Fuzzy 4-sphere, matrix models and higher spin

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Based on arXiv:1704.02863 & 1707.00885 with H. C. Steinacker

Outline

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Motivation

Long-term goal: develop framework for **quantum geometry**

$\rightarrow$ accommodates gravity and quantum mechanics

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→ accommodates gravity and quantum mechanics

- **IKKT matrix model** good starting point
- fully covariant 4-dim quantum spaces
- best-known example: **basic fuzzy 4-sphere**

  → leads to fuzzy higher spin theory

  Q: Does it contain gravity?
- Can generalise construction of fuzzy 4-sphere

  → interesting extra dimensions

Outline

Basic fuzzy 4-sphere S^4_N

semi-classical limit: $\mathrm{SU}(4) \cong \mathrm{SO}(6)$ coadj. orbit of $\Lambda{=}(N,0,0)$

$$\mathcal{O}_\Lambda = \{\mathrm{Ad}_g(\Lambda) | g \in \mathrm{SU}(4)\} \cong \mathbb{C}P^3 \subset \mathbb{R}^{15}$$

- 15 coordinate functions m_{ab}
- Hopf map $S^2 \hookrightarrow \mathbb{C}P^3 \to S^4$
- S^4 via projection of $\mathbb{C}P^3$ to $\mathbb{R}^5$ with coordinate functions $x^a = m^{a6}$, s.t. $x_a x^a = \frac{N^2}{4}$

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fuzzy geometry:

- irrep. $\mathcal{H}_\Lambda$, then $\mathrm{SO}(6)$ generators M_{ab}
- projection now realised by $X_a = M_{a6}$ s.t. $X_a X^a = \frac{1}{4} N(N+4)\mathbb{1}$

Classification of fluctuation modes

IKKT matrix model

$$S = \frac{1}{g^2}\mathrm{tr}\left(-[Y_a, Y_b][Y^a, Y^b] + 2\mu^2 Y_a Y^a\right)$$

$\rightarrow$ **fluctuations** $Y_a = X_a + \mathcal{A}_a$ around S^4_N background X_a

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classically: $\mathcal{C}(\mathbb{C}P^3) \cong \oplus_s \mathcal{C}^s$, with $\mathcal{C}^s \cong \oplus_{n\geqslant 0}(n+s, 2s)_{\mathrm{SO}(5)}$

- functions: $\mathcal{C}^s \ni \phi^s \cong \{\text{fct. on } S^4\} \otimes \begin{array}{|c|c|c|c|}\hline 1 & 2 & \cdot\cdot & s \\ \hline 1 & 2 & \cdot\cdot & s \\ \hline\end{array}$

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- vector modes: decompose $(1,0) \otimes (n, 2s)$

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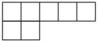
fuzzy: cut-off $\sim N$ in maximal spin

Overview 1707.00885

We found

- fluctuations $\leftrightarrow$ Young diagrams 

  $\to$ "higher spin valued" functions on S^4_N
- IKKT action $\to$ action for fuzzy higher spin theory
- analysis of spin 2 sector

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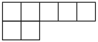
- fluctuations $\leftrightarrow$ Young diagrams 

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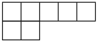
- fluctuations $\leftrightarrow$ Young diagrams 

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But way out might be **generalised fuzzy S^4**!

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Generalised fuzzy 4-sphere S^4_Λ

semi-classical limit: $\mathrm{SU}(4) \cong \mathrm{SO}(6)$ coadj. orbit of $\Lambda{=}(N, 0, n)$

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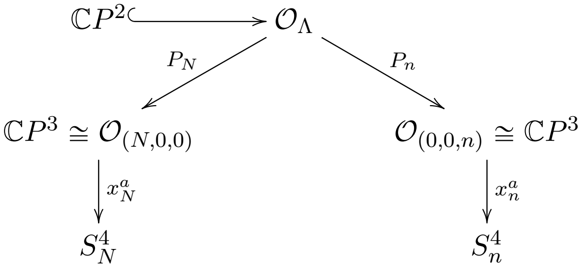
intricate geometry



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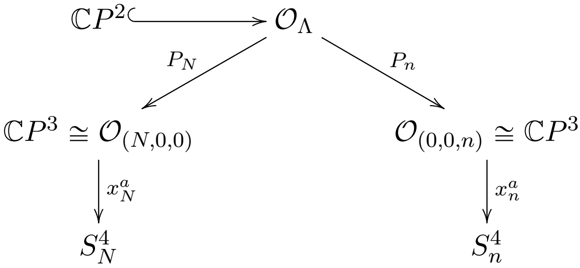
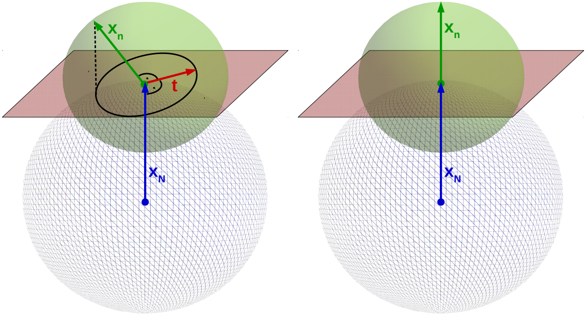


$\rightarrow$ need **suitable coordinates**!

Set of coordinates

Various view points:

- $S^4_N \times S^4_n$ **structure**: coordinates x_N, x_n
  s.t. $x_N \cdot x_N = \frac{N^2}{4}$, $x_n \cdot x_n = \frac{n^2}{4}$, and $x_n \cdot x_n = \delta$
- $\mathbb{C}P^2$ **fibre:** additional coordinate t



Overview 1704.02863

- Description of classical $\mathcal{O}_\Lambda$ and introduced coordinates
- **Fuzzy analogues** X_N, X_n, T plus their algebra
- Found **solutions** for IKKT matrix model of form

$$\begin{pmatrix} X_N + X_n \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} X_N - X_n \\ T \end{pmatrix}$$

- Investigated effective metric

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Summary and outlook

What's done?

- Classification fluctuation modes on basic fuzzy S^4_N.
- Analysis of spin 2 modes on S^4_N.
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- **Explicit matrix model solutions** with 10 matrices.

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What's to be done?

- Classification of fluctuation modes on S^4_Λ.
- Analysis of spin 2 modes on S^4_Λ.

  $\rightarrow \exists$ **physical graviton?**