Vector-Like Quarks

at the Origin of Light Quark Masses and mixing

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Question:

In view of the strong hierarchy of quark masses, can one conclude that the small mixing in the quark sector (i.e. $V_{CKM} \approx \mathbb{1}$) is "Natural"?

Answer: Absolutely not!!

A novel flavour fine-tuning problem in the SM:

Contrary to "conventional wisdom" in the SM, with no extra-symmetries the "natural value" of $|V_{13}|^2 + |V_{23}|^2$ is large, of order one:

$$|V_{13}|^2 + |V_{23}|^2 = O(1)$$

This is to be compared with the experimental value:

$$|V_{13}|^2 + |V_{23}|^2 \cong 1.6 \times 10^{-3}$$

In order to show the fine-tuning problem, let us consider the extreme chiral (EC) limit, where

$$m_d = m_s = 0 \;; \quad m_b \neq 0$$

$$m_u = m_c = 0 \;; \quad m_t \neq 0$$

In the EC limit the general quark mass matrices can be written:

$$M_d = U_L^{d\dagger} \begin{bmatrix} 0 & & \\ & 0 & \\ & & m_b \end{bmatrix} U_R^d$$

$$M_u = U_L^{u\dagger} \begin{bmatrix} 0 & & \\ & 0 & \\ & & m_t \end{bmatrix} U_R^u$$

where $U_L^{d,u}$, $U_R^{d,u}$ are arbitrary unitary matrices. The ordering of the eigenvalues in the diagonal matrices has no physical meaning!!

Taking into account that in the EC limit the first 2 generations are massless, one can make an arbitrary redefinition of the light quark masses through a unitary transformation of the type

$$W_{u,d} = \begin{bmatrix} X_{u,d} & 0 \\ 0 & 1 \end{bmatrix}$$

$X_{u,d} \rightarrow$ 2×2 unitary matrices.

Under this transformation, V^0 transforms as :

$$\boxed{V^0 \rightarrow V' = W_u^+ V^0 W_d}$$

One has the freedom to choose $X_{u,d}$ at will, to diagonalise the 2×2 upper left sector of V', leading to $V'_{12} = V'_{21} = 0$.

So, one has :

$$V' = \begin{bmatrix} V'_{11} & 0 & V'_{13} \\ 0 & V'_{22} & V'_{23} \\ V'_{31} & V'_{32} & V'_{33} \end{bmatrix}$$

Unitarity of V' leads then to :

$$V'^{*}_{13} V'_{23} = 0$$

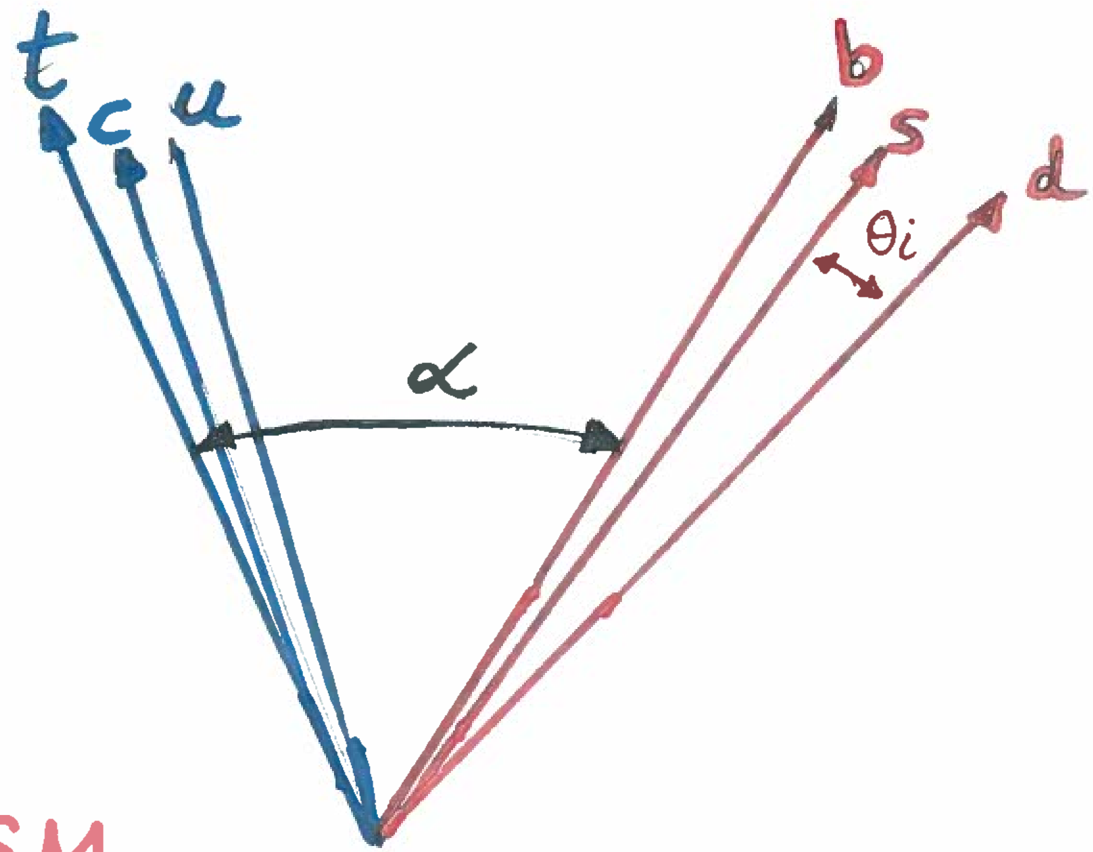
One can then choose, without loss of generality, $V'_{13} = 0$, $V'_{31} = 0$

V^{CKM} becomes then an orthogonal matrix

$$V^{CKM} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha \\ 0 & -s_\alpha & c_\alpha \end{bmatrix}$$

Important point — This mixing is meaningful even in the EC limit and it is arbitrary.

Schematic picture of the Novel Fine-Tuning Problem



In the SM,
The angle α is in general of order (1)
and indipendent of quark masses
hierarchy!

In the literature, the hypothesis of "anarchy" has been used to "justify small mixing in the quark sector. This argument is necessarily Wrong!

There is always the hidden assumption of Flavour alignment!

Tentative implications

- This novel fine-tuning problem provides further motivation to introduce some flavour symmetry in the SM which could guarantee

$$V_{CKM} \simeq 1$$

- The root of the problem is the fact that in SM, Y_u, Y_d are totally independent complex matrices

A comment :

This flavour fine-tuning problem in the SM, may provide motivation for considering left-right symmetric theories like e.g

$$SU(2)_L \times SU(2)_R \times U(1)$$

with only one bi-doublet, one has

$$M_u \propto M_d \Rightarrow V_{CKM} = 1$$

An invariant measure of alignment in flavour space.

Suppose that one has quark mass matrices M_u, M_d, with hierarchical mass eigenvalues, written in a arbitrary weak-basis (WB)

How can one have an invariant measure of alignment?

By "invariant measure of alignment", we mean the following: Given an arbitrary set of matrices M_u, M_d one looks for a weak-basis invariant which provides an order of magnitude of:

$$|V_{13}^{CKM}|^2 + |V_{23}^{CKM}|^2$$

Consider the following WB invariant:

$$A \equiv \frac{1}{2}\,\mathrm{tr}\,B^2 \quad ; \quad B \equiv h_d - h_u$$

where $$h_d = \frac{H_d}{\mathrm{tr}[H_d]} \quad ; \quad h_u = \frac{H_u}{\mathrm{tr}[H_u]}$$

where $$H_{u,d} \equiv \left(M_{u,d} M_{u,d}^{\dagger}\right)$$

By construction $\mathrm{tr}\,h_d = \mathrm{tr}\,h_u = 1$

In the EC limit, where $M_{u,d}$ have rank one

$$A = |V_{23}|^2 + |V_{13}|^2 .$$

The invariant A gives a measure of the size of the mixing even when the first two generations acquire mass.

$$A \approx |V_{23}|^2 + |V_{13}|^2 + O(m_s/m_b)^4$$

A simple discrete symmetry, introduced in the context of the SM, which leads to alignment and $V^{CKM} = \mathbb{1}$

$$Q^0_{L_1} \to e^{i\tau} Q^0_{L_1} \; ; \; Q^0_{L_2} \to e^{-2i\tau} Q^0_{L_2} \; ; \; Q^0_{L_3} \to e^{-i\tau} Q^0_{L_3}$$

$$d^0_{R_1} \to e^{-i\tau} d^0_{R_1} \; ; \; d^0_{R_2} \to e^{-i\tau} d^0_{R_2} \quad d^0_{R_3} \to e^{-i2\tau} d^0_{R_3}$$

$$u^0_{R_1} \to e^{i\tau} u^0_{R_1} \quad u^0_{R_2} \to e^{i\tau} u^0_{R_2} \; ; \; u^0_{R_3} \to u^0_{R_3}$$

The Yukawa couplings are :

$$\mathcal{L}_Y = -\left[\overline{Q}^0_{L_i} \phi \, Y_d \, d^0_{R_j} + \overline{Q}^0_{L_i} \tilde{\phi} \, Y_u \, u^0_{R_j} \right] + \text{h.c.}$$

With the discrete symmetry, the couplings become :

$$Y_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix} ; \quad Y_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

How to generate masses for the light-quark generations?

Possible answer: introduce vector-like quarks (VLQ).

(VLQ) are "cousins" of right-handed neutrinos (ν_R). Both VLQ and ν_R have gauge invariant masses.

For example:

$$\overline{D}_L \, M_D \, D_R$$

$$\nu_R^T \, C M_R \, \nu_R$$

We know that it is "natural" to consider $M_D, M_R \gg v$ but the actual value is not known

Seven reasons to consider vector-like quarks

1. They provide a self-consistent framework with naturally small violations of 3×3 unitarity of V^{CKM}.

2. Lead to naturally small Flavour Changing Neutral currents (FCNC) mediated by Z_μ

$$\text{New Physics in} \left\{ \begin{array}{l} B_d - \bar{B}_d \text{ mixing} \\ B_s - \bar{B}_s \text{ mixing} \\ K^0 - \bar{K}^0 \text{ mixing} \end{array} \right.$$

3. Provide the simplest framework to have Spontaneous CP Violation, with a vacuum phase generating a <u>non-trivial CKM phase</u>.

4. Provide New Physics contributions to $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings

5. Provide a simple solution to the Strong CP problem, which <u>does not require Axions</u>

6. May contribute to the understanding of the observed pattern of fermion masses and mixing.

7. Provide a framework where there is a common origin for all CP violations:

(i) CP violation in the Quark Sector

(ii) CP Violation in the Lepton Sector, detectable through neutrino oscillations

$U_{e3} \neq 0 \longrightarrow$ Great News

(iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis.

Comment :

There is nothing "strange" in having deviations of 3×3 unitarity. The PMNS matrix in the leptonic sector in the context of type-one seesaw (νSM) is not 3×3 unitary !!

Generation of realistic quark masses and mixings through the introduction of vector-like quarks:

$3 \quad Q = -1/3 \quad$ VLQ $\quad D_L, D_R$

$3 \quad Q = 2/3 \quad$ VLQ $\quad U_L, U_R$

We only include terms which are <u>allowed</u> by by gauge invariance and the discrete symmetry. <u>Will not introduce soft-breaking terms</u>

Extension of the symmetry to the full Lagrangian.

$$D^0_{L_1} \to e^{-i3\tau} D^0_{L_1} \; ; \; D^0_{L_2} \to e^{-2i\tau} D^0_{L_2} \; ; \; D^0_{L_3} \to e^{-i\tau} D^0_{L_3}$$

$$D^0_{R_1} \to e^{-i2\tau} D^0_{R_1} \; ; \; D^0_{R_2} \to e^{-i3\tau} D^0_{R_2} \; ; \; D^0_{R_3} \to D^0_{R_3}$$

$$u^0_{L_1} \to e^{-i\tau} u^0_{L_1} \; ; \; u^0_{L_2} \to u^0_{L_2} \; ; \; u^0_{L_3} \to e^{-i\tau} u^0_{L_3}$$

$$u^0_{R_1} \to u^0_{R_1} \; ; \; u^0_{R_2} \to e^{-i\tau} u^0_{R_2} \; ; \; u^0_{R_3} \to e^{-i2\tau} u^0_{R_3}$$

A complex scalar singlet is also introduced, which transforms as: $S \to e^{-i\tau} S$

	$d^0_{R_1}(-\tau)$	$d^0_{R_2}(-\tau)$	$d^0_{R_3}(-2\tau)$	$D^0_{R_1}(-2\tau)$	$D^0_{R_2}(-3\tau)$	$D^0_{R_3}(0)$
$\overline{Q}^0_{L_1}(-\tau)$	•	•	•	•	•	$-\tau$
$\overline{Q}^0_{L_2}(2\tau)$	•	•	•	•	$-\tau$	•
$\overline{Q}^0_{L_3}(\tau)$	•	•	$-\tau$	$-\tau$	•	•
$\overline{D}^0_{L_1}(3\tau)$	•	•	τ	τ	1	•
$\overline{D}^0_{L_2}(2\tau)$	τ	τ	1	1	$-\tau$	•
$\overline{D}^0_{L_3}(\tau)$	1	1	$-\tau$	$-\tau$	•	τ

Similar structure for up sector

Mass terms written in a compact form

$$\mathcal{L}_M = -\left(\bar{d}^0_L \quad \bar{D}^0_L\right) \mathcal{M}_d \begin{bmatrix} d^0_R \\ D^0_R \end{bmatrix} - \left[\bar{u}^0_L \quad \bar{U}^0_L\right] \mathcal{M}_u \begin{bmatrix} u^0_R \\ U^0_R \end{bmatrix}$$

$$\mathcal{M}_d \equiv \left[\begin{array}{c:c} m_d & \omega_d \\ \hdashline X_d & M_d \end{array}\right]$$

$m_d, \omega_d \longrightarrow$ of order v

$X_d, M_d \longrightarrow$ of order $\langle S \rangle \equiv V$ and invariant mass terms.

similar for up quarks.

The 6×6 matrix $\mathcal{M}_d, (\mathcal{M}_u)$ are diagonalized by

$$\begin{bmatrix} d_L^0 \\ D_L^0 \end{bmatrix} = \begin{bmatrix} A_{dL} \\ B_{dL} \end{bmatrix} d_L = \mathcal{U}_{dL}\, d_L$$

similar for up quarks

L. Lavoura, GCB (1986)

L. Lavoura, J.P. Silva (1993)

A_{dL}, B_{dL} are 3×6 matrices

$$\mathcal{U}_{dL}^\dagger \mathcal{M}_d\, \mathcal{U}_{dR} = \mathrm{diag}\,(d_d^2, D_d^2)$$

charged currents:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \left(\bar{u}_L^0 \gamma^\mu d_L^0\right) W_\mu = -g \left(\bar{u}_L V \gamma^\mu d_L\right) W_\mu$$

$\boxed{V = A_{uL}^\dagger A_{dL}}$, with u_L, d_L running from 1 to 6

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W}\left[\frac{1}{2}\left(\bar{u}_L W_u \gamma^\mu u_L - \bar{d}_L W_d \gamma^\mu d_L\right) - \sin^2\theta_W \cdot \left(\frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d\right)\right] Z_\mu$$

$$W_d = V^\dagger V \;;\; W_u = V V^\dagger \;;\; V \equiv \left(A_{uL}^\dagger A_{dL}\right)$$

$$\mathcal{L}_H = -\frac{h}{v}\left[\bar{d}_L W_d \mathcal{D}_d d_R + \bar{u}_L W_u \mathcal{D}_u u_R\right] + h.c.$$

There are Z-mediated and h-mediated neutral currents which violate flavour (FCNC) but they are both naturally suppressed.

$$\mathcal{U}_{qL} = \begin{pmatrix} K_q & R_q \\ S_q & T_q \end{pmatrix}$$

with $q = u, d$; K_q, R_q, S_q, T_q are 3×3 matrices

$$K_q K_q^\dagger = \mathbb{1} - R_q R_q^\dagger$$

$$R_q \approx \frac{(m_q X_q^\dagger + \omega_q M_q^\dagger) T_q}{D_q^2} \approx (m/M)$$

$$K_q^\dagger K_q = 1 - S_q^\dagger S_q$$

$$S_q \approx \left(\frac{X_q m_q^\dagger + M_q \omega_q^\dagger}{X_q X_q^\dagger + M_q M_q^\dagger} \right) K_q \approx m/M$$

$\mathcal{U}^q_{L,R}$ will be determined through an exact numerical diagonalization. But is useful to derive an effective Hermitian quark mass matrix

$$K_q^{-1} H^q_{eff} K_q = \text{diag.}(d^2)$$

$$\boxed{H^q_{eff} = (m_q m_q^\dagger + \omega_q \omega_q^\dagger) - (m_q X_q^\dagger + \omega_q M_q^\dagger)(X_q X_q^\dagger + M_q M_q^\dagger)^{-1} \times (X_q m_q^\dagger + M_q \omega_q^\dagger)}$$

Realistic Examples

masses at M_Z scale in GeV

$$m_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3,\ldots \end{bmatrix} \; ; \quad m_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 185,\ldots \end{bmatrix}$$

$$w_d = \begin{bmatrix} 0 & 0 & 0.06\ldots \\ 0 & 0.399 & 0 \\ 0.399 & 0 & 0 \end{bmatrix} \; ; \quad w_u = \begin{bmatrix} 0 & 0 & 0.009\ldots \\ 0 & 3.7\ldots & 0 \\ 185,\ldots & 0 & 0 \end{bmatrix}$$

$$M_d = \begin{bmatrix} 767 & & 0 \\ & 1535 & \\ 0 & & 1842 \end{bmatrix} \; ; \quad M_u = \begin{bmatrix} 1295 & & 0 \\ & 1481 & \\ 0 & & 2221 \end{bmatrix}$$

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$$X_d = \begin{bmatrix} 0 & 0 & -115,.. \\ -262,-i & 46,\dots & 460,-230i \\ 486,\dots & 0 & 368,\dots \end{bmatrix} ; \quad X_u = \begin{bmatrix} 0 & 0 & 68,\dots \\ -212,\dots & -185,\dots & 0 \\ 416,\dots & 0 & 0 \end{bmatrix}$$

$$|V| = \left[\begin{array}{ccc:c} 0.97\dots & 0.22\dots & 0.003\dots & \\ 0.22 & .97.. & 0.04\dots & \text{very small} \\ 0.008 & 0.039 & 0.98 & \\ \hdashline \text{very small} & & 0.15 & \text{very small} \end{array}\right]$$

Conclusions

- There is a novel fine-tuning problem which provides motivation to introduce a flavour symmetry which leads to $V^{CKM} = \mathbb{1}$ in leading order
- Vector-like quarks may generate light quark masses and a realistic V^{CKM}