

Minima of multi-Higgs potentials with triplets of $\Delta(3n^2)$ and $\Delta(6n^2)$

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Minima of multi-Higgs potentials

Multi-Higgs models: well motivated BSM scenario

Great to improve our understanding of CP violation (CPV)

- 1 Method to test for Spontaneous CP Violation (SCPV)
- 2 Method to find the vacuum expectation values (VEVs)
- 3 For each potential, find the minima and find if there is SCPV

Based on

IdMV, King, Luhn, Neder

<http://arxiv.org/abs/1603.06942>

<http://arxiv.org/abs/1704.06322>

<http://arxiv.org/abs/1706.07606>

See also:

Branco, Rebelo, Silva-Marcos

<http://arxiv.org/abs/hep-ph/0502118>

Davidson, Haber

<http://arxiv.org/abs/hep-ph/0504050>

Gunion, Haber

<http://arxiv.org/abs/hep-ph/0506227>

CP in the scalar sector

Ogreid, Osland, Rebelo

<http://arxiv.org/abs/1701.04768>

(uses Higgs basis)

Talks here by Ogreid (2HDM/3HDM), Osland (3HDM with S_3)

Aftermath of Greek Night

Greek Mo(u)rning after...



Standard form

$$V = \phi^{*a} Y_a^b \phi_b + \phi^{*a} \phi^{*c} Z_{ac}^{bd} \phi_b \phi_d$$

For n Higgs doublets $H_{i\alpha} = (h_{i,1}, h_{i,2})$, where $\alpha = 1, 2$ denotes the $SU(2)_L$ index and i goes from 1 to n

$$\phi = (\varphi_1, \varphi_2, \dots, \varphi_{2n-1}, \varphi_{2n}) = (h_{1,1}, h_{1,2}, \dots, h_{n,1}, h_{n,2})$$

Invariance under symmetries is encoded in the Y and Z tensors

For example, $SU(2)_L$ invariance, $-m^2 \sum_{\alpha} h_{\alpha} h^{*\alpha}$:

$$Y_1^1 = Y_2^2 = -m^2, Y_1^2 = Y_2^1 = 0$$

Complex conjugation

Complex conjugation raises/lower indices:

$$\begin{aligned}\phi_a &\mapsto (\phi_a)^* \equiv \phi^{*a} \\ \phi^{*a} &\mapsto (\phi^{*a})^* \equiv \phi_a\end{aligned}$$

$V = V^*$ therefore

$$(Y_b^a)^* = Y_a^b$$

and

$$(Z_{bd}^{ac})^* = Z_{ac}^{bd}$$

Basis change (V)

$$\phi_a \mapsto V_a^{a'} \phi_{a'}$$

$$\phi^{*a} \mapsto \phi^{*a'} V_{a'}^{\dagger a}$$

$$Y_a^b \mapsto V_a^{a'} Y_{a'}^{b'} V_{b'}^{\dagger b}$$

$$Z_{ac}^{bd} \mapsto V_a^{a'} V_c^{c'} Z_{a'c'}^{b'd'} V_{b'}^{\dagger b} V_{d'}^{\dagger d}$$

General CP transformation (X)

$$\begin{aligned}\phi_a &\mapsto \phi^{*a'} X_{a'}^a \\ \phi^{*a} &\mapsto X_a^{\dagger a'} \phi_{a'}\end{aligned}$$

E.g. for 3 scalars (ϕ_1, ϕ_2, ϕ_3) one may have:

$$X_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$$

Top 3

Top 3 reasons to use General CP transformation instead of Generalised/Generalized CP transformation

- 1 It is invariant... Under spelling changes (U.K./U.S.)
- 2 Gustavo Branco (CFTP) won't get upset
- 3 Better for (terrible) jokes

The General (with stars)



Clearly the best notation, as we use stars for conjugation...

Potential $\Delta(6n^2)$, $n > 3$

$$V_{\Delta(6n^2)}(\varphi) = V_0(\varphi) \equiv -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

V_0 is invariant under both CP_0 and CP with X_{23} ($i = 1, 2, 3$)

A_4 potential, CP conserving with X_{23}

$$V_{A_4}(\varphi) = V_0(\varphi) + c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ + c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right),$$

$V_{A_4}(\varphi)$ is invariant under CP with X_{23} for any c
 $(\phi_1, \phi_2, \phi_3) \mapsto X_{23}(\phi^{*1}, \phi^{*2}, \phi^{*3}) = (\phi^{*1}, \phi^{*3}, \phi^{*2})$

Potential $\Delta(27)$

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ + d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right)$$

$V_{\Delta(27)}(\varphi)$ is in general CP Violating (CPV)

Invariant under both CP_0 and CP with X_{23} only if d is real.

Generally



The general idea is that this should be general knowledge.

Basis invariants (no V s, absolutely no X s)

Basis invariants: contract all the indices
(this cancels V and X dependence)

Examples

$$Z_{ab}^{ab} \text{ and } Z_{ba}^{ab}$$

Of 24 possibilities with two Z , just two independent ones, e.g.:

$$Z_{bd}^{ab} Z_{ac}^{cd} \text{ and } Z_{cd}^{ab} Z_{ab}^{cd}$$

Many invariants are equivalent, or products of smaller invariants

From basis invariants to CPIs

Many basis invariants are CP-even (all examples so far)

Construct **CP-odd invariant** (CPI)

$$\mathcal{I} = I - I^*$$

Complex conjugation swaps upper and lower indices:
diagrams and other technique help find cases with $I \neq I^*$

These CPIs can test if a potential has explicit CPV
(without worrying about X s)

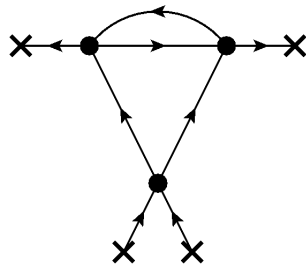
SCPIs

$$\mathcal{J} = J - J^*$$

Involves the VEVs $v_a \equiv \langle \phi_a \rangle$

These SCPIs can test if a VEV has SCPV
(in a specific CP conserving potential)

$$J^{(3,2)} \equiv Z_{a_4 a_5}^{a_1 a_2} Z_{a_2 a_6}^{a_3 a_4} Z_{a_7 a_8}^{a_5 a_6} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$

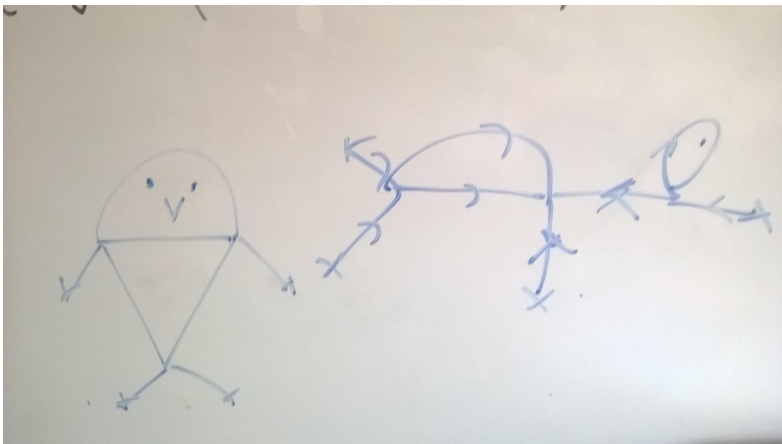


The real (well, complex) Penguin diagram!

$$J^{(3,2)} \equiv Z_{a_4 a_5}^{a_1 a_2} Z_{a_2 a_6}^{a_3 a_4} Z_{a_7 a_8}^{a_5 a_6} v_{a_1} v_{a_3} v^{*a_7} v^{*a_8} =$$



Whiteboard...



SCPI for $V(H_1, H_2)$ with CP_0

SCPV makes sense if potential is CP conserving

$$\mathcal{J}_{CP_0}^{(3,2)} = F(a_1, a_2, b, b', c_1, c_2, d, \langle h_{i,j} \rangle) [\langle h_{2,1} \rangle \langle h_{1,1} \rangle^* + \langle h_{2,2} \rangle \langle h_{1,2} \rangle^* - h.c.]$$

SCPV depends on **relative phase** between $\langle H_1 \rangle$ and $\langle H_2 \rangle$
(note $\langle h_{1,1} \rangle = \langle h_{2,1} \rangle = 0$ for charge preserving VEVs)

Potential $\Delta(27)$ (reminder)

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ + d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right)$$

$V_0(\varphi)$ has two quartics, with coefficients r and s

$V_{\Delta(27)}(\varphi)$ has CPV in general, invariant under CP_0 if d is real

SCPI for $V_{\Delta(27)}(\varphi)$ (general structure)

$$\mathcal{J}^{(3,2)} = A(d)Q(|v_i|) + B(d, s)R(v_i)$$

$A(d) = 0$ for some CP syms.,

$R(v_i) \neq 0$ for VEVs that violate those

$B(d, s) = 0$ for some other CP syms.,

$Q(|v_i|) \neq 0$ for VEVs that violate those

SCPI for $V_{\Delta(27)}(\varphi)$

$$\begin{aligned} \mathcal{J}^{(3,2)} &= \frac{1}{4}(d^{*3} - d^3)Q(|v_i|) \\ &+ \frac{1}{2}(dd^{*2} - 2d^*s^2 + d^2s)(v_2v_3v_1^{*2} + v_1v_3v_2^{*2} + v_1v_2v_3^{*2}) \\ &- \frac{1}{2}(d^2d^* - 2ds^2 + d^{*2}s)(v_2^*v_3^*v_1^2 + v_1^*v_3^*v_2^2 + v_1^*v_2^*v_3^2). \end{aligned}$$

Impose trivial CP, CP_0 : forces $\text{Arg}(d) = 0$

SCPI for $V_{\Delta(27)}(\varphi)$ with CP_0

$$\mathcal{J}_{CP_0}^{(3,2)} = \frac{1}{2}(d^3 - 2ds^2 + d^2s)$$

$$\left[(v_2 v_3 v_1^{*2} + v_1 v_3 v_2^{*2} + v_1 v_2 v_3^{*2}) - (v_2^* v_3^* v_1^2 + v_1^* v_3^* v_2^2 + v_1^* v_2^* v_3^2) \right]$$

VEVs: $\omega \equiv e^{i2\pi/3}$ is a geometrical phase in the following VEVs

$\langle \varphi \rangle = (1, \omega, \omega^2)$ conserves CP

$\langle \varphi \rangle = (\omega, 1, 1)$ violates CP with a geometrical phase:
Spontaneous Geometrical CP Violation (SGCPV)

Decreasing symmetry

Using a method of decreasing symmetry, found minima for potentials with two triplets of $\Delta(3n^2)$ and $\Delta(6n^2)$ (with $n = 2, 3$ and $n > 3$)

For one triplet of $A_4, S_4, \Delta(27), \Delta(54)$, our simple method reproduces results in I.P. Ivanov, C.C. Nishi

<https://arxiv.org/abs/1410.6139>

For $\Delta(6n^2)$ with $n > 3$ we get

$$v_1(1, 0, 0), \quad v_2(1, 1, 0), \quad v_3(1, 1, 1)$$

as follows...

Minima for one triplet of $\Delta(6n^2)$, $n > 3$

$$V_0(\varphi) \equiv -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$V_0 = V_{U(3)} + V_{\Delta(6\infty^2) \times U(1)}$$

$U(3)$ - single orbit connected by the continuous symmetry

Orbit split

$$V_{\Delta(6\infty^2)\times U(1)} = \mathbf{s}(\varphi_1\varphi^{*1}\varphi_1\varphi^{*1} + \varphi_2\varphi^{*2}\varphi_2\varphi^{*2} + \varphi_3\varphi^{*3}\varphi_3\varphi^{*3})$$

splits the single $U(3)$ orbit into 3 classes of directions

$$\left\{ \begin{pmatrix} e^{i\eta} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ e^{i\eta} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ e^{i\eta} \end{pmatrix} \right\}, \left\{ \begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ 0 \end{pmatrix}, \text{permut.} \right\}, \left\{ \begin{pmatrix} e^{i\eta} \\ e^{i\zeta} \\ e^{i\theta} \end{pmatrix}, \text{permut.} \right\}.$$

2 generation example

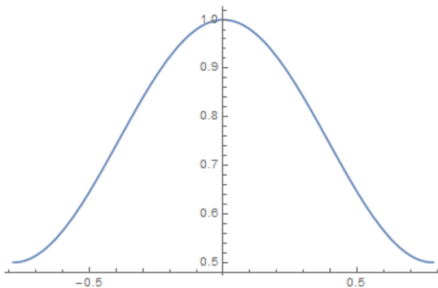
Extrema of $|\varphi_1|^4 + |\varphi_2|^4$ for fixed magnitude v :

Positive: $\propto (1, 1)/\sqrt{2} \rightarrow V \sim +2v^4/4$

Negative: $\propto (0, 1) \rightarrow V \sim -v^4$

2 generation example plot

```
Plot[Cos[θ]^4 + Sin[θ]^4, {θ, -π/4, π/4}]
```



```
Minimize[Cos[θ]^4 + Sin[θ]^4, θ]
```

```
{Cos[2 ArcTan[1 - √2]]^4 + Sin[2 ArcTan[1 - √2]]^4, {θ → -2 ArcTan[1 - √2]}}
```

```
FullSimplify[Pi/4 == -2 ArcTan[1 - √2]]
```

```
True
```

Minima for two triplets of $\Delta(3n^2)$, $n > 3$

For two triplets, method reveals some candidate VEVs like

$$(e^{i\eta}, 0, 0), (e^{i\eta'}, 0, 0) \rightarrow (1, 0, 0), (1, 0, 0)$$

$$(e^{i\eta}, e^{i\zeta}, e^{i\theta}), (e^{i\eta'}, e^{i\zeta'}, e^{i\theta'}) \rightarrow (1, 1, 1), (1, e^{i\zeta'}, e^{i\theta'}).$$

For last case, phases are physical.

Minimize phase-dependent part of potential to fix the phases.

Minima with geometrical phases

Found new VEVs with geometrical phases

$$(1, 1, 1), (1, \omega, \omega^2) \text{ and } (1, 1, 1), (1, \omega^2, \omega).$$

Do they SCPV?

Use SCPI to test for SCPV

$$\begin{aligned}
 \mathcal{J}^{(3,2)} = & -\frac{1}{16} \tilde{s}_2 [\tilde{r}_2 (-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2 + 3\tilde{r}_2) - \tilde{s}_3^2] W_{CP_0} \\
 & - \frac{1}{8} i \tilde{s}_3 [\tilde{s}_3^2 - 3\tilde{r}_2^2] W_{CP_{23}} \\
 & - \frac{1}{16} i \tilde{s}_2 \tilde{s}_3 [-4s - 4s' + 2\tilde{s}_1 - \tilde{s}_2] (\dots)
 \end{aligned}$$

where

$$W_{CP_0} \equiv [(v_1 v_1'^* v_2' v_2^* + v_2 v_2'^* v_3' v_3^* + v_3 v_3'^* v_1' v_1^*) - h.c.]$$

and CP_0 forces $\tilde{s}_3 = 0$

New SGCPV

$$W_{CP_0} \equiv [(v_1 v_1'^* v_2' v_2^* + v_2 v_2'^* v_3' v_3^* + v_3 v_3'^* v_1' v_1^*) - h.c.]$$

$$W_{CP_0}^{(3,2)}[v(1, 1, 1), v'(1, \omega, \omega^2)] = 3(\omega - \omega^2)v^2 v'^2 \neq 0$$

Found new cases of SGCPV in $\Delta(3n^2) \times CP_0$, for 6HDM

Summary

- Developed formalism for CPIs and SCPIs
- Methods for explicit and spontaneous CP violation are valid for any potential when brought to standard form
- Verified the CP properties of 3HDM and 6HDM symmetric under $\Delta(3n^2)$ and $\Delta(6n^2)$ groups, for new minima and found new cases with SGCPV

Potential 2HDM

$$\begin{aligned} V(H_1, H_2) = & m_1^2 H_1^\dagger H_1 + m_{12}^2 e^{i\theta_0} H_1^\dagger H_2 + m_{12}^2 e^{-i\theta_0} H_2^\dagger H_1 + m_2^2 H_2^\dagger H_2 + \\ & + a_1 \left(H_1^\dagger H_1 \right)^2 + a_2 \left(H_2^\dagger H_2 \right)^2 \\ & + b \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + b' \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) + \\ & + c_1 e^{i\theta_1} \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_1 \right) + c_1 e^{-i\theta_1} \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 \right) + \\ & + c_2 e^{i\theta_2} \left(H_2^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) + c_2 e^{-i\theta_2} \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 \right) + \\ & + d e^{i\theta_3} \left(H_1^\dagger H_2 \right)^2 + d e^{-i\theta_3} \left(H_2^\dagger H_1 \right)^2 . \end{aligned}$$

Potentials $\Delta(3n^2)$, A_4 , $\Delta(27)$

$$V_0(\varphi) = V_{\Delta(3n^2)}(\varphi) = -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$$V_{A_4}(\varphi) = V_0(\varphi) + c \left(\varphi_1 \varphi_1 \varphi^{*3} \varphi^{*3} + \varphi_2 \varphi_2 \varphi^{*1} \varphi^{*1} + \varphi_3 \varphi_3 \varphi^{*2} \varphi^{*2} \right) \\ + c^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_3 + \varphi^{*2} \varphi^{*2} \varphi_1 \varphi_1 + \varphi^{*3} \varphi^{*3} \varphi_2 \varphi_2 \right) ,$$

$$V_{\Delta(27)}(\varphi) = V_0(\varphi) + d \left(\varphi_1 \varphi_1 \varphi^{*2} \varphi^{*3} + \text{cycl.} \right) \\ + d^* \left(\varphi^{*1} \varphi^{*1} \varphi_3 \varphi_2 + \text{cycl.} \right)$$

Potential $\Delta(3n^2)$

$$V_{\Delta(3n^2)}(\varphi) = -m_\varphi^2 \sum_i \varphi_i \varphi^{*i} + r \left(\sum_i \varphi_i \varphi^{*i} \right)^2 + s \sum_i (\varphi_i \varphi^{*i})^2$$

$V_{\Delta(3n^2)}(\varphi)$ is invariant under both CP_0 and CP with X_{23}

Potential $\Delta(27)$, doublets

$$\begin{aligned}
 V_0(H) = & -m_h^2 \sum_{i,\alpha} h_{i\alpha} h^{*i\alpha} + \sum_{i,j,\alpha,\beta} \left[r_1 (h_{i\alpha} h^{*i\alpha}) (h_{j\beta} h^{*j\beta}) + r_2 (h_{i\alpha} h^{*i\beta}) (h_{j\beta} h^{*j\alpha}) \right] \\
 & + s \sum_{i,\alpha,\beta} (h_{i\alpha} h^{*i\alpha}) (h_{i\beta} h^{*i\beta})
 \end{aligned}$$

$$V_{\Delta(27)}(H) = V_0(H) + \sum_{\alpha,\beta} \left[d \left(h_{1\alpha} h_{1\beta} h^{*2\alpha} h^{*3\beta} + \text{cycl.} \right) + \text{h.c.} \right]$$

Potential $\Delta(3n^2)$, $n > 3$ with two triplets

$$\begin{aligned}
 V_1(\varphi, \varphi') = & + \tilde{r}_1 \left(\sum_i \varphi_i \varphi^{*i} \right) \left(\sum_j \varphi'_j \varphi'^{*j} \right) + \tilde{r}_2 \left(\sum_i \varphi_i \varphi'^{*i} \right) \left(\sum_j \varphi'_j \varphi^{*j} \right) \\
 & + \tilde{s}_1 \sum_i \left(\varphi_i \varphi^{*i} \varphi'_i \varphi'^{*i} \right) \\
 & + \tilde{s}_2 \left(\varphi_1 \varphi^{*1} \varphi'_2 \varphi'^{*2} + \varphi_2 \varphi^{*2} \varphi'_3 \varphi'^{*3} + \varphi_3 \varphi^{*3} \varphi'_1 \varphi'^{*1} \right) \\
 & + i \tilde{s}_3 \left[(\varphi_1 \varphi'^{*1} \varphi'_2 \varphi^{*2} + \text{cycl.}) - (\varphi^{*1} \varphi'_1 \varphi'^{*2} \varphi_2 + \text{cycl.}) \right].
 \end{aligned}$$

$$V_{\Delta(3n^2)}(\varphi, \varphi') = V_0(\varphi) + V'_0(\varphi') + V_1(\varphi, \varphi'),$$