Is dark matter really different from modified gravity?¹

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¹based on X. Calmet, I. K. [arXiv:1702.03832]

Classification of theories

2 Dark matter

• Fields are dummy variables

$${\cal A}=\int {\cal D}\phi {
m e}^{i{\cal S}[\phi]}$$

- Physics doesn't depend on the parametrization
- Classification?

- ullet Mathematically a theory is parametrized by $\mathcal{T}=(\phi^1_{lpha_1},\ldots,\phi^n_{lpha_n},S)$
- \bullet $\mathcal{T}_1 \sim \mathcal{T}_2$ iff there is a frame (choice of variables) where
 - 1) they have the same #(degrees of freedom)
 - 2) and the same action
- The theories might look different at first sight
- The set of theories is divided into equivalence classes
- Algorithm
 - Identify all degrees of freedom
 - Verify how they couple to each other

• Degrees of freedom can be found by linearizing the EOM $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$

$$\mathcal{D}_{lphaeta\mu
u} \mathbf{h}^{\mu
u} = \mathcal{T}_{lphaeta} \implies \mathcal{P}_{lphaeta\mu
u} = \mathcal{D}_{lphaeta\mu
u}^{-1}$$

• The position of the poles reveals the degrees of freedom

- The dynamics can be compared in two ways:
 - Lagrangian level: suitable transformations that map one theory to the other (e.g. field redefinitions)
 - EOM level: checking if EOMs can be matched

• Example: f(R)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} f(R) + \mathcal{L}_M \right)$$

ullet Conformal transformation $ilde{g}_{\mu
u}=f'(R)g_{\mu
u}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left(rac{1}{16\pi G} \tilde{R} - rac{1}{2} \tilde{g}^{\mu
u} \partial_{\mu}\phi \partial_{\nu}\phi - V(\phi)
ight)
onumber \ + \int d^4x \sqrt{-\tilde{g}} F^{-2}(\phi) \mathcal{L}_M(F^{-1}(\phi) \tilde{g}_{\mu
u}, \psi_M),$$

where

$$\phi \equiv \sqrt{\frac{3}{16\pi G}} \log F,$$
$$F(\phi) \equiv f'(R(\phi)).$$

- More generally, $f(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})$ is equivalent to GR plus a scalar χ and a non-physical massive spin-2 field $\tilde{\pi}_{\mu\nu}$
- Generalized Pauli-Fierz conditions

$$\begin{split} \tilde{\nabla}^{\mu}\left(\tilde{\pi}_{\mu\nu}-g_{\mu\nu}\tilde{\pi}\right)&=0,\\ \tilde{\pi}-{m_2}^{-2}\left[\left(\tilde{\nabla}\chi\right)^2+2{m_0}^2\left(1-e^{-\chi}\right)^2\right]&=0. \end{split}$$

- Does that mean that *ANY* physical theory is equivalent to GR? NO!
- But conservative theories are
- This doesn't cover theories that:
 - has different symmetries than GR
 - has different spacetime structure

Classification of theories

2 Dark matter

- One-to-one map between modified gravity and modifications in the matter sector
- Is it possible to differentiate between modified gravity and particle dark matter?
- Different interpretations of the same phenomenon

Classification of theories

2 Dark matter

- We proposed a classification framework for the gravitational dynamics
- Assuming Diff invariance, we showed modified gravity = GR + new fields
- Corollary: particle dark matter = modified gravity dark matter
- Other applications might include GWs, dark energy and so on
- Note: this doesn't explain dark matter, just show the equivalence between the two approaches