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COST Action MP 1405
Quantum Structure of Spacetime

Noncommutative Scalar Quasinormal Modes of the ReissnerNordström Black Hole

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M. Dimitrijević Ćirić, N. Konjik and A. Samsarov, arXiv:1708.04066

Motivation

NC geometry$\rightarrow$
-Quantum gravity near massive BH
-QNM to see perturbations of BH
-Gravitational waves are measurable$\rightarrow$ QNM are also

Our goal
-Calculate effects of NC deformation on QNM spectrum of RN BH
-Semiclassical approximation (similar to Hawking radiation): gravitational field is undeformed while the scalar field (perturbation) is NC

NC space-time form the angular twist

NC deformation is introduced via twist formalism [Aschieri, Dimitrijević, Kulish, Lizzi, Wess, 2009]
Twist $\mathcal{F}$ is invertible bidifferential operator from the universal enveloping algebra of the symmetry algebra

We work in 4D and deform the space-time by the following Abelian twist

$\mathcal{F} = \mathrm{e}^{-\frac{i}{2}\theta_{ab}X^a \bigotimes X^b}$
$[X^a, X^b] = 0$, a,b=1,2 Angular twist in curved coordinates
$X_1 = \partial_0$ and $X_2 = \partial_\varphi$
-Semiclasical aproach that metric tensor $g_{\mu\nu}$ does not depend on t and φ coordinates: X_1 and X_2 are the Killing vector fields [Aschieri, Castelani, 2009]
-Basis one-forms $\star$ -commute with functions
-Hodge dual remains the same as in commutative case

NC charged scalar field in RN background

The action (based on $U(1)\star$ theory) is given by

$$S[\hat{\phi}, \hat{A}] \;=\; \int d^4x \sqrt{-g} \star \left(g^{\mu\nu} \star D_\mu \hat{\phi}^+ \star D_\nu \hat{\phi} - \mu^2 \hat{\phi}^+ \star \hat{\phi} \right)$$

$D_\mu \hat{\phi} = \partial_\mu \hat{\phi} - iA_\mu \star \hat{\phi}$ and $g_{\mu\nu}$ is RN metric

We expand the action using SW map and $\star$ product

Seiberg-Witten map for fields

$$\hat{\phi} = \phi - \frac{1}{4}\theta^{\rho\sigma}A_\rho(\partial_\sigma\phi + D_\sigma\phi),$$
$$\hat{A}_\mu = A_\mu - \frac{1}{2}\theta^{\rho\sigma}A_\rho(\partial_\sigma A_\mu + F_{\sigma\mu})$$

After expanding the action and varying with respect to Φ^+ we obtain the EOM for Φ

$$g^{\mu\nu}\Big(D_\mu D_\nu\phi - \Gamma^\lambda_{\mu\nu}D_\lambda\phi\Big) - \frac{1}{4}\theta^{\alpha\beta}g^{\mu\nu}\Big(D_\mu(F_{\alpha\beta}D_\nu\phi) - \Gamma^\lambda_{\mu\nu}F_{\alpha\beta}D_\lambda\phi$$
$$-2D_\mu(F_{\alpha\nu}D_\beta\phi) + 2\Gamma^\lambda_{\mu\nu}F_{\alpha\lambda}D_\beta\phi - 2D_\beta(F_{\alpha\mu}D_\nu\phi)\Big) = 0$$

$D_\mu\phi = \partial_\mu\phi - iA_\mu\phi \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\Gamma^\lambda_{\mu\nu}$-Christoffel symbol for RN metric

RN metric tensor is

$$g_{\mu\nu} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & -\frac{1}{f} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{bmatrix}$$

with $f = 1 - \frac{2MG}{r} + \frac{Q^2 G}{r^2}$
Q-charge of RN BH M-mass of RN BH
Two horisons: $r_{\pm} = MG \pm \sqrt{M^2 G^2 - GQ^2}$
Non-zero components of gauge fields are $A_0 = -\frac{qQ}{r}$ i.e. $F_{r0} = \frac{qQ}{r^2}$
q-charge of scalar field

EOM for scalar field in RN space-time

$$\Big(\frac{1}{f}\partial_t^2 - \Delta + (1-f)\partial_r^2 + \frac{2MG}{r^2}\partial_r + 2iqQ\frac{1}{rf}\partial_t - \frac{q^2Q^2}{r^2 f}\Big)\phi$$
$$+\frac{aqQ}{r^3}\Big((\frac{MG}{r} - \frac{GQ^2}{r^2})\partial_\varphi + rf\partial_r\partial_\varphi\Big)\phi = 0$$

where $a = \theta^{t\varphi}$

Assuming ansatz $\phi_{lm}(t,r,\theta,\varphi) = R_{lm}(r)e^{-i\omega t}Y_l^m(\theta,\varphi)$ we got equation for radial part follows

$$fR''_{lm} + \frac{2}{r}(1-\frac{MG}{r})R'_{lm} - \Big(\frac{l(l+1)}{r^2} - \frac{1}{f}(\omega - \frac{qQ}{r})^2\Big)R_{lm}$$
$$-ima\frac{qQ}{r^3}\Big((\frac{MG}{r} - \frac{GQ^2}{r^2})R_{lm} + rfR'_{lm}\Big) = 0$$

QNM

-Special solutions of this equation

-Damped oscillations of a perturbed black hole (Im part of ω important for dumping)

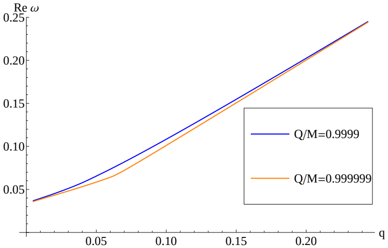
A set of the boudary condition which leads to this solution is the following: at the horizon the QNMs are purely incoming, while in the infinity the QNMs are purely outgoing

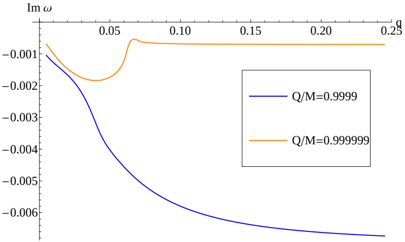
-We work in near extremal approximation: $r_+ \sim r_-$, that is $M \sim Q$

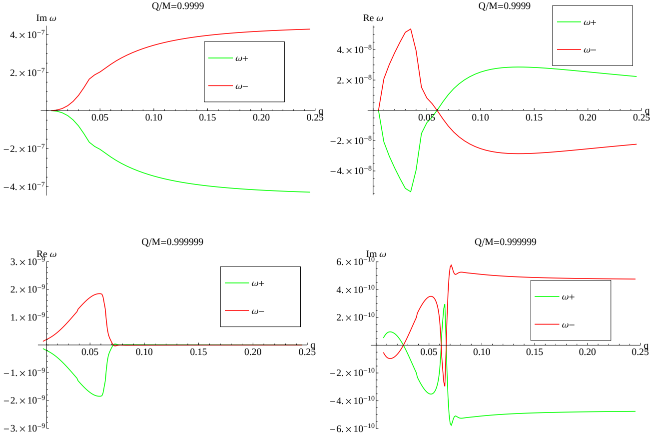
In that case we obtain condition from which the ω follow

$$\frac{\Gamma(1-2i\sigma)\Gamma(-2i\sigma-2\tilde{\rho})}{\Gamma(\frac{1}{2}-i\sigma-ik-\tilde{\rho})\Gamma(\frac{1}{2}-i\sigma+ik-i\Omega-\tilde{\rho})\Gamma(\frac{1}{2}-i\sigma-i\kappa)}$$
$$=-\frac{\Gamma(1+2i\sigma)\Gamma(2i\sigma+2\tilde{\rho})\tau^{-2\tilde{\rho}}}{\Gamma(\frac{1}{2}+i\sigma-ik+\tilde{\rho})\Gamma(\frac{1}{2}+i\sigma+ik-i\Omega+\tilde{\rho})\Gamma(\frac{1}{2}+i\sigma-i\kappa)}$$
$$\times\Big(-2i\sqrt{\omega^2-\mu^2}\; r_+\tau\Big)^{-2i\sigma}$$

This condition we solved numerically using Wolfram Mathematica






Q/M=0.9999
Im ω
ω+
ω−
q
Q/M=0.9999
Re ω
ω+
ω−
q
Q/M=0.999999
Re ω
ω+
ω−
q
Q/M=0.999999
Im ω
ω+
ω−
q

Discussion

NC effects on QNM of RN BH:

-Zeeman like splitting of the QNM spectrum (ω depends on m magnetic number)

-Duality between J (rotating, commutative BH) and a (static, NC BH)[Bufalo,Tureanu 2015],[Gupta, Jurić, Samsarov 2017]

Future work:

- Back reaction of NC scalar field on the geometry, corections on horisons, entrophy...

-QNM for spinors, vectors...

Thank you!!!