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Noncommutative Wilson lines in higher-spin theory and correlation functions of conserved currents for free conformal fields

based on arXiv :1705.03928
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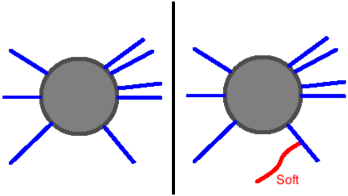
Training School : Quantum Spacetime and Physics models

Motivations

- They appear in string theory
- They are in Wigner's classification
- Various no-go



- HS/CFT : Vasiliev's minimal bosonic model is conjectured to be dual to the free $O(N)$ vector model.

Higher spin gravity

- *Frønsdal, 1978*
  Generalisation of linearised metric gravity
  - $\varphi_{(\mu_1 \ldots \mu_s)}$
  - $\delta_\epsilon \varphi = \partial_{(\mu_1} \epsilon_{\mu_2 \ldots \mu_s)}$
  - $F_{\mu_1 \ldots \mu_s} := \Box \varphi_{(\mu_1 \ldots \mu_s)} + \ldots \approx 0$
- *Lopatin & Vasiliev, 1988*
  Generalisation of linearised frame gravity
  - $\omega^{(s,t)} = \omega_{a_1 \ldots a_{s-1}, b_1 \ldots b_t}$
  - $\delta_\epsilon \omega = D_\omega \epsilon$
  - $D_\omega \omega^{(s,t)} \approx \delta_{t,s-1} hhC$
- *Fradkin & Vasiliev, 1987*
  Cubic and quartic actions around AdS_4
- *Vasiliev, 1989*
  Non-linear equations of motions

Vasiliev's formalism

- Introduce a Weyl algebra to build generating functions

$$Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot{\alpha}}) \qquad [Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha}\underline{\beta}}$$

$$M_{ab} = -\frac{1}{8}\left((\sigma_{ab})^{\alpha\beta} y_\alpha y_\beta + (\bar{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}}\bar{y}_{\dot{\beta}}\right)$$

$$P_a = -\frac{\lambda}{4}(\sigma_a)^{\alpha\dot{\alpha}} y_\alpha \bar{y}_{\dot{\alpha}}$$

- Another Weyl algebra

$$Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}}) \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha}\underline{\beta}}$$

- Normal ordered $\star$-product

$$[Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0 \qquad Y_{\underline{\alpha}} \star Z_{\underline{\beta}} = Y_{\underline{\alpha}} Z_{\underline{\beta}} - iC_{\underline{\alpha}\underline{\beta}}$$

- Klein operators $\hat{\kappa}$ and $\hat{\bar{\kappa}}$

$$\hat{\kappa} \star \hat{f}(x, z, -\bar{z}, y, \bar{y}) = \hat{f}(x, -z, -\bar{z}, -y, \bar{y}) \star \hat{\kappa} \qquad \hat{\kappa} \star \hat{\kappa} = 1$$

Vasiliev's bosonic model

- Master Fields on $\mathcal{X}_4 \times \mathcal{Z}_4 \times \mathcal{Y}_4$

$$\widehat{A}(Z,Y|x) = \mathrm{d}x^\mu \widehat{A_\mu} + \mathrm{d}Z^{\underline{\alpha}} \widehat{A_{\underline{\alpha}}} \qquad \hat{\Phi}(Z,Y|x)$$

$$\widehat{A} \star (\hat{\kappa}\hat{\bar{\kappa}}) = (\hat{\kappa}\hat{\bar{\kappa}}) \star \widehat{A} \qquad \hat{\Phi} \star (\hat{\kappa}\hat{\bar{\kappa}}) = (\hat{\kappa}\hat{\bar{\kappa}}) \star \hat{\Phi}$$

- Vasiliev equations

$$\widehat{\mathrm{d}}\widehat{A} + \widehat{A} \star \widehat{A} - \frac{i}{4}\left(e^{i\theta}\,\widehat{\Phi} \star \hat{\kappa}\,\mathrm{d}z^\alpha \mathrm{d}z_\alpha + e^{-i\theta}\,\widehat{\Phi} \star \hat{\bar{\kappa}}\,\mathrm{d}\bar{z}^{\dot{\alpha}}\mathrm{d}\bar{z}_{\dot{\alpha}}\right) = 0$$

$$\widehat{\mathrm{d}}\widehat{\Phi} + \widehat{A} \star \widehat{\Phi} - \widehat{\Phi} \star \hat{\kappa} \star \widehat{A} \star \hat{\kappa} = 0$$

- The system is Cartan integrable
- It has a gauge symmetry for $\hat{\epsilon}(Z,Y|x)$

$$\delta_{\hat{\epsilon}}\widehat{A} = \widehat{\mathrm{d}}\hat{\epsilon} + [\widehat{A}, \hat{\epsilon}]_\star \qquad \delta_{\hat{\epsilon}}\hat{\Phi} = -\hat{\epsilon} \star \hat{\Phi} + \hat{\Phi} \star \hat{\kappa} \star \epsilon \star \hat{\kappa}$$

Soltuions

- Solution AdS_4

$$\hat{\Phi}^{(0)} = 0 \qquad \widehat{A}^{(0)} = \mathrm{d}x^{\mu}\Omega_{\mu}(Y|x)$$

- First order in perturbation theory
  - $\Phi(Y)$
  - Solve everything on extended
  - *Vasiliev, 1989* Setting $Z = 0$ at the end gives the Central On Mass Shell Theorem
- *Boulanger, Kessel, Skvorstsov & Taronna 2016*
  Second order in perturbation theory : Highly non-local vertices
- Various exact solutions
  Sezgin&Sundell, 2005 : Inflaton-like
  Didenko&Vasiliev, 2009 : Black-hole-like
  Iazeolla & Sundell, 2011 : 6 families of solutions

Observables in Vasiliev's bosonic model

- The gauge symmetry is of the Yang-Mills form
  Ambjørn, Makeenko, Nishimura & Szabo, 2000
  Gross, Hashimoto & Ithzaki, 2000
- Wilson Lines

$$W_{\mathcal{C}}(x, Z; Y) := P \exp \int_0^1 \mathrm{d}\sigma \left(\dot{\xi}^{\mu}(\sigma) \widehat{A}_{\mu}(\sigma) + \dot{\xi}^{\underline{\alpha}}(\sigma) \widehat{A}_{\underline{\alpha}}(\sigma) \right)$$

- Observables from Wilson Line

$$\xi^{\mu}(0) = \xi^{\mu}(1) = 0 \quad \xi^{\underline{\alpha}}(0) = 0 \quad \xi^{\underline{\alpha}}(1) = 2M^{\underline{\alpha}} = 2\, C^{\underline{\alpha}\underline{\beta}} M_{\underline{\beta}}$$

$$\widetilde{O}_{\mathcal{C}}\,(M|x) := \int \mathrm{d}^4 Z\; \mathrm{d}^4 Y\; \left[\hat{O}(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \star e^{iMZ} \right]$$

- Observables from straight Wilson Lines

$$L_{2M} : [0,1] \to \mathcal{X}_4 \times \mathcal{Z}_4 : \sigma \to (0, 2\sigma M^{\underline{\alpha}})$$

Observables in Vasiliev's bosonic model

- Generic adjoint element

$$\hat{O}_{n_0,t;\underline{\alpha}(K)} := (\Phi \star \hat{\kappa})^{\star n_0} \star \left(\hat{\kappa}\hat{\bar{\kappa}}\right)^{\star t} \star \left(\widehat{S}_{\underline{\alpha}}\right)^{\star K}$$

  with deformed oscillator $\widehat{S}_{\underline{\alpha}} := Z_{\underline{\alpha}} - 2i\widehat{A}_{\underline{\alpha}}$
- Straight Wilson Line

$$\exp_{\star}\left(iM\widehat{S}\right) = W_{\mathcal{C}}(x, Z; Y) \star \exp(iMZ)$$

- Observables

$$\mathcal{I}_{n_0,t}(M) = \int d^4Z\, d^4Y\ (\Phi \star \hat{\kappa})^{\star n_0} \star \left(\hat{\kappa}\hat{\bar{\kappa}}\right)^{\star t} \star \exp_{\star}\left(iM\widehat{S}\right)$$

- *Colombo & Sundell, 2012*
  Zero-form charges, proposed as building blocks for free energy

Pre-amplitudes

- Leading order zero-form charges

$$\mathcal{I}^{(n_0)}_{n_0,t}(M) = \int \mathrm{d}^4 Z\ \mathrm{d}^4 Y\ \ (\Phi \star \hat{\kappa})^{\star n_0} \star \left(\hat{\kappa}\hat{\bar{\kappa}}\right)^{\star t} \star e^{i\mu z} \star e^{-i\bar{\mu}\bar{z}}$$

- Quasi-amplitudes

$$\mathcal{Q}_{n_0,t}(\Phi_i|M) := \int \mathrm{d}^4 Z\ \mathrm{d}^4 Y \sum_{\mathrm{perm.}\Phi_j} \mathop{\bigstar}_{i=1}^{n_0} (\Phi_i \star \hat{\kappa}) \star \left(\hat{\kappa}\hat{\bar{\kappa}}\right)^{\star t} \star e^{i\mu z} \star e^{-i\bar{\mu}\bar{z}}$$

- Pre-amplitudes

$$\begin{aligned}\mathcal{A}_{n_0,t}(\Phi_i|M) :&= \int \mathrm{d}^4 Z\ \mathrm{d}^4 Y\ \mathop{\bigstar}_{i=1}^{n_0} (\Phi_i \star \hat{\kappa}) \star \left(\hat{\kappa}\hat{\bar{\kappa}}\right)^{\star t} \star e^{i\mu z} \star e^{-i\bar{\mu}\bar{z}} \\ &= \delta^2(\mu)^{1-e}\delta^2(\bar{\mu})^{1-t}\mathcal{A}_{n_0,t}(\Phi_i)\end{aligned}$$

- Smearing function

$$\mathcal{A}^{\mathcal{V}}_{n_0,t}(\Phi_i) = \int \mathrm{d}^4 M\ \tilde{\mathcal{V}}(M)\mathcal{A}_{n_0,t}(\Phi_i|M) = \tilde{\mathcal{V}}_{n_0,t}\,\mathcal{A}_{n_0,t}(\Phi_i)$$

Correlators from zero-form charges

- *Giombi & Yin, 2009*
  Bulk to boundary propagator for master field Φ, with Neumann b.c.

$$\mathcal{K}_i(x_0, x_i, \chi_i|Y) := K_i e^{iy\Sigma_i\bar{y}} \sum_{\sigma_i=\pm 1} \left(e^{i\theta} e^{i\sigma_i \bar{\nu}_i \bar{\Sigma}_i y} + e^{-i\theta} e^{i\sigma_i \nu_i \Sigma_i \bar{y}} \right)$$

- Plugging them in the pre-amplitudes gives

$$\mathcal{A}_{n_0,t}\left(\mathcal{K}_i\right) = \beta_{n_0,t} \exp\left(-\frac{i}{4}\sum_{i=1}^{n_0} Q_i\right)\left(\prod_{i=1}^{n_0}\frac{1}{|x_{i,i+1}|}\right)$$

$$\times\left((\cos\theta)^{n_0}\prod_{i=1}^{n_0}\cos\left(\tfrac{1}{2}P_{i,i+1}\right) - (-1)^t(\sin\theta)^{n_0}\prod_{i=1}^{n_0}\sin\left(\tfrac{1}{2}P_{i,i+1}\right)\right)$$

$$P_{i,i+1} = \chi_i\,\sigma^r\,\check{x}_{i,i+1}\,\chi_{i+1} \qquad Q_i = \chi_i\,\sigma^r\,\left(\check{x}_{i,i+1} - \check{x}_{i,i-1}\right)\,\chi_i$$

- Consistent with *Didenko & Skvortsov, 2012*

Free U(N) model : conserved currents

- Propagators

$$\langle \phi^i(x)\phi^j(y)\rangle = 0 = \langle \phi^*_i(x)\phi^*_j(y)\rangle \quad \langle \phi^i(x)\phi^*_j(y)\rangle = c_1 \frac{\delta^i{}_j}{|x-y|^{d-2}}$$

- *Craigie, Dobrev & Todorov 1985, Giombi & Yin 2009*
  Higher-spin currents :

$$\sum_{s=0}^{\infty} a_s J_{\mu(s)}(x)\,(\epsilon^\mu)^s = J(x,\epsilon) = \phi^*_i(x) f\left(\epsilon, \overleftarrow{\partial}, \overrightarrow{\partial}\right)\phi^i(x)$$

  Conserved and traceless :

$$\partial^2_\epsilon f(\epsilon,u,v) = 0 \qquad \partial_\epsilon\cdot(\partial_u+\partial_v)|_{u^2=v^2=0}$$

$$f(u,v,\epsilon) = \sum_{s,k} b_s \binom{s}{k} \frac{\Gamma(s+\frac{d-2}{2})\Gamma(\frac{d-2}{2})}{\Gamma(k+\frac{d-2}{2})\Gamma(s-k+\frac{d-2}{2})}(-\epsilon\cdot u)^k(\epsilon\cdot v)^{s-k}$$

Free U(N)

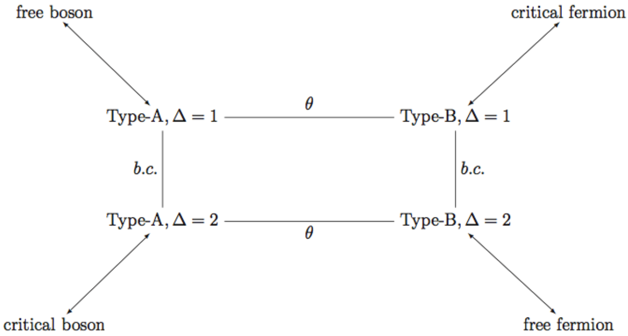
- Polarisation vector $(\chi_i)_\alpha\,(\bar{\chi}_i)_{\dot{\alpha}} = \epsilon_i^\mu\,(\sigma_\mu)_{\alpha\dot{\alpha}}$
- A single Wick contraction yields

$$\langle J_1, \ldots, J_{n_0}\rangle_{\text{cyclic}} = \prod_{i=1}^{n_0} \exp\left(-\gamma Q_i\right) \sum_{c_i} \frac{\left(\gamma P_{i,i+1}\right)^{2c_i}}{c_i!\,\Gamma(c_i + \frac{d-2}{2})}\,|x_{i,i+1}|^{2-d}$$

- Consistent with *Sleight & Taronna 2016, Gelfond & Vasiliev 2016*
- The results match on both sides
- The truncation to the $O(N)$ is equivalent to the minimal bosonic projection
- The equivalence holds at the level of cyclic structures

HS/CFT Dualities

- Sezgin & Sundell, 2002
  Klebanov & Polyakov, 2002



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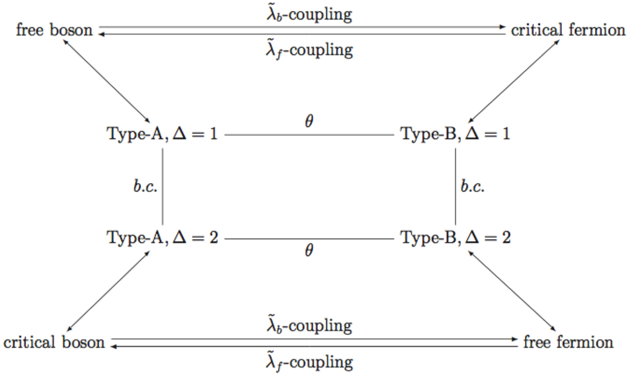

- *Sezgin, Skvortsov & Zhu 2017*

Scope

- Beyond linear order
- Different boundary conditions
- Parity-breaking term
- Exact solution
- $(d+1)$-dimensional Vasiliev's theory
- 3 dimensional Prokushkin-Vasiliev theory
- Supersymmetry

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Thank you for your attention !

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