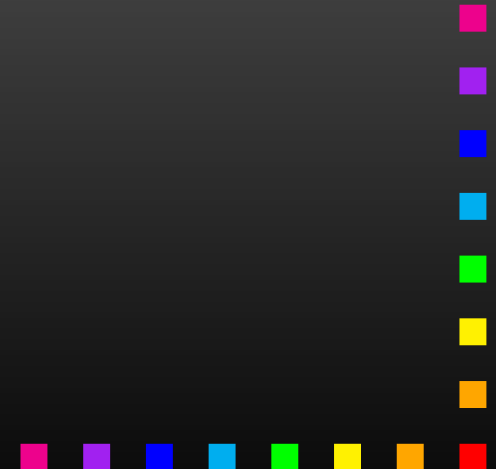


FormCalc and FeynArts

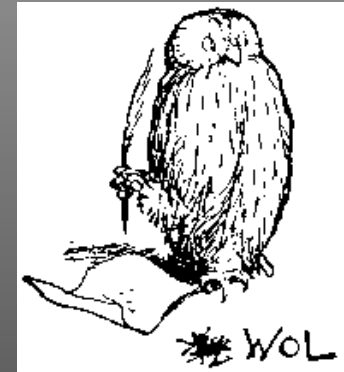
Thomas Hahn

Max-Planck-Institut für Physik
München



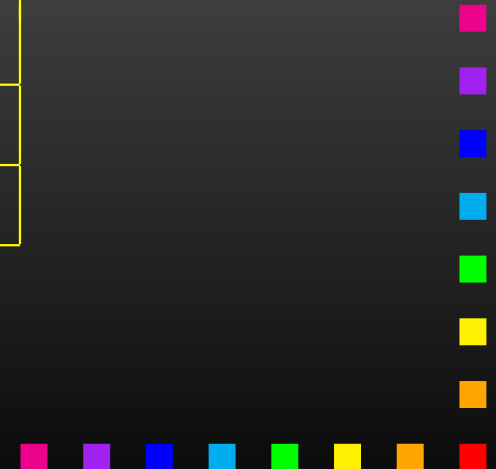
Contradiction

An Introduction is to introduce people, but FeynArts and Friends have already been introduced to you. So this is the opposite. When we asked Pooh what the opposite of an Introduction was, he said “The what of a what?” which didn’t help us as much as we had hoped, but luckily Owl kept his head and told us that the Opposite of an Introduction, my dear Pooh, was a Contradiction.



... actually, a Distinction:

FeynArts/FormCalc	FeynHiggs
Generic answer to an arbitrary question	Concrete answer to a specific question
Paint & brush	Finished painting
Generator generator	Generator



One-loop since mid-1990s

Automated NLO computations is an industry today, with many packages becoming available in the last decade:

- GoSam, MadGraph5_aMC@NLO, OpenLoops, HELAC-NLO, BlackHat, Rocket, ...

Here: **FeynArts (1991) + FormCalc (1995)**

FormCalc was doing largely the same as FeynCalc (1992) but used FORM for the time-consuming tasks, hence the name FormCalc.

- Feynman-diagrammatic method,
- Analytic calculation as far as possible ('any' model),
- Generation of code for the numerical evaluation of the squared matrix element.

FeynArts + FormCalc also used as 'engine' in SARAH, SloopS.



Package Types

‘Production’



MG5_aMC@NLO
GoSam
OpenLoops

‘Exploration’



FormCalc
FeynCalc
Package-X

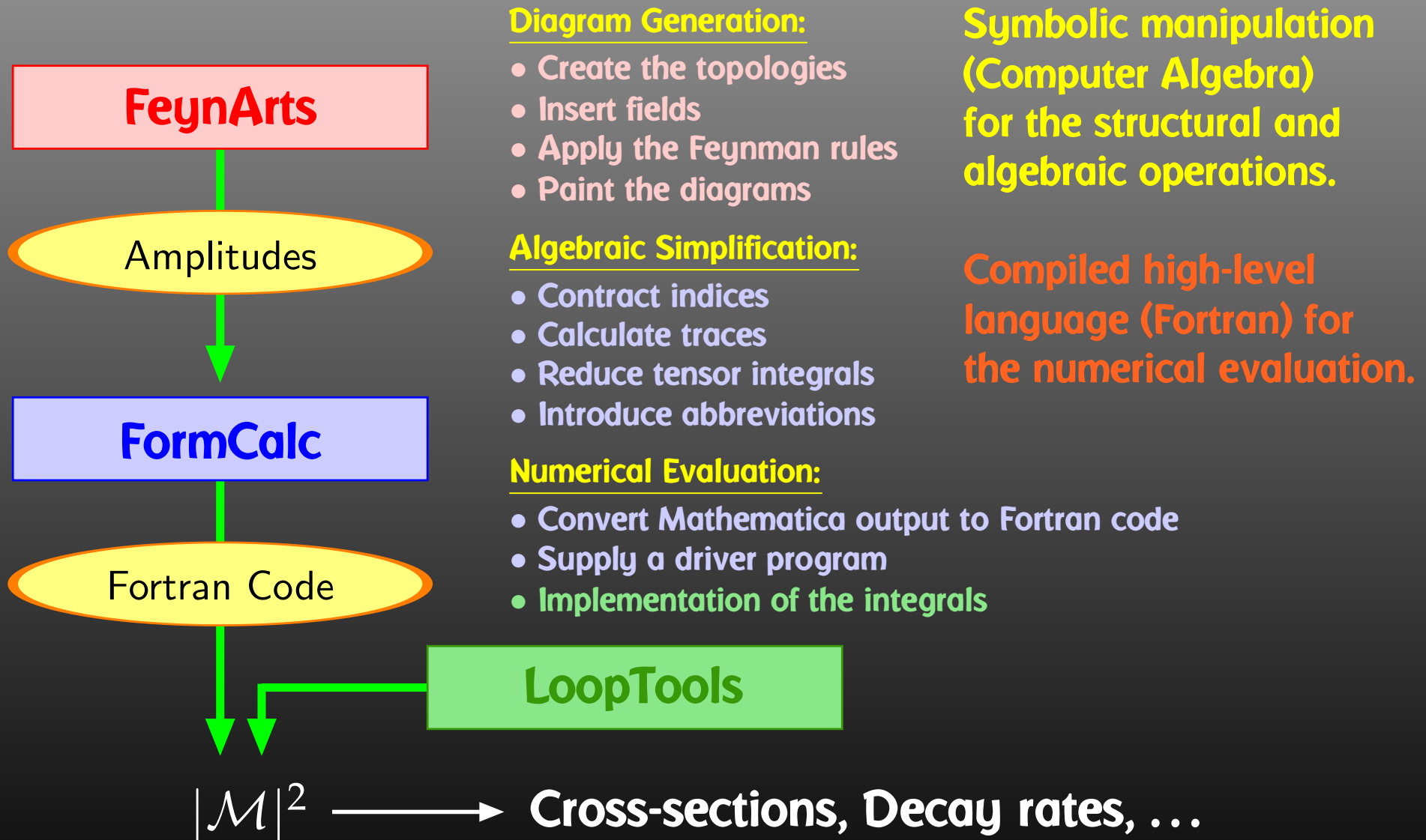
‘Specific’



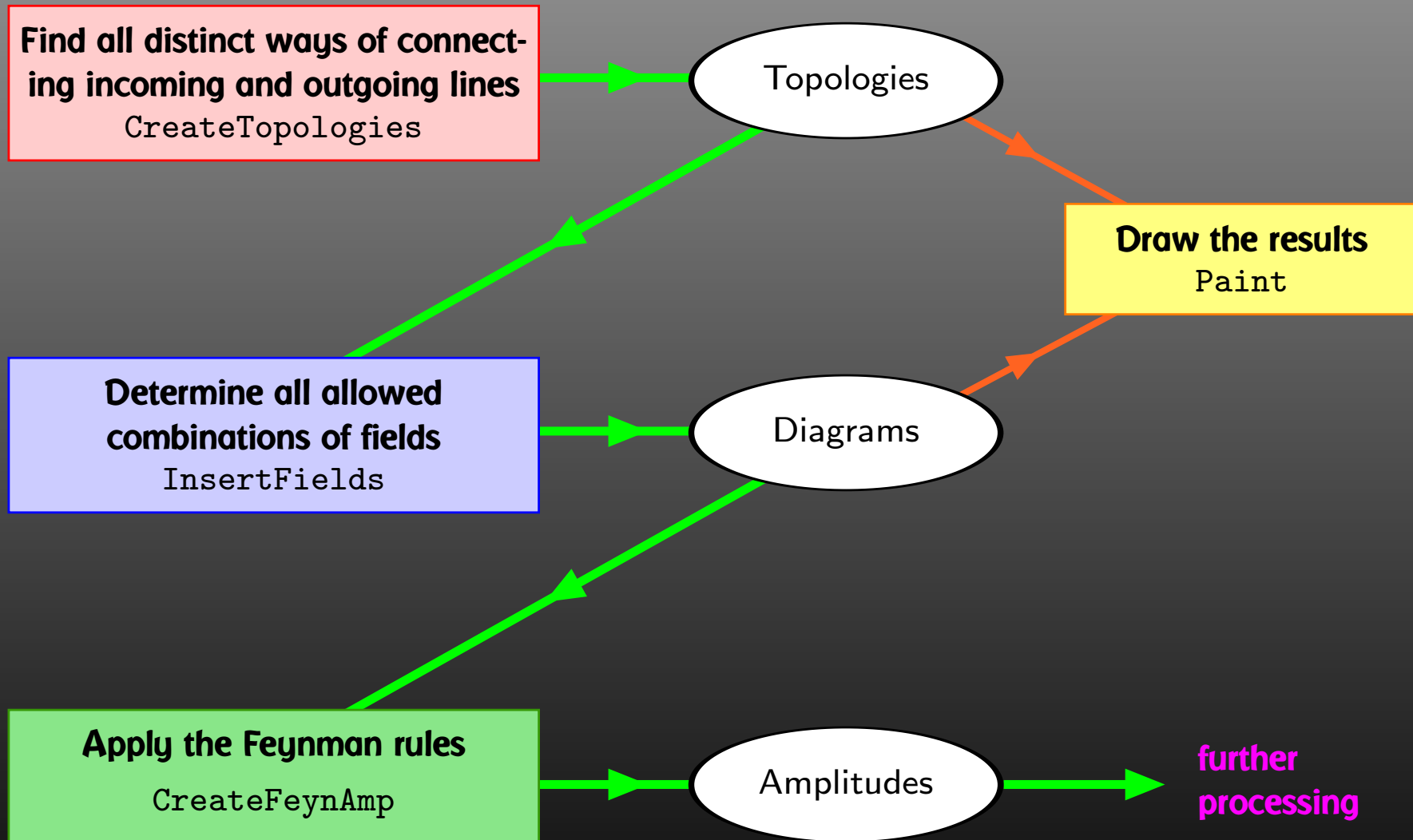
FeynHiggs
DarkSUSY
Prospino



Diagram Evaluation with FeynArts and FormCalc



FeynArts



Three Levels of Fields

Generic level, e.g. F, F, S

$$C(F_1, F_2, S) = G_L \mathbb{P}_L + G_R \mathbb{P}_R \quad \mathbb{P}_{R,L} = (\mathbb{1} \pm \gamma_5)/2$$

Kinematical structure completely fixed, most algebraic simplifications (e.g. tensor reduction) can be carried out.

Classes level, e.g. $-F[2], F[1], S[3]$

$$\bar{\ell}_i \nu_j G : \quad G_L = -\frac{i e m_{\ell,i}}{\sqrt{2} \sin \theta_w M_W} \delta_{ij}, \quad G_R = 0$$

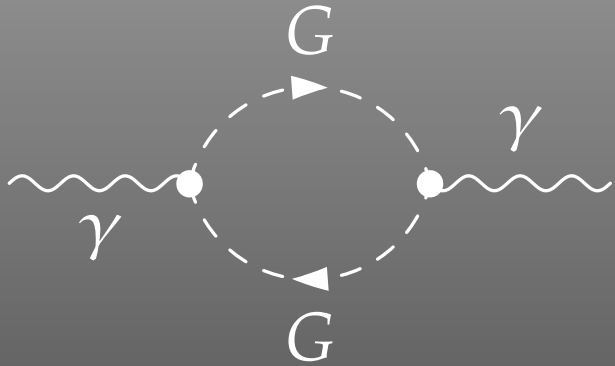
Coupling fixed except for i, j (can be summed in do-loop).

Particles level, e.g. $-F[2, \{1\}], F[1, \{1\}], S[3]$

insert fermion generation (1, 2, 3) for i and j

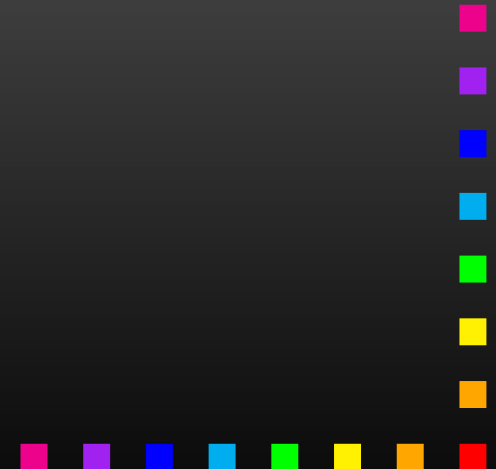


Sample CreateFeynAmp output

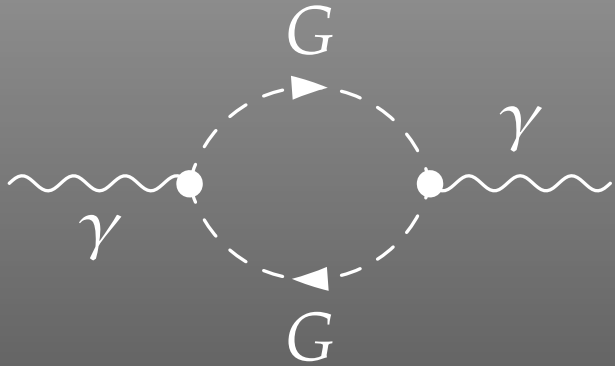


= FeynAmp[identifier,
loop momenta,
generic amplitude,
insertions]

GraphID[Topology == 1, Generic == 1]

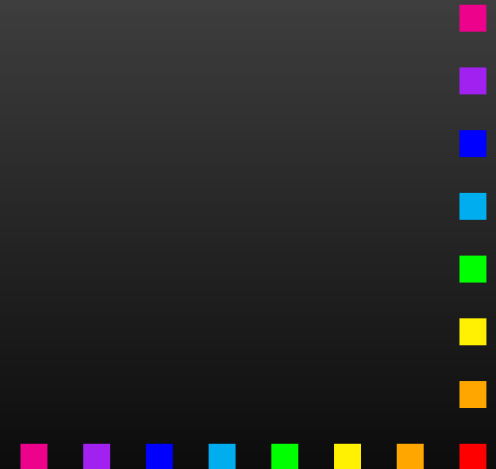


Sample CreateFeynAmp output

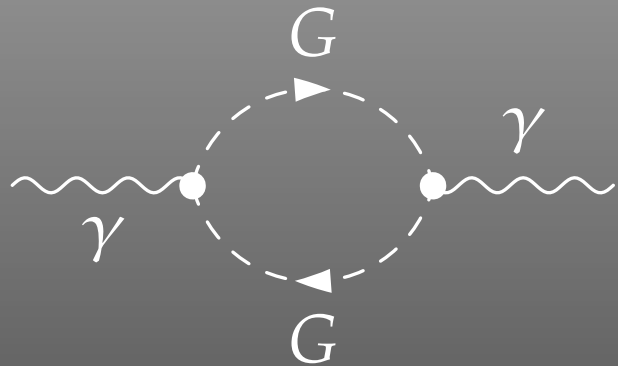


```
= FeynAmp[ identifier ,  
            loop momenta ,  
            generic amplitude ,  
            insertions ]
```

Integral[q1]



Sample CreateFeynAmp output



= FeynAmp [*identifier* ,
loop momenta ,
generic amplitude ,
insertions]

$\frac{1}{32 \text{ Pi}^4}$ RelativeCFprefactor

FeynAmpDenominator [$\frac{1}{q1^2 - \text{Mass}[S[\text{Gen3}]]^2}$,
 $\frac{1}{(-p1 + q1)^2 - \text{Mass}[S[\text{Gen4}]]^2}$] loop denominators

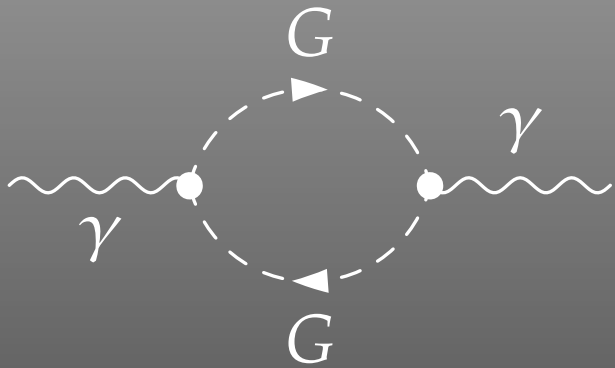
$(p1 - 2q1)[\text{Lor1}] (-p1 + 2q1)[\text{Lor2}]$ kin. coupling structure

$\text{ep}[V[1], p1, \text{Lor1}] \text{ep}^*[V[1], k1, \text{Lor2}]$ polarization vectors

$G_{SSV}^{(0)}[(\text{Mom}[1] - \text{Mom}[2])[\text{KI1}[3]]]$
 $G_{SSV}^{(0)}[(\text{Mom}[1] - \text{Mom}[2])[\text{KI1}[3]]]$, coupling constants

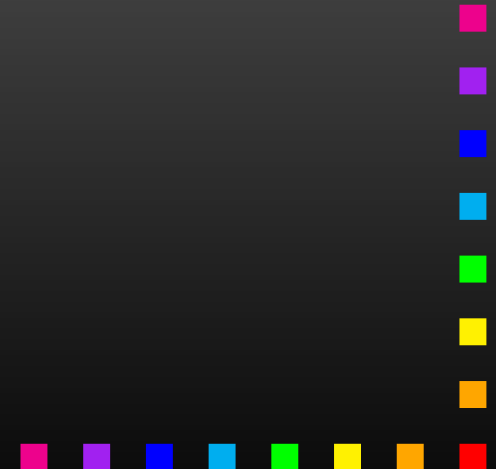


Sample CreateFeynAmp output



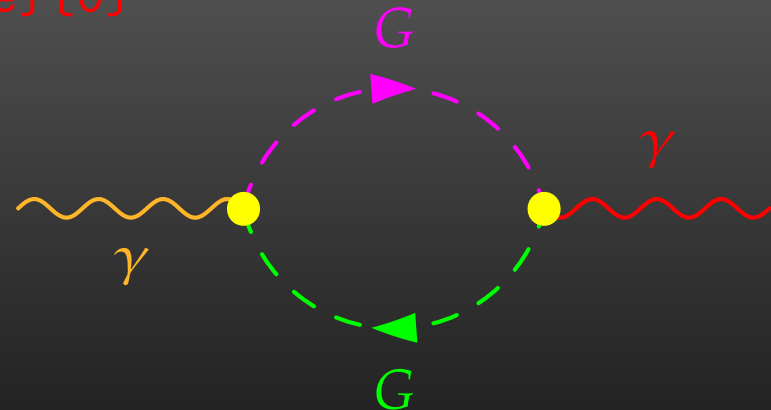
= FeynAmp[*identifier*,
loop momenta,
generic amplitude,
insertions]

```
{ Mass[S[Gen3]],
  Mass[S[Gen4]],
  GSSV(0)[(Mom[1] - Mom[2])[KI1[3]]],
  GSSV(0)[(Mom[1] - Mom[2])[KI1[3]]],
  RelativeCF } ->
Insertions[Classes][{MW, MW, I EL, -I EL, 2}]
```

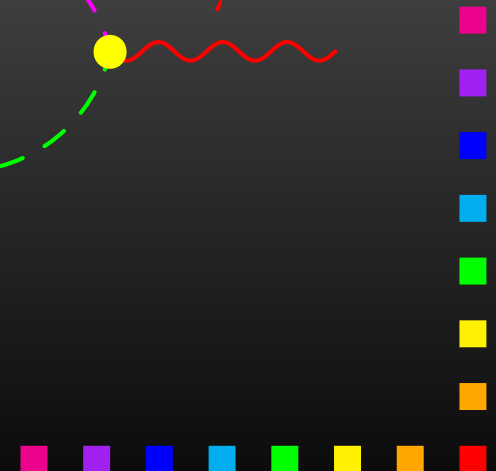


Sample Paint output

```
\begin{feynartspicture}(150,150)(1,1)
\FADiagram{}
\FAProp(6.,10.)(14.,10.)(0.8,){ScalarDash}{-1}
\FALabel(10.,5.73)[t]{G}
\FAProp(6.,10.)(14.,10.)(-0.8,){ScalarDash}{1}
\FALabel(10.,14.27)[b]{G}
\FAProp(0.,10.)(6.,10.)(0.,){Sine}{0}
\FALabel(3.,8.93)[t]{\gamma}
\FAProp(20.,10.)(14.,10.)(0.,){Sine}{0}
\FALabel(17.,11.07)[b]{\gamma}
\FAVert(6.,10.){0}
\FAVert(14.,10.){0}
\end{feynartspicture}
```



Technically: uses its own PostScript prologue.



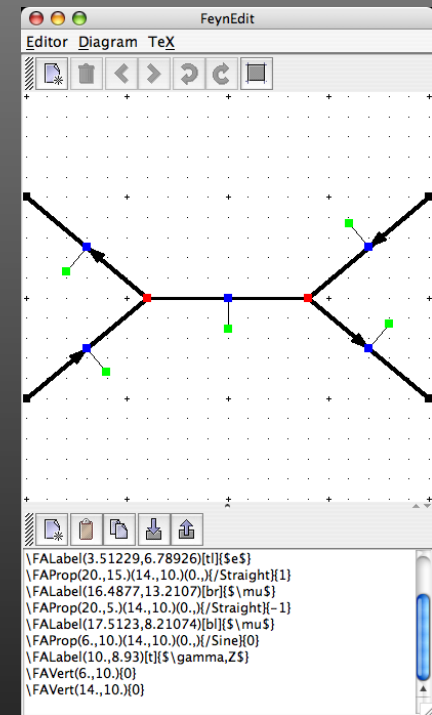
Editing Feynman Diagrams

The elements of the diagram are easy to recognize and it is straightforward to **make changes e.g. to the label text** using any text editor.

It is less straightforward, however, to alter the geometry of the diagram, i.e. to **move vertices and propagators**.

The **FeynEdit** tool lets the user:

- copy-and-paste the \LaTeX code into the lower panel of the editor,
- visualize the diagram,
- modify it using the mouse, and finally
- copy-and-paste it back into the text.



Algebraic Simplification

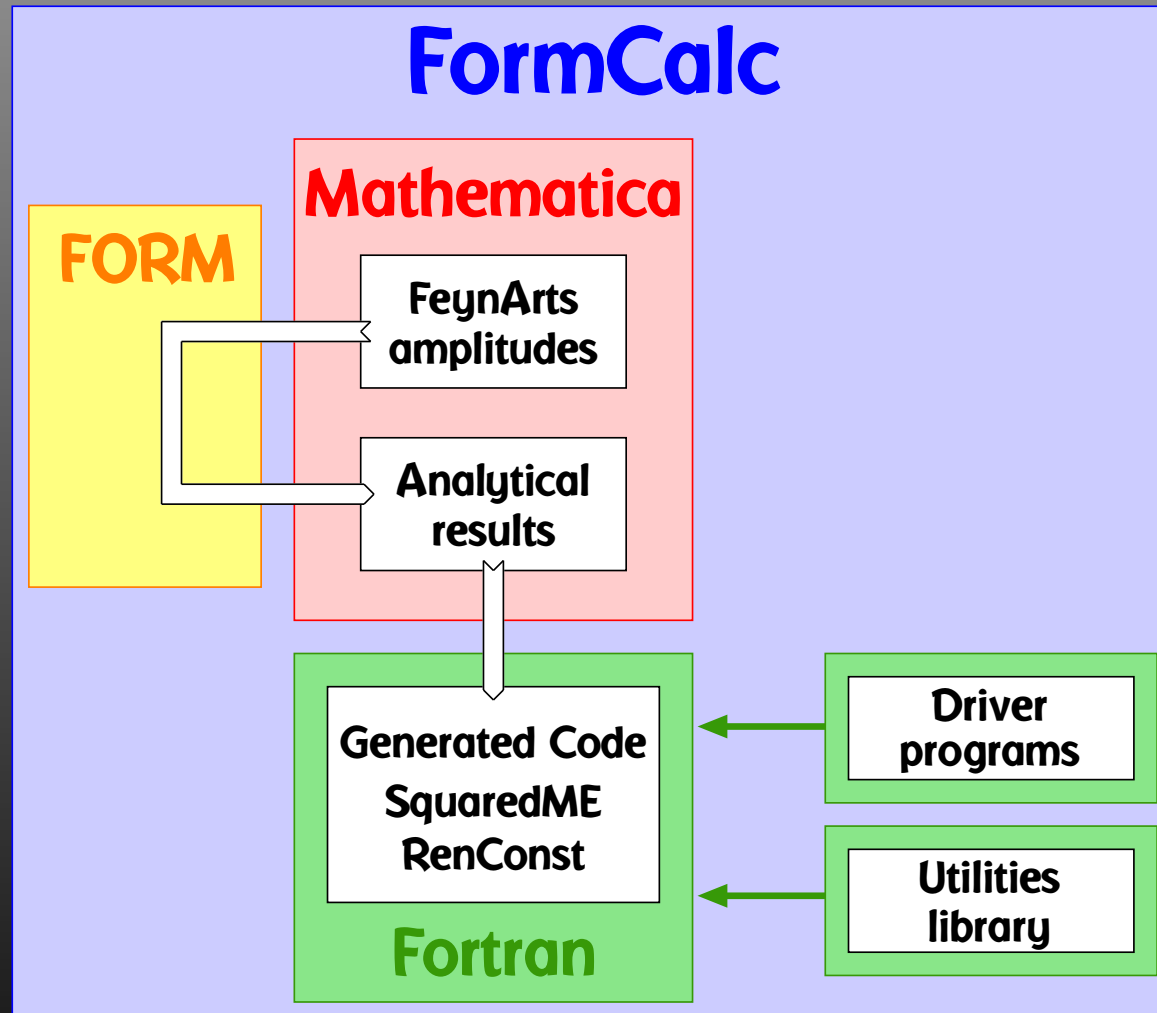
The amplitudes of `CreateFeynAmp` are in **no good shape for direct numerical evaluation.**

A number of steps have to be done analytically:

- **contract indices as far as possible,**
- **evaluate fermion traces,**
- **perform the tensor reduction / separate numerators,**
- **add local terms arising from D ·(divergent integral),**
- **simplify open fermion chains,**
- **simplify and compute the square of $SU(N)$ structures,**
- **“compactify” the results as much as possible.**



FormCalc Internals



FormCalc Output

A typical term in the output looks like

$$\begin{aligned} & C0i[cc12, MW2, MW2, S, MW2, MZ2, MW2] * \\ & (-4 \text{ Alfa2 } MW2 \text{ CW2/SW2 } S \text{ AbbSum16 } + \\ & 32 \text{ Alfa2 } \text{ CW2/SW2 } S^2 \text{ AbbSum28 } + \\ & 4 \text{ Alfa2 } \text{ CW2/SW2 } S^2 \text{ AbbSum30 } - \\ & 8 \text{ Alfa2 } \text{ CW2/SW2 } S^2 \text{ AbbSum7 } + \\ & \text{ Alfa2 } \text{ CW2/SW2 } S (T-U) \text{ Abb1 } + \\ & 8 \text{ Alfa2 } \text{ CW2/SW2 } S (T-U) \text{ AbbSum29 }) \end{aligned}$$

 = loop integral

 = kinematical variables

 = constants

 = automatically introduced abbreviations



Abbreviations

Outright factorization is usually out of question.
Abbreviations are necessary to reduce size of expressions.

$$\text{AbbSum29} = \text{Abb2} + \text{Abb22} + \text{Abb23} + \text{Abb3}$$

$$\text{Abb22} = \text{Pair1} \text{Pair3} \text{Pair6}$$

$$\text{Pair3} = \text{Pair}[e[3], k[1]]$$

The full expression corresponding to **AbbSum29** is

$$\begin{aligned} & \text{Pair}[e[1], e[2]] \text{Pair}[e[3], k[1]] \text{Pair}[e[4], k[1]] + \\ & \text{Pair}[e[1], e[2]] \text{Pair}[e[3], k[2]] \text{Pair}[e[4], k[1]] + \\ & \text{Pair}[e[1], e[2]] \text{Pair}[e[3], k[1]] \text{Pair}[e[4], k[2]] + \\ & \text{Pair}[e[1], e[2]] \text{Pair}[e[3], k[2]] \text{Pair}[e[4], k[2]] \end{aligned}$$

Categories of Abbreviations

- Abbreviations are **recursively defined** in several levels.
- When generating code, FormCalc introduces another set of abbreviations for the **loop integrals**.

In general, the **abbreviations are thus costly in CPU time**. It is key to a decent performance that the abbreviations are separated into different **Categories**:

- **Abbreviations that depend on the helicities,**
- **Abbreviations that depend on angular variables,**
- **Abbreviations that depend only on \sqrt{s} .**

Correct execution of the categories guarantees that **almost no redundant evaluations** are made and makes the generated code essentially as fast as hand-tuned code.



External Fermion Lines

An amplitude containing **external fermions** has the form

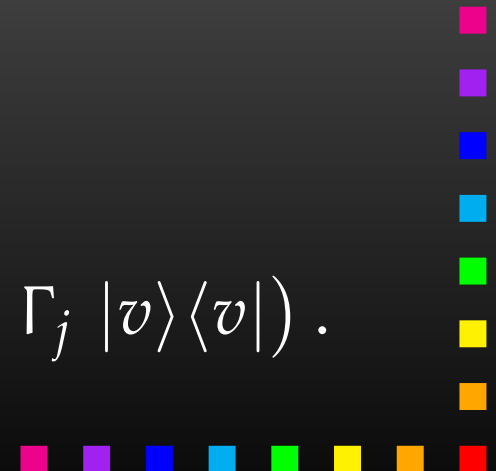
$$\mathcal{M} = \sum_{i=1}^{n_F} c_i F_i \quad \text{where} \quad F_i = \text{(Product of)} \langle u | \Gamma_i | v \rangle .$$

n_F = number of fermionic structures.

Textbook procedure: **Trace Technique**

$$|\mathcal{M}|^2 = \sum_{i,j=1}^{n_F} c_i^* c_j F_i^* F_j$$

where $F_i^* F_j = \langle v | \bar{\Gamma}_i | u \rangle \langle u | \Gamma_j | v \rangle = \text{Tr}(\bar{\Gamma}_i | u \rangle \langle u | \Gamma_j | v \rangle \langle v |)$.



Problems with the Trace Technique

PRO: Trace technique is independent of any representation.

CON: For n_F F_i 's there are n_F^2 $F_i^* F_j$'s.

Things get worse the more vectors are in the game:
multi-particle final states, polarization effects . . .

Essentially $n_F \sim (\# \text{ of vectors})!$ because all
combinations of vectors can appear in the Γ_i .

Solution: Use Weyl-van der Waerden spinor formalism to
compute the F_i 's directly.



Fermion Chains

FormCalc uses Dirac (4-component) spinors in most of the algebra (extension to D dim more obvious).

Move to 2-comp. Weyl spinors for the numerical evaluation:

$$\langle u|_4 \equiv (\langle u_+|_2, \langle u_-|_2), \quad |v\rangle_4 \equiv \begin{pmatrix} |v_- \rangle_2 \\ |v_+ \rangle_2 \end{pmatrix}.$$

Every **chiral Dirac chain** maps onto a single Weyl chain:

$$\langle u| \mathbb{P}_L \gamma_\mu \gamma_\nu \cdots |v\rangle_4 = \langle u_-| \bar{\sigma}_\mu \sigma_\nu \cdots |v_\pm\rangle_2,$$

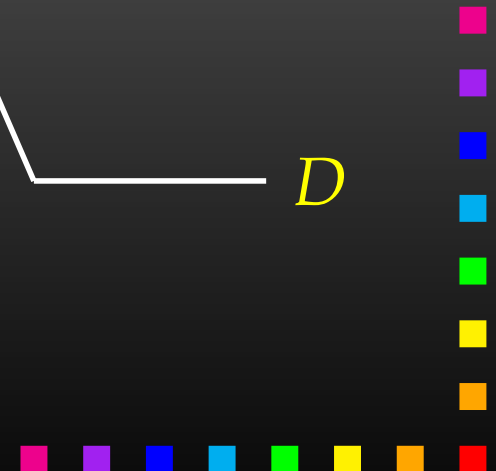
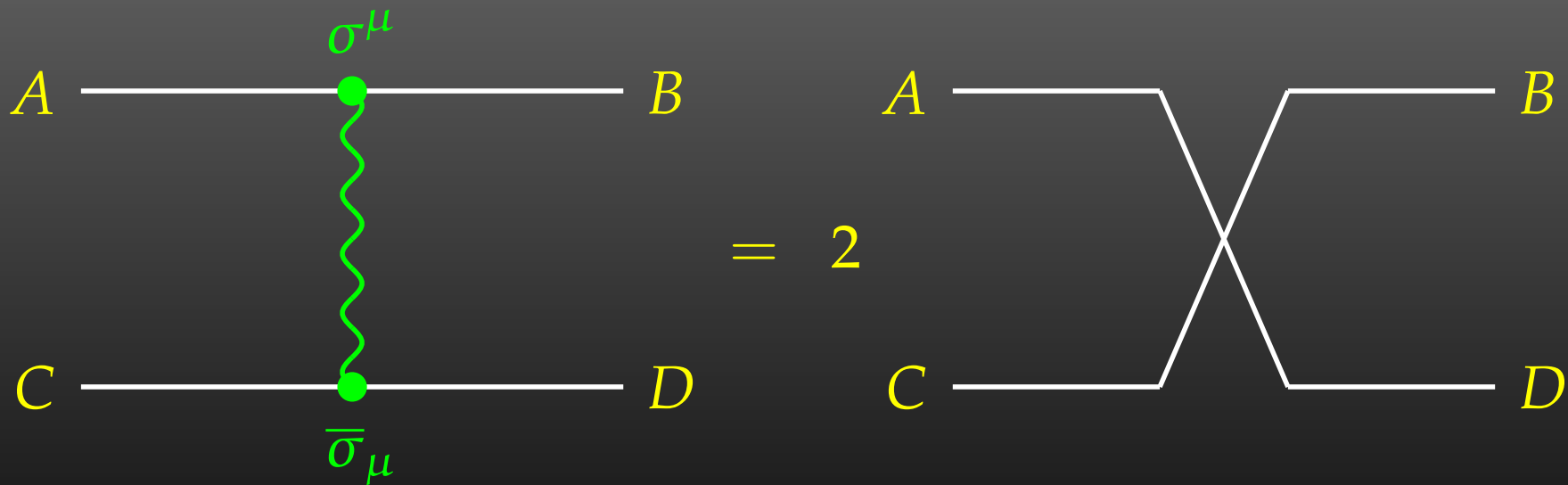
$$\langle u| \mathbb{P}_R \gamma_\mu \gamma_\nu \cdots |v\rangle_4 = \langle u_+| \sigma_\mu \bar{\sigma}_\nu \cdots |v_\mp\rangle_2.$$

FORM-like notation: $\langle u| \sigma_\mu \bar{\sigma}_\nu \sigma_\rho |v\rangle k_1^\mu \varepsilon_2^\nu k_3^\rho \equiv \langle u| k_1 \bar{\varepsilon}_2 k_3 |v\rangle.$

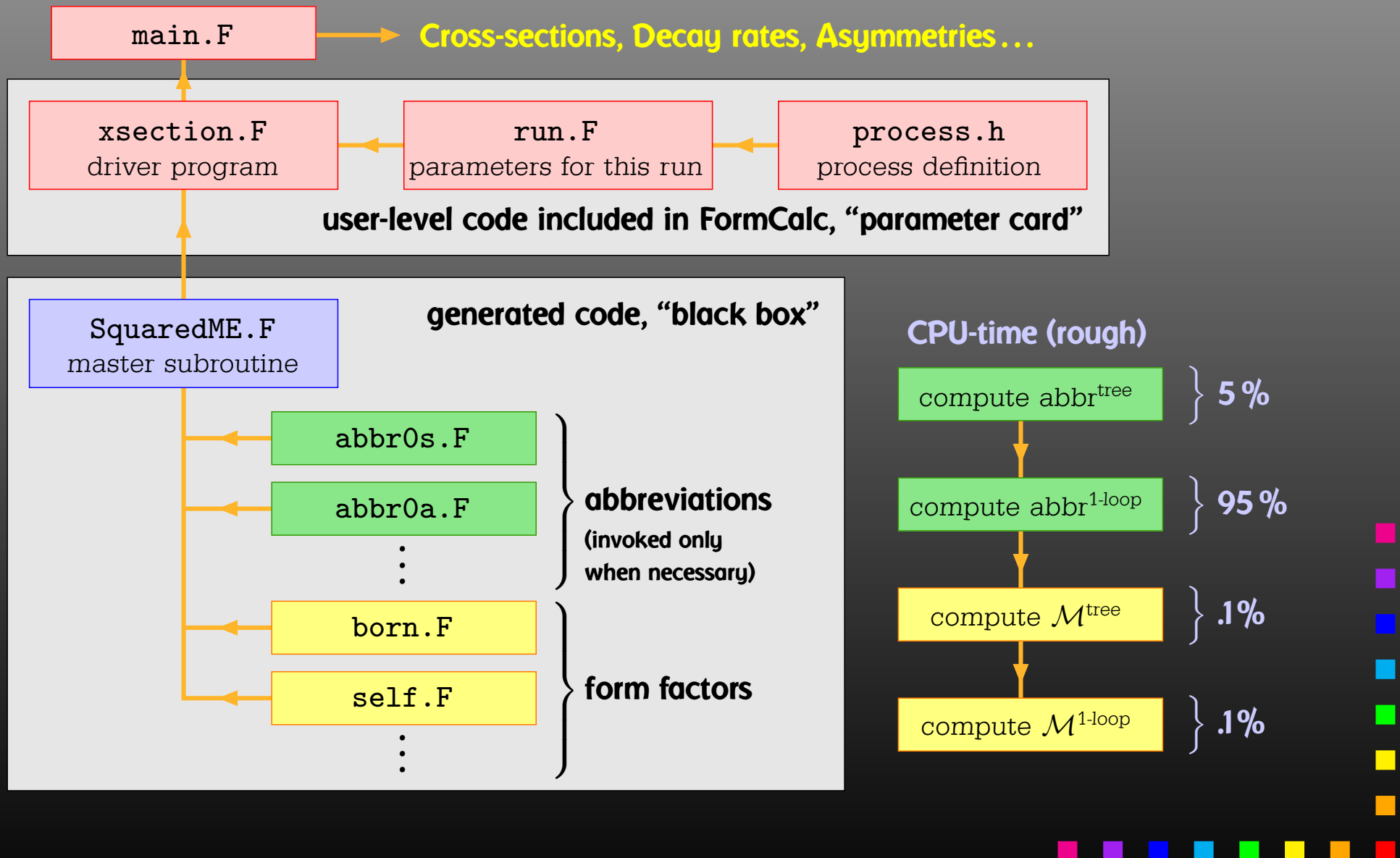
Fierz Identities

With the Fierz identities for sigma matrices it is possible to **remove all Lorentz contractions** between sigma chains, e.g.

$$\langle A | \sigma_\mu | B \rangle \langle C | \bar{\sigma}^\mu | D \rangle = 2 \langle A | D \rangle \langle C | B \rangle$$



Numerical Evaluation



Mathematica Interface

The **Mathematica Interface** turns the generated **stand-alone Fortran code** into a **Mathematica function for evaluating the cross-section or decay rate** as a function of user-selected model parameters. Think of:

```
ContourPlot[sigma[TB, MA0], {TB, 5}, {MA0, 250}]
```

Changes in code (run.F):

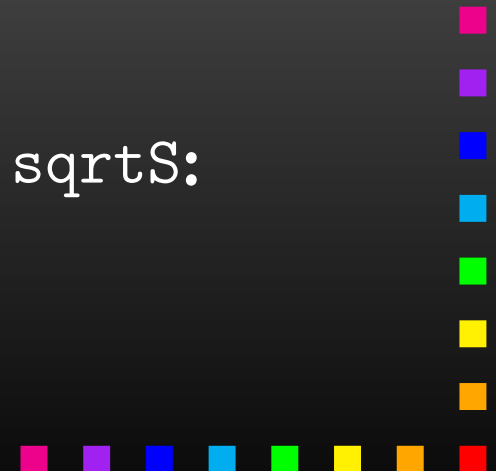
```
TB = 5 → call MmaGetReal(TB)
```

Compile, load in Mathematica with

```
Install["run"]
```

Compute e.g. a differential cross-section at $\sqrt{s} = \text{sqrtS}$:

```
run[sqrtS, TB, MA0, ...]
```



The Model Files

One has to set up, once and for all, a

- **Generic Model File** (seldomly changed) containing the generic part of the couplings,

Example: the FFS coupling

$$C(F, F, S) = G_L \mathbb{P}_L + G_R \mathbb{P}_R = \vec{G} \cdot \begin{pmatrix} \mathbb{P}_L \\ \mathbb{P}_R \end{pmatrix}$$

```
AnalyticalCoupling[s1 F[j1, p1], s2 F[j2, p2], s3 S[j3, p3]]  
== G[1][s1 F[j1], s2 F[j2], s3 S[j3]] .  
  { NonCommutative[ ChiralityProjector[-1] ],  
    NonCommutative[ ChiralityProjector[+1] ] }
```

The Model Files

One has to set up, once and for all, a

- **Classes Model File** (for each model)
declaring the particles and the allowed couplings

Example: the $\bar{\ell}_i \nu_j G$ coupling in the Standard Model

$$\vec{G}(\bar{\ell}_i, \nu_j, G) = \begin{pmatrix} G_- \\ G_+ \end{pmatrix} = \begin{pmatrix} -\frac{i e m_{\ell,i}}{\sqrt{2} \sin \theta_w M_W} \delta_{ij} \\ 0 \end{pmatrix}$$

```
C[ -F[2,{i}], F[1,{j}], S[3] ]  
== { {-I EL Mass[F[2,{i}]]/(Sqrt[2] SW MW) IndexDelta[i, j]},  
      {0} }
```

Included Model Files

Model Files presently available for FeynArts:

- **SM [w/QCD], normal and background-field version.**
All one-loop counter terms included.
- **MSSM [w/QCD].**
All one-loop counter terms included.
- **Two-Higgs-Doublet Model.**
Counter terms not included yet.
- **ModelMaker utility generates Model Files from the Lagrangian.**
- **“3rd-party packages” FeynRules and LanHEP generate Model Files for FeynArts and others.**
- **SARAH package derives SUSY Models.**



Partial (Add-On) Model Files

FeynArts distinguishes

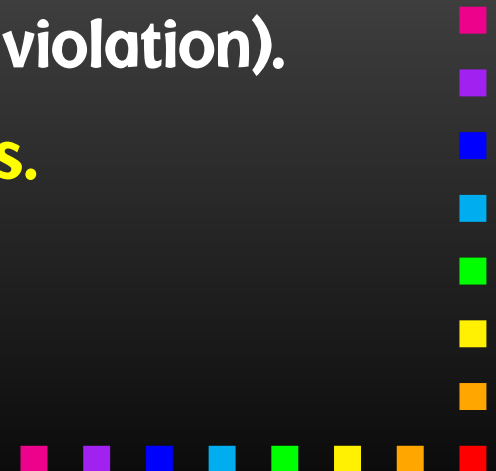
- **Basic Model Files** and
- **Partial (Add-On) Model Files.**

Basic Model Files, e.g. SM.mod, MSSM.mod, can be modified by Add-On Model Files. For example,

```
InsertFields[..., Model -> {"MSSMQCD", "FV"}]
```

This loads the Basic Model File MSSMQCD.mod and modifies it through the Add-On FV.mod (non-minimal flavour violation).

Model files can thus be built up from several parts.



Tweaking Model Files

Or, How to efficiently make changes in an existing model file.

Bad: Copy the model file, modify the copy. – Why?

- It is typically not very transparent what has changed.
- If the original model file changes (e.g. bug fixes), these do not automatically propagate into the derivative model file.

Better: Create a new model file which reads the old one and modifies the particles and coupling tables.

- `M$ClassesDescription` = list of particle definitions,
- `M$CouplingMatrices` = list of couplings.



Tweaking Model Files

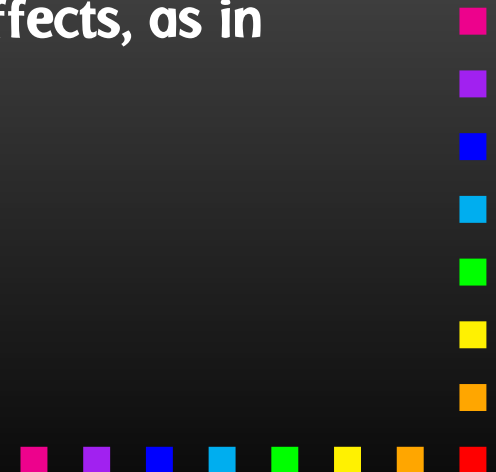
Example: Introduce **enhancement factors** for the $b-\bar{b}-h_0$ and $b-\bar{b}-H_0$ Yukawa couplings in the MSSM.

```
EnhCoup[ (lhs:C[F[4,{g_,_}], -F[4,_], S[h:1|2]]) == rhs_ ] :=  
  lhs == Hff[h,g] rhs  
EnhCoup[other_] := other  
  
M$CouplingMatrices = EnhCoup/@ M$CouplingMatrices
```

To see the effect, make a printout with the WriteTeXFile utility of FeynArts.

The `Hff[h,g]` can be defined to include e.g. resummation effects, as in

```
double precision Hff(2,3)  
data Hff /6*1/  
Hff(1,3) = 1 - CA/(SA*TB)*Delta_b  
Hff(2,3) = 1 + SA/(CA*TB)*Delta_b
```



Linear Combinations of Fields

FeynArts can automatically linear-combine fields, i.e. one can specify the **couplings in terms of gauge rather than mass eigenstates**. For example:

```
M$ClassesDescription = { ...,  
  F[11] = {...,  
    Indices -> {Index[Neutralino]},  
    Mixture -> ZNeu[Index[Neutralino],1] F[111] +  
              ZNeu[Index[Neutralino],2] F[112] +  
              ZNeu[Index[Neutralino],3] F[113] +  
              ZNeu[Index[Neutralino],4] F[114]} }
```

Since F[111]...F[114] are not listed in M\$CouplingMatrices, they drop out of the model completely.



Linear Combinations of Fields

Higher-order mixings can be added, too:

```
M$ClassesDescription = { ...,  
  S[1] = {...},  
  S[2] = {...},  
  S[10] == {...,  
    Indices -> {Index[Higgs]},  
    Mixture -> UHiggs[Index[Higgs],1] S[1] +  
              UHiggs[Index[Higgs],2] S[2],  
    InsertOnly -> {External, Internal}} }
```

This time, $S[10]$ and $S[1], S[2]$ appear in the coupling list (including all mixing couplings) because all three are listed in `M$CouplingMatrices`.

Due to the `InsertOnly`, $S[10]$ is inserted only on tree-level parts of the diagram, not in loops.



Not the Cross-Section

Or, How to get things the Standard Setup won't give you.

Example: extract the Wilson coefficients for $b \rightarrow s\gamma$.

```
tops = CreateTopologies[1, 1 -> 2]
ins = InsertFields[tops, F[4,{3}] -> {F[4,{2}], V[1]}]
vert = CalcFeynAmp[CreateFeynAmp[ins], FermionChains -> Chiral]

mat[p_Plus] := mat/@ p

mat[r_. DiracChain[s2_Spinor, om_, mu_, s1:Spinor[p1_, m1_, _]]] :=
  I/(2 m1) mat[r DiracChain[sigmunu[om]]] +
  2/m1 r Pair[mu, p1] DiracChain[s2, om, s1]

mat[r_. DiracChain[sigmunu[om_]], SUNT[Col1, Col2]] :=
  r 07[om]/(EL MB/(16 Pi^2))

mat[r_. DiracChain[sigmunu[om_]], SUNT[Glu1, Col2, Col1]] :=
  r 08[om]/(GS MB/(16 Pi^2))

coeff = Plus@@ vert //. abbr /. Mat -> mat

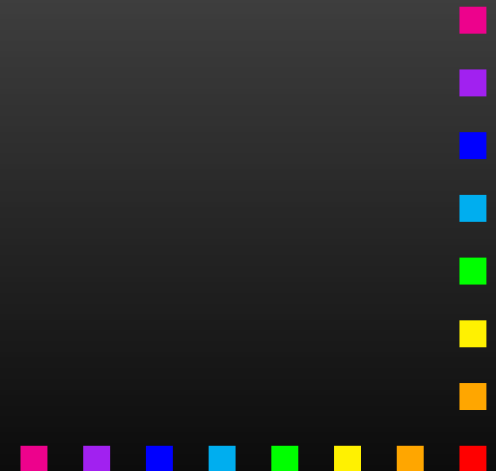
c7 = Coefficient[coeff, 07[6]]
c8 = Coefficient[coeff, 08[6]]
```

Not the Cross-Section

Using FormCalc's output functions it is also pretty straightforward to **generate your own Fortran code:**

```
file = OpenFortran["bsgamma.F"]  
  
WriteString[file,  
  SubroutineDecl["bsgamma(C7,C8)"] <>  
  "\tdouble complex C7, C8\n" <>  
  "#include \"looptools.h\"\n"]  
  
WriteExpr[file, {C7 -> c7, C8 -> c8}]  
  
WriteString[file, "\tend\n"]  
  
Close[file]
```

More details in [hep-ph/0607049](https://arxiv.org/abs/hep-ph/0607049).



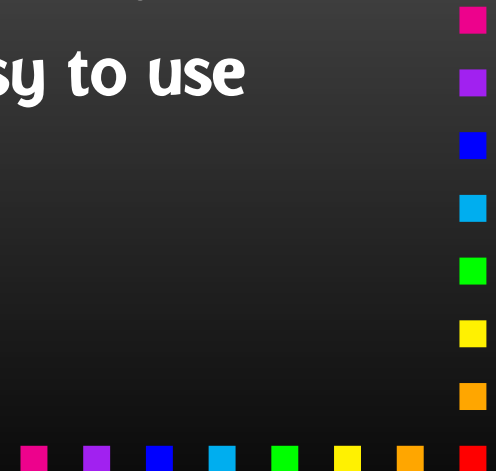
More Complex Calculations

Often special requirements:

- **Resummations** (e.g. hbb in MSSM),
- **Approximations** (e.g. gaugeless limit),
- **K-factors**,
- **Nontrivial renormalization.**

Software design so far:

- Mostly **'monolithic'** (one package does everything).
- Often controlled by **parameter cards**, not easy to use beyond intended purpose.
- May want to/must use other packages.



Universality w.r.t. Loop Order

✓ **Model** → **Model file** → **Diagrams**

Diagram Generation

✓ **Fermion algebra (traces, Dirac eq)**

Symbolic Simplification

✓ **Color algebra ($SU(N)$ traces)**

✗ **Perform tensor reduction, or
Isolate integrals for OPP**

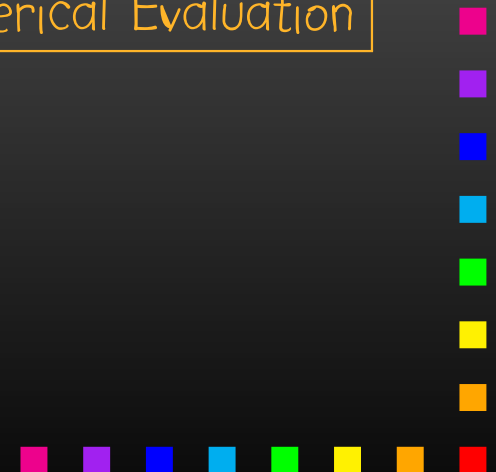
✓ **Other simplifications (e.g. Fierz, abbreviations)**

✓ **Code generation (Fortran, C/C++)**

Numerical Evaluation

✓ **Phase-space integration**

✗ **Evaluation of loop integrals**



Example: $\mathcal{O}(\alpha_t^2)$ MSSM Higgs-mass corrections

Hollik, Paßehr 2014

Template for calculation (2L, nontrivial model + renorm.):

- Break calculation into **several steps**.
- Implement each step as **independent program** (invoked from command line).
- In lieu of ‘in vivo’ debugging **keep detailed logs**.
- Coordinate everything through a **makefile**.
- No single control program (e.g. single Mathematica session) like in package’s demo programs.

More details in arXiv:1508.00562.



Scripting Mathematica

Efficient batch processing with Mathematica:

Put everything into a script, using **sh's Here documents**:

```
#!/bin/sh ..... Shell Magic
math << \_EOF_ ..... start Here document (note the \)
  << FeynArts'
  << FormCalc'
  top = CreateTopologies[...];
  ...
\_EOF_ ..... end Here document
```

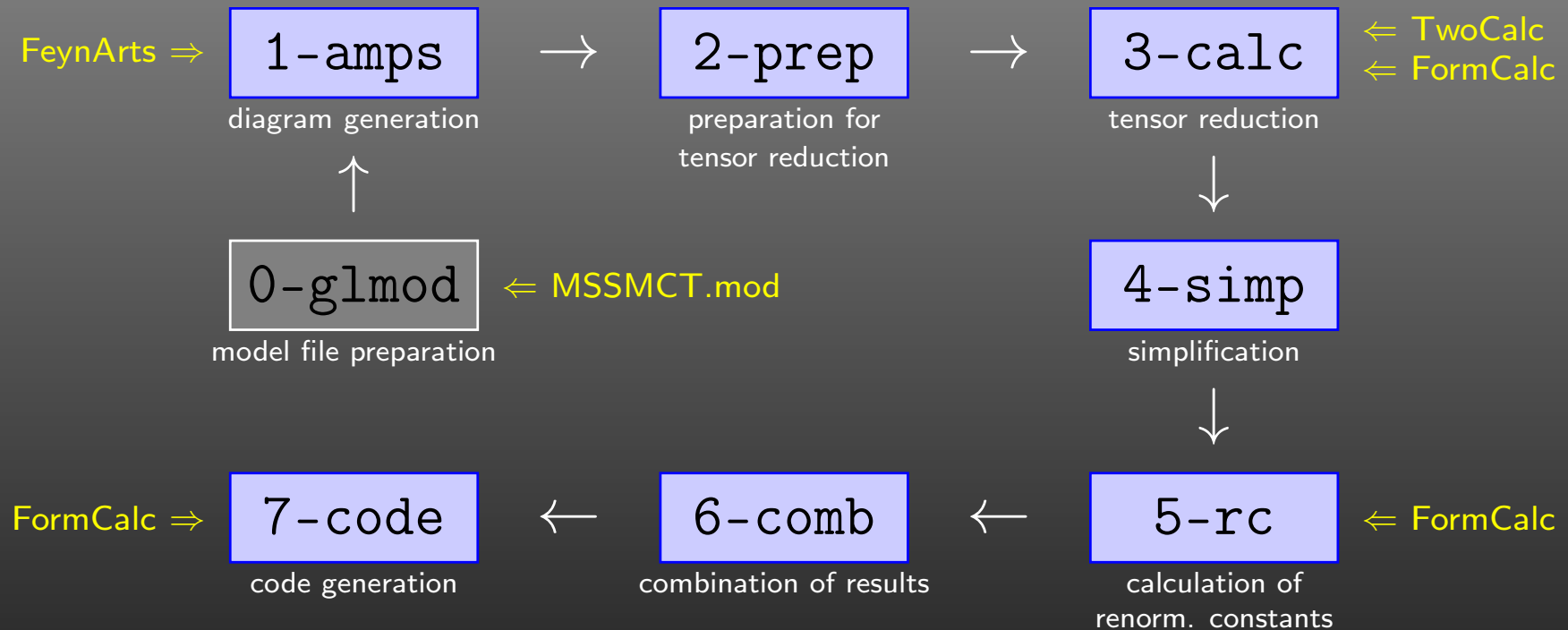
Everything between “<< *tag*” and “*tag*” goes to Mathematica as if it were typed from the keyboard.

Note the “\” before *tag*, it makes the shell pass everything literally to Mathematica, without shell substitutions.



Steps of the Calculation

Calculation split into 7 (8) steps:



Example: Step 0 = Take gaugeless limit

Gaugeless approximation:

- ① Set gauge couplings $g, g' = 0 \Rightarrow M_W, M_Z = 0$.
- ② Keep finite weak mixing angle.
- ③ Keep $\frac{\delta M_W^2}{M_W^2}$ and $\frac{\delta M_Z^2}{M_Z^2}$ finite.

Must set $m_b = 0$ so that $\mathcal{O}(\alpha_t^2)$ corrections form supersymmetric and gauge-invariant subset.

Most efficient to **modify Feynman rules**:

- Load MSSMCT.mod model file.
- Modify couplings, remove zero ones.
- Write out MSSMCTgl.mod model file.



Finally

- There are many packages for tree-level and increasingly also 1L calculations available.
- For **‘standard tasks’** (e.g. cross-section computation) largely automated “model to events” **toolchains exist.**
- Other tasks requiring evaluation of Feynman diagrams are not so well automated (and may never be).
- Packages like FeynArts, FormCalc, FeynCalc, Package-X provide an **“exploration toolkit” for unusual models, unusual renormalizations, package building, ...**
- Long-term strategy: maybe best use Unix philosophy **“Do one thing and do it well”** - modular components for individual tasks + stick together by script.

