On the Cosmological Frame Problem

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Based on

– S. Karamitsos, D. Burns and A.P., Nucl. Phys. B907 (2016) 785

– S. Karamitsos and A.P., `arXiv:1706.07011`

Motivation:

Frame Problems in the History of Science (from WIKIPEDIA)

- **Geocentric versus Heliocentric System**

  Geocentric: Anaximander (6c BC), . . . , Plato (4c BC),
  Aristotle (3c BC), Ptolemy (2c AD), . . .

  Heliocentric: Aristarchus (3c BC), Seleucus (2c BC), . . . ,
  Kopernicus (15c AD), Kepler (16c AD), Galileo (16c AD), . . .

- **Absolute versus Relative/Local Inertial Frame in Gravitation**

  Absolute: Newton (17c BC), . . .

  Relative: Einstein (20c BC), . . .

- **Einstein versus Jordan Frame in Inflationary Cosmology** [this talk]

– Einstein versus Jordan Frame

Action in Enstein Frame: $S^{\rm EF}[g_{\mu\nu},\varphi] = \int_x \Big[-\frac{1}{2}M_P^2 R \,+\, \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\Big]$

Action in Jordan Frame: $S^{\rm JF}[\tilde{g}_{\mu\nu},\widetilde{\varphi}] = \int_x \Big[-\,\frac{1}{2}f(\widetilde{\varphi})\widetilde{R} \,+\, \frac{1}{2}(\partial_\mu\widetilde{\varphi})^2 - \widetilde{V}(\widetilde{\varphi})\Big]$

Frame equivalence $\quad\Longrightarrow\quad S^{\rm JF}[\tilde{g}_{\mu\nu},\widetilde{\varphi}] \;=\; S^{\rm EF}[g_{\mu\nu},\varphi]$ [R. H. Dicke '62]

– Einstein versus Jordan Frame

Action in Enstein Frame: $S^{\rm EF}[g_{\mu\nu},\varphi] = \int_x \left[-\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right]$

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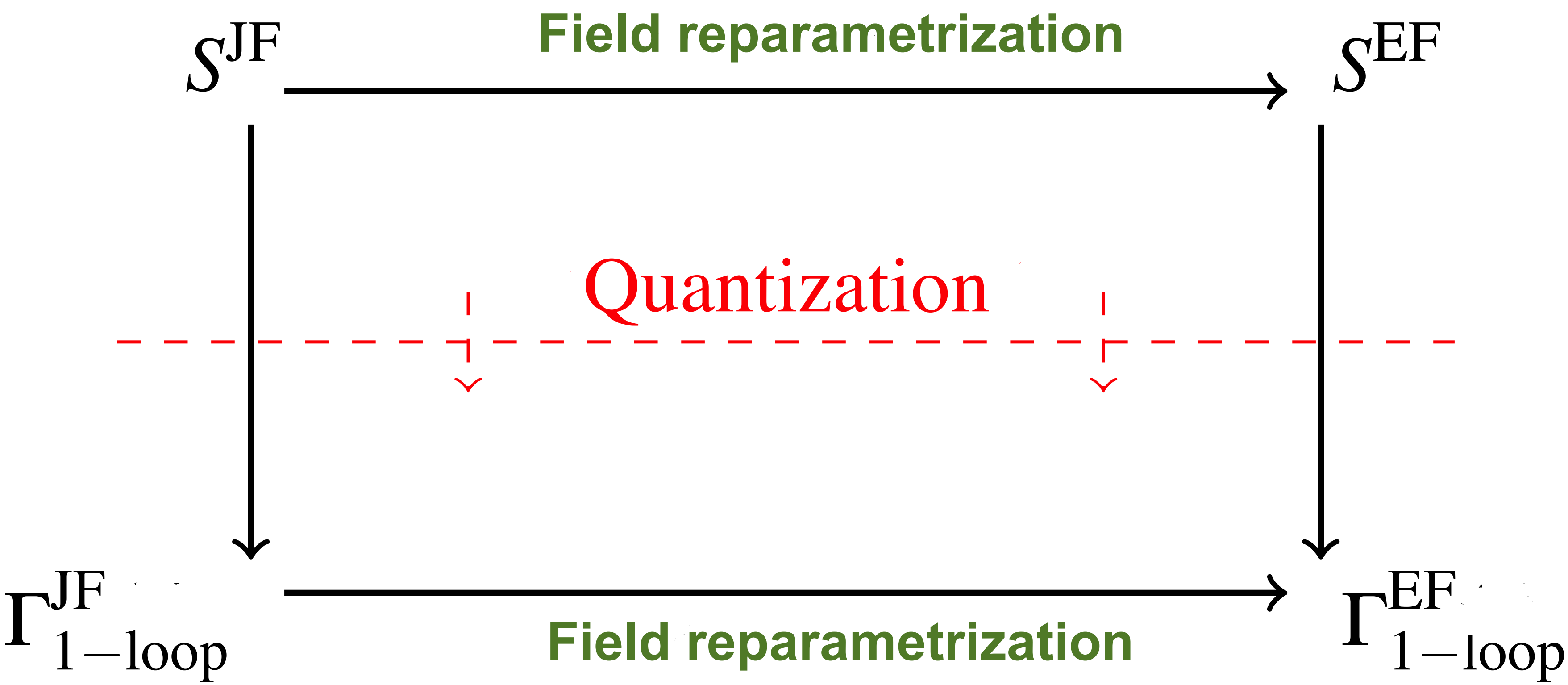
Frame equivalence $\Longrightarrow$ $S^{\rm JF}[\tilde{g}_{\mu\nu},\widetilde{\varphi}] = S^{\rm EF}[g_{\mu\nu},\varphi]$ [R. H. Dicke '62]



$\Gamma^{\rm JF}_{1-\rm loop}[\tilde{g}_{\mu\nu},\widetilde{\varphi}] \neq \Gamma^{\rm EF}_{1-\rm loop}[g_{\mu\nu},\varphi]$: Effective action is **frame dependent**.

– Partial list of references

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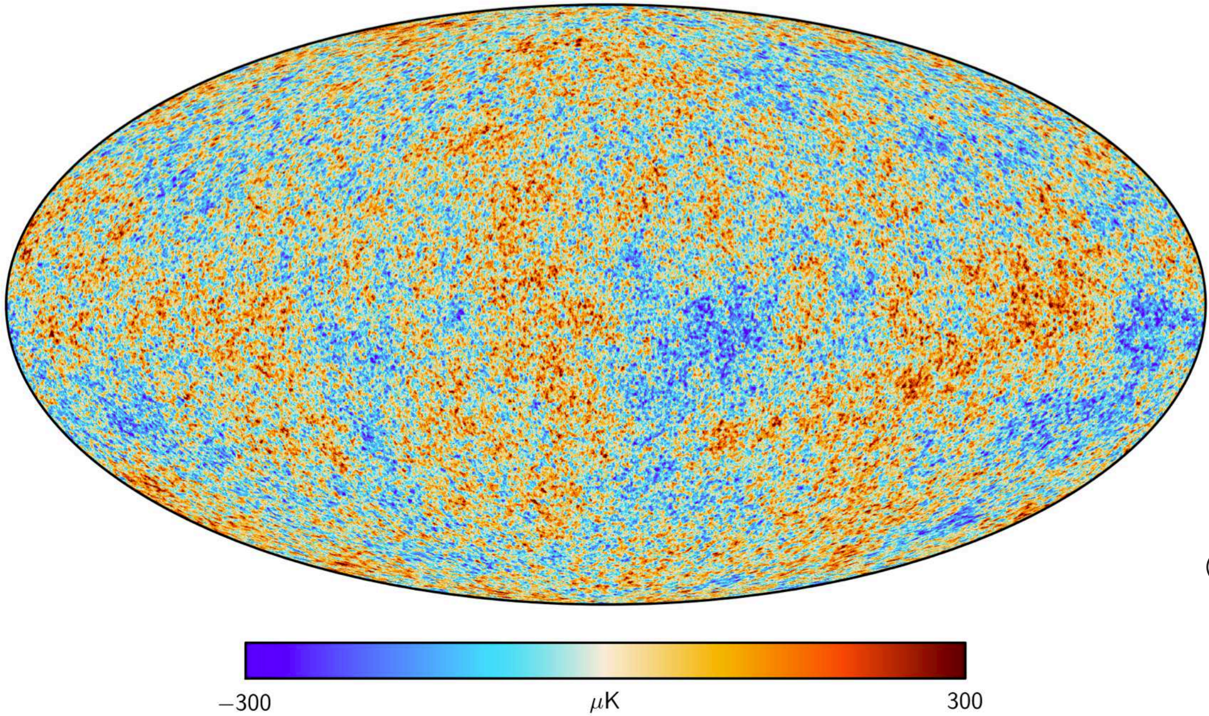
Outline:

- A Profound View of Our Cosmos
- Frame Covariance in Curved Field Space
- Frame Covariant Quantum Perturbations
- Frame Invariant Cosmological Observables
- Two-Field Models
- Beyond the Tree Approximation
- Conclusions and Future Directions

• A Profound View of Our Cosmos

Standard Big-Bang Cosmology cannot naturally explain the *flatness* of our Universe and the largeness of its *causal horizon*.



$$\frac{\delta T}{T} \lesssim 10^{-4}$$

Inflation provides a natural solution to these problems of our Cosmos, as well as a quantitative framework for explaining the CMB anisotropies.

Interesting class of viable inflationary models: Scalar-Curvature Multifield Theories, including Higgs Inflation.

• Frame Covariance in Curved Field Space

– Scalar-Curvature Multifield Theories in Jordan Frame

$$S \;=\; \int d^4x\sqrt{-g}\,\left[-\frac{f(\varphi)}{2}R \;+\; \frac{1}{2}k_{AB}(\varphi)\,g^{\mu\nu}(\nabla_\mu\varphi^A)(\nabla_\nu\varphi^B) \;-\; V(\varphi)\right],$$

$S = S\big[g_{\mu\nu}, \varphi, f(\varphi), k(\varphi), V(\varphi)\big]$: *classical action*,

$f(\varphi),\; k_{AB}(\varphi), V(\varphi)$: *model functions*.

Frame transformations consist of

(i) *Conformal transformations*

$$g_{\mu\nu} \;\to\; \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\ , \qquad\qquad \varphi^A \;\to\; \widetilde{\varphi}^A = \Omega^{-1}\varphi^A$$

(ii) *Field reparametrizations*

$$\varphi^A \;\to\; \varphi^{\widetilde{A}} = \varphi^{\widetilde{A}}(\varphi)\ , \qquad\qquad \frac{d\widetilde{\varphi}^{\widetilde{A}}}{d\varphi^B} \;=\; \Omega^{-1}K^{\widetilde{A}}_{\ B}(\varphi)$$

Under a frame transformation: $\varphi^A \mapsto \widetilde{\varphi}^{\widetilde{A}} = \widetilde{\varphi}^{\widetilde{A}}(\boldsymbol{\varphi})$, the model functions transform as

$$\begin{aligned}
\tilde{f}(\widetilde{\boldsymbol{\varphi}}) &= \Omega^{-2} f(\boldsymbol{\varphi})\,,\\
\tilde{k}_{\widetilde{A}\widetilde{B}}(\widetilde{\boldsymbol{\varphi}}) &= \Big[k_{AB} - 6f(\ln\Omega)_{,A}(\ln\Omega)_{,B} + 3f_{,A}(\ln\Omega)_{,B} + 3(\ln\Omega)_{,A}f_{,B}\Big]K^A_{\ \widetilde{A}}K^B_{\ \widetilde{B}}\,,\\
\widetilde{V}(\widetilde{\boldsymbol{\varphi}}) &= \Omega^{-4} V(\boldsymbol{\varphi})\,.
\end{aligned}$$

Functional form of the classical action S remains invariant:

$$\boxed{S[\tilde{g}_{\mu\nu}, \widetilde{\boldsymbol{\varphi}}, \tilde{f}(\widetilde{\boldsymbol{\varphi}}), \tilde{k}(\widetilde{\boldsymbol{\varphi}}), \widetilde{V}(\widetilde{\boldsymbol{\varphi}})] \;=\; S[g_{\mu\nu}, \boldsymbol{\varphi}, f(\boldsymbol{\varphi}), k(\boldsymbol{\varphi}), V(\boldsymbol{\varphi})]}$$

Models related by a frame transformation define an *equivalence class*

$\Longrightarrow$ Our starting point for *Frame Covariance*.

Introduce *Field Space Metric*:

$$G_{AB} \equiv \frac{k_{AB}}{f} + \frac{3}{2}\,\frac{f_{,A}f_{,B}}{f^2}\,.$$

Frame Tensors $X^{A_1\ldots A_p}_{B_1\ldots B_q}$ in Field Space:

$$\widetilde{X}^{\widetilde{A}_1\ldots\widetilde{A}_p}_{\widetilde{B}_1\ldots\widetilde{B}_q} = \Omega^{-(w_X+p-q)}\left(K^{\widetilde{A}_1}_{\ A_1}\ldots K^{\widetilde{A}_p}_{\ A_p}\right) X^{A_1\ldots A_p}_{B_1\ldots B_q} \left(K^{B_1}_{\ \widetilde{B}_1}\ldots K^{B_q}_{\ \widetilde{B}_q}\right).$$

X	conformal weight (w_X)	scaling dimension (d_X)
dx^μ	0	0
$d\varphi^A$	0	1
$d\varphi_A$	0	-1
$g_{\mu\nu}$	-2	-2
$g^{\mu\nu}$	2	2
N_L, a	-1	-1
$\mathcal{H} \equiv (\mathcal{D}_t a)/a$	1	1
f	2	2
G_{AB}	0	-2
G^{AB}	0	2
$U \equiv V/f^2$	0	0
$X^{A_1A_2\ldots A_p}_{B_1B_2\ldots B_q}$	w_X	$w_X + p - q$
$\nabla_A X^{A_1A_2\ldots A_p}_{B_1B_2\ldots B_q}$	w_X	$w_X - 1 + p - q$
$\mathcal{D}_\lambda X^{A_1A_2\ldots A_p}_{B_1B_2\ldots B_q}$	$w_X - d_{\delta\lambda}$	$w_X - d_{\delta\lambda} + p - q$

Fully Frame Covariant Differentiation:

$$\nabla_C X^{A_1\dots A_p}_{B_1\dots B_q} \;\equiv\; X^{A_1\dots A_p}_{B_1\dots B_q,C} \;-\; \frac{w_X}{2}\,\frac{f_{,C}}{f}X^{A_1\dots A_p}_{B_1\dots B_q}$$
$$+\;\Gamma^{A_1}_{CD}X^{D\dots A_p}_{B_1\dots B_q} \;+\;\cdots\;+\;\Gamma^{A_p}_{CD}X^{A_1\dots D}_{B_1\dots B_q}$$
$$-\;\Gamma^{D}_{B_1C}X^{A_1\dots A_p}_{D\dots B_q} \;-\;\cdots\;-\;\Gamma^{D}_{B_qC}X^{A_1\dots A_p}_{B_1\dots D}\;.$$

Frame-Invariant Number of e-folds:

$$N(t) \;=\; -\int_{t_{\rm end}}^{t} dt'\,\mathcal{H}(t')\;.$$

$t_{\rm end}$: *time of* end *of inflation.*

Frame Covariant Cosmological Equations:

– Scalar-field equations: $\mathcal{D}_t\mathcal{D}_t\varphi^A \;+\; 3\mathcal{H}\,(\mathcal{D}_t\varphi^A) \;+\; f\,G^{AB}U_{,B} \;=\; 0\;.$

– Friedmann equation: $\mathcal{H}^2 \;=\; \frac{1}{3}\left[\,\frac{1}{2}G_{AB}(\mathcal{D}_t\varphi^A)(\mathcal{D}_t\varphi^B) \;+\; fU\right]\;.$

• Frame Covariant Quantum Perturbations

– Metric perturbations:

$$g_{\mu\nu}dx^\mu dx^\nu \;=\; (1+2\Phi)N_L^2\, dt^2 \;-\; a^2\big[(1-2\Phi)\delta_{ij} + h_{ij}\big]dx^i dx^j$$

– Frame Covariant *Mukhanov–Sasaki* variables:

$$Q^A \;=\; \delta\varphi^A \;+\; \frac{\mathcal{D}_t\varphi^A}{\mathcal{H}}\,\Phi$$

– Line element in the *Field Space*: $d\sigma^2 \;=\; G_{AB}\, d\varphi^A\, d\varphi^B$

– Vielbeins along Curvature ($||$) and Isocurvature ($\perp$) direction(s):

$$e^A_\sigma \;=\; \frac{\mathcal{D}_t\varphi^A}{\mathcal{D}_t\sigma}\,, \qquad e^A_s \;=\; -\,\frac{s^A{}_B\, U^{,B}}{\sqrt{s^{AB}\,U_{,A}U_{,B}}}\,,$$

with $s^A{}_B \;\equiv\; \delta^A_B \;-\; e^A_\sigma e^\sigma_B$.

– Comoving adiabatic ($\mathcal{R}$) and entropic ($\mathcal{S}^{(i)}$) perturbations:

$$\mathcal{R} \equiv \frac{\mathcal{H}}{\mathcal{D}_t \sigma} Q^\sigma , \qquad \mathcal{S}^{(i)} \equiv \frac{\mathcal{H}}{\mathcal{D}_t \sigma} Q^{s_i} .$$

– Scalar power spectrum $P_\mathcal{R}$: $2\pi^2 p^{-3} P_\mathcal{R}\, \delta(\mathbf{p}+\mathbf{q}) \equiv \langle \mathcal{R}_\mathbf{p} | \mathcal{R}_\mathbf{q} \rangle$

– Frame-invariant scalar and tensor power spectra:

$$P_\mathcal{R} = \frac{1}{8\pi^2} \frac{\mathcal{H}^2}{f(\varphi)\bar{\epsilon}_H} , \qquad P_T = \frac{2}{\pi^2} \frac{\mathcal{H}^2}{f(\varphi)} ,$$

with $\bar{\epsilon}_H \equiv -(\mathcal{D}_N \mathcal{H})/\mathcal{H}$.

– Power spectrum modified by Entropy Transfer effects:

$$P_\mathcal{R}(t) \approx P_\mathcal{R}(t_*) \left(1 + T^2_{\mathcal{RS}}\right),$$

t_* : time at horizon exit
t : time at observation.

• Frame Invariant Cosmological Observables

– Cosmological observables:

$$
n_{\mathcal{R}} - 1 \equiv \frac{d\ln P_{\mathcal{R}}}{d\ln k}\bigg|_{k=a\mathcal{H}}, \quad n_T \equiv \frac{d\ln P_T}{d\ln k}\bigg|_{k=a\mathcal{H}}, \qquad r \equiv \frac{P_T}{P_{\mathcal{R}}},
$$

$$
\alpha_{\mathcal{R}} \equiv \frac{dn_{\mathcal{R}}}{d\ln k}\bigg|_{k=a\mathcal{H}}, \quad \alpha_T \equiv \frac{dn_T}{d\ln k}\bigg|_{k=a\mathcal{H}}, \quad f_{NL} \equiv \frac{5}{6}\frac{N^{,A}N^{,B}\nabla_A\nabla_B N}{(N_{,A}N^{,A})^2}.
$$

– Frame-invariant potential slow-roll parameters:

$$
\bar{\epsilon}_{U,1} \equiv \frac{1}{2}(\ln U)_{,A}\, G^{AB}\, (\ln U)_{,B}\ ,
$$

$$
\vdots
$$

$$
\bar{\epsilon}_{U,n} \equiv -\, (\ln \bar{\epsilon}_{U,n-1})_{,A}\, G^{AB}\, (\ln U)_{,B}\ ,
$$

with $\mathcal{D}_t\mathcal{D}_t\varphi^A \ll \mathcal{H}\,(\mathcal{D}_t\varphi^A)$, and $\bar{\epsilon}_U \equiv \bar{\epsilon}_{U,1}$, $\bar{\eta}_U \equiv \bar{\epsilon}_{U,2}$ and $\bar{\xi}_U \equiv \bar{\epsilon}_{U,3}$.

– Cosmological Observables in the slow-roll approximation:

$$n_{\mathcal{R}} = 1 - 2\bar{\epsilon}_U + \bar{\eta}_U - \mathcal{D}_N \ln\left(1 + T_{\mathcal{RS}}^2\right), \qquad r = 16\bar{\epsilon}_U\left(1 + T_{\mathcal{RS}}^2\right)^{-1},$$

$$\alpha_{\mathcal{R}} = -2\bar{\epsilon}_U\bar{\eta}_U - \bar{\eta}_U\bar{\xi}_U + \mathcal{D}_N\mathcal{D}_N \ln\left(1 + T_{\mathcal{RS}}^2\right), \qquad \alpha_T = -2\bar{\epsilon}_U\bar{\eta}_U\ .$$

– Isocurvature perturbation(s) $\mathcal{S}$ evolve outside the horizon as follows:

$$\mathcal{D}_N\mathcal{R} \ = \ -2\,\omega\,\mathcal{S}\,, \qquad \mathcal{D}_N\mathcal{S} \ = \ -\,B\,\mathcal{S}\ .$$

– Entropy Transfer depends on the norm ω of the *acceleration vector*:

$$\omega^A \ \equiv \ \mathcal{D}_N\left(\frac{\mathcal{D}_t\varphi^A}{\mathcal{D}_t\,\sigma}\right) \ \approx \ (\ln U)^{,B}\left[\frac{(\ln U)^{,A}}{\sqrt{2\,\bar{\epsilon}_U}}\right]_{,B}\ .$$

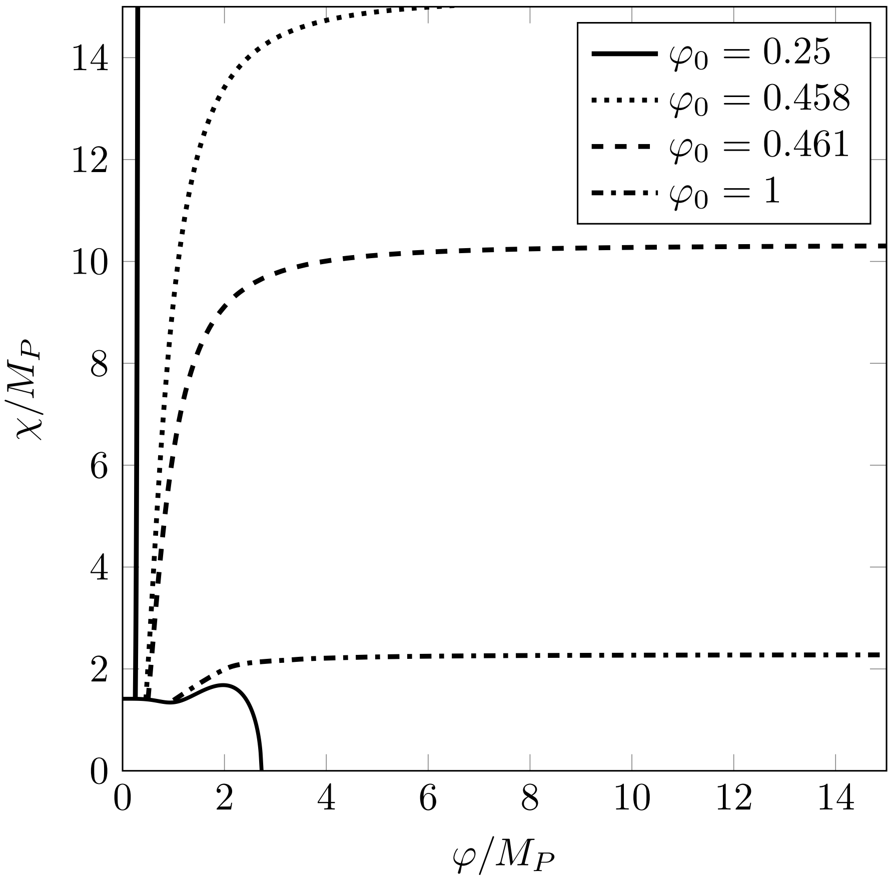
– Entropy Transfer function $T_{\mathcal{RS}}$ becomes

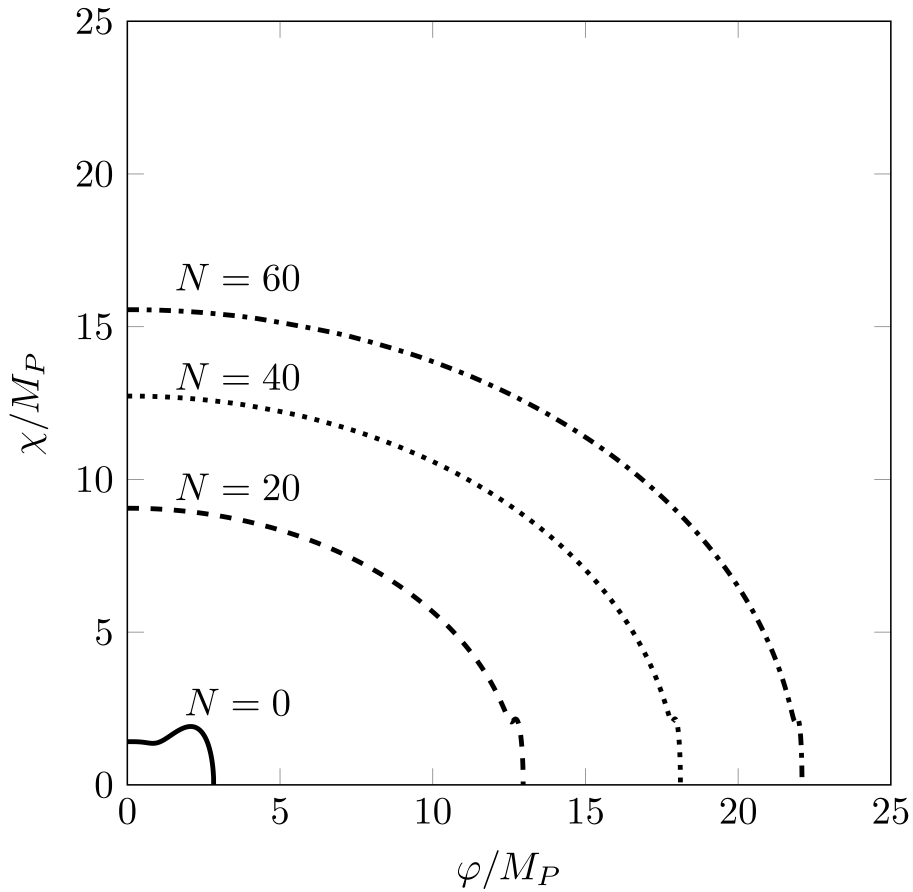
$$T_{\mathcal{RS}}(N_*, N) \ \approx \ \frac{A_*}{B_*}\left[e^{-B_*(N-N_*)} - 1\right].$$

• Two-Field Models

– Minimal Model:

$$\mathcal{L} \;=\; \frac{M_P^2 R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda\varphi^4}{4} - \frac{m^2\chi^2}{2}$$

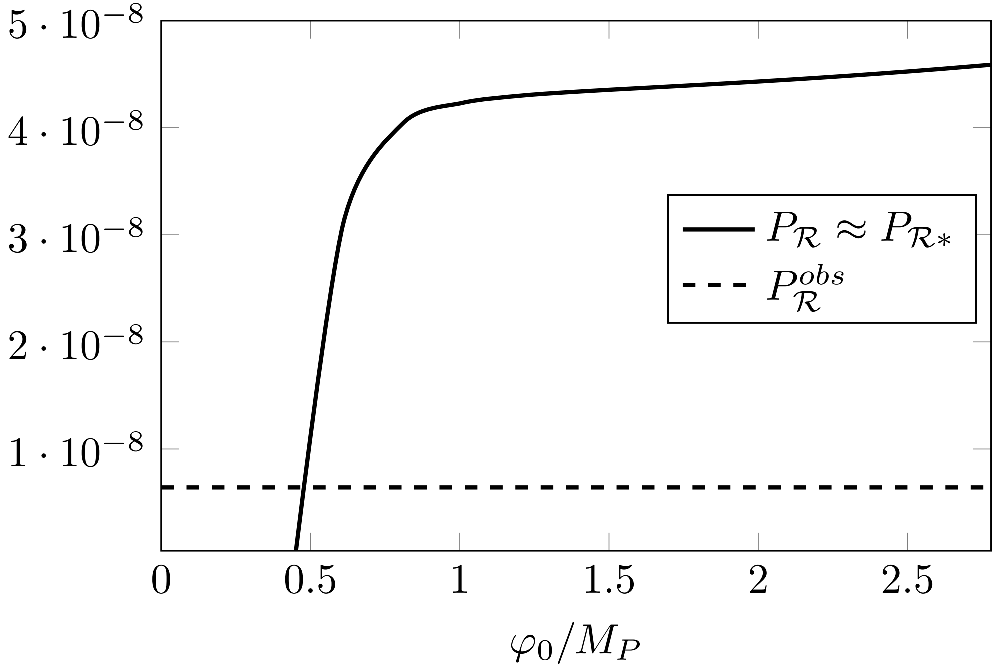


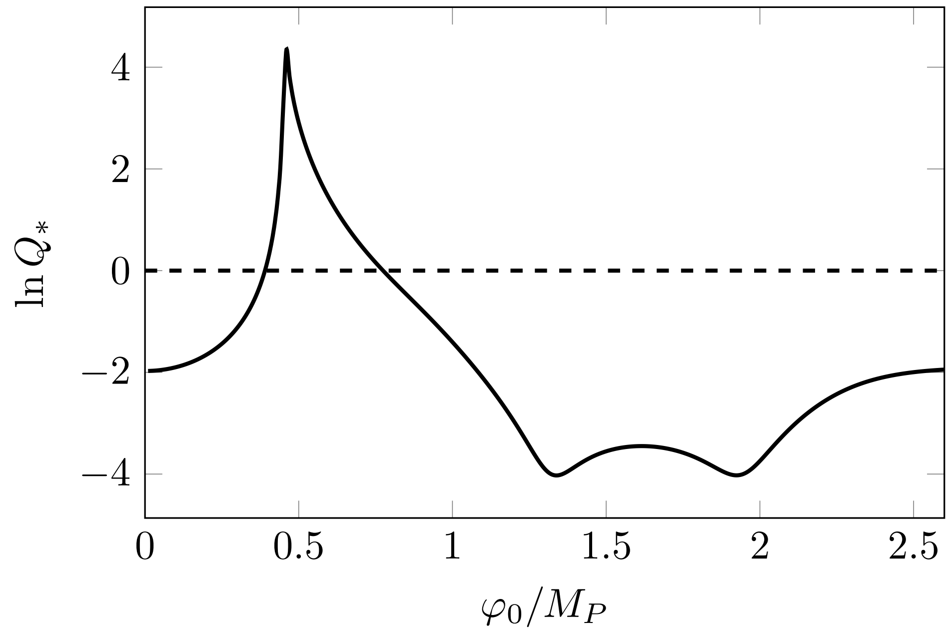


– Predictions in the Minimal Model

[S. Karamitsos, A.P., arXiv:1706.07011]

$m/M_P = 10^{-6}$ and $\lambda = 10^{-12}$:

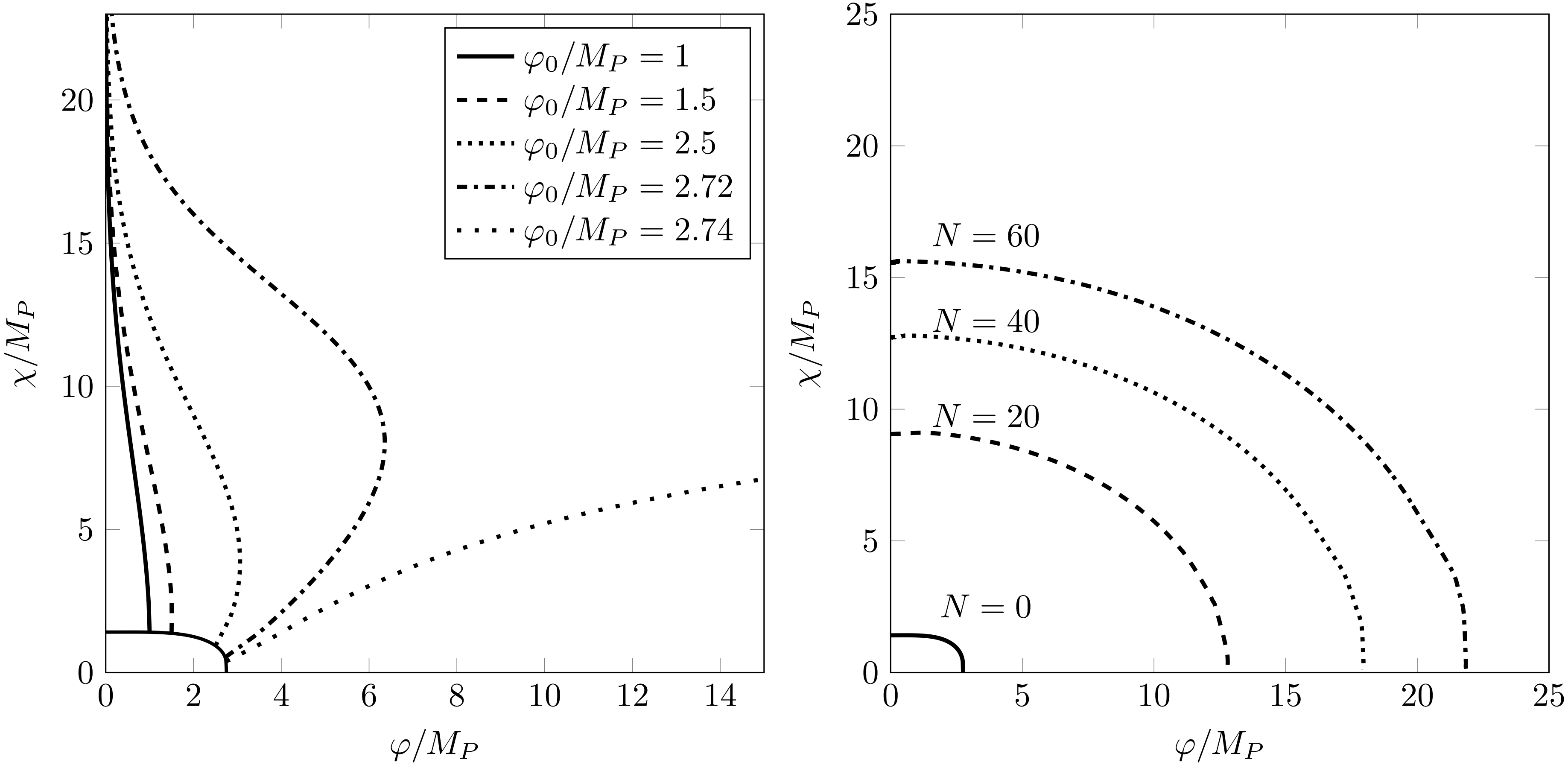




	$\varphi_0/M_P = 0.498$	PLANCK 2015
r	0.501	≤ 0.12 (95% CL)
$n_{\mathcal{R}}$	0.906	0.968 ± 0.006 (68% CL)
$\alpha_{\mathcal{R}}$	-0.00288	-0.003 ± 0.008 (68% CL)
α_T	-0.0019	-0.000167 ± 0.000167 (68% CL)
f_{NL}	-0.0000509	0.8 ± 5.0 (68% CL)

– Non-Minimal Model

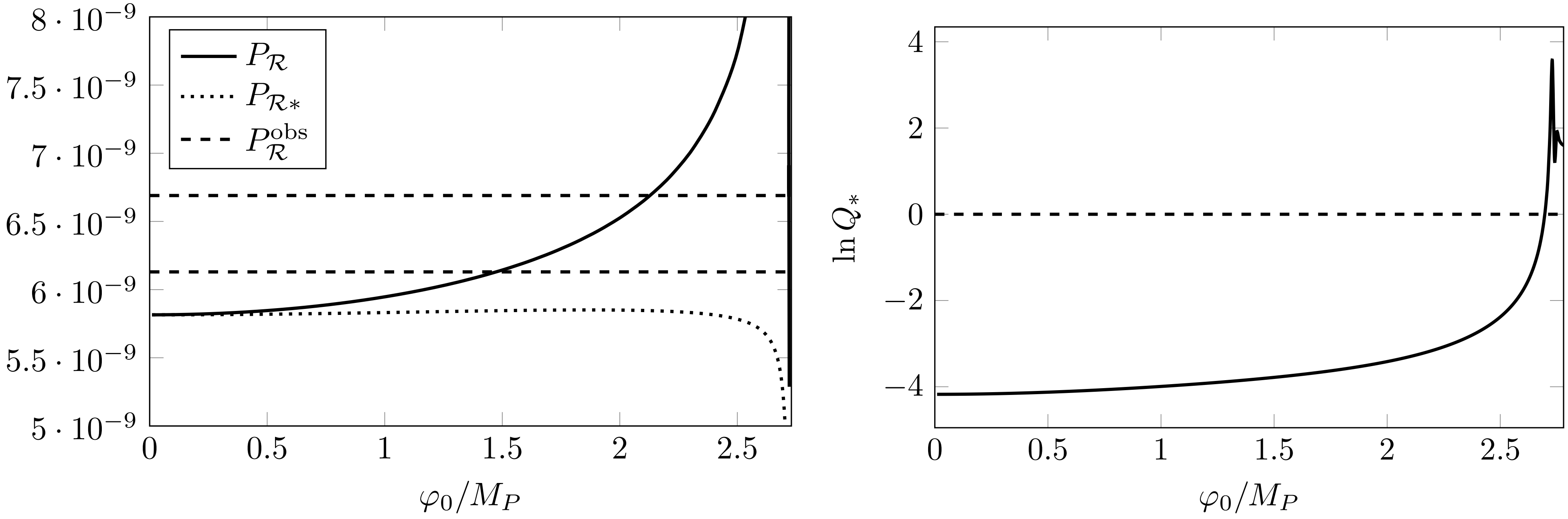
$$\mathcal{L} = \frac{(M_P^2 + \xi\varphi^2)R}{2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}(\nabla\chi)^2 - \frac{\lambda(\varphi^2 - v^2)^2}{4} - \frac{m^2\chi^2}{2}$$



– Predictions in the Non-Minimal Model

[S. Karamitsos, A.P., arXiv:1706.07011]

$m/M_P = 5.6 \times 10^{-6}$, $\lambda = 10^{-12}$ and $\xi \approx 0.01$:



	$\varphi_0/M_P = 1.885^{+0.243}_{-0.411}$	PLANCK 2015
r	$0.12056^{+0.005}_{-0.005}$	≤ 0.12 (95% CL)
$n_{\mathcal{R}}$	$0.949^{+0.005}_{-0.003}$	0.968 ± 0.006 (68% CL)
$\alpha_{\mathcal{R}}$	$-0.0003^{+0.0001}_{-0.00008}$	-0.008 ± 0.008 (68% CL)
α_T	$-0.000276^{+0.000003}_{-0.000003}$	-0.000155 ± 0.00016 (68% CL)
f_{NL}	$0.033^{+0.00017}_{-0.0002}$	0.8 ± 5.0 (68% CL)

• Beyond the Tree Approximation

– Standard 1PI Effective action ($\Gamma[\boldsymbol{\varphi}] = W[\boldsymbol{J}(\boldsymbol{\varphi})] - J_a\varphi^a$):

$$\exp\left(\frac{i}{\hbar}\Gamma[\boldsymbol{\varphi}]\right) \;=\; \int [\mathcal{D}\boldsymbol{\phi}]\, \mathcal{M}[\boldsymbol{\phi}]\, \exp\left\{\frac{i}{\hbar}\Big[S[\boldsymbol{\phi}] + \Gamma_{,a}\big(\varphi^a - \phi^a\big)\Big]\right\},$$

with $\Gamma_{,a} \;\equiv\; \delta\Gamma[\boldsymbol{\varphi}]/\delta\varphi^a \;=\; -\, J_a$.

– $\Gamma[\boldsymbol{\varphi}]$ is **not** frame-invariant, i.e.

$$\Gamma[\boldsymbol{\varphi}] \;\neq\; \widetilde{\Gamma}[\widetilde{\boldsymbol{\varphi}}]\,,$$

because φ^a is **not** a frame covariant vector.

– Vilkovisky–De Witt formalism: $\varphi^a - \phi^a \;\longrightarrow\; \sigma^a(\boldsymbol{\varphi},\boldsymbol{\phi})$.

Under $\boldsymbol{\varphi} \to \widetilde{\boldsymbol{\varphi}} = \widetilde{\boldsymbol{\varphi}}(\boldsymbol{\varphi})$, $\sigma^a(\boldsymbol{\varphi},\boldsymbol{\phi})$ transforms as a vector.

Under $\boldsymbol{\phi} \to \widetilde{\boldsymbol{\phi}} = \widetilde{\boldsymbol{\phi}}(\boldsymbol{\phi})$, $\sigma^a(\boldsymbol{\varphi},\boldsymbol{\phi})$ is a (frame-invariant) scalar.

– Vilkovisky–De Witt 1PI effective action:

$$\exp\left(\frac{i}{\hbar}\Gamma[\varphi]\right) \;=\; \int[\mathcal{D}\phi]\,\mathcal{M}[\phi]\,\exp\left\{\frac{i}{\hbar}\Big[S[\phi]+\nabla_a\Gamma[\varphi]\,\sigma^a(\varphi,\phi)\Big]\right\},$$

with $\mathcal{M}[\phi] \;\equiv\; \sqrt{\det\mathcal{G}_{ab}}$, and a configuration field space metric:

$$\mathcal{G}_{ab} \;\equiv\; G_{AB}\,\delta(x_A-x_B) \;=\; \left(\frac{k_{AB}}{f}+\frac{3}{2}\frac{f_{,A}f_{,B}}{f^2}\right)\delta(x_A-x_B)\,.$$

– $\hbar$-Expansion of the Frame-Invariant Vilkovisky–De Witt effective action:

$$\Gamma[\varphi] \;=\; \sum_n \hbar^n\Gamma_n[\varphi] \;=\; \widetilde{\Gamma}[\widetilde{\varphi}]\,,$$

with $\Gamma_0[\varphi] \;=\; S[\varphi]$, and

$$\Gamma_1[\varphi] \;=\; -\frac{i}{2}\,\mathrm{tr}\ln\mathcal{G}_{ab}+\frac{i}{2}\,\mathrm{tr}\ln\Big(\nabla_a\nabla_b\,S[\varphi]\Big) \;=\; \frac{i}{2}\,\mathrm{tr}\ln\Big(\nabla^a\nabla_b\,S[\varphi]\Big)\,.$$

• Conclusions

- **Frame Covariant Approach to Multifield Inflation in Scalar-Curvature Theories.**

- **Use of Differential Geometry in Curved Multifield Space to covariantly compute Curvature and Isocurvature Perturbations.**

- **Entropic Effects significant in Non-Minimal Two-Field Models.**

- **Full Frame Covariance beyond the Tree Approximation by virtue of the Vilkovisky–De Witt Formalism.**

• Future Directions

- **Extension of the Frame Covariant Approach to $F(R)$ and $F(\varphi, R)$ Theories.**

- **Inclusion of Matter and Quantum Gravity Effects.**

- **Higher-Loop Impact of Curved Field Space on Cosmological Observables.**

- **Consideration of Curved Field Space Effects in the Standard Model and Beyond.**

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