Low Q^2 Boundary Conditions for DGLAP Equations Dictated by Quantum Statistical Mechanics Functions

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Summary

1. Phenomenological motivation for the Quantum Statistical Parton Model
2. Consequences of the Parton Model sum rules, the QCD equilibrium conditions and of the diffractive contributions
3. Parton Distributions proposed in 2002 by C. Bourrely, FB and J. Soffer and the extension to the transverse degrees of freedom
4. Comparison with the HERA fit and NNPDF for the light fermions and gluons distributions
5. Comparison with the standard parametrisation
6. Conclusion

Main content of this talk

Phenomenological facts:

1. Role of Pauli principle to account for the isospin asymmetry in proton sea

$$\bar{d}(x) > \bar{u}(x)$$

x is the proton momentum fraction

→ Proton Parton Distribution Functions seems to obey to quantum statistical mechanics

2. Quantum statistical mechanics implies that the fermion parton distributions are the product of Fermi Dirac functions of the variables, which appear in the sum rules for the longitudinal momentum and the transverse energy sum rules.

The equilibrium for the elementary QCD processes:

A. Relates the "potentials" of the valence partons and their antiparticles

B. Fix a Planck distribution for the gluons

C. For the low x behaviour of the structure functions an additional contribution of a diffractive isoscalar unpolarized term is required, that correspond to the presence of an infinite number of partons

Main content of this talk

3. The statistical approach has a successful predictive power

- for the isospin and spin asymmetries of the sea

$$\bar{d}(x) > \bar{u}(x), \quad \Delta\bar{u}(x) > 0 > \Delta\bar{d}(x)$$

- in decoupling the contributions of the valence partons and their antiparticles

- in describing the x dependance of the ratios

$$\frac{F_2^n(x)}{F_2^p(x)}, \quad \frac{\Delta u(x)}{u(x)}, \quad \frac{\Delta d(x)}{d(x)}$$

- also the Boltzmann behaviour exp(-x/$\bar{x}$) for x larger than the higher potential, $X_{(u\uparrow)}$ is in good agreement with experiment

Main content of this talk

4. The parametrization implied by the statistical approach has several advantages with respect to the standard ones : $Ax^B(1-x)^C P(x)$

    - It describes at the same time the unpolarized and polarized distributions

    - It relates the distributions of the valence partons and of their antiparticles as a consequence of the QCD equilibrium conditions

    - Gives a better information for the regions, where the experimental knowledge is scarce.

Phenomenological motivations for the Statistical Parton Distributions

- The isospin asymmetry in the sea of the proton

$$\bar{d}(x) > \bar{u}(x)$$

advocated many years ago by Niegawa and Sisiki and by Feynman and Field as a consequence of Pauli principle and confirmed by the defect in the Gottfried sum rule and by the larger Drell-Yan production of muon pairs in pn scattering than in pp scattering.

- The correlation between the first moments of the valence partons and the shapes of their x distributions is the one expected for a quantum gas:

Broader shapes for higher first moments

- the dramatic decrease at high x of the ratio $\frac{F_2^n(x)}{F_2^n(x)}$ as a consequence of a similar behaviour of the ratio $\frac{d(x)}{u(x)}$
- the increasing with x of the positive ratio $\frac{\Delta u(x)}{u(x)}$
- and the decreasing with x of the negative ratio $\frac{\Delta d(x)}{d(x)}$

Variables to be used for the Statistical Parton Distributions

- the usual choice of the energy as the variable to be used in statistical mechanics follows from its appearing in the constraint which fixes the total available energy

$$\Sigma n_i \varepsilon_i = E$$

- The resulting Boltzmann expression

$$e^{\left(-\frac{\varepsilon_i}{KT}\right)}$$

- is modified by quantum statistics into the Bose-Einstein and Fermi-Dirac expressions

$$\frac{1}{e^{\left(-\frac{\epsilon_i - \epsilon_0}{KT}\right)} \pm 1}$$

The proper variable for the Statistical Parton Distributions

- The role of Pauli principle suggests to write Fermi-Dirac functions for the quarks in the variable x, which is the one appearing in the parton model sum rules for the proton

$$\Sigma_i \int_0^1 x p_i(x) = 1$$

$$\int_0^1 [u(x) - \bar{u}(x)] dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1$$

$$\int_0^1 [\Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x)] = \frac{G_A}{G_V} = 1.26$$

- Remember that the usual choice of the energy as the variable appearing in statistical mechanics follows from its appearing in the constraint, which fixes the total energy available

The “Potentials” and the “Temperature”

- We write the Fermi-Dirac expressions for the valence partons defined by their flavor (u or d) and spin along the proton momentum

$$\frac{1}{e^{\frac{(x-X_q)}{\bar{x}}} \pm 1}$$

- where x and X_q play the role of the ”temperature” and the ”potentials” respectively and

$$q = u^{\uparrow}, d^{\downarrow}, u^{\downarrow}, d^{\uparrow}$$

- we expect decreasing values for the “potentials”

$$X_{u^{\uparrow}},\ X_{d^{\downarrow}},\ X_{u^{\downarrow}},\ X_{d^{\uparrow}}$$

The Melosh transformation

- The transverse degrees of freedom of the quarks in the hadrons play an important role, since they account for the generator of the transformation from constituent to current quarks in the infinite p_z frame found at CERN in March 1970

$$Z = (\vec{W} \times \vec{M})_z$$

with W a SU(3) singlet of the adjoint representation (35) of SU(6) and M a vector with respect to the orbital momentum L

- In 1973 Melosh wrote the Wigner rotation of the spin of the quarks for its projection along the momenta of the quark and the containing hadron, the angle, for which the tangent is

$$\frac{(\vec{\sigma} \times \vec{p})_z}{p_0 + p_z + m}$$

The Transverse Energy sum rule

- The transverse distributions have been fixed by a sum rule for the transverse energy, defined as the difference between the energy and the momentum. For the initial hadron it is given by

$$P_0 - P_z$$

approximately equal for large P_z to $\dfrac{M^2}{2P_z}$

- For a massless parton, which carries the fraction x of the hadron momentum and with transverse momentum p_T the transverse energy is given by

$$\frac{p_T^2}{p_z + \sqrt{p_z^2 + p_T^2}} = \frac{p_T^2}{P_z(x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}})}$$

- By multiplying by $2P_z$ we get the sum rule with M^2 in the right hand side. By taking P_z the momentum of the initial hadron in the frame of the final hadrons, one gets (neglecting terms in $(xM)^2$)

$$P_z^2 = \frac{Q^2}{4x(1-x)}$$

The Parton Trasverse Distributions

- This implies the following dependance on x and p^2_T for the non- diffractive part of x q(x)

$$\frac{A' x^{b-1}}{(\mu)^2 f(x, X_q) g(x, p_T^2, Y_q)} \quad \begin{cases} f(x, X_q) = e^{\left(\frac{x - x_q}{\bar{x}}\right)} + 1 \\ g(x, p_T^2, X_q) = e^{\left(\frac{2P_T^2}{x + \sqrt{x^2 + \frac{p_T^2}{P_z^2}}} - Y_q\right)} + 1 \end{cases}$$

where Y_q is the ”transverse potential. With the transformation

$$P_T^2 = \frac{\mu^2 \eta (x + \sqrt{x^2 + \frac{p^2}{P_z^2}})}{2}$$

we obtain the integral in η of $\frac{1 + \frac{2\eta(\mu)^2(1-x)}{Q^2}}{e^{(\eta - Y_q)} + 1}$ which gives rise to

$$[\ln(1 + e^{Y_q}) + \frac{2(1-x)\mu^2}{Q^2} \mathrm{Li}_2(-\mathrm{e}^{\mathrm{Y_q}})]$$

for the factor that multiplies $\frac{A' x^b}{e^{\left(\frac{x - X_q}{\bar{x}}\right)} + 1}$

The Parton Trasverse Distributions

- For the polarized distributions the Melosh-Wigner rotation implies the absence of the second term.

- The parameter $(\mu)^2$ will be fixed by the transverse energy sum rule to be 0.200 GeV^2 and is proportional to the denominator of the gaussian form assumed by the transverse distribution for p^2_T larger than $\mu^2 x Y_q$

The QCD equilibrium conditions

- By requiring equilibrium for the two elementary QCD processes, the emission of a gluon by a fermion parton and the conversion of the gluon into a $q\bar{q}$ pair with opposite helicity, one has the important consequence to have a vanishing potential for the gluons of both helicities and opposite values for the potentials for a quark and its antiparticle with opposite helicity

  - the Bose-Einstein expression for the gluons xG(x) turns into a Planck form

$$\frac{1}{e^{\left(\frac{x}{\bar{x}}\right)}-1} \quad \text{and} \quad \Delta G(x) = 0$$

  - quark and antiquark contributions in the e. m. DIS disentangled thanks to the relation:

$$X_q^h + X_{\bar{q}}^{-h} = 0$$

  - for the unpolarized distributions the disentangling is obtained from the obvious conditions

$$u - \bar{u} = 2$$

$$d - \bar{d} = 1$$

  - for the polarized distributions the equilibrium conditions allow to determine the polarization of the light antiquarks from the knowledge of the shapes of the valence quark distributions

The Diffractive Contribution

- At small x parton distribution is dominated by a diffractive contribution, probably a consequence of the gluon distribution, implying an infinite number of partons

$q_D(x)$ proportional to $x^{(-1.25)}$

- To be consistent with the parton sum rules

$$u - \bar{u} = 2$$

$$d - \bar{d} = 1$$

$$\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} = \frac{G_A}{G_V} = 1.26$$

the diffractive contribution should be the same for the particles and the antiparticles and should not contribute to the Bjorken sum rule

The Diffractive Contribution

- To reproduce data one had to modify the Fermi-Dirac function with the factor $A\tilde{X}_q x^b$ and add a diffractive contribution

$$\frac{A\tilde{X}_q x^b}{e^{\left(\frac{x-\tilde{X}_q}{\bar{x}}\right)}+1}+\frac{\tilde{A}x^{\tilde{b}}}{e^{\left(\frac{x}{\bar{x}}\right)}+1}$$

  - $\bar{x}$ plays the role of the “temperature”
  - $\tilde{X}_q$ is the potential of the parton depending on its flavor and helicity
  - diffractive contribution isoscalar and unpolarized to avoid an infinite contribution to the parton model sum rules (since $\tilde{b}$ = -0.25 is negative)

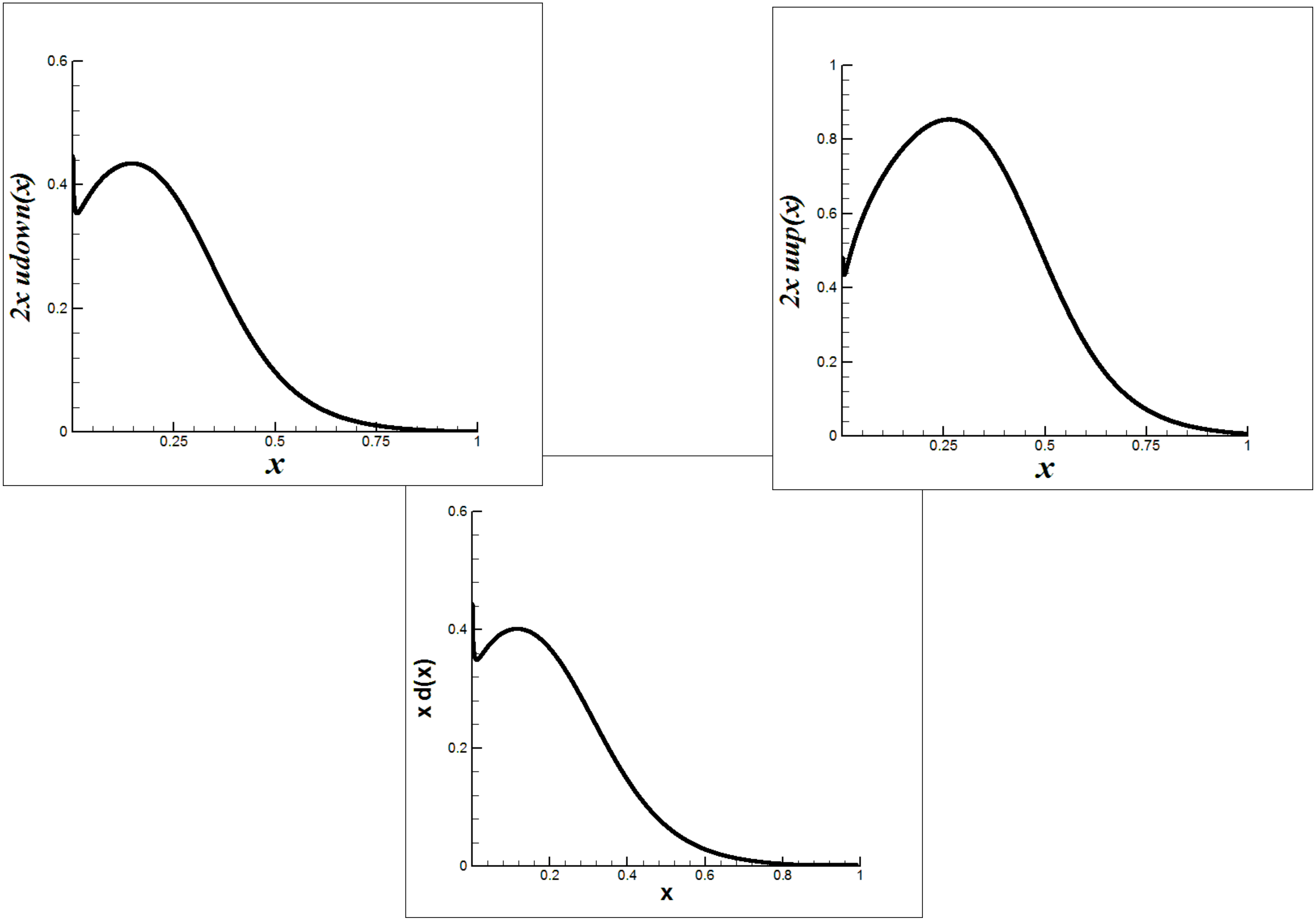
The Description of the Statistical parton Distributions and the Comparison with the HERA Fit for the Unpolarized Ddistributions

- Some years ago a joint analysis of the DIS data measured in the H_1 and ZEUS experiments has been performed to give the unpolarized parton distributions and Jacques Soffer immediately realized the similarity with the statistical distributions

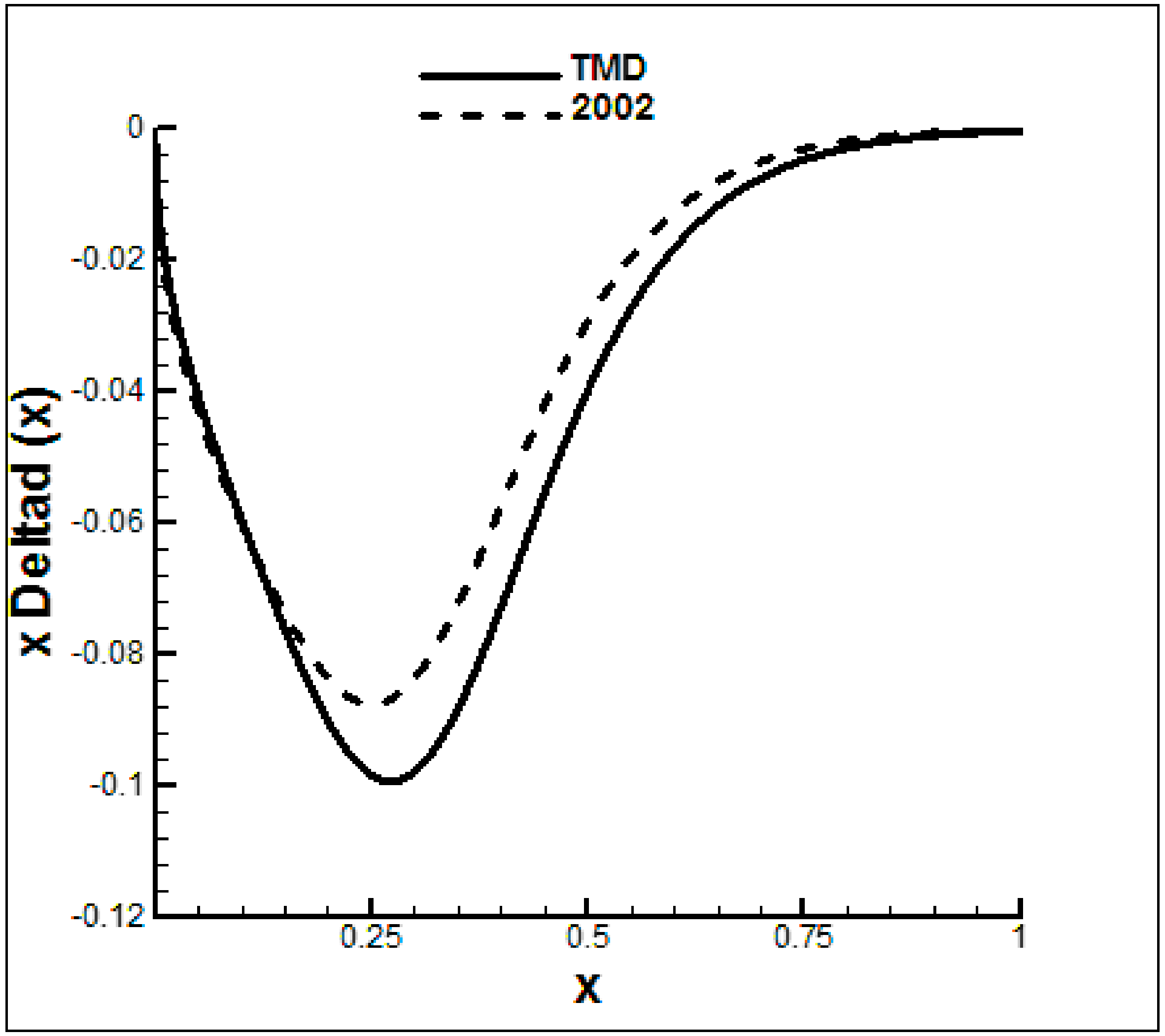
- To perform a check for the quantum statistical parton distributions, we determine the parameters introduced in order to reproduce the Hera result for the unpolarized distributions of the light parton fermions, while for the polarized ones we require to reproduce the expressions found in 2002, which have been successful to describe the polarized structure functions $g^{p,d,He^3}(x)$ and the production of $W^{\pm}$ weak bosons

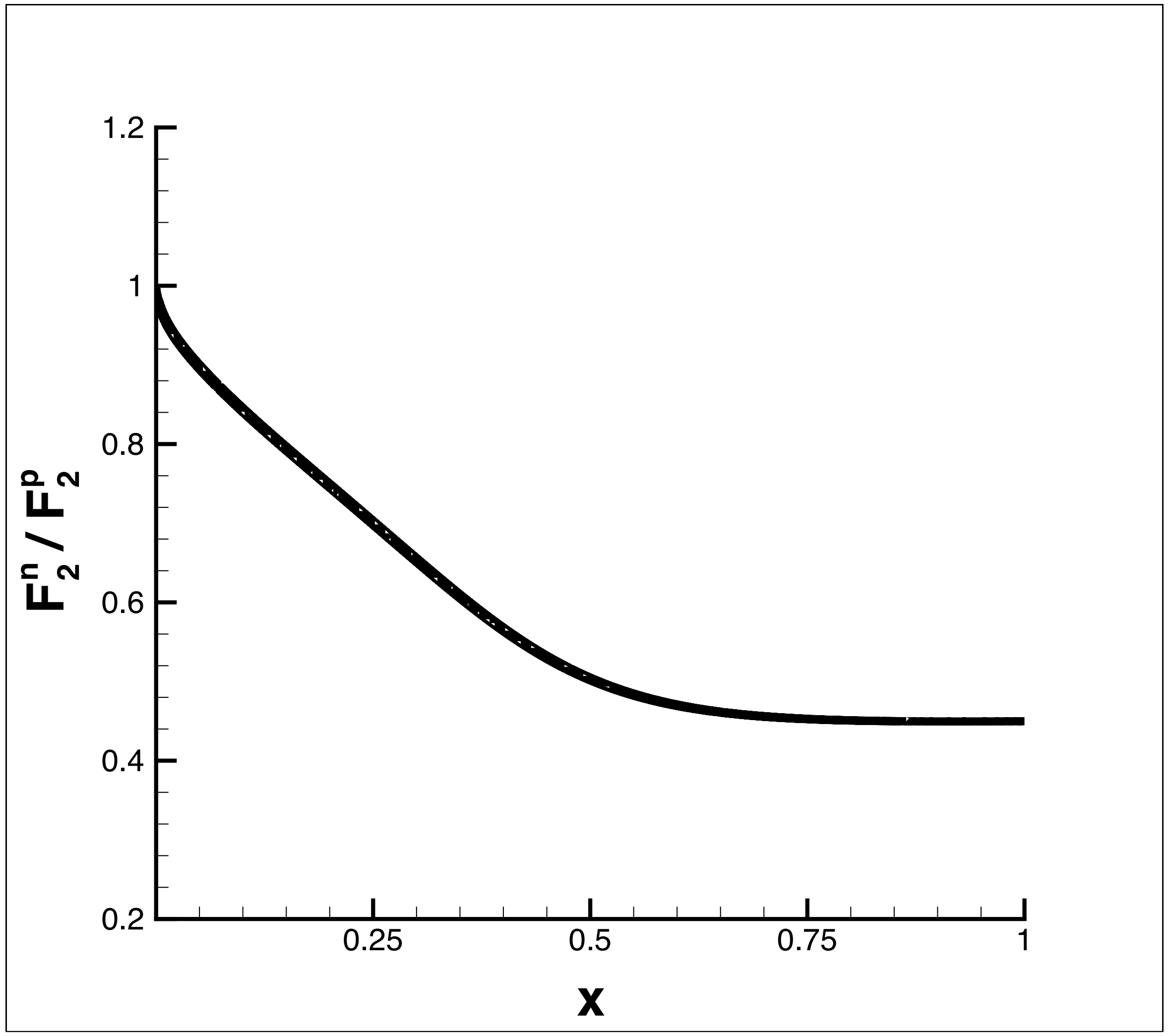
- The description of 2002 polarized distributions is shown in the following figures and the parameters are compared with the ones determined in that paper

The Description of the Statistical parton Distributions

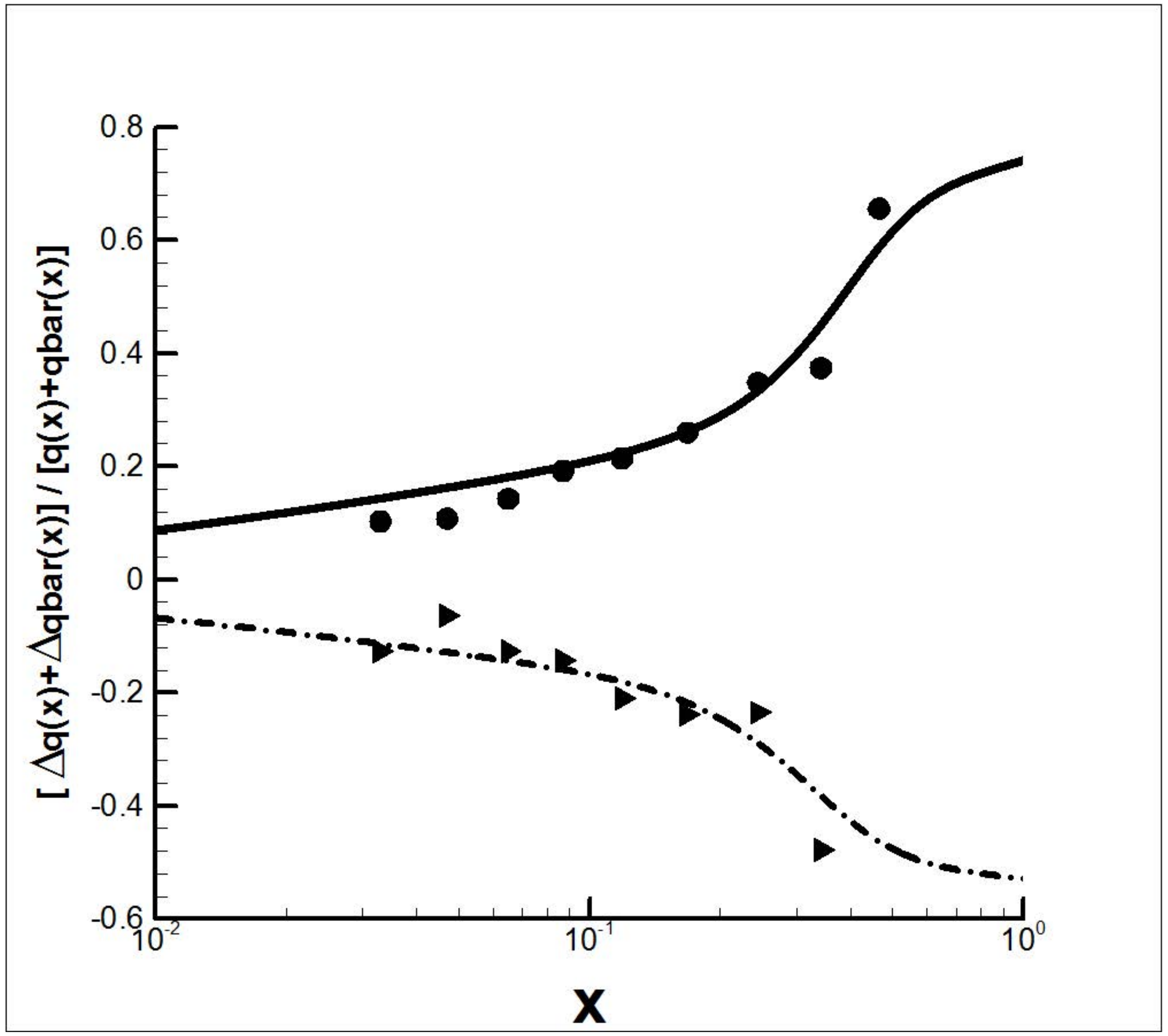


The Description of the Statistical parton Distributions





The Description of the Statistical parton Distributions



- Our results on $\dfrac{\Delta u(x)+\Delta\bar{u}(x)}{u(x)+\bar{u}(x)}$ (solid) and $\dfrac{\Delta d(x)+\Delta\bar{d}(x)}{d(x)+\bar{d}(x)}$ (dashed dot) at $Q^2 = 4(\text{GeV})^2$

  in comparison to the HERMES data at $Q^2 = 2.5\ \text{GeV}^2$

- The agreement is very good for both the comparisons for the fermions.

The Comparison of the Parameters in the 2002 paper with HERA fit

- From a selected choice of Deep Inelastic Scattering we have been able to determine the small number of parameters introduced, getting the following values:

	2002	2015
$\tilde{X}_{u\uparrow}$	0.46188	0.446
$\tilde{X}_{d\downarrow}$	0.30174	0.320
$\tilde{X}_{u\downarrow}$	0.29766	0.297
$\tilde{X}_{d\uparrow}$	0.22775	0.222
b	0.40962	0.43
$\tilde{A}$	0.08318	0.070
$\tilde{b}$	-0.25347	-0.240
$\bar{x}$	0.09907	0.102

- In the second column we report the values of the parameters obtained by demanding that our expressions reproduce the H_1–ZEUS fit

- The values of the parameters agree, according to an analysis by Claude Bourrely, within 2 maximizing the entropy defined by J. Cleymans and D. Worku in a 2012 paper

The Comparison with NNPDF

- A good test for the statistical parton distributions is provided by the comparison with NNPDF with the parameters fixed by the comparison with HERA
- It is instructive the comparison of the three square differences between statistical, NNPDF and Hera divide by the square of NNPDF result integrated on ranges of the x variable for u, d and gluons for unpolarized distributions:

Parton	Range	$\int_{x_1}^{x_2} \frac{[p_{st} - p_{NNPDF}]^2}{[p_{NNPDF}]^2}$	$\int_{x_1}^{x_2} \frac{[p_{HERA} - p_{NNPDF}]^2}{[p_{NNPDF}]^2}$	$\int_{x_1}^{x_2} \frac{[p_{st} - p_{HERA}]^2}{[p_{NNPDF}]^2}$
u	(0.0 - 0.2)	0.00177	0.00057	0.00356
u	(0.2 - 0.5)	0.00046	0.0001	0.00025
u	(0.5 - 0.7)	0.000077	0.00084	0.0013
u	(0.7 - 0.8)	0.0109	0.00057	0.00356
u	(0.8 - 0.9)	319	16.2	478
d	(0.0 - 0.2)	0.00116	0.0002	0.00078
d	(0.2 - 0.5)	0.00193	0.0067	0.0042
d	(0.5 - 0.6)	0.00048	0.008	0.012
d	(0.6 - 0.7)	0.018	0.0074	0.025
g	(0.0 - 0.7)	0.0675	0.29	0.54

The Comparison with NNPDF

- Interestingly enough for u in the range (0.5,0.8), for d in the range (0.2,0.7) and for gluons in the range (0,0.7) the agreement with NNPDF is better than the one of HERA and the one with HERA.

- As long as for the strong disagreement with both NNPDF and HERA for u above 0.8 one should say that at $Q^2 = 4GeV^2$, \$ $M'^2 = M^2 + Q^2 (1/x - 1)$ is < 1.9, just in the region of the Δ resonance, away from the deep inelastic regime and the large numbers come from the fast decrease at high x induced by the factor $(1 - x)^C$: the fact that the limit of the ratio d(x)/u(x) at large x supplied by the Boltzmann limit, $\exp\{-x/\bar{x}\}$ is in perfect agreement with the value found by Orwell, Accardi and Melnitchuk is a further point in favour of the hypothesis that the low Q^2 boundary condition for DGLAP equation is fixed by quantum statistical mechanics.

- As long as for the light sea the isospin and spin asymmetries are automatically predicted in sign and order oif magnitude from the QCD equilibrium conditions.

The Comparison with the Standard Form for Parton Distributions

- Despite the fact that x = 0 ($Q^2 = 0$) and the neighborough of x = 1 (elastic and resonance production) are not in the domain of DIS, the standard parametrization for parton distribution has the following form:

$$Ax^B(1-x)^C$$

with A, B and C fixed by the comparison with experiment for each parton distribution and a separate analysis for unpolarized and polarized distributions

- Sometimes to improve the agreement with data some polynomial factor is introduced

- Indeed the diffractive component has a singular power behaviour near x = 0, while the valence partons, which dominate the intermediate and the high x regions have a different (more soft) power behaviour at small x, while the positive value of C gives rise to a decrease with x and also to a different weight for the valence partons, 2 (u and d) for the unpolarized distributions and 4 if one considers also the polarized ones

The difference between the Statistical and the Standard Parton Distributions

- For the statistical distributions the decrease at high x is naturally explained by the Boltzmann behaviour of the parton distributions for x larger than the “potential” of each parton

$$\exp\left(\frac{-x}{\bar{x}}\right)$$

- The variation of the ratios between the different valence parton distributions

$$\frac{d(x)}{u(x)},\ \frac{\Delta u(x)}{u(x)}\ and\ \frac{\Delta d(x)}{d(x)}$$

is concentrated in the range between the lowest and highest potential

$$X_{d\uparrow},\ X_{u\uparrow}$$

while in the Boltzmann regime their ratios vary more slowly

- This behaviour is the opposite for the standard parametrization, for which the effect of the different exponents for the power $(1 - x)^{c}$ becomes more important as x approaches 1

Comparison with d(1)/u(1)

- The ratio $F_2^n(x)/F_2^p(x)$ at high x depends on the ratio d(x)/u(x) in the same region

- The difficult to obtain the neutron unpolarized structure function at high x is related to the Fermi motion of the two nucleons in the deuton, which makes very problematic to get it from the ones measured for the proton and for the deuton. So to get the ratio d(x)/u(x) in that region is not a trivial task. The small statistics and the choice of the standard parametrization give rise to a big uncertainty on that ratio. In the statistical approach the free parameters, from which that ratio depends, the "temperature" and the "longitudinal and transverse potentials"

  $$\bar{x},\ X_q\ and\ Y_q$$

  are fixed in regions, the intermediate x region (0.22,0.46), where the statistics is large and the systematic errors are small. The perfect agreement of the prediction for

  $$\frac{d(1)}{u(1)} = 0.22$$

  with the result of the careful analysis by Orwell, Accardi and Mecnitchouk is a good confirm for the statistical parton distributions

Disadvantages of the Standard Parametrization

- The form $Ax^B(1-x)^C P(x)$ for the different parton distributions has the disadvantage that the high x behaviour for each distribution is fixed by the exponent C, which comes out different for the different valence quarks with the consequence that the limit d(x)/u(x) for $x \to 1$ comes out or 0 or infinity. Indeed in the fit of the joined Hera-Zeus group parameter C is larger for u than for d, while for the sea it is still smaller in such a way that it dominates in that limit. To comply with the experimental behaviour of the ratio d(x)/u(x) they introduce for the parton u the ad-hoc factor $(1 + 9.7x^2)$.

The Gluon Distribution: THE PLANCK FORM

- The equilibrium conditions fix the "potentials" for the gluon to vanish for both helicities, which implies

$\Delta G(x) = 0$ and a Planck form: $$xG(x) = \frac{A_g x}{e^{\left(\frac{x}{\bar{x}}\right)} - 1}$$

where the exponent 1 for the power follows by the idea that the hadron is a black body cavity for the chromomagnetic radiation and A_g is fixed by the sum rule for the longitudinal momentum

- Indeed the fact that HERA data show that xG(x) is growing at small x for $Q^2 = 1.9(\mathrm{GeV})^2$ and decreasing at $Q^2 = 10(\mathrm{GeV}^2$ suggests that the Q^2, where it is stationary, will not be so different from $4(\mathrm{GeV})^2$

In fact $B_H(Q^2) = -0.0257$

The Comparison between Gluon Distributions

- The standard form

$$Ax^B(1-x)^C$$

implies that the decreasing at high x depends on the exponent C and gets faster at increasing x, while the Planck form, as soon as one can neglect the −1 in the denominator, has a more regular behaviour

$$e^{\left(\frac{-x}{\bar{x}}\right)}$$

- Since the gluon distribution in DIS has influence on the logarithmic scaling violation, a method to establish the degree of agreement of the Planck distribution with the experimental information obtained at HERA is to compare at $Q^2 = 4$:

$$\int_{0.0}^{0.2} xG(x)dx = 0.36 \qquad \text{with} \qquad \int_{0}^{0.2} \frac{A_g x}{e^{\left(\frac{x}{\bar{x}}\right)}-1}dx = 0.34$$

$$\int_{0.2}^{1} xG(x)dx = 0.05 \qquad \int_{0.2}^{1} \frac{A_g x}{e^{\left(\frac{x}{\bar{x}}\right)}-1}dx = 0.125$$

Standard or Planck ?

- The agreement is good for

$$\int_0^{0.2} xG(x)dx$$

  in the range, where most gluons are concentrated, while for x larger than 0.2 HERA gives a faster decrease

- Since for the fermion partons the decrease at high x is better described by the statistical distributions, it is legitimate to make the conjecture that the fast decrease at high x advocated by HERA is more a consequence of their parametrization than of the experimental evidence

Comparison with NNPDF and HERA: up

	$\int dx(\frac{xu_{TMD}-xu_{NNPDF}}{xu_{NNPDF}})^2$	$\int dx(\frac{xu_{HERA}-xu_{NNPDF}}{xu_{NNPDF}})^2$	$\int dx(\frac{xu_{TMD}-xu_{HERA}}{xu_{NNPDF}})^2$
0.00001-0.1	0.000719015	0.000554071	0.00238961
0.1-0.2	0.00104956	0.0000261622	0.00117524
0.2-0.3	0.000417635	0.0000460843	0.000193605
0.3-0.4	0.0000430725	$9.69007*10^{-6}$	0.0000161791
0.4-0.5	$1.32506*10^{-6}$	0.0000581102	0.0000438131
0.5-0.6	0.0000128613	0.0000501951	0.0000914226
0.6-0.7	0.0000643858	0.000790763	0.00121047
0.7-0.8	0.0108695	0.0160115	0.0527942
0.8-0.9	318.897	16.1839	478.302
0.00001-0.9	318.91	16.2015	478.36

Comparison with NNPDF and HERA: down

	$\int dx(\frac{xd_{TMD} - xd_{NNPDF}}{xd_{NNPDF}})^2$	$\int dx(\frac{xd_{HERA} - xd_{NNPDF}}{xd_{NNPDF}})^2$	$\int dx(\frac{xd_{TMD} - xd_{HERA}}{xd_{NNPDF}})^2$
0.00001-0.1	0.000935016	0.0000869746	0.000522174
0.1-0.2	0.00023197	0.000122482	0.000264989
0.2-0.3	0.000886122	0.00106942	0.0000129748
0.3-0.4	0.00093035	0.00220476	0.00037315
0.4-0.5	0.00011196	0.00343388	0.00384319
0.5-0.6	0.00048035	0.00806401	0.0122964
0.6-0.7	0.0179844	0.0745385	0.0248437
0.00001-0.7	0.0215601	0.0895201	0.0421566

Comparison with NNPDF and HERA: ubar

	$\int dx(\frac{x\bar{u}_{TMD} - x\bar{u}_{NNPDF}}{x\bar{u}_{NNPDF}})^2$	$\int dx(\frac{x\bar{u}_{HERA} - x\bar{u}_{NNPDF}}{x\bar{u}_{NNPDF}})^2$	$\int dx(\frac{x\bar{u}_{TMD} - x\bar{u}_{HERA}}{x\bar{u}_{NNPDF}})^2$
0.00001-0.1	0.00403639	0.000211	0.00287137
0.1-0.2	0.00508968	0.000418277	0.00736075
0.2-0.3	0.00660197	0.0055041	0.0240166
0.3-0.4	0.0119411	0.0112131	0.0462101
0.4-0.5	0.0222474	0.00533699	0.0475496
0.5-0.6	0.0363909	0.00570592	0.0190559
0.6-0.7	0.0512114	0.0666935	0.0032279
0.7-0.8	0.0629684	0.185068	0.032574
0.8-0.9	0.0665441	0.132565	0.0269948
0.00001-0.9	0.267031	0.412716	0.209861

Comparison with NNPDF and HERA: dbar

	$\int dx(\frac{x\bar{d}_{TMD} - x\bar{d}_{NNPDF}}{x\bar{d}_{NNPDF}})^2$	$\int dx(\frac{x\bar{d}_{HERA} - x\bar{d}_{NNPDF}}{x\bar{d}_{NNPDF}})^2$	$\int dx(\frac{x\bar{d}_{TMD} - x\bar{d}_{HERA}}{x\bar{d}_{NNPDF}})^2$
0.00001-0.1	0.00298472	0.00327952	0.000219856
0.1-0.2	0.00550765	0.00289295	0.000889416
0.2-0.3	0.00213423	0.0108324	0.0174773
0.3-0.4	0.0223143	0.359481	0.207696
0.00001-0.4	0.0329409	0.376486	0.226283

Comparison with NNPDF and HERA: gluon

	$\int dx(\frac{xg_{TMD} - xg_{NNPDF}}{xg_{NNPDF}})^2$	$\int dx(\frac{xg_{HERA} - xg_{NNPDF}}{xg_{NNPDF}})^2$	$\int dx(\frac{xg_{TMD} - xg_{HERA}}{xg_{NNPDF}})^2$
0.00001-0.1	0.00190835	0.000736242	0.00384873
0.1-0.2	0.00713221	0.000340814	0.0090403
0.2-0.3	0.0173979	0.00838282	0.0490799
0.3-0.4	0.0161694	0.0328795	0.0943154
0.4-0.5	0.00873125	0.063188	0.118073
0.5-0.6	0.0035346	0.0869633	0.125171
0.6-0.7	0.00393113	0.0982558	0.140749
0.7-0.8	0.120633	0.0882559	0.372319

Conclusion

1. The agreement with the Hera distributions with the form dictated by the quantum statistical approach for the fermion parton distributions is an impressive confirm of the validity of the proposal in the 2002 paper with the improved theoretical foundation achieved with the extension of the transverse degrees of freedom and with the consideration of the Melosh-Wigner rotation
2. The similarity of the values of the parameters with the ones found in the previous work supports the validity of the statistical approach
3. As long as the p_T dependance in the Boltzmann limit, neglecting the power dependance and with the gaussian approximation for the exponential we get the behaviour

$$\sqrt{p_T} e^{\frac{-2p_T}{\mu\sqrt{\bar{x}}}}$$

with an “effective temperature” 49MeV, smaller than the range proposed in the paper by Cleymans, Lykasov, Sorin and Teryaev, 120 – 150MeV , but the important quantum effect gives rise to a harder p_T distribution

Conclusion

4. The decrease at high x and the ratios between the different valence partons seem to be better described by the statistical distribution than by the standard distributions

$$Ax^B(1-x)^C$$

In fact the ratios change more fastly in the range (0.22, 0.46) than above 0.46

5. An attractive feature of the statistical model is that the parameters are fixed by regions of x, where there is a large statistics and small systematic errors, small x for the two parameters needed for the diffractive term, the intermediate region (0.22, 0.46) for the ones associated to the valence partons, which fix both the high x Boltzmann behaviour proportional to $e^{\left(\frac{-x}{\bar{x}}\right)}$ and the disentangling of the valence partons and their antiparticles
6. As long as for the gluons at high x of the Planck form is in better agreement with the "parametrization independent" of NNPDF than the standard parametrization $Ax^B(1-x)^C$ proposed by HERA

Conclusion

7. A crucial test will be provided by the measurement at high x of $\bar{d}(x)/\bar{u}(x)$, for which a previous experiment gave a weird behaviour abruptly decreasing based on uncertain data.

8. For the spin and isospin asymmetries of the sea, $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$ the result found by NNPDF appears very weird with changes of sign, while the Boltzmann behaviour predicted by the statistical model shows a more regular behaviour

Conclusion

9. At high x the Boltzmann behaviour seems to better reproduce the distributions than the commmonly used factors $(1-x)^c$ for the valence quarks and for their antiparticles as well as for the gluons

10. The comparison with HERA and NNPDF shows that, despite the free parameters of the statistical distributions have been fited to agrre with HERA, in the central region of x, in the region of the "potentials" of the valence partons, they often agree better with NNPDF than HERA. This shows once again that the parametrization of the statistical distributions, which has a theoretical foundation, is more suitable than the standard one to predict the distributions before a better experimental knowledge is achieved. Typical the case of the ratio $\bar{d}(x)/\bar{u}(x)$ where the more recent and precise experiment has confirmed the pattern predicted fifteen years ago