Lattice check of dynamical fermion mass generation

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• Theoretical proposal

- **R. Frezzotti and G.C. Rossi**
Nonperturbative mechanism for elementary particle mass generation
PRD 92 (2015) 054505

- **R. Frezzotti and G.C. Rossi**
Dynamical mass generation
PoS LATTICE2013 (2014) 354

• & other relevant publications

- **R. Frezzotti. M. Garofalo and G.C. Rossi**
Nonsupersymmetric model with unification of electroweak and strong interactions
PRD 93 (2016) 105030

- **S. Capitani** *et al.*
Check of a new non-perturbative mechanism for elementary fermion mass generation
PoS LATTICE2016 (2016) 212

- **S. Capitani** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup
PoS LATTICE2017

- **S. Capitani** *et al.*
Testing a non-perturbative mechanism for elementary fermion mass generation: Numerical results
PoS LATTICE2017

- **F. Pittler**
Spectral statistics of the Dirac operator near a chiral symmetry restoration in a toy model
PoS LATTICE2017

Overview

- Widely accepted *incompleteness* of the **SM** -for a number of fundamental phenomena not satisfactorily or not at all described by/within it- has motivated a vast variety of ingenous and original **N**ew **P**hysics proposals **BSM**.
  However, this task is proved to be non-trivial perhaps due to the fact that **SM** is a renormalisable theory.
- **SM** describes elemenentary particle masses employing the symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}}$.
- The *hierarchy* pattern of fermion masses (but also Higgs mass un-natural feature) lack deep understanding, they are rather accomodated by fitting to experimental data.

Overview

- **Dynamical generation of fermion masses**

  - Similar physics effect which generates $\langle \bar{q}q \rangle \neq 0$
  - where dynamical χSB *triggered* by an explicit χSB term i.e. fermion mass or Wilson term.
  - In massless LQCD with Wilson term Non-Perturbative contribution ($\propto \Lambda_{QCD}$) is accompanied by an $1/a$ divergent term.
  - Separation of the two effects requires an infinite fine tuning ($\rightarrow$ *naturalness problem).*

- *Proposal*: QCD extended to a theory with **enriched symmetry** for tackling *naturalness problem*.

Overview

- **Dynamical generation of fermion masses**
  - owing to a NP mechanism triggered by a Wilson-like (naively irrelevant) chiral breaking term.
- **Simplest toy-model where the mechanism can be realised**:
  - $SU(N_f = 2)$ doublet of strongly ($SU(3)_c$) interacting fermions coupled to scalars via Yukawa and Wilson-like terms
  - physics depends crucially on the phase (Wigner or NG)
  - enhanced symmetry (naturalness à la t'Hooft) leads to $\langle \Phi \rangle$-independence of fermion masses
- The intrinsic NP character of the mechanism requires lattice numerical investigation of the toy model.
- The proposed mechanism can be falsified/verified.

Theoretical setup

- Toy-model: QCD$_{N_f=2}$ + Scalar field + Yukawa + Wilson

  $L_{toy} = L_{kin}(Q, A, \Phi) + V(\Phi) + L_Y(Q, \Phi) + L_W(Q, A, \Phi)$, with:

$$\begin{aligned}
L_{kin}(Q, A, \Phi) &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{Q}_L \gamma_\mu D_\mu Q_R + \bar{Q}_R \gamma_\mu D_\mu Q_L + \frac{1}{2} \mathrm{Tr}\left[\partial\Phi^\dagger \partial\Phi\right] \\
V(\Phi) &= \frac{1}{2}\mu^2 \mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{1}{4}\lambda\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 \\
L_Y(Q, \Phi) &= \eta\left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi Q_L\right) \\
L_W(Q, A, \Phi) &= \rho \frac{b^2}{2}\left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \bar{Q}_R \overleftarrow{D}_\mu \Phi^\dagger D_\mu Q_L\right)
\end{aligned}$$

  (where $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + i g_s \lambda^a A^a_\mu$, $D_\mu = \partial_\mu - i g_s \lambda^a A^a_\mu$)

- Q: fermion $SU(2)$ doublet coupled to $SU(3)$ gauge field and to scalar field through Yukawa and Wilson terms.

- b^{-1}: UV cutoff.

Theoretical setup *(contd.)*

- $\chi_L \times \chi_R$ transformations are symmetry of L_{toy}:

$$\chi_L : \tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) \qquad \chi_R : \tilde{\chi}_R \otimes (\Phi \to \Omega_R \Phi)$$

$$\tilde{\chi}_L : Q_L \to \Omega_L Q_L, \qquad \tilde{\chi}_R : Q_R \to \Omega_R Q_R,$$

$$\bar{Q}_L \to \bar{Q}_L \Omega_L^\dagger \qquad \bar{Q}_R \to \bar{Q}_R \Omega_R^\dagger$$

$$\Omega_L \in SU(2)_L \qquad \Omega_R \in SU(2)_R$$

- **Exact symmetry** $\chi \equiv \chi_L \times \chi_R$ acting on fermions and scalars $\Rightarrow$ *NO* power divergent mass terms.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations are *not* a symmetry for *generic* (non-zero) η and ρ.
- P, C, T, gauge invariance ... are symmetries & power counting renormalisation.

Theoretical setup *(contd.)*

- The shape of $V(\Phi)$ determines crucially the physical implications of the model
- When the scalar potential $V(\Phi)$ has one minimum
  - $\chi_L \times \chi_R$ is realized à la Wigner.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations generate Schwinger-Dyson Eqs (unrenormalised).
- They get renormalised after considering the operator mixing procedure.
- PT operator mixings $\rightarrow$ *NO* $\tilde{\chi}-$SSB phenomenon occurs $\rightarrow$ *NO* NP fermion mass generation

Theoretical setup *(contd.)*

- **Critical Model**: $\tilde{\chi}$-symmetry restoration occurs when the Yukawa term is compensated by the Wilson term. This takes place (in the Wigner phase) at a certain value of the Yukawa coupling.
- In fact, for $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) get
  $\partial_\mu \langle \tilde{Z}_{\tilde{J}} J_\mu^{L,i}(x) O(0) \rangle = (\eta - \overline{\eta}(\eta; g_0^2, \rho, \lambda)) \langle [\bar{Q}_L \tau^i \Phi Q_R - h.c.](x) O(0) \rangle + O(b^2)$
  (SDE renrm/tion here analogous to chiral SDE renrm/tion in Bochicchio *et al. NPB 1985*)

▶ *enforce* the current $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation $\Longrightarrow$
$\eta - \overline{\eta}(\eta; g_0^2, \rho, \lambda) = 0 \quad \rightarrow \quad \eta_{cr}(g_0^2, \rho, \lambda)$.

- The Low-Energy effective action (in the Wigner phase) reads

$$\Gamma^{Wig}_{\mu_\Phi^2 > 0} = \frac{1}{4}(F \cdot F) + \bar{Q} \not{D} Q + (\eta - \eta_{cr})(\bar{Q}_L \Phi Q_R + \text{h.c.}) + \frac{1}{2} \mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \mathrm{V}_{\mu_\Phi^2 > 0}(\Phi)$$

- in the critical theory ($\tilde{\chi}$ is a symmetry, up to $O(b^2)$)
  - ▶ Scalars decoupled (up to cutoff effects) from quarks and gluons.
  - ▶ no fermionic mass ($m_Q = 0$ up to $O(b^2)$).

Numerical investigation

- **Lattice simulation details**
  - Lattice discretization, $L_{latt.}$, with exact χ-symmetry.
  - Use naive fermions with symmetric covariant derivative, $\tilde{\nabla}_\mu$, throughout.
  - We limit our first study to the quenched approximation
  - Quenching: independent generation of gauge (U) and scalar (Φ) configurations.

    $\Rightarrow$ it is quite certain that the mechanism under investigation, if confirmed, survives quenching.

    $\Rightarrow$ Naive fermions are relatively cheap and fine with quenched approximation. (For an unquenched study one might employ staggered or domain-wall or overlap fermions)

Numerical investigation

- **Lattice simulation details**

  - To avoid "exceptional configurations" ($\rightarrow$ due to fermions zero modes) introduce twisted mass regulator $L_{latt.} + i\mu_Q \bar{Q}\gamma_5\tau^3 Q$.

    (Frezzotti, Grassi, Sint and Weisz, JHEP 2001)

    $\Rightarrow$ at a cost of soft breaking of $\chi_L \times \chi_R$, symmetry recovered after an extrapolation to $\mu_Q \rightarrow 0$.

  - Locally smeared Φ in $\bar{Q} D_{lat}[U, \Phi] Q$ for noise reduction.

Numerical investigation

- **Lattice simulation parameters**

  - ⋆ simulations at **two** values of the lattice spacing
  - ⋆ $\beta = 5.75$ ($b = 0.15$ fm) & $\beta = 5.85$ ($b = 0.12$ fm)
  - ⋆ $L/b = 16$ & $T/b = 40$
  - ⋆ use lattice scale $r_0 = 0.5$ fm (motivated from QCD, for illustration)
    Guagnelli, Sommer and Wittig NPB 535 (1998) & Necco and Sommer NPB 622 (2002)
  - ⋆ $\rho = 1.96$ in the Wigner & NG phase
    (for checking the validity of the mechanism it is sufficient to set some reasonable value $\neq 0$)
  - ⋆ choose scalar field parameters by imposing conditions on $(r_0 M_\sigma)^2$, λ_R and $(r_0 v_R)^2$
  - ⋆ **statistics:** #configs (gauge $\times$ scalar) $\sim 240 - 480$

    @ several values of the Yukawa coupling η (and μ_Q).

▶ Determination of η_{cr} in the Wigner phase

- Compute correlation function

$$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x)\ \tilde{D}^{S3}(y)\rangle$$

where

$$\begin{aligned}
\tilde{J}_0^{V3}(x) &= \tilde{J}_0^{L\,3}(x) + \tilde{J}_0^{R\,3}(x) \\
\tilde{J}_0^{L/R\,3}(x) &= \frac{1}{2}\left[\bar{Q}_{L/R}(x-\hat{0})\gamma_0 \frac{\tau_3}{2} U_0(x-\hat{0}) Q_{L/R}(x) + \bar{Q}_{L/R}(x)\gamma_0 \frac{\tau_3}{2} U_0^\dagger(x-\hat{0}) Q_{L/R}(x-\hat{0})\right] \\
\tilde{D}^{S3}(y) &= \bar{Q}_L(y)\left[\Phi, \frac{\tau^3}{2}\right] Q_R(y) - \bar{Q}_R(y)\left[\frac{\tau^3}{2}, \Phi^\dagger\right] Q_L(y)
\end{aligned}$$

- Renormalised Schwinger-Dyson eqs of $\tilde{V}^3$-type (in the form of a would be $\tilde{\chi}$-WTI):

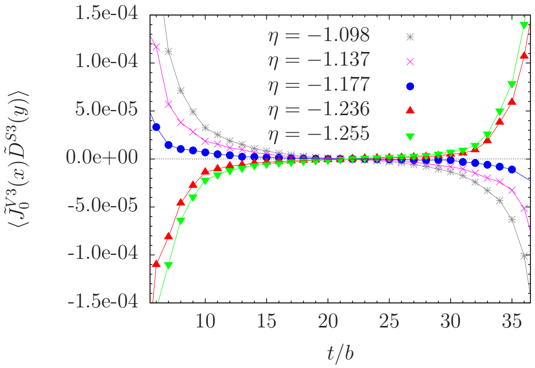
$$\partial_\mu \tilde{J}_\mu^{V3} = (\eta-\eta_{cr})\tilde{D}^{S3} + O(b^2) \quad \text{and} \quad \langle 0|\,\partial_0 \tilde{J}_0^{V3}\,|M_S\rangle \equiv f_{M_S} M_{M_S}^2$$

▶ Determination of η_{cr} in the Wigner phase

In the Wigner phase at $\eta = \eta_{cr}$ the *restored $\tilde{\chi}$-symmetry* is realised à la Wigner (*first example in a local setting*) and leads to vanishing correlation function

$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x)\ \tilde{D}^{S3}(y)\rangle$

i.e. vanishing matrix element: $\mathrm{ME} = \langle 0|\tilde{J}_0^{V3}|M_S\rangle\langle M_S|\tilde{D}^{S3}|0\rangle \xrightarrow{\eta\to\eta_{cr}} 0$



(example at $\beta = 5.85$ and $a\mu_Q = 0.0224$)

▶ Determination of η_{cr} in the Wigner phase

An accurate determination of η_{cr} is obtained employing the "WI" ratio i.e. compute:

$$r_{WI} = \frac{\partial_0 \langle \tilde{J}_0^{V3}(x) \tilde{D}^{S3}(y) \rangle}{\langle \tilde{D}^{S3}(x) \tilde{D}^{S3}(y) \rangle}$$

at several values of η (and μ_Q) and extrapolate to $r_{WI} \to 0$:

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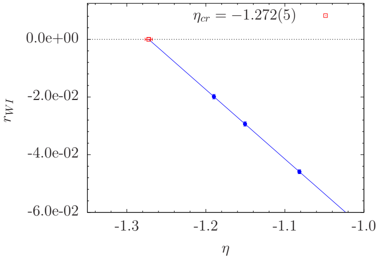
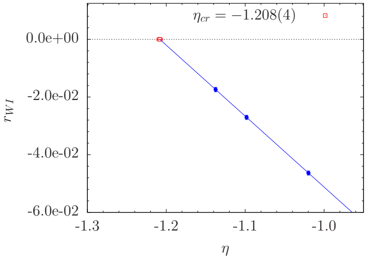
$[\beta = 5.75]$



$[\beta = 5.85]$



(r_{WI} in latt. units after extrapolating to $a\mu_Q = 0$)

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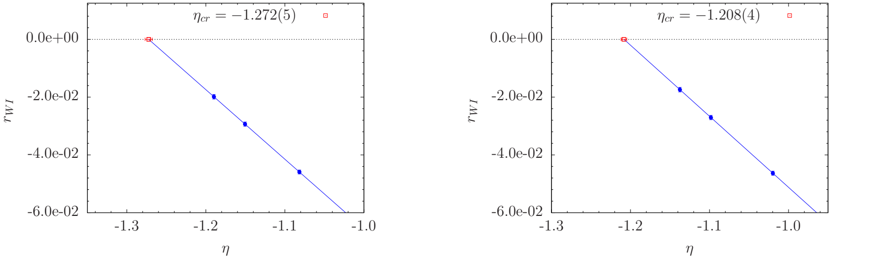
at various values of η (and μ_Q) and extrapolate to $r_{WI} \to 0$:

$[\beta = 5.75]$ $[\beta = 5.85]$



$\eta_{cr} = -1.272(5)$ @ $\beta = 5.75$ & $\eta_{cr} = -1.208(4)$ @ $\beta = 5.85$

$\Rightarrow$ A few per mille *statistical* error for η_{cr} determination.

(Comparable *systematic* uncertainties - Preliminary results!)

▶ Features and properties of the toy-model in NG-phase

- $V(\Phi)$ of mexican hat shape $\to \chi_L \times \chi_R$ realised à la NG.
- $\chi_L \times \chi_R$ spontaneously broken: $\Phi = v + \sigma + i\vec{\tau}\vec{\pi}, \qquad \langle\Phi\rangle = v \neq 0.$
- $L_W(Q, A, \Phi) = \frac{\rho b^2}{2}\left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \text{h.c.}\right) \underset{a\leftrightarrow b}{\overset{r\leftrightarrow b v\rho}{\sim}} L_W^{QCD}(Q, A) = -\frac{ar}{2}\left(\bar{Q}_L D^2 Q_R + \text{h.c.}\right).$
- In the *critical* theory $\eta = \eta_{cr}$:
  - ▶ the (Yukawa) mass term, $v\bar{Q}Q$, gets cancelled.
  - ▶ $\tilde{\chi}-$ breaking due to residual $O(b^2 v)$ effects is expected to trigger dynamical χSB.

▶ Features and properties of the toy-model in NG-phase

$\Longrightarrow$ Look for dynamically generated fermion mass:

- **NP** mass term has to be $\chi_L \times \chi_R$ invariant (and under chiral variation can be accomodated in the $\tilde{\chi}$ WTI's).

  Note that a term like $m[\bar{Q}_L Q_R + \bar{Q}_R Q_L]$ is not $\chi_L \times \chi_R$ invariant.

- At *generic* η, two $\tilde{\chi}$ breaking operators are expected to arise:

  Yukawa induced + dynamically generated ($\leftarrow$ conjecture)

- $\Gamma^{NG} = \ldots + (\eta - \eta_{cr})(\bar{Q}_L \langle \Phi \rangle Q_R + \text{h.c.}) + c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.})$ where

$$\mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v+\sigma)\mathbb{1} + i\vec{\tau}\vec{\varphi}}{\sqrt{v^2 + 2v\sigma + \sigma^2 + \vec{\varphi}\vec{\varphi}}} \simeq \mathbb{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \ldots$$

  and $\Lambda_s \equiv$ RGI NP mass scale.

  – $\mathcal{U}$ is a non-analytic function of Φ, but transforms like Φ under $\chi_L \times \chi_R$; obviously $\mathcal{U}$ can not be defined in the Wigner phase ($\langle \Phi \rangle = 0$) $\rightarrow$ no NP mass or mixings in the Wigner phase.

  – Note that (χ-inv. term): $c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.}) \simeq c_1 \Lambda_s \bar{Q} Q + \ldots$

▶ Features and properties of the toy-model in NG-phase

- Work at the same lattice parameters (β, λ, ρ and volume) as in the Wigner phase
- Compute WTI quark mass: $m^{WTI} = \frac{\partial_0 \langle \widetilde{J}_0^{A\pm}(x) P^{\pm}(y) \rangle}{\langle P^{\pm}(x) P^{\pm}(y) \rangle}$ in the NG-phase (where $P^{\pm} = \bar{Q}\gamma_5 \tau^{\pm} Q$ - pseudoscalar density) & M_{ps} from $\langle P^{\pm}(x) P^{\pm}(y) \rangle$.

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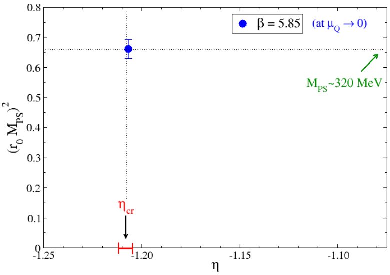

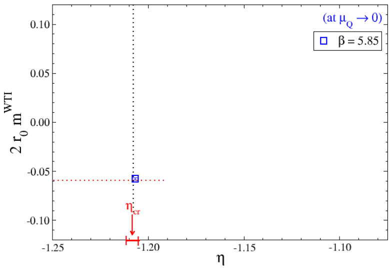

$$\text{At } \eta = \eta_{cr} : \begin{cases} M_{PS} \neq 0 \\ m^{WTI} \neq 0, \quad \text{Note } m^{WTI} = (\eta - \eta_{cr})v + c_1\Lambda_s \xrightarrow{\eta = \eta_{cr}} m^{WTI} = c_1\Lambda_s \end{cases}$$

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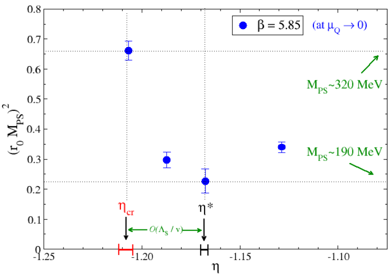

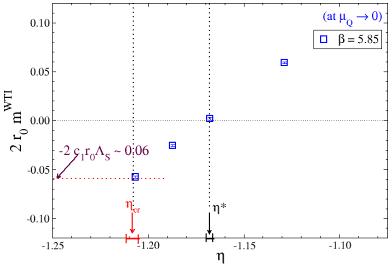

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- m^{WTI} cancels at $\eta^* = \eta_{cr} - c_1\Lambda_s/v \Rightarrow \eta_{cr} \neq \eta^* \leftrightarrow c_1\Lambda_s \neq 0$
- At $\eta = \eta_{cr}$ for $\beta = 5.85$: $M_{PS} \sim 320\,\text{MeV}$ and $m^{WTI}_{\rm bare} \neq 0$.

(Preliminary results!)

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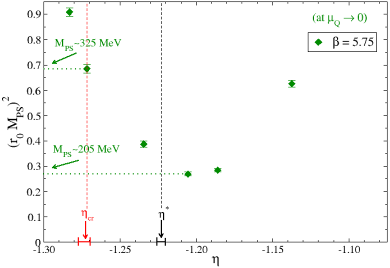
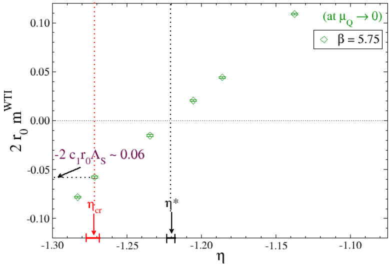
- Work at the same lattice parameters (β, λ, ρ and volume) as in the Wigner phase





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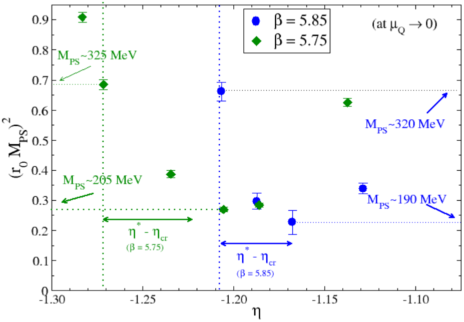
▶ Towards the CL: scaling behaviour

- $(r_0 M_{\rm PS})^2$ against η at two β-values:



▶ Towards the CL: scaling behaviour

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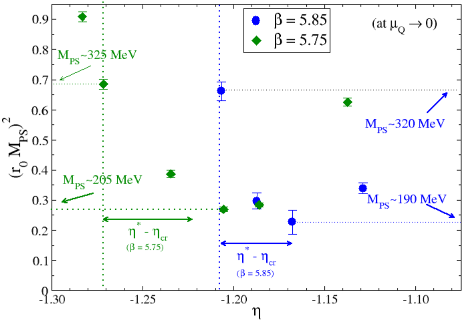
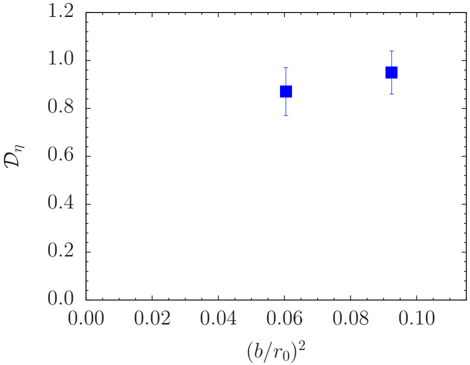


$$
\text{At } \eta = \eta_{cr} : \begin{cases} M_{\rm PS} : & \text{(very) small cutoff effects } \to M_{\rm PS} \neq 0 \text{ at } \mathbf{CL} \\ m^{WTI} : & \text{renormalised may differ by 5-10\% wrt bare, but} \\ & \text{observed small cutoff effects } \to \text{ quite certain } m^{WTI} \neq 0 \text{ at } \mathbf{CL} \end{cases}
$$

Towards the CL: scaling behaviour

- $c_1\Lambda_S \neq 0 \Leftrightarrow (\eta^* - \eta_{cr}) \neq 0$
- $(\eta^* - \eta_{cr})$ has to be renormalised ...
- Consider renormalised quantity: $\mathcal{D}_\eta = (\eta^* - \eta_{cr}) d(r_0 M_{PS})^2/d\eta|_{\eta_{cr}}$



- Quite certainly $\mathcal{D}_\eta \neq 0$ at **CL**.

Conclusions & Outlook

- We have presented a toy-model that exemplifies a *novel* NP mechanism for fermion mass generation.
  R. Frezzotti and G.C. Rossi, PRD 2015, [arXiv:1402.0389 [hep-lat]]

- The **toy model** is a non-Abelian gauge model with an SU($N_f = 2$)-doublet of strongly interacting fermions coupled to scalars through Yukawa and Wilson-like terms: at the *critical point*, where (fermion) $\tilde{\chi}$ invariance is recovered in Wigner phase (up to UV-effects) the model is *conjectured* to give rise in NG phase to dynamical $\tilde{\chi}$-SSB and hence to non-perturbative fermion mass generation.

- The main physical implications of the conjecture above can be *verified/falsified* by numerical simulations of the toy-model (rather cheap in the quenched approximation).

Conclusions & Outlook

- A study at two values of the lattice spacing ($\sim$ 0.12 and 0.15 fm) in the quenched approximation has been presented.

- We have shown that the critical value of the Yukawa coupling in the Wigner phase at which $\tilde{\chi}$ is restored can be accurately determined. Then we explored the effects of dynamical SSB of the (restored) $\tilde{\chi}$-symmetry in the NG phase which look very well compatible with the generation of a non-zero (effective) fermion mass and $M_{PS} \sim O(\Lambda_s)$ at the CL.

- These findings might be checked and verified at a finer value of the lattice spacing in order to get even more solid confirmation for the persistence of the dynamical mass generation mechanism in the continuum limit.

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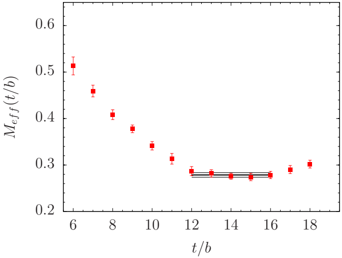
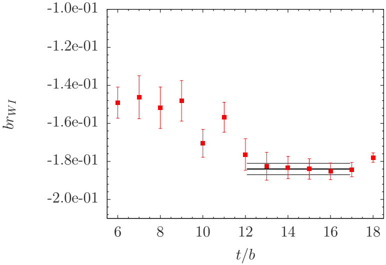
Thank you for your attention!

Extra slides

• Eucledian time behaviour for $M_{eff}(t)$ associated to the correlation function $C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x)\ \tilde{D}^{S3}(y)\rangle$ & r_{WI} (example case $\eta = -1.020$ @ $a\mu_Q = 0.0224$ in Wigner phase & $\beta = 5.85$):

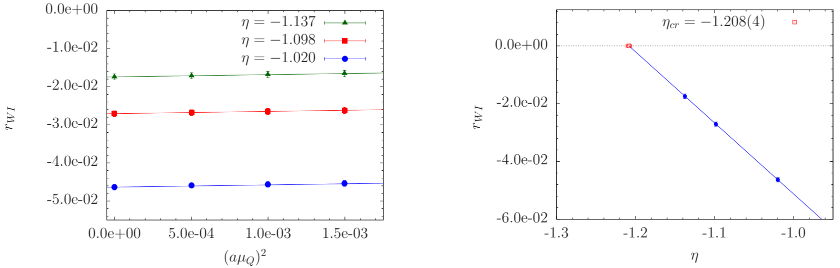




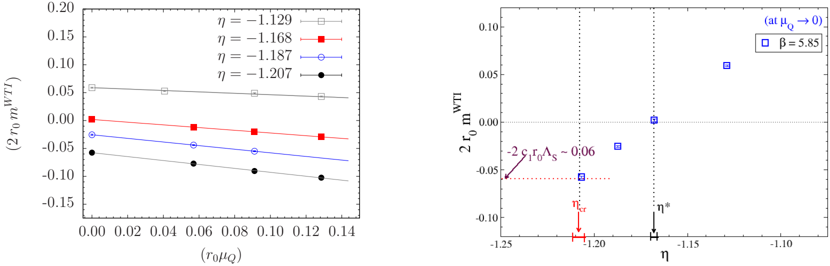
Extrapolation in $\mu_Q = 0$ and in η of

$$r_{WI} = \frac{\partial_0 \langle \tilde{J}_0^{V3}(x) \tilde{D}^{S3}(y) \rangle}{\langle \tilde{D}^{S3}(x) \tilde{D}^{S3}(y) \rangle}$$

in order to determine η_{cr} where r_{WI} (in latt. units) vanishes ($\beta = 5.85$):



Extrapolation in $\mu_Q = 0$ (left panel) and η-dependence (right panel) for $2r_0 m^{WTI} = \frac{2r_0\partial_0\langle 0|\widetilde{J}_0^{A\pm}|M_{PS^\pm}\rangle}{\langle 0|P^\pm|M_{PS^\pm}\rangle}$ in the NG-phase ($\beta = 5.85$).



Extrapolation in $\mu_Q = 0$ (left panel) and η-dependence (right panel) for $(r_0 M_{\rm PS})^2$ ($\beta = 5.85$)