QUANTUM MATTER ON A CLASSICAL SPACETIME AS A FUNDAMENTAL THEORY

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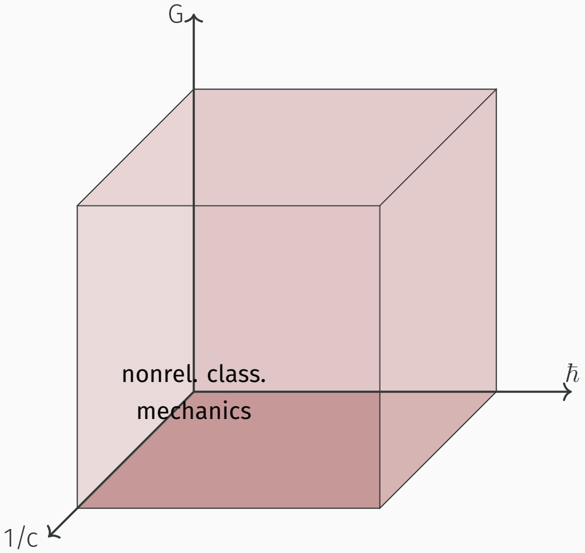
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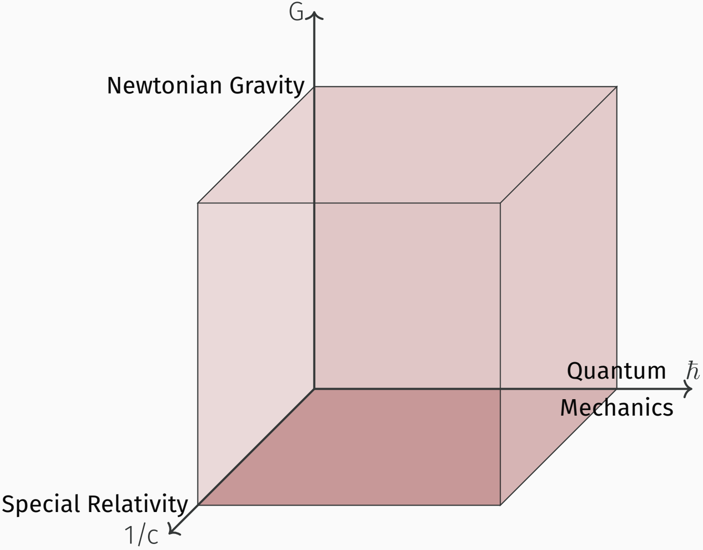
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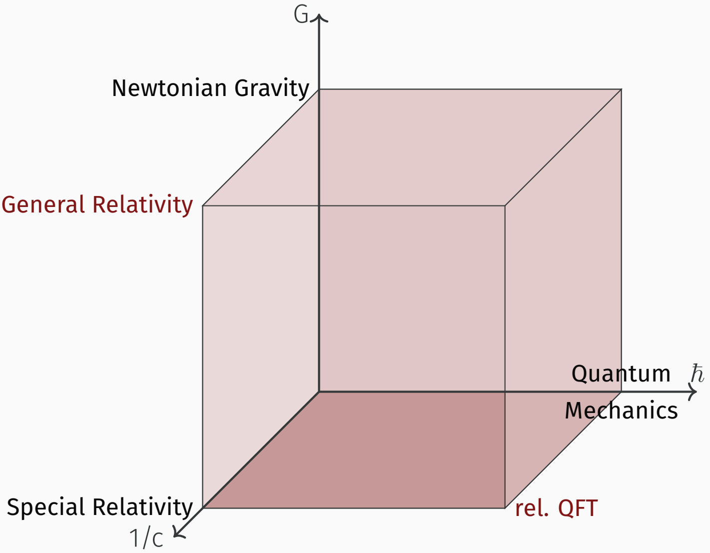
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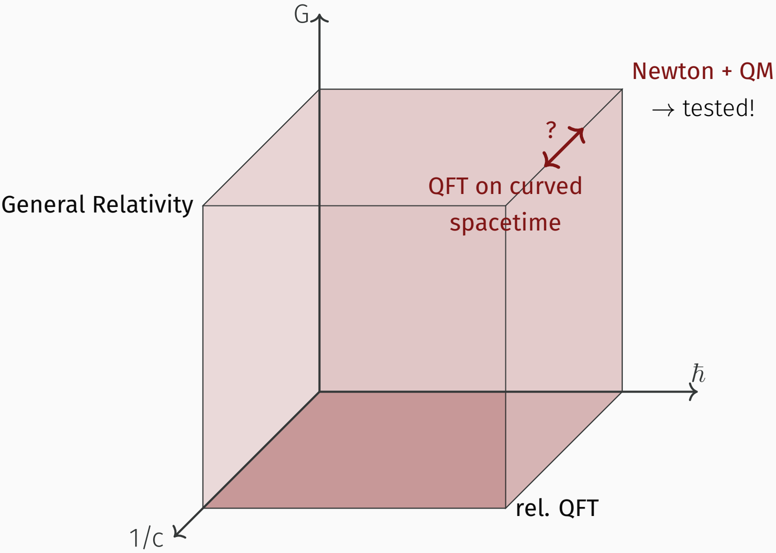
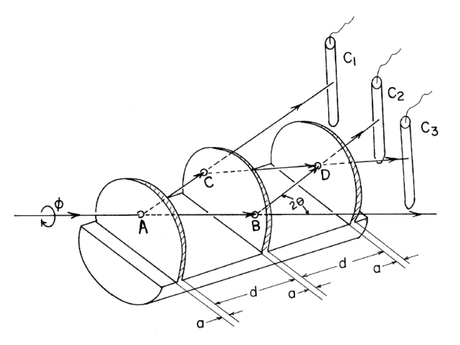
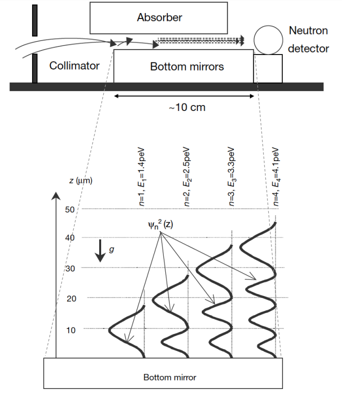


external gravitational field

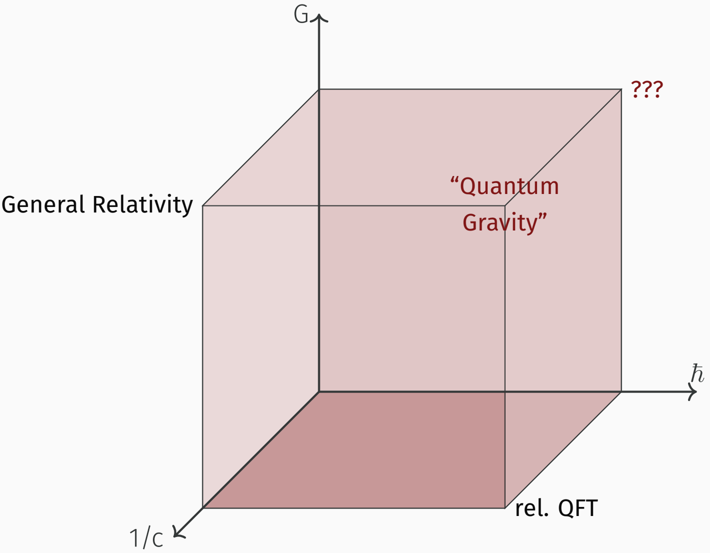
$$i\hbar\,\dot\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 - m\,g\,x\right)\psi$$



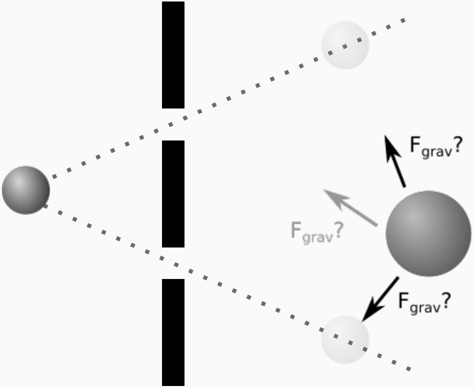


Colella, Overhauser, Werner (1975)

- *external* field (Earth)
- *Newtonian* gravity



quantum matter as gravity **source**



What is the gravitational field of a superposition state?

SOLUTION 1: QUANTISED GRAVITY

In analogy to electrodynamics:
The field sourced by a superposition state is itself in a superposition
$\Rightarrow$ **superposition of two spacetimes**

Problems:

- **Nonrenormalisability** of gravity as a field theory:
  $\rightarrow$ gravity must be different in **some** respect
  $\rightarrow$ there is no fully consistent theory of quantum gravity (yet?)
- How to **identify** points in **different** spacetimes?
  $\rightarrow$ quantum matter on curved spacetime is not a conceptually consistent theory **even** in the Newtonian, low energy limit of the double slit experiment

SOLUTION 2: SEMI-CLASSICAL GRAVITY

- Quantum fields living **on** spacetime and dynamics **of** spacetime are two conceptually very different things
- Take (classical) GR seriously
  (and leave it to experiments, at which point it might brake down):
  spacetime is a 4-dim. manifold with quantum matter living on it

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}}_{\text{spacetime (class.)}} \overset{\lightning}{=} \underbrace{\frac{8\pi G}{c^4}\, \hat{T}_{\mu\nu}}_{\text{matter (quantum)}}$$

- **Quantisation of gravity:** spacetime is "quantum" in some way
  At low energies: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with "quantum field" $\hat{h}_{\mu\nu}(x)$
- **Gravitisation of QM:** replace $\hat{T}_{\mu\nu}$ by a classical object, e. g. $\langle \hat{T}_{\mu\nu} \rangle$

NEWTONIAN SEMI-CLASSICAL GRAVITY

In the weak-field nonrelativistic limit: $\hat{\rho} = m\hat{\psi}^\dagger \hat{\psi}$

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = \tfrac{8\pi G}{c^4} \langle \psi |\, \hat{T}_{\mu\nu}\, | \psi \rangle \quad \rightarrow \quad \nabla^2 V = 4\pi G \langle \psi |\, \hat{\rho}\, | \psi \rangle$$

$$\hat{H}_{\text{int}} = -\tfrac{1}{2} \int \mathrm{d}^3 r\, h_{\mu\nu} \hat{T}^{\mu\nu} \quad \rightarrow \quad \hat{H}_{\text{int}} = \int \mathrm{d}^3 r\, V \hat{\rho}$$

Results in the **Schrödinger-Newton equation** (here for one particle)

$$\mathrm{i}\hbar\, \dot{\psi}(t, \mathbf{r}) = \left(-\frac{\hbar^2}{2m} \nabla^2 - G m^2 \int \mathrm{d}^3 r' \, \frac{|\psi(t, \mathbf{r}')|^2}{|\mathbf{r} - \mathbf{r}'|} \right) \psi(t, \mathbf{r})$$

$\Rightarrow$ Nonlinear Schrödinger equation

Realistic systems for testing SN are **not** single particles:

$$i\hbar\dot{\Psi}_N(\mathbf{r}^N) = \left[-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\Delta_{\mathbf{r}_i} + V_{\text{linear}}(\mathbf{r}^N) + V_{\text{G}}[\Psi_N(\mathbf{r}^N)]\right]\Psi_N(\mathbf{r}^N)$$

$$V_{\text{G}}[\Psi_N(\mathbf{r}^N)] = -G\sum_{i=1}^{N}\sum_{j=1}^{N} m_i m_j \int \frac{\left|\Psi_N(\mathbf{r}'^N)\right|^2}{\left|\mathbf{r}_i - \mathbf{r}'_j\right|}\,\mathrm{d}V'^N$$

Centre of mass equation (**approx.**), separation $\Psi_N = \psi \otimes \chi_{N-1}$:

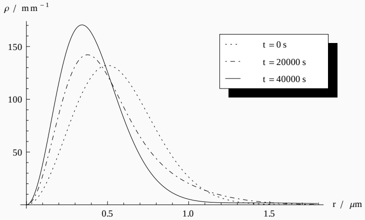
$$i\hbar\,\dot{\psi}(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2M}\nabla^2 + V^{\text{ext.}}_{\text{lin.}} - G\int \mathrm{d}^3r'\,\left|\psi(t,\mathbf{r}')\right|^2\, I_\rho(\mathbf{r}-\mathbf{r}')\right)\psi(t,\mathbf{r})$$

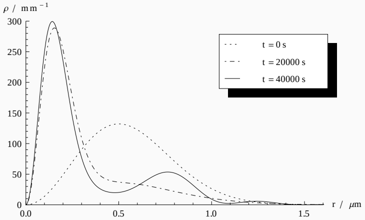
$$I_\rho(\mathbf{d}) = \int \mathrm{d}^3x\,\mathrm{d}^3y\,\frac{\rho(\mathbf{x})\rho(\mathbf{y}-\mathbf{d})}{|\mathbf{x}-\mathbf{y}|} \quad \text{(where } \rho \text{ is given by } \left|\chi_{N-1}\right|^2\text{)}$$

EXPERIMENTAL TESTS

wave-function $\gg$ particle size $\Rightarrow$ $\rho \approx \delta(\mathbf{r}_{cm})$ $\Rightarrow$ $I_\rho(d) \approx 1/|d|$:

$$i\hbar\,\dot\psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2\int d^3r'\,\frac{|\psi(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|}\right)\psi(t,\mathbf{r})$$



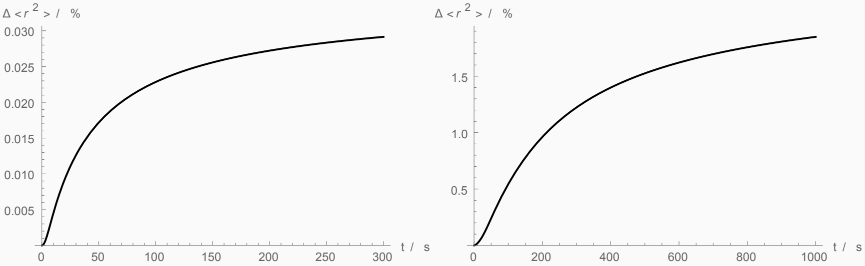


$\rho = 4\pi\, r^2\ |\psi|^2$ for masses of 7×10^9 u and 10^{10} u

Problem: time scale (order of **hours**!)

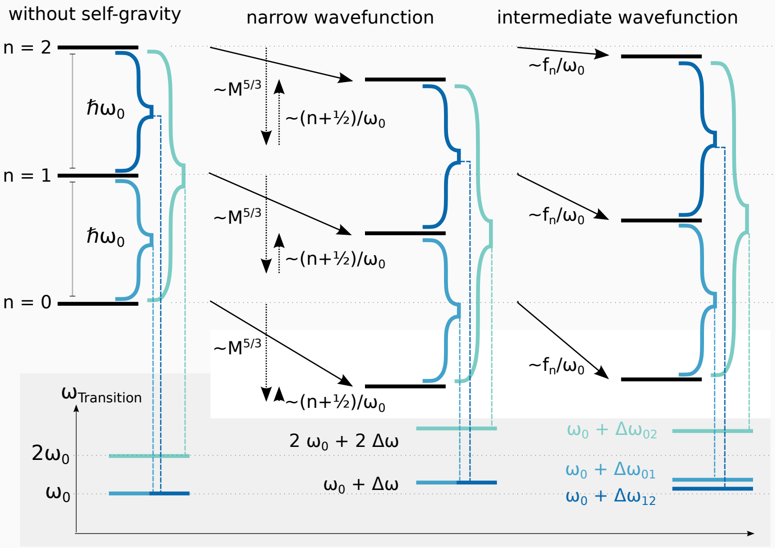
For narrower wave-functions (here $\mathcal{O}(10\text{ nm}) \lesssim$ particle size):
approximate ODE (assume: Gaussian wave-packet remains Gaussian)

$$\frac{\mathrm{d}^3}{\mathrm{d}t^3}\langle r^2\rangle = -3\omega_{\mathrm{SN}}^2 f(\langle r^2\rangle)\,\frac{\mathrm{d}}{\mathrm{d}t}\langle r^2\rangle$$



rel. deviation from standard Schrödinger evolution for $m = 10^9$ u and 10^{10} u

$\Rightarrow$ 1 % deviation after 200 s $\quad\rightarrow$ **maybe in space?**


without self-gravity
narrow wavefunction
intermediate wavefunction
n = 2
n = 1
n = 0
ħω0
ħω0
~M5/3
~M5/3
~M5/3
~(n+½)/ω0
~(n+½)/ω0
~(n+½)/ω0
~fn/ω0
~fn/ω0
~fn/ω0
ωTransition
2ω0
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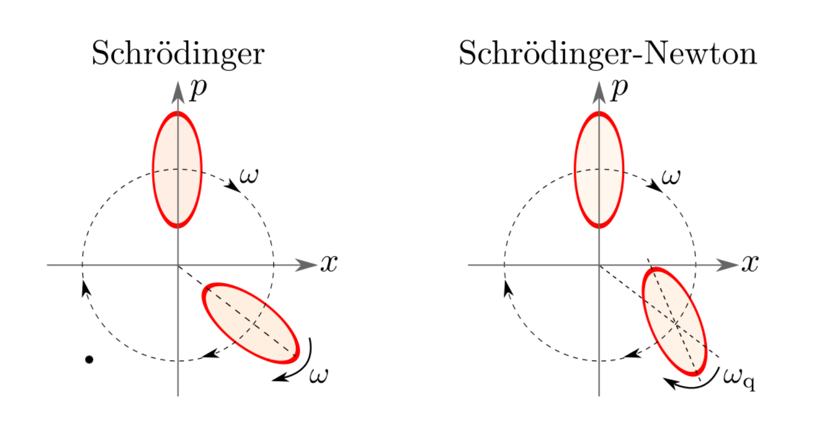
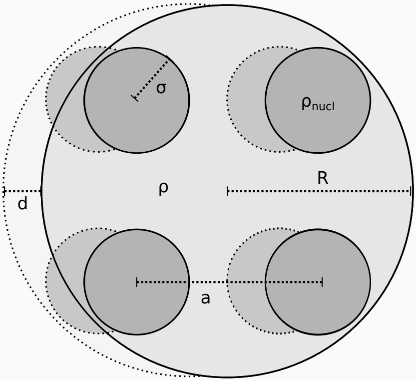
ω0 + Δω
ω0 + Δω02
ω0 + Δω01
ω0 + Δω12



Yang et al. PRL 110 (2013) 170401



- the relevant radius is σ (localisation of the nuclei)
- effective mass density $\rho_{\text{nucl}} \sim 10^3 \rho$
- $\omega_{\text{SN}} = \sqrt{\frac{G m_{\text{atom}}}{\sigma^3}} \sim \sqrt{G\rho_{\text{nucl}}} \sim 1$ Hz for osmium

Need **ground state** cooling for:
mass $\sim 10^{15}$ u (μm sized) particle
trapped at $\mathcal{O}(10\,\text{Hz})$

CONCLUSIONS

- Testing the Schrödinger-Newton equation is feasible
- However, only tests the specific semi-classical coupling

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle$$

- Other approaches for semi-classical coupling possible:
  - Source gravity from stochastic CSL collapse events[1]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} \left(\langle \hat{T}_{\mu\nu} \rangle + \delta T_{\mu\nu} \right)$$

    $\rightarrow$ yields **linear** master equation (no superluminal signalling problem)
    $\rightarrow$ experimental predictions less clear (has free parameters)
    $\rightarrow$ stochastic field of unexplained origin
  - Stochastic gravity[2]

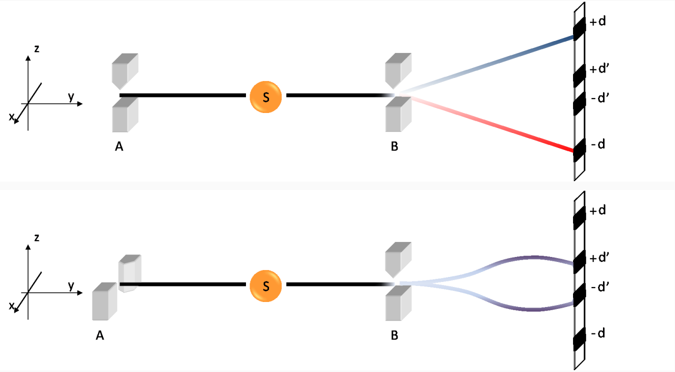
[1] Tilloy & Diósi, PRD 93, 024026 (2016)
[2] Hu & Verdaguer, Living Rev. Relativ. 11, 3 (2008)

Thank You!

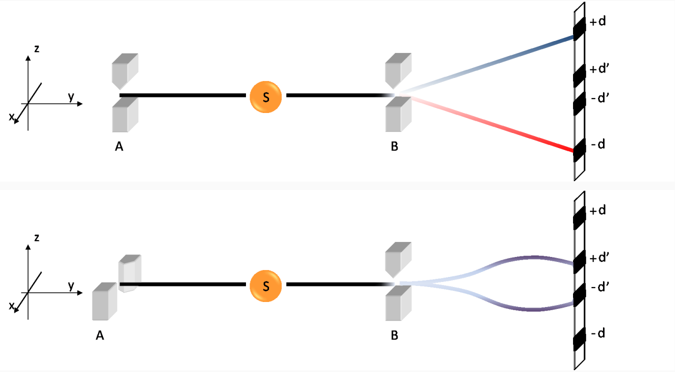
Questions?

ADDITIONAL SLIDES

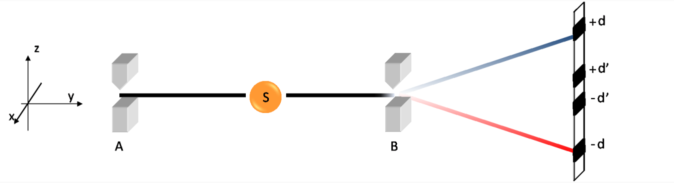
- An instantaneous collapse **violates** divergence freedom of Einstein's equations

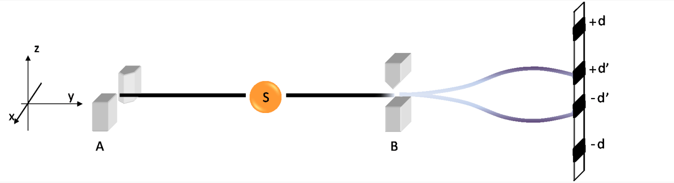


- An instantaneous collapse **violates** divergence freedom of Einstein's equations
- A macroscopic superposition of *here* and *there* "collapses" **in the middle** (rather than 50:50 here **or** there)

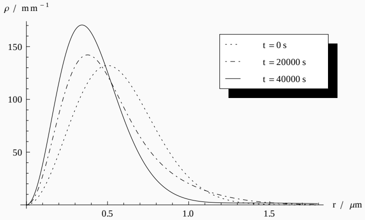


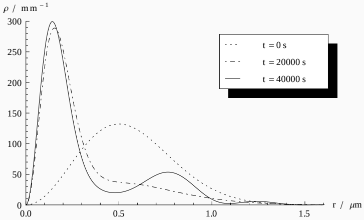
- An instantaneous collapse **violates** divergence freedom of Einstein's equations
- A macroscopic superposition of *here* and *there* "collapses" **in the middle** (rather than 50:50 here **or** there)
- With the standard collapse: **faster-than-light signalling**





In the wide wave function limit: one-particle SN equation





$\rho = 4\pi\, r^2\ |\psi|^2$ for masses of 7×10^9 u and 10^{10} u

- For a mass of $\sim 10^{10}$ u and a wave packet size of about 500 nm a significant deviation is visible after several *hours*
- **Scaling law:** with $\psi(t, \mathbf{x})$ for mass m, a solution for mass $\mu\, m$ is obtained as $\mu^{9/2}\psi(\mu^5 t, \mu^3 \mathbf{x}) \quad \Rightarrow \quad$ e. g. 10^{11} u at 0.5 nm would show an effect in *less than a second* **but** must remain in wide wave function regime (Os at 10^{10} u has 100 nm diameter)

Assumption: a **Gaussian** wave packet stays approximately **Gaussian**

The free spreading of a Gaussian wave packet and spherical particle can be approximated by a third order ODE for the width $u(t) = \langle r^2 \rangle(t)$:

$$\dddot{u}(t) = -3\omega_{\mathrm{SN}}^2 \, f(u(t)) \, \dot{u}(t)$$

with $\omega_{\mathrm{SN}} = \sqrt{Gm/R^3} \sim \sqrt{G\rho}$, initial conditions

$$u(0) = u_0 \,, \qquad \dot{u}(0) = 0 \,, \qquad \ddot{u}(0) = \frac{9\hbar^2}{2m^2 \, u_0} - \omega_{\mathrm{SN}}^2 \, g(u_0) \, u_0 \,,$$

and the functions (with u in units of R)

$$f(u) = \operatorname{erf}\left(\sqrt{\frac{3}{u}}\right) + \sqrt{\frac{u}{3\pi}} \left(u - \frac{7}{2} - \frac{324 - 162u - 35u^4 + 70u^5}{70\, u^4} \, \mathrm{e}^{-3/u} \right)$$

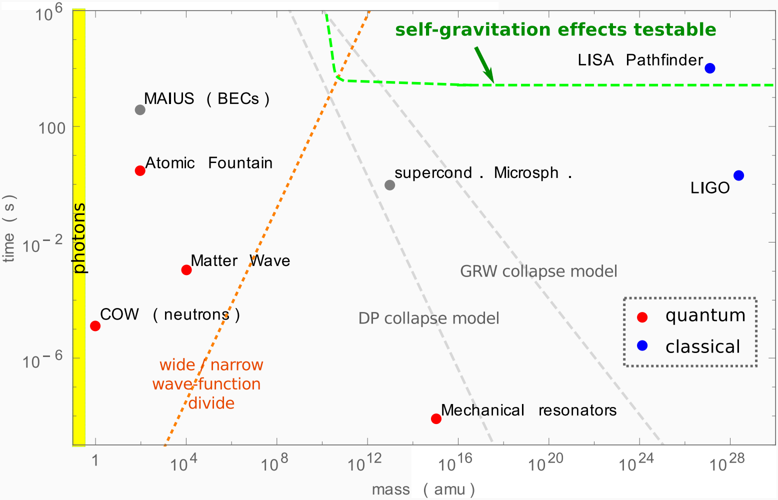
$$g(u) = \operatorname{erf}\left(\sqrt{\frac{3}{u}}\right) + \sqrt{\frac{u}{3\pi}} \left(\frac{2}{3} u - 3 + \frac{486 + 105u^3 - 70u^4}{105u^3} \, \mathrm{e}^{-3/u} \right)$$

$$u(t) \approx u_0 + \frac{1}{2}\,\ddot{u}(0)\,t^2$$

- *exact* without self-gravity term
- deviates from usual evolution by dependence on $g(u_0)$ in

$$\ddot{u}(0) = \frac{9\hbar^2}{2m^2\,u_0} - \omega_{\mathrm{SN}}^2\,g(u_0)\,u_0$$

- stationarity condition $\ddot{u}(0) = 0$ yields (pessimistic) estimate for the scales where self-gravity becomes important
- Assume **osmium** particle initially trapped with ω_0
  $\Rightarrow$ characteristic time scale $\tau = \omega_0^{-1}, \quad u_0 = 3\hbar\,\tau/m$
- $\ddot{u}(0) = 0$ determines characteristic (m, τ) graph
- limit $g(u) \to 1$ for $u \to 0$ yields $\tau(m) = \text{const.}$ for large m



green line intuitively: free wave-function would have increased by 25% but maintains its width due to self-gravity

For a homogeneous sphere:

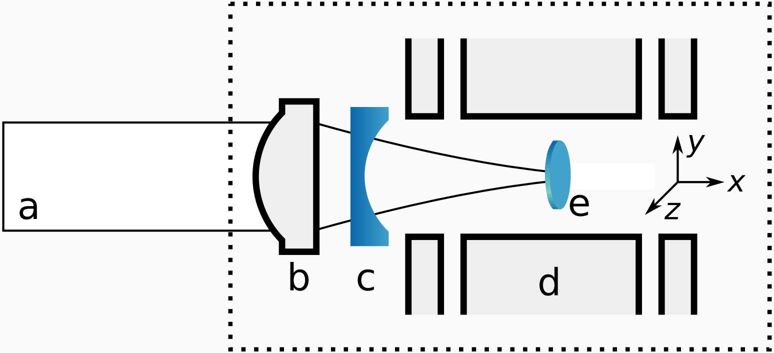
$$I_\rho(d) = -\frac{M^2}{R} \times \begin{cases} \frac{6}{5} - 2\left(\frac{d}{2R}\right)^2 + \frac{3}{2}\left(\frac{d}{2R}\right)^3 - \frac{1}{5}\left(\frac{d}{2R}\right)^5 & (d \leq 2R) \\ \frac{R}{d} & (d > 2R) \end{cases}$$

- different behaviour for narrow and wide wave functions
- enhancement of $\mathcal{O}(10^3)$ for narrow wf. in crystalline matter

$$\omega_{SN} = \sqrt{\frac{G m_{atom}}{\sigma^3}}$$

Material	m_{atom} / u	ρ / g cm^{-3}	σ / pm	ω_{SN} / s^{-1}
Silicon	28.086	2.329	6.96	0.096
Tungsten	183.84	19.30	3.48	0.695
Osmium	190.23	22.57	2.77	0.996
Gold	196.97	19.32	4.66	0.464

Note: ω_{SN} enters **squared** in the evolution equation
$\Rightarrow$ osmium two orders of magnitude better than silicon


a
b
c
d
e
y
x
z