i, \quad (m^2)_j^i = m_j^2 \delta_j^i \quad \text{for all p and q.}$$

We find the the following soft scalar-mass sum rule, also to all-loops

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction =0 for universal choice

$$(m_i^2 + m_j^2 + m_k^2) / MM^\dagger = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4)$$

Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman; Kobayashi, Kubo, Zoupanos

Also satisfied in certain class of orbifold models, where massive states are organized into $N = 4$ supermultiples

Several aspects of Finite Models have been studied

- ▶ **$SU(5)$ Finite Models studied extensively**

Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M., Kapetanakis, Zoupanos; etc

- ▶ One of the above coincides with a non-standard Calabi-Yau
 $SU(5) \times E_8$

Greene et al; Kapetanakis, M.M., Zoupanos

- ▶ Finite theory from compactified string model also exists (albeit not good phenomenology)

Ibáñez

- ▶ Criteria for getting finite theories from branes

Hanany, Strassler, Uranga

- ▶ $N = 2$ finiteness

Freere, Mezincescu and Yao

- ▶ Models involving three generations

Babu, Enkhbat, Gogoladze

- ▶ Some models with $SU(N)^k$ **finite** \iff **3 generations, good phenomenology with $SU(3)^3$**

Ma, M.M, Zoupanos

- ▶ Relation between commutative field theories and finiteness studied

Jack and Jones

- ▶ Proof of conformal invariance in finite theories

Kazakov

$SU(5)$ Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3 \bar{\mathbf{5}} + 3 \mathbf{10} + 4 \{ \mathbf{5} + \bar{\mathbf{5}} \} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- ▶ The soft scalar masses obey a sum rule
- ▶ At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM
- ▶ At the same time finiteness is broken
- ▶ The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{ \mathbf{5} + \bar{\mathbf{5}} \}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the **24**

The superpotential which describes the two models takes the form

$$\begin{aligned}
 W = & \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 \\
 & + g_{23}^d \mathbf{10}_2 \bar{\mathbf{5}}_3 \bar{H}_4 + g_{32}^d \mathbf{10}_3 \bar{\mathbf{5}}_2 \bar{H}_4 + \sum_{a=1}^4 g_a^f H_a \mathbf{24} \bar{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3
 \end{aligned}$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries
 We will do a partial reduction, only third generation

The finiteness relations give at the M_{GUT} scale

Model A

- ▶ $g_t^2 = \frac{8}{5} g^2$
 - ▶ $g_{b,\tau}^2 = \frac{6}{5} g^2$
 - ▶ $m_{H_u}^2 + 2m_{10}^2 = M^2$
 - ▶ $m_{H_d}^2 + m_{\frac{5}{5}}^2 + m_{10}^2 = M^2$
- ▶ **3 free parameters:**
 $M, m_{\frac{5}{5}}^2$ and m_{10}^2

Model B

- ▶ $g_t^2 = \frac{4}{5} g^2$
 - ▶ $g_{b,\tau}^2 = \frac{3}{5} g^2$
 - ▶ $m_{H_u}^2 + 2m_{10}^2 = M^2$
 - ▶ $m_{H_d}^2 - 2m_{10}^2 = -\frac{M^2}{3}$
 - ▶ $m_{\frac{5}{5}}^2 + 3m_{10}^2 = \frac{4M^2}{3}$
- ▶ **2 free parameters:**
 $M, m_{\frac{5}{5}}^2$

Phenomenology

The gauge symmetry is broken below M_{GUT} , and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT} , below that is the MSSM.

- ▶ Fix the value of $m_\tau \Rightarrow \tan \beta \Rightarrow M_{top}$ and m_{bot}
- ▶ We assume a unique susy breaking scale
- ▶ The LSP is neutral
- ▶ The solutions should be compatible with radiative electroweak breaking
- ▶ No fast proton decay

We also

- ▶ Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- ▶ Include radiative corrections to bottom and tau, plus resummation (**very important!**)
- ▶ Estimate theoretical uncertainties

We look for the solutions that satisfy the following constraints:

- ▶ Right masses for top and bottom (top has been long found)
fact of life
- ▶ B physics observables
fact of life

FeynHiggs

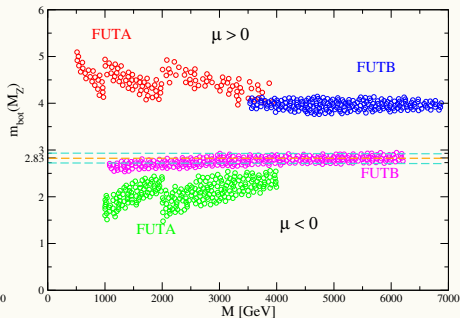
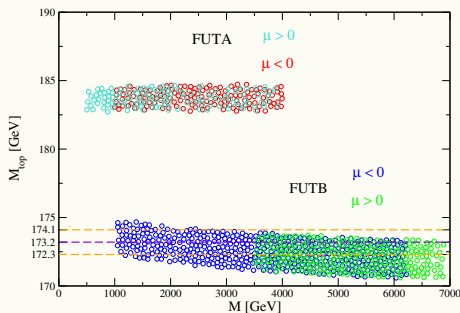
The lightest MSSM Higgs boson mass

$$M_H = \sim 121 - 126 \text{ GeV, 2007}$$

The SUSY spectrum

FeynHiggs, FUT

TOP AND BOTTOM MASS



FUTA: $M_{top} \sim 182 \sim 185$ GeV FUTB: $M_{top} \sim 172 \sim 174$ GeV

Theoretical uncertainties $\sim 4\%$

Δb and $\Delta \tau$ included, resummation done

FUTB $\mu < 0$ favoured

Experimental data

- ▶ We use the experimental values of M_H to compare with our previous results ($M_H = \sim 121 - 126 \text{ GeV}$, 2007) and put extra constraints $M_H^{exp} = 126 \pm 2 \pm 1$ and $M_H^{exp} = 125 \pm 3$ 2 GeV theoretical, 1 GeV experimental
- ▶ We also use the BPO constraints

$$\text{BR}(b \rightarrow s\gamma)_{SM/MSSM} : |BR_{bsg} - 1.089| < 0.27$$

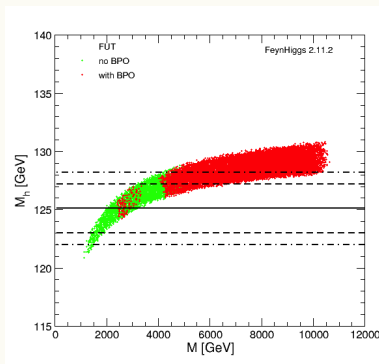
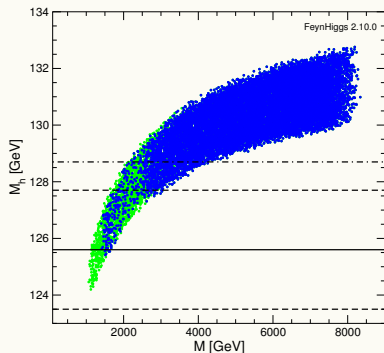
$$\text{BR}(B_u \rightarrow \tau\nu)_{SM/MSSM} : |BR_{b\tau\nu} - 1.39| < 0.69$$

$$\Delta M_{B_s}^{SM/MSSM} : 0.97 \pm 20$$

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 1.4) \times 10^{-9}$$

- ▶ We can now restrict (partly) our boundary conditions on M

Higgs mass



FUTB

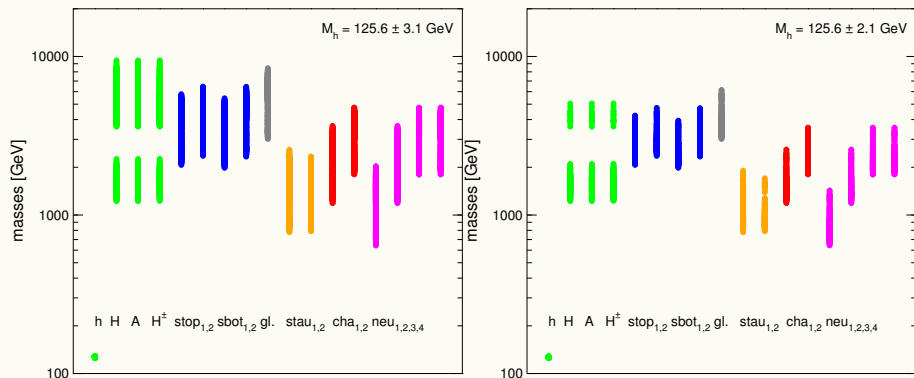
constrained by $M_{Higgs} \sim 126 \pm 3$ (2013) and 125.1 ± 3 GeV (2015)

blue and red points satisfy B Physics constraints 2013 and 2016,
2016 with 2-loop m_{top} corrections

Uncertainties ± 3 GeV (FeynHiggs)

Heinemeyer, M.M., Tracas, Zoupanos (2013) (2016)

S-SPECTRUM



SUSY spectrum with B physics constraints

Challenging for LHC

Heinemeyer, M.M., Zoupanos (2014)

Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

- ▶ $M_{top} \sim 173 \text{ GeV}$ 4% $M_{top}^{exp} = (173.2 \pm 0.9)\text{GeV}$
- ▶ $m_{bot}(M_Z) \sim 2.8 \text{ GeV}$ 8% $m_{bot}^{exp}(M_Z) = (2.83 \pm 0.10)\text{GeV}$
- ▶ $M_{Higgs} \sim 125 \text{ GeV}(\pm 3\text{GeV})$ $M_{Higgs}^{exp} = 125.1 \pm 1$
- ▶ $\tan \beta \sim 44 - 46$
- ▶ s-spectrum $\gtrsim 500 \text{ GeV}$ consistent with exp bounds

In progress

- ▶ 3 families with discrete symmetry under way
- ▶ neutrino masses via \mathcal{R}

Reduction of couplings in the MSSM

The superpotential

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2$$

with soft breaking terms,

$$-\mathcal{L}_{SSB} = \sum_{\phi} m_{\phi}^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^3 \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c.} \right] \\ + [h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \text{h.c.}],$$

then, reduction of couplings implies

$$\beta_{Y_{t,b,\tau}} = \beta_{g_3} \frac{dY_{t,b,\tau}}{dg_3}$$

Boundary conditions at the unification scale

$$\frac{Y_t^2}{4\pi} = c_1 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi} \right)^2 \quad (2)$$

$$\frac{Y_b^2}{4\pi} = p_1 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi} \right)^2 \quad (3)$$

are given by

$$c_1 = \frac{1}{3} + \frac{71}{525} \frac{\alpha_1}{\alpha_3} + \frac{3}{7} \frac{\alpha_2}{\alpha_3} + \frac{1}{35} \frac{Y_\tau^2/4\pi}{\alpha_3},$$
$$p_1 = \frac{1}{3} + \frac{29}{525} \frac{\alpha_1}{\alpha_3} + \frac{3}{7} \frac{\alpha_2}{\alpha_3} - \frac{6}{35} \frac{Y_\tau^2/4\pi}{\alpha_3}$$

Y_τ not reduced, its reduction gives imaginary values

c_2 and p_2 also found (long expressions not shown)

Soft breaking terms

The reduction of couplings in the SSB sector gives the following boundary conditions at the unification scale

$$Y_t^2 = c_1 g_3^2 + c_2 g_3^4 / (4\pi) \quad \text{and} \quad Y_b^2 = p_1 g_3^2 + p_2 g_3^4 / (4\pi)$$
$$h_{t,b} = -c_{t,b} M Y_{t,b},$$

At the unification scale $c_t = c_b = 1$ and we recover

$$m_{H_u}^2 + m_Q^2 + m_{t^c}^2 = M^2,$$
$$m_{H_d}^2 + m_Q^2 + m_{b^c}^2 = M^2,$$

M is unified gaugino mass

Corrections

- ▶ Same as with Yukawas the τ coupling cannot be reduced
- ▶ Possible to calculate the corrections coming from $\alpha_1, \alpha_2, g_\tau$ to sum rule

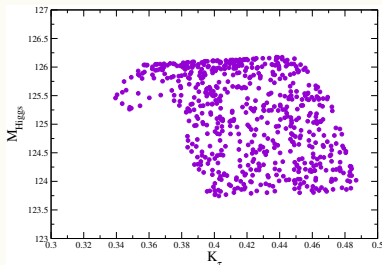
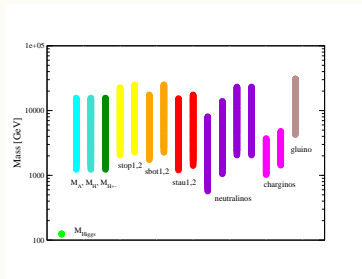
$$c_t = \frac{N_t}{D} \quad c_b = \frac{N_b}{D}$$

$$N_t = 240 + 400p_1 + 39\rho_1^2 + 71p_1\rho_1^2 + 135\rho_2^2 + 225p_1\rho_2^2 - 15p_1\sigma_h\sqrt{\rho_\tau}$$

$$N_b = 240 + 400c_1 + 21\rho_1^2 + 29c_1\rho_1^2 + 135\rho_2^2 + 225c_1\rho_2^2 + 45\sigma_h\sqrt{\rho_\tau} + 90c_1\sigma_h\sqrt{\rho_\tau}$$

$$D = 15(9 + 18p_1 + 18c_1 + 35c_1p_1)$$

Before corrections to the sum rule (2013,2014)



SUSY spectrum and the Higgs mass

Reduction of the third generation of quark masses at M_{GUT} , before corrections to the sum rule and with one-loop corrections from m_t to the Higgs mass

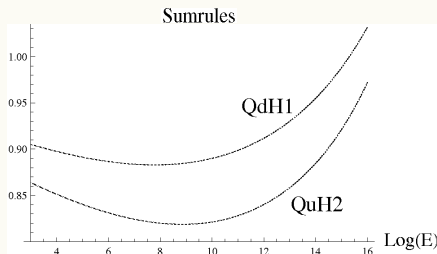
Corrections to sum rule

Corrections to the sum-rule come from tau and first two families, they are scale dependent.

$$M_3^2 (c_{H_u}^2 + c_Q^2 + c_{t^c}^2) = M_3^2 \frac{N_{SR1}}{D_{SR}},$$

$$M_3^2 (c_{H_d}^2 + c_Q^2 + c_{b^c}^2) = M_3^2 \frac{N_{SR2}}{D_{SR}},$$

Gluino mass M_3 equal to unified gaugino mass M at M_{GUT}



At the unification scale they are $< 3 - 5\%$, at other scales they can be larger ($< 17\%$)

Sum rules

$$S_1 = -\frac{S_{1Num}}{2S_{Den}}, \quad S_2 = -\frac{S_{2Num}}{S_{Den}}$$

$$\begin{aligned} S_{1Num} = & 2160F_u - 360F_dG_b^2 - 360F_{H_d}G_b^2 + 2160F_uG_b^2 + 612F_u\rho_1 - \\ & 102F_dG_b^2\rho_1 - 102F_{H_d}G_b^2\rho_1 + 612F_uG_b^2\rho_1 + 360F_u\rho_\tau - \\ & 60F_dG_b^2\rho_\tau + 180F_uG_b^2\rho_\tau + 84F_u\rho_1\rho_\tau - 11F_dG_b^2\rho_1\rho_\tau + 27F_uG_b^2\rho_1\rho_\tau + \\ & 12F_{H_u}(30(6 + \rho_\tau) + \rho_1(51 + 7\rho_\tau)) + 3G_b^2(5(12 + \rho_\tau) + \rho_1(17 + \rho_\tau)) + \\ & 2F_Q(6(30(6 + \rho_\tau) + \rho_1(51 + 7\rho_\tau)) + G_b^2(60(15 + \rho_\tau) + \rho_1(255 + 11\rho_\tau))) \end{aligned}$$

$$\begin{aligned} S_{2Num} = & 1080F_Q - 180F_{H_u}G_t^2 + 900F_QG_t^2 - 180F_uG_t^2 + 306F_Q\rho_1 - 51F_{H_u}G_t^2\rho_1 + \\ & 255F_QG_t^2\rho_1 - 51F_uG_t^2\rho_1 + 18F_{H_d}(1 + G_t^2)(60 + 17\rho_1) + 180F_Q\rho_\tau - 30F_{H_u}G_t^2\rho_\tau + \\ & 150F_QG_t^2\rho_\tau - 30F_uG_t^2\rho_\tau - 9F_{H_u}\rho_1\rho_\tau + 33F_Q\rho_1\rho_\tau + 18F_u\rho_1\rho_\tau - 16F_{H_u}G_t^2\rho_1\rho_\tau + \\ & 26F_QG_t^2\rho_1\rho_\tau + 11F_uG_t^2\rho_1\rho_\tau + 3F_d(1 + G_t^2)(60(6 + \rho_\tau) + \rho_1(102 + 11\rho_\tau)) \end{aligned}$$

$$\begin{aligned} S_{Den} = & 72(1 + G_t^2)(30(6 + \rho_\tau) + r1(51 + 7\rho_\tau)) + \\ & 2G_b^2(9(720 + 204r1 + 60\rho_\tau + 11\rho_1\rho_\tau) + G_t^2(6300 + 1785\rho_1 + 510\rho_\tau + 92\rho_1\rho_\tau)) \end{aligned}$$

where

$$F_Q = 2c_t^2G_t^2 + 2c_b^2G_b^2 - \frac{32}{3} - 6\rho_2^3 - \frac{2}{15}\rho_1^3$$

$$F_u = 4c_t^2G_t^2 - \frac{32}{3} - \frac{32}{15}\rho_1^3$$

$$F_d = 4c_b^2G_b^2 - \frac{32}{3} - \frac{8}{15}\rho_1^3$$

$$F_{H_u} = 6c_t^2G_t^2 - 6\rho_2^3 - \frac{6}{5}\rho_1^3$$

$$F_{H_d} = 6c_b^2G_b^2 + 2\rho_{h_r}^2 - 6\rho_2^3 - \frac{6}{5}\rho_1^3$$

At GUT scale only few free parameters:

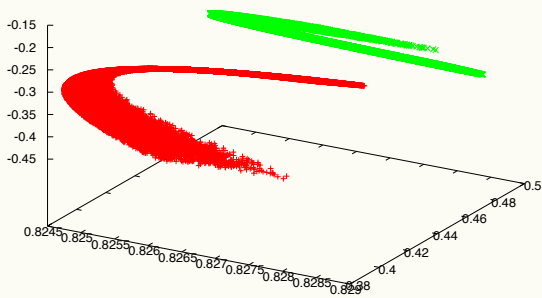
$Y_\tau = \kappa_\tau g^2$ in gauge-Yukawa sector

M, μ, B, c_τ in soft breaking sector

- ▶ Tau mass fixes κ_τ , its value enters in corrections to top and bottom mass
- ▶ Interesting: adding two-loop corrections from τ and gauge couplings of first two families improves top and bottom mass values
- ▶ μ, B fixed by radiative electroweak symmetry breaking
- ▶ BPO constrains M
- ▶ Sum rule completely determined at any scale, with corrections coming from τ coupling
- ▶ $h_\tau = c_\tau M Y_\tau$ so far unconstrained

Sum Rules

"../data2/c-ntsqd-b9-b11-2s.dat" using 89:91:94 +
"../data2/c-ntsqd-b9-b11-2s.dat" using 89:90:95 x



$$C_Q = M_Q^2/M^2, C_{H_u, H_d} = M_{H_u, H_d}^2, C_{t^c, t^b} = M_{t^c, t^b}^2$$

$$M^2 = M_Q^2 + M_{bc}^2 + M_{H_d}^2$$

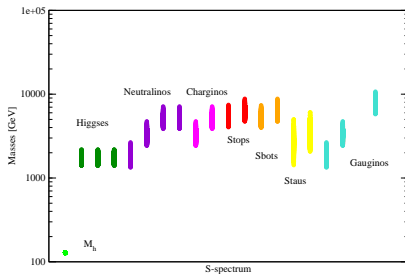
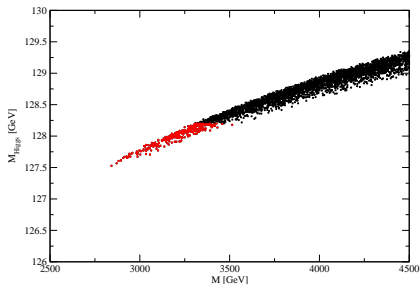
dependence on c_T and k_T

$$M^2 = M_Q^2 + M_{t^c}^2 + M_{H_u}^2$$

dependence on k_T

Preliminary

Higgs mass expected to move down slightly with new version of FeynHiggs



All points comply with BPO constraints

Red ones with DM constraints also

Very heavy spectrum comes out naturally

Results in Reduced MSSM

- ▶ Possible to have reduction of couplings in MSSM, third family of quarks
- ▶ Up to now only attempted in SM or in GUTs
- ▶ Reduced system further constrained by phenomenology: compatible with quark masses with $\mu < 0$
- ▶ Large $\tan \beta$
- ▶ Higgs mass $\sim 127 \sim 130$ GeV, might move down with latest FeynHiggs
- ▶ Heavy susy spectrum $M_{LSP} \geq 1$ TeV

Summary

- ▶ Reduction of couplings: powerful principle implies Gauge Yukawa Unification
- ▶ **Finiteness, interesting and predictive principle**
⇒ **reduces greatly the number of free parameters**
- ▶ **completely** finite theories
i.e. including the SSB terms, that satisfy a sum rule among soft breaking scalars
- ▶ **Successful prediction for top quark and Higgs boson mass**
- ▶ Confronting the $SU(5)$ FUT models with low-energy precision data **does** distinguish among models
⇒ **FUTB favoured**

Possible to have reduction of couplings in MSSM (RMSSM) third family of quarks

Satisfy BPO constraints (not trivial)

Also satisfies the sum rule, which can be determined up to tau corrections They share features

- ▶ Large $\tan \beta$
- ▶ Heavy SUSY spectrum: $LSP \gtrsim 1 \text{ TeV}$
- ▶ Prediction for Higgs mass
 - ▶ FUTB $M_h \sim 126 \text{ GeV}$
 - ▶ RMSSM $M_h \sim 128 \text{ GeV}$
- ▶ Results have to be revisited with new theoretical corrections and new version of FeynHiggs

These results hint at an underlying reduction of couplings in the third generation with SUSY