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(New) Physics Phenomena and F-GUTs

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$\mathcal{A}$

Experimental Facts

... focus in two properties

Beyond the **Standard Model**:

▲ I) Massive Neutrinos

Evidence: Neutrino Oscillations

▲ II) Lepton Flavour Universality (LFU) Violation:

Evidence? Anomalies in B-decays ?

$\mathcal{A}_1$

Neutrinos

▲ neutrino oscillations tightly connected to non-zero neutrino masses and the mixing

▲ Old data ($\sim$ 15 yrs ago) consistent with simple Tri-Bimaximal mixing

$$V_{TB} = V_l^\dagger V_\nu = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

▲ ... theoretical interpretation $\rightarrow$ invariance under some discrete group:

$$S_4,\ A_4,\ Z_2 \times Z_2,\ A_5, \ldots$$

▲ Recent data show that the actual case is far more complicated...

Neutrino data: parametrization of mixing angles

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{23}s_{12}s_{13} - c_{12}s_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

Experimental data (3σ range) of the angles ($c_{ij} \equiv \cos\theta_{ij}$)

$$\begin{aligned} \sin^2\theta_{12} &= [0.259 - 0.359] \\ \sin^2\theta_{23} &= [0.331 - 0.637] \\ \sin^2\theta_{13} &= [0.0169 - 0.0313] \neq 0 \\ \delta &= 0.77\pi - 1.36\pi \end{aligned}$$

$\mathcal{A}_2$

Anomalous B decays

▲ Lepton Flavour Universality (LFU):

SM: EW couplings of leptons to gauge bosons flavour independent:

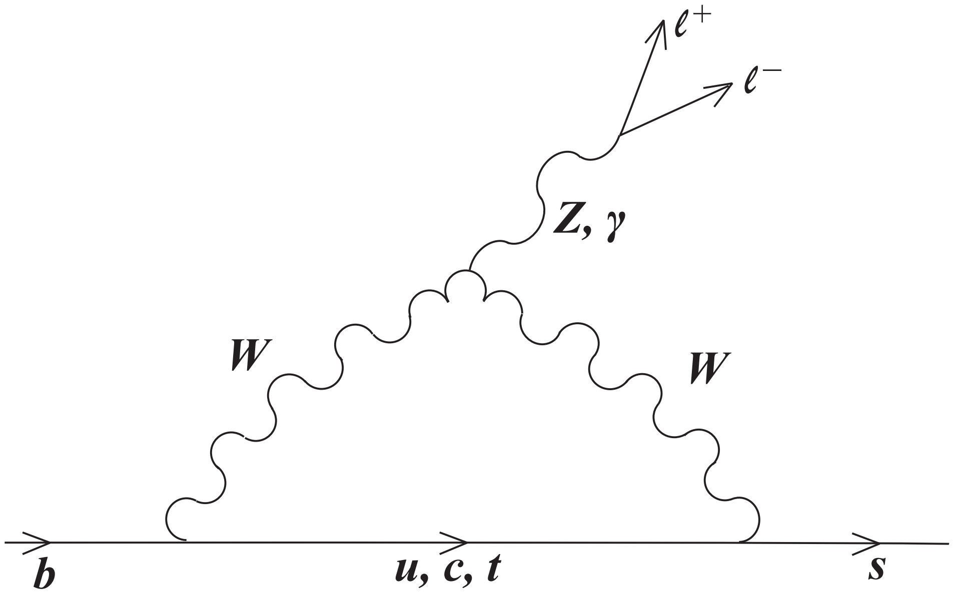
example:

$$\Gamma(\tau^- \to \mu^- + \bar{\nu}_\mu + \nu_\tau) = \Gamma(\tau^- \to e^- + \bar{\nu}_e + \nu_\tau),\ e.t.c...$$

▲ **Sensitive interactions to test LFU :**

⇉ decays involving quark fields such as $b \to s\ell^+\ell^-$, $\ell = e, \mu, \tau$

in SM: $b \to s\gamma$ *at one-loop:*



Suitable candidates to test LFU:

B mesons such as $B^+ = \bar{b}u$ and $B^0 = \bar{b}d$

$$B^+ \quad \to \quad K^+ \ell^+ \ell^-$$

$$B^0 \quad \to \quad K^{*0} \ell^+ \ell^-$$

(K^{*0} *reconstructed in the final state* $K^+\pi^-$)

Ratios of branching ratios in SM expected to be ~ 1:

$$R_{X_{ij}} = \frac{\mathrm{BR}(B \to X^+ \ell_i^+ \ell_i^-)}{\mathrm{BR}(B \to X^+ \ell_j^+ \ell_j^-)} \approx 1 \quad : \; i, j = e, \mu, \tau; \; X = K^+, K^0, \cdots$$

LHCb Experimental evidence:

$$R_K = \frac{\mathrm{BR}(B \to K^+ \mu^+ \mu^-)}{\mathrm{BR}(B \to K^+ e^+ e^-)} = 0.745 \pm 0.09(stat) \pm 0.036(syst)$$

integrated over $1\mathrm{GeV}^2 < q^2 < 6\ \mathrm{GeV}^2$ (dilpeton invariant mass^2)
(... *away from* $B^+ \to J/\psi(\ell^+\ell^-)K^+ \ldots$)

$$R_{K^*} = \frac{\mathrm{BR}(B \to K^* \mu^+ \mu^-)}{\mathrm{BR}(B \to K^* e^+ e^-)} \approx \begin{cases} 0.660 & (2m_\mu)^2 < q^2 < 1.1\mathrm{GeV}^2 \\ 0.685 & 1.1\mathrm{GeV}^2 < q^2 < 6\mathrm{GeV}^2 \end{cases}$$

Both ratios → defficit in same direction!

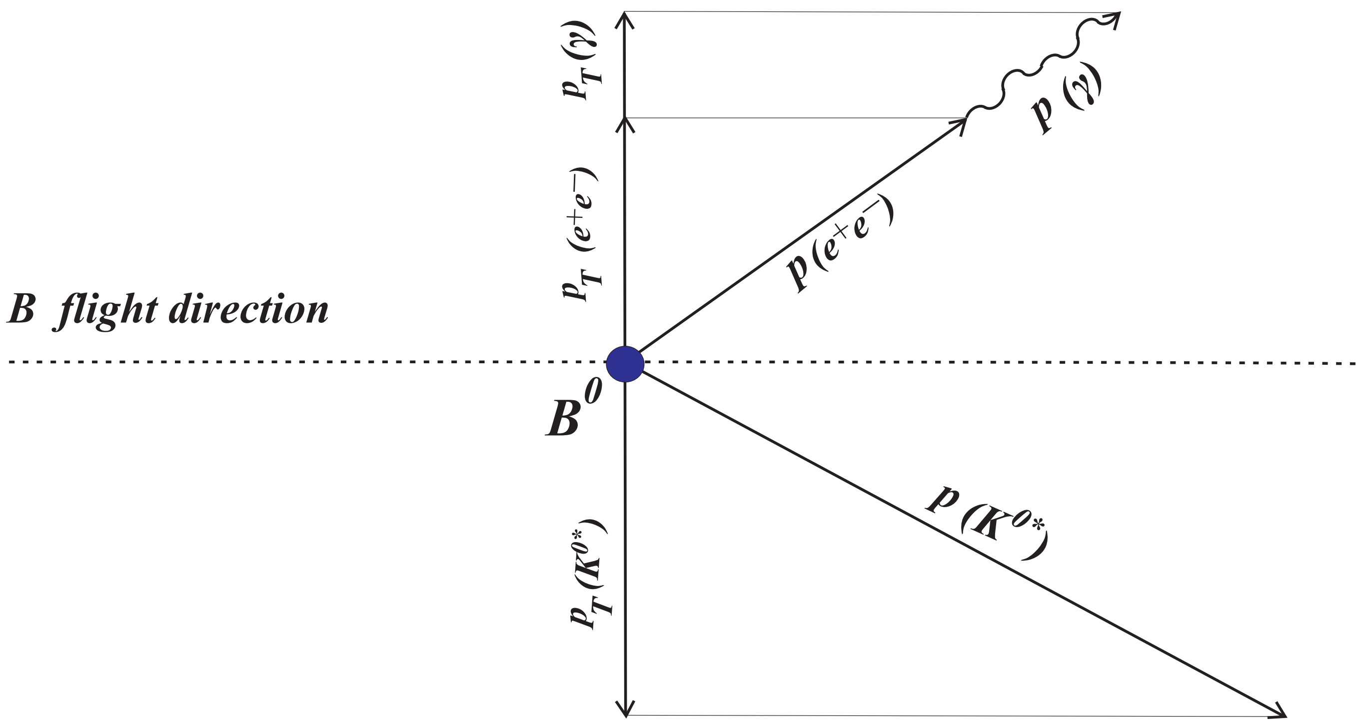
▲ Unexplained in SM ▲

experimental data … against expectations?…

▲ LHCb more efficient for $B \to K^{*}\mu^{+}\mu^{-}$

▲ $B \to K^{*}e^{+}e^{-}$: significant reduction due to bremsstralung

Bremsstralung effect recovery (evaluation of p_T difference): (LHCb 1705.05802, and talk of Monica Pepe Altarelli)



Other deviations

- Defficit of differential branching fraction $B_s \to \phi\mu^+\mu^-$:

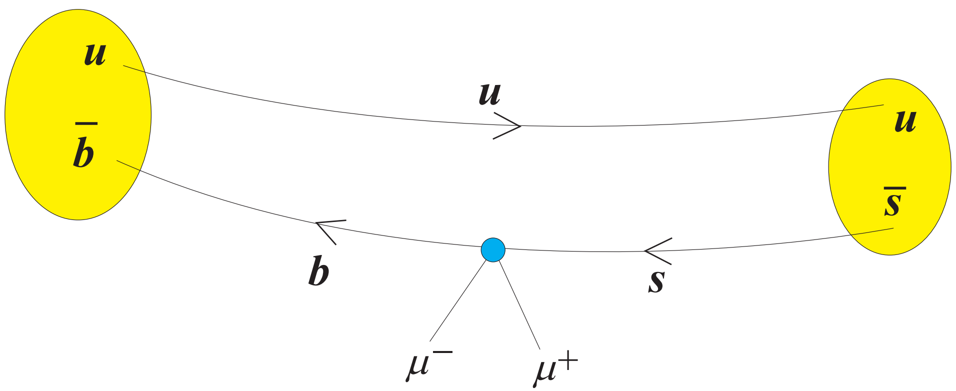
$$\frac{d\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-)}{dq^2} \;\; ; (\phi \to K^+K^-)$$

$(B_s = \bar{b}s, \phi = \bar{s}s)$

New Physics *solution to the puzzle...required!*

Generate new contributions to the process :

$$\boldsymbol{B} \to \boldsymbol{K}\,\mu^-\mu^+$$



SM & BSM Contributions

parametrised in terms of the effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha}{4\pi} \sum_k \left(C_k(\mu) \mathcal{O}_k(\mu) + C'_k(\mu) \mathcal{O}'_k(\mu) \right) \qquad (1)$$

data in accordance with contributions of semi-leptonic operators:

$$\mathcal{O}_9 = \bar{s}\gamma_\lambda P_L b\, \bar{\ell}\gamma^\lambda \ell \qquad \mathcal{O}'_9 = \bar{s}\gamma_\lambda P_R b\, \bar{\ell}\gamma^\lambda \ell$$

$$\mathcal{O}_{10} = \bar{s}\gamma_\lambda P_L b\, \bar{\ell}\gamma^\lambda \gamma_5 \ell \qquad \mathcal{O}'_{10} = \bar{s}\gamma_\lambda P_R b\, \bar{\ell}\gamma^\lambda \gamma_5 \ell$$

$P_L = \frac{1}{2}(1-\gamma_5),\ P_R = \frac{1}{2}(1+\gamma_5)$
Dimensionless parameters C_k in $\mathcal{H}_{eff} \to$ **Wilson** coefficients:

$$C_k(\mu) = C_k(\mu)^{SM} + C_k(\mu)^{BSM}$$

Possible Solutions

▲ $\mathcal{A}$: **Negative** New Physics contributions 25% to C_9.

$${C_9}^{NP} \approx -1.07$$

▲ $\mathcal{B}$: Contributions along the $SU(2)$-invariant direction $C_9 = -C_{10}$, ($V - A$-type)

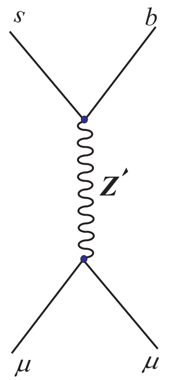
$${C_9}^{NP} \approx -0.53, \; {C_{10}}^{NP} \approx 0.53$$

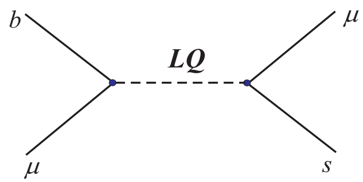
BSM Extensions involve:

▲ 1.) Z' neutral gauge boson (*coupled differently to fermion familes and vectorlike fields*) (*see talks by Steve King and M. Quiros*)

▲ 2.) New Particles (Leptoquarks...) (*Crivellin talk, 1709.00692*)

Graphs: Z' boson and Leptoquark contributions to $b \to s\mu^+\mu^-$





focus: Z' contribution to $B^+ \to K^+\mu^+\mu^-$ (*see e.g. 1511.07447*)

$$J'^0_\lambda \quad = \quad g_\mu\, \bar{\mu}\gamma_\lambda \mu + g_t\, \bar{t}\gamma_\lambda P_L t + g_q \bar{q}\gamma_\lambda P_L q$$

$$+(g_t - g_q)\, V^*_{ts} V_{tb}\, \bar{s}\gamma_\lambda P_L b + \cdots$$

g_q *assumed equal for* $q = u, d, c, s$ *to suppress FCNC. Then:*

$$C_9^{NP} = -\frac{\pi g_\mu (g_t - g_q)}{2\sqrt{2} G_F M^2_{Z'} c^2_W \alpha} \approx -\frac{\pi g_\mu (g_t - g_q)}{c^2_W (M_{Z'}/(2\,\mathrm{TeV}))^2}$$

... ~ -1 for natural values of g_t, g_q, g_μ and $M_{Z'} \sim$ few TeV.

thus... minimal requirement:

... Z' explains the anomalous B-decays if :

Z' couplings to 2^{nd} and 3^{rd} families are different!

$\mathcal{B}$

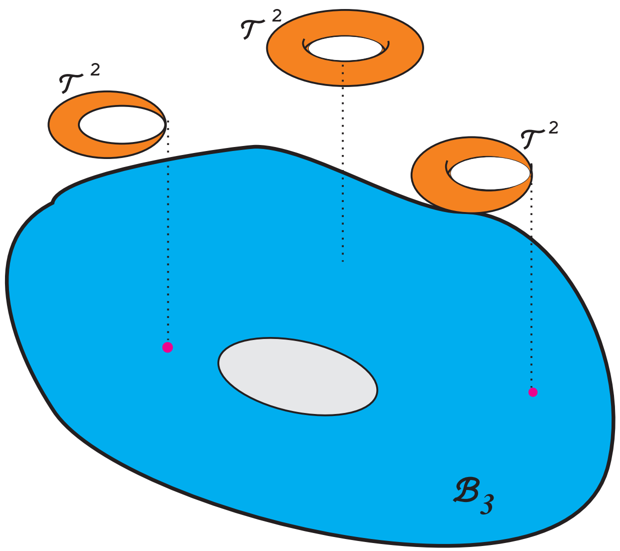
F-theory Models

F-theory defined on $\mathcal{R}^{3,1} \times \mathcal{X}$

$\mathcal{X}$, *elliptically* fibered **CY** 4-fold over B_3

Fibration described by $\mathcal{W}$eierstraß $\mathcal{E}$quation :

$$y^2 \quad = \quad x^3 + f(z)\,x + g(z) \;\rightarrow (\text{“}torus\text{”})$$



Topol. and geom. properties of $\mathcal{X}$ depend on $f(z), g(z)$

Conjecture (*hep-th/9602022*)

$\Downarrow$

CY_4-**geometric singularities** $\rightleftarrows$ **gauge symmetries**

$\Downarrow$

singularities classified in terms of $\mathcal{A}\,\mathcal{D}\,\mathcal{E}$ Lie groups

(**Kodaira**$\sim$ 1960)

Kodaira classification of singularities: w.r.t. vanishing order of f, g and discriminant Δ: ($y^2 = x^3 + fx + g$, $\Delta = 4f^3 + 27g^2$)

ord(f)	ord(g)	ord(Δ)	fiber type	Singularity
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

A Class of 'semi-local' constructions

▲ F-GUTs embedded in maximal exceptional group:

$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}$$

Present choice: embedding of SM in minimal GUT F-SU(5):

$$\begin{aligned} \mathcal{E}_8 \quad &\to \quad SU(5) \times SU(5)_\perp \to \mathcal{C} = SU(5)_\perp \\ &\to \quad SU(5) \times U(1)^4 \end{aligned}$$

Spectral Cover description: $SU(5)_\perp \to$ described by Cartan roots:

$$t_i \; = \; SU(5)_\perp - \text{roots} \to \; \sum_{i=1}^{5} t_i = 0$$

Fluxes:

$SU(5)$ Chirality

$SU(5)$ Symmetry Breaking

Splitting of $SU(5)$-reps

Two types of fluxes: (*parametrised with integers* M_i, N_j)

M_{10}, M_5:

associated with flux-restrictions on $U(1)$'s$\in$ $SU(5)_\perp$:

(*recall that* $E_8 \supset SU(5)_{GUT} \times SU(5)_\perp \supset SU(5)_{GUT} \times U(1)^4_\perp$)

M_i determine the chirality of complete $10, 5 \in SU(5)$.

N_Y:

related to Cartan generators of $SU(5)_{GUT}$.

They are taken along $U(1)_Y \in SU(5)_{GUT}$ and **split** $SU(5)$-reps.

$SU(5)$ chirality from $U(1)_\perp$ Flux

$U(1)_\perp$–Flux on SM reps $\in$ **10**'s:

$$\#10 - \#\overline{10} = \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} & = & M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} & = & M_{10} \\ n_{(1,1)_1} - n_{(1,1)_{-1}} & = & M_{10} \end{cases}$$

$U(1)_\perp$– Flux on SM reps $\in$ **5**'s:

$$\#5 - \#\overline{5} = \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} & = & M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} & = & M_5 \end{cases}$$

SM chirality form Hypercharge Flux

$U(1)_Y-$**Flux**-splitting of **10**'s:

$$
\begin{aligned}
n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= M_{10} \\
n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= M_{10} - N_{Y_{10}} \\
n_{(1,1)_{1}} - n_{(1,1)_{-1}} &= M_{10} + N_{Y_{10}}
\end{aligned}
$$

$U(1)_Y-$ **Flux**-splitting of **5**'s:

$$
\begin{aligned}
n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} &= M_{5} \\
n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_{5} + N_{Y_{5}}
\end{aligned}
$$

Hyper-Flux Doublet-Triplet splitting :

$U(1)_Y-$ **Flux**-splitting of $\mathbf{5_{H_u}}$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 + 1 = 1 \; (H_u)$$

$U(1)_Y-$ **Flux**-splitting of $\mathbf{\bar{5}_{H_d}}$ $\rightarrow$:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 = 0$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N_{Y_5} = 0 - 1 = -1 \; (H_d)$$

Fluxes and Anomaly Cancellation

M_j, N_{Y_j} subject to geometry/anomaly cancellation restrictions

Anomaly cancellation

$$\mathcal{A}_{SU(3)^2-U(1)},\ \mathcal{A}_{SU(2)^2-U(1)},\ \mathcal{A}_{U(1_Y)^2-U(1)}$$

$$\mathcal{A}_{U(1_Y)-U(1)^2}$$

equivalent to geometric constraints * : ($q_n = n$-irrep $U(1)$ *'charge'*)

$$\begin{aligned} \sum_{\Sigma_{10}} q_{10_j} N_{10_j} + \sum_{\Sigma_5} q_{5_i} N_{5_i} &= 0 \\ 3\sum_{\Sigma_{10}} q^2_{10_j} N_{10_j} + \sum_{\Sigma_5} q^2_{5_i} N_{5_i} &= 0 \end{aligned} \tag{2}$$

* (*see works of Dudas-Palti, Marsano, Weigand,...*)

$\mathcal{C}$

Proposed interpretations within **F-GUTs**

$\mathcal{C}_1$

Anomalies of B decays

Interpretation in F-theory GUTs

- Z' must couple differently to families.
- $\rightarrow$ ... scenario naturally realised in an F-theory framework

$$E_8 \supset SU(5) \times SU(5)_\perp \rightarrow SU(5) \times U(1)_\perp^4$$

Cartan generators $\Leftrightarrow$ $U(1)_\perp$

$$\begin{aligned} H_1 &= \frac{1}{2}\mathrm{diag}(1,-1,0,0,0), \\ H_2 &= \frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,-2,0,0), \\ H_3 &= \frac{1}{2\sqrt{6}}\mathrm{diag}(1,1,1,-3,0), \\ H_4 &= \frac{1}{2\sqrt{10}}\mathrm{diag}(1,1,1,1,-4), \end{aligned}$$

recall that spectral surface $\mathcal{C}_5 \leftrightarrow U(5)_\perp$ *described by:*

$$\sum_{k=0}^{5} b_k t^k = 0$$

▲ *Topological properties encoded* $\in$ b_k *coeffs*

however:

▲ *description of* EFT *model relies on roots* t_i

solutions $t_i(b_k)$: a non-trivial monodromy subgroup of Weil group:

$$W(SU(5)_\perp) = S_5 : \; S_4, A_4, Z_2 \text{ etc}$$

▲ *Simplest case* Z_2 reduces $U(1)_\perp^4 \to U(1)_\perp^3$:

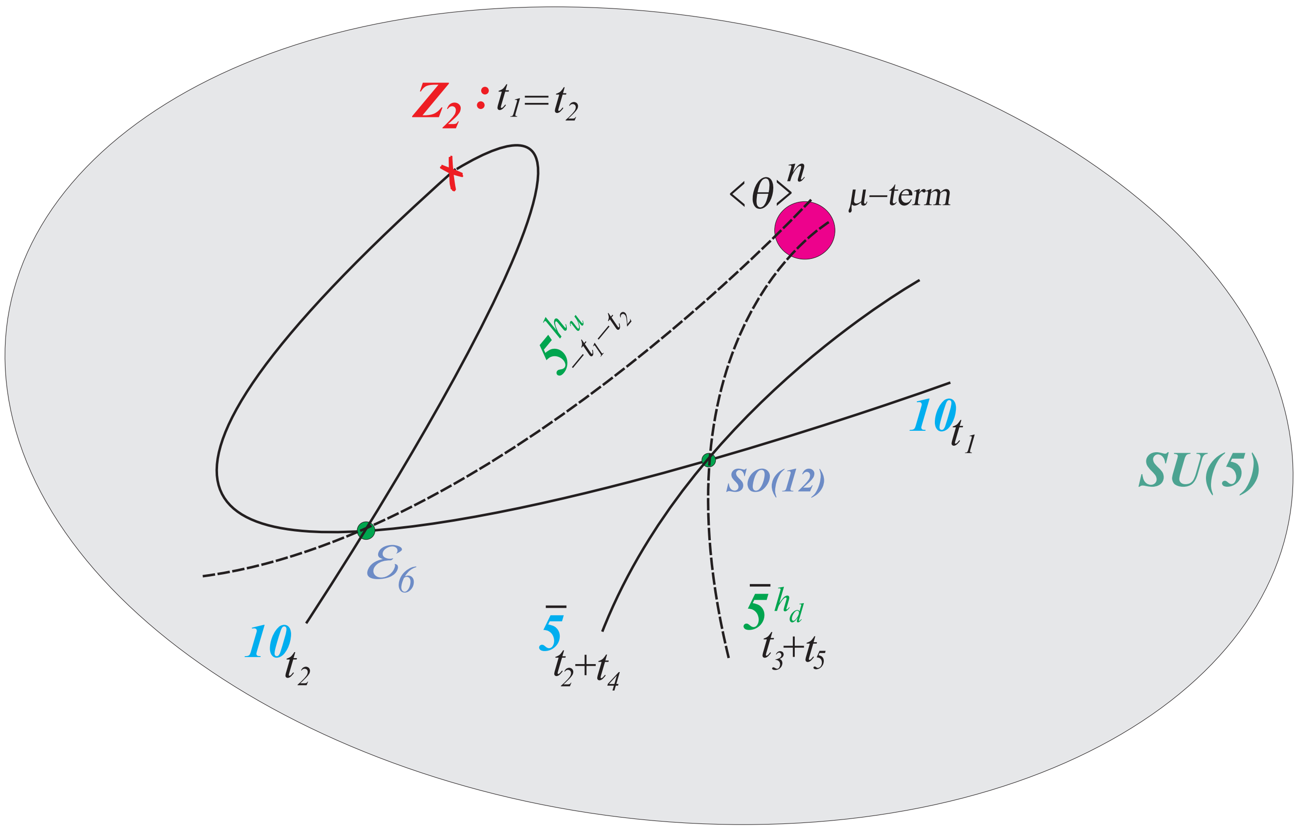
$$Z_2 : \; t_1 \leftrightarrow t_2 \Rrightarrow U(1)_\perp^3$$

example of a Z_2 model (GKL & GG Ross 1009.6000)

$SU(5), U(1)_i$	SM spectrum	Exotics	R-parity
$10_i, t_i$	Q_i, u_i^c, d_i^c	$-$	$-$
$\bar{5}_1, t_3+t_4$	d_1^c, ℓ_1	$-$	$-$
$\bar{5}_2, t_1+t_3$	d_2^c, ℓ_2	$-$	$-$
$\bar{5}_3, t_1+t_4$	d_3^c, ℓ_3	$-$	$-$
$5_{H_u}, -2t_1$	H_u	D	$+$
$\bar{5}_{H_d}, t_3+t_5$	H_d	$-$	$+$
$5_x, -(t_1+t_5)$	$-$	$(H_{u_i}, D_i)_{i=1,\ldots,n}$	$+$
$\bar{5}_{\bar{x}}, t_4+t_5$	$-$	$D^c + (H_{d_i}, D_i^c)_{i=1,\ldots,n}$	$+$
$\theta_{12,21}, 0$		S (singlet)	$-$

(for phenomenological aspects see 1706.08372)

Geometric picture of a generic Z_2 model



A Z' with Non-Universal Gauge-Lepton couplings

M. Crispim-Romão, S.F. King, G.K.L.

in preparation

Assumed Model: $SU(5)$ *with* $t_1 \leftrightarrow t_2$ *monodromy*

choosing a convenient basis for $U(1)_i$*'s* $\perp$ *to* $SU(5)$:

$$E_8 \rightarrow E_6 \times U(1)_\perp^2 \tag{3}$$

$$\rightarrow SO(10) \times U(1)_\psi \times U(1)_\perp^2 \tag{4}$$

$$\rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \times U(1)_\perp^2 . \tag{5}$$

Unbroken generators after *imposing a* Z_2 *monodromy* :

$$Q_\chi \propto \text{diag}[-1,-1,-1,-1,4] \in E_6$$

$$Q_\psi \propto \text{diag}[1,1,1,-3,0] \in E_6$$

$$Q_\perp \propto \text{diag}[1,1,-2,0,0] \tag{6}$$

▲ $U(1)'$ must be combination of unbroken generators:

$$Q = c_1 Q_\chi + c_2 Q_\chi + c_3 Q_\perp$$

▲ must respect anomaly cancellation conditions.

Additional Conditions on c_i coeffs and M_{10_j}, M_{5_i} :

$$c_1^2 + c_2^2 + c_3^2 = 1, \ \sum_j M_{10_j} = -\sum_i M_{5_i} = 3$$

▲ 3rd family Q_3' differently charged under $U(1)'$

▲ preferably $Q_1' = Q_2'$ in quark sector (to suppress FCNCs)

Results: **Plenty of solutions:**

▲ cases with an additional $5 + \bar{5}$ a pair

▲ cases with an additional $10 + \overline{10}$

▲ models with complete vector-like family $(10 + \overline{10}) + (5 + \bar{5})$

An example: *... search underway for a successful model...*

Curve Name	$\sqrt{10}\,Q_3$	SM content
5_{H_u}	$\frac{3}{2}$	H_u
5_1	-1	L
5_2	$\frac{3}{2}$	H_d
5_3	$\frac{1}{4}$	L
5_4	-1	d^c
5_5	$-\frac{9}{4}$	$d^c + L$
5_6	$\frac{1}{4}$	d^c
10_t	$-\frac{3}{4}$	$Q + 2u^c$
10_2	$\frac{7}{4}$	$-$
10_3	-$\frac{3}{4}$	$Q + 2e^c$
10_4	$\frac{1}{2}$	$Q + u^c + e^c$

$\mathcal{C}_2$

Symmetries for Neutrinos

... a wide class of **Discrete Groups** $PSL_2(p)$, p **prime**

▲ **Requirements:** ▲

▲ ... of physical interest only those with $3 - dim.$ representations

▲ *GUT and "perpendicular"-group embedded in max. sym.* E_8*:*

$$E_8 \quad \supset \quad SU(5) \times SU(5)_\perp \tag{7}$$

$\rightarrow PSL_2(p)$

- must be subgroups of $\mathbf{SU(5)_\perp} \rightarrow p \leq 11$
- must have 3-d representations $(m_\nu \rightarrow 3 \times 3) \rightarrow p \leq 7$

$$\Downarrow$$

$$PSL_2(7) \in SU(3)$$

rich structure, less explored, viable candidate!

▲ *Construction of 3-d. irreducible representation of* $PSL_2(7)$
(*E.G.Floratos, GKL arXiv:1511.01875*)

$$\mathfrak{a}^2 = \mathfrak{b}^3 = (\mathfrak{a}\mathfrak{b})^7 = ([\mathfrak{a}, \mathfrak{b}])^4 = I$$

Method: use of Weil's Metaplectic Representation
(based on *Balian & Itzykson* Acad. Dc. Paris 303 (1986).)

Defining $\eta = e^{2\pi i/7}$, (7^{th} *root of unity*)

$$\mathfrak{a} \to A^{[3]} \quad = \quad \frac{i}{\sqrt{7}} \begin{pmatrix} \eta^2 - \eta^5 & \eta^6 - \eta & \eta^3 - \eta^4 \\ \eta^6 - \eta & \eta^4 - \eta^3 & \eta^2 - \eta^5 \\ \eta^3 - \eta^4 & \eta^2 - \eta^5 & \eta - \eta^6 \end{pmatrix}$$

and

$$\mathfrak{b} \to B^{[3]} \quad = \quad \frac{i}{\sqrt{7}} \begin{pmatrix} \eta - \eta^4 & \eta^4 - \eta^6 & \eta^6 - 1 \\ \eta^5 - 1 & \eta^2 - \eta & \eta^5 - \eta \\ \eta^2 - \eta^3 & 1 - \eta^3 & \eta^4 - \eta^2 \end{pmatrix}$$

Application to neutrino mixing:

Invariance of M_ν *under* $PSL_2(7)$ *(sub)group* A_i

$$[M, A_i] = 0$$

$\Rightarrow$ common eigenvectors, $\rightarrow$ mixing matrix.

Observation: $PSL_2(7)$ generators have Latin square structure:

$$U \quad \propto \quad \begin{pmatrix} r_1 & r_2 & r_3 \\ r_2 & r_3 & r_1 \\ r_3 & r_1 & r_2 \end{pmatrix}$$

Imposing conditions: orthogonality, unitarity , . . . , roots satisfy:

$$x^3 + x^2 - r_1 r_2 r_3 = 0$$

for $PSL_2(7)$, $r_1 r_2 r_3 = \frac{1}{7}$

classification of all 168 elements : (*Aliferis, GKL Vlachos*)

Example: *The following elements give the correct mixing*
(*commuting with* $[M_\nu, U_1] = 0, \ [M_\ell, U_2] = 0]$ *respectively*)

$$U_1 = \begin{pmatrix} r_3 & -r_1 & -r_2 \\ -r_1 & r_2 & r_3 \\ -r_2 & r_3 & r_1 \end{pmatrix}, \ U_2 = \begin{pmatrix} 0 & 0 & -e^{\frac{6\pi i}{7}} \\ e^{-\frac{2\pi i}{7}} & 0 & 0 \\ 0 & e^{-\frac{4\pi i}{7}} & 0 \end{pmatrix}$$

$$U_\nu = \begin{pmatrix} 0.802e^{0.57i} & 0.577e^{2.39i} & 0.153e^{-1.27i} \\ 0.366e^{0.1065i} & 0.577e^{-0.87i} & 0.729e^{-0.35i} \\ 0.471e^{-1.66i} & 0.577e^{3.05i} & 0.667e^{0.64i} \end{pmatrix}$$

Comparison with experimental data:

▲ $\theta_{12}, \theta_{23}, \theta_{13}$ in agreement with experimental values.

▲ θ_{13} *automatically* non-zero (*see arXiv:1612.06161*)

F-theory models :

⇓⇓⇓

Geometric interpretation of GUTs

Calculability, form handful of topological properties, natural

Doublet-Triplet splitting...

Prediction of Vector-like pairs and singlets ...

viable models for New Physics

such as ...Z''s and B-physics, resonances, events at a few TeV...

Discrete Symmetries interpreting the Neutrino data naturally

incorporated in E_8 singularity