

*Quasiparticle picture*  
*from the*  
*Bekenstein bound*

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Testing Fundamental Physics Principles

Corfu, Greece

Ref: G.Acquaviva, A.I., M.Scholtz, arXiv:1704.00345 (Ann Phys, *tbp*)

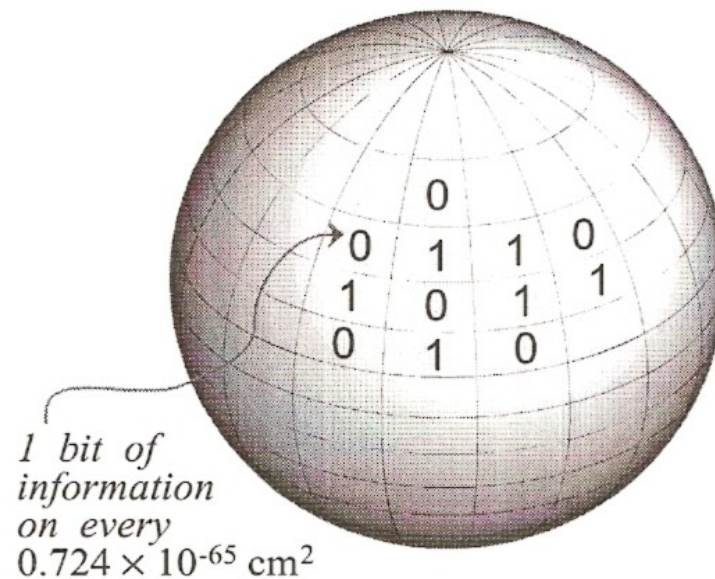
# Quasiparticle picture

It is widely accepted

$$S \leq S_{\text{BH}} = \frac{1}{4} \frac{\partial V}{\ell_P^2},$$

where  $\partial V \equiv A_{\text{EH}}$ .

One possible interpretation is that, when a BH is formed, the “X level” has been reached.



- If

$$S \leq S_{BH} = \frac{1}{4} \frac{A}{\ell_P^2}$$

means that fundamental degrees of freedom  $X$  exist, then

$$g_{\mu\nu} \text{ AND } \phi$$

both emerge from  $X$

- Then, in general:

- a) particles we call elementary are, in fact, quasiparticles, and
- b) there is field-geometry entanglement

- Different configurations of  $X$  may give rise to the same  $g_{\mu\nu}$  but then yield different  $\phi$ s

$$(g_{\mu\nu}, \phi), \dots, (g_{\mu\nu}, \phi')$$

- Thus, even if

$$g_{\mu\nu}^{<BH} = g_{\mu\nu}^{>BH} \equiv g_{\mu\nu}$$

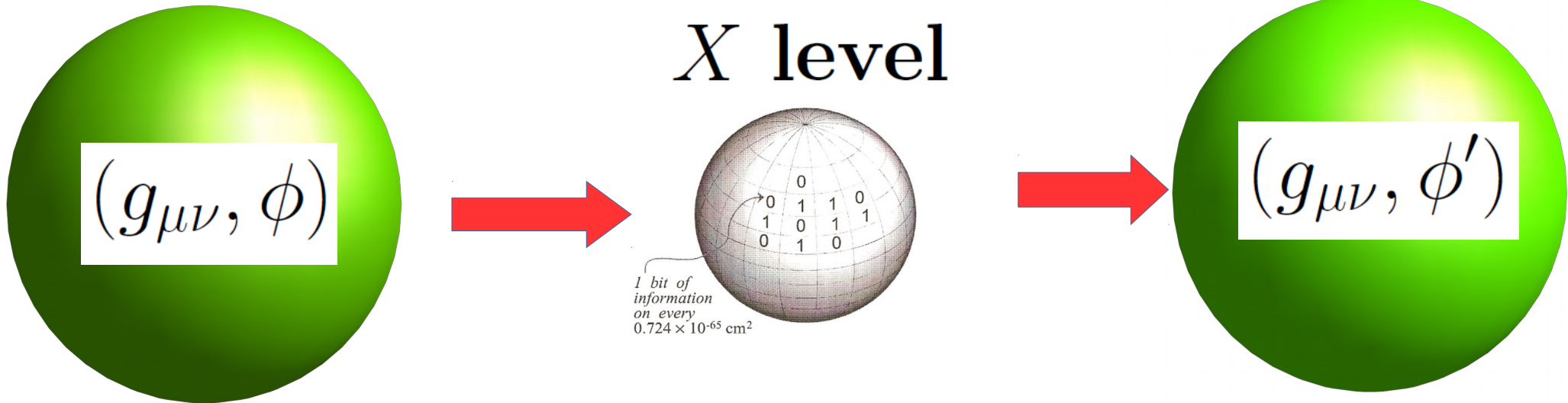
the emerging quantum fields  $\phi \neq \phi'$  and live in different Hilbert spaces.

- Since the  $X$ s rearrange, even a unitary evolution at the  $X$  level leads to information loss for  $\phi$ !

Thus BHs are (the only!) drivers of phase transitions between different “emergent” arrangements of the  $X$  level

Before formation

After evaporation



$$S \leq 2\pi \frac{RE}{\hbar c} \longrightarrow S|_{E=E_S} = \pi R_S^2 \frac{c^3}{\hbar G} = \frac{1}{4} \frac{A_{EH}}{l_P^2} \longrightarrow S \leq 2\pi \frac{RE}{\hbar c}$$

On the other hand, the following is widely accepted

Take

$$H = H_A^m \otimes H_B^n$$

and  $U|\psi_0\rangle \in H$  a random state, with associated  $\rho_A(U)$ , and  $S_{m,n}(U)$

The average entanglement entropy of  $A$

$$S_{m,n} = \langle S_{m,n}(U) \rangle$$

and the average information contained in  $A$

$$I_{m,n} = \ln m - S_{m,n}$$

Page conjectured

$$I_{m,n} = \ln m + \frac{m-1}{2n} - \sum_{k=n+1}^{mn} \frac{1}{k}$$

for  $m < n$

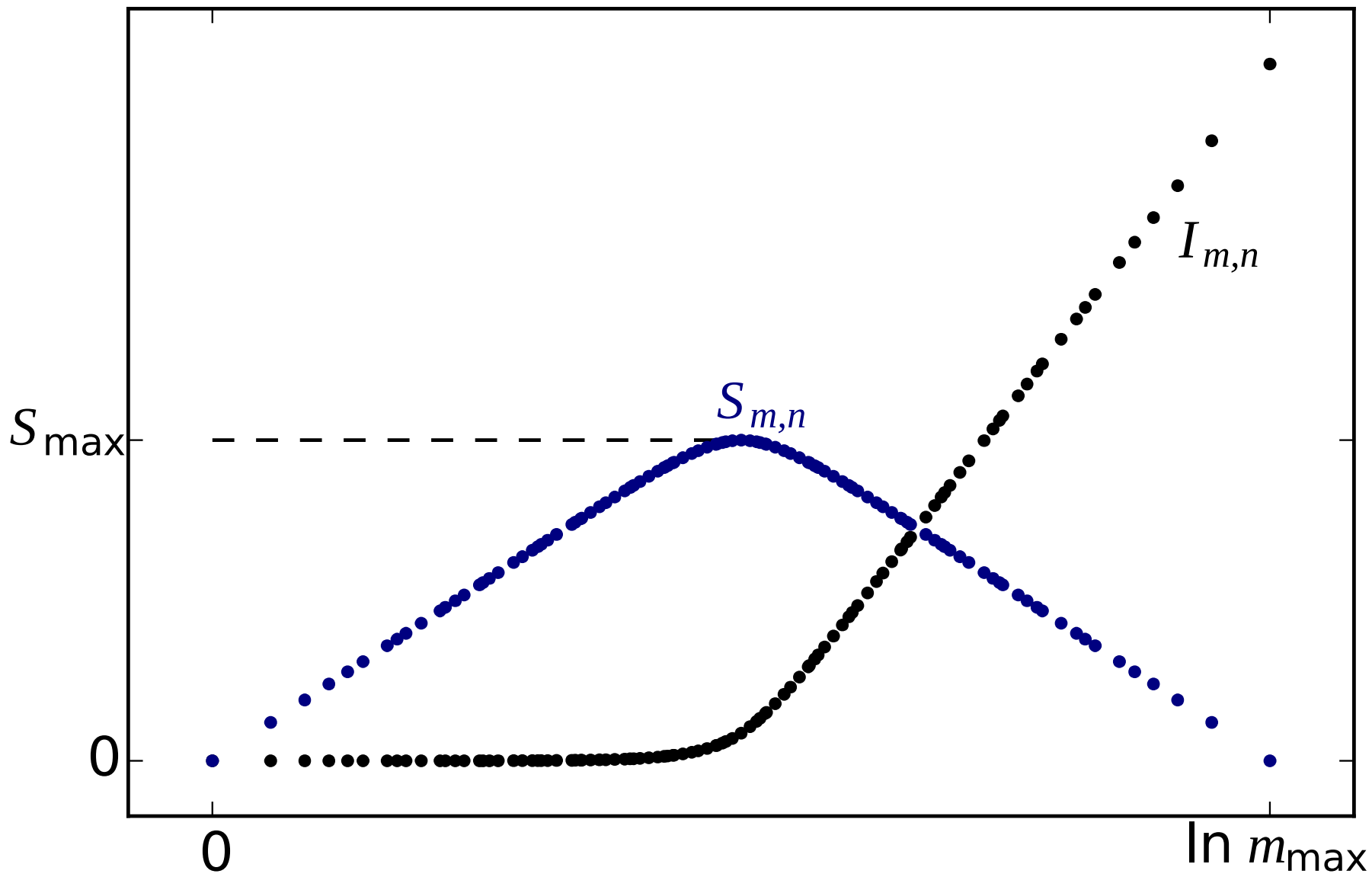
Applied to BH evaporation:  $A$  corresponds to the states under the horizon,  $B$  corresponds to the radiation

When the BH is formed,  $n = 1$  and  $m = \dim H$ :  $S_{m,n}$  is zero

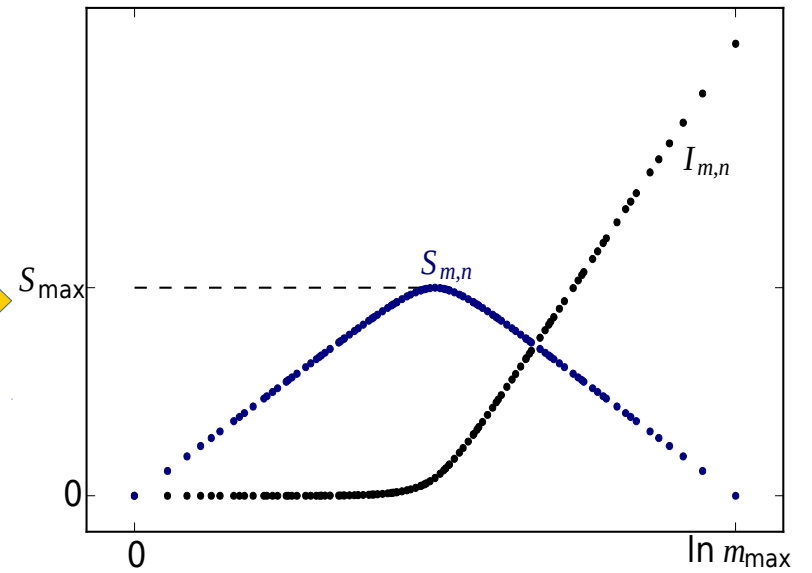
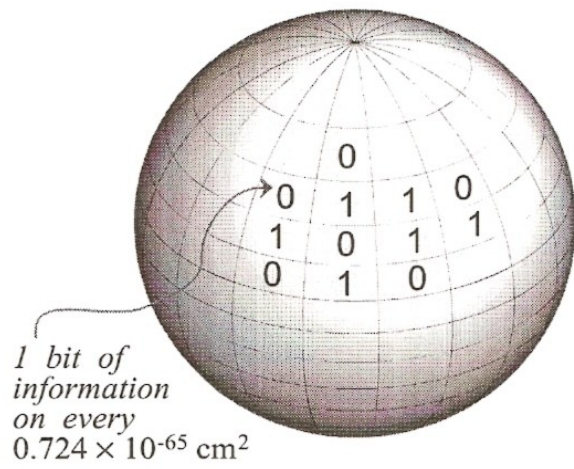
As the BH evaporates,  $n$  increases and  $m$  decreases ( $m n = \text{const.}$ ):  
 $S_{m,n}$  increases

At some stage (approximately half time,  $t_{Page}$ )  $I_{m,n}$  starts to leak from the BH:  $S_{m,n}$  decreases

When the BH fully evaporates,  $m = 1$  and  $n = \dim H$ :  
 $S_{m,n}$  returns to zero

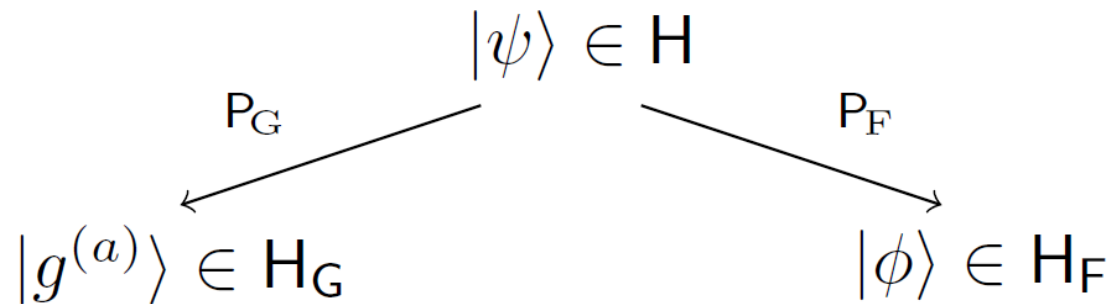






# Modeling BH evaporation

The Hilbert space  $\mathbb{H}$  for the  $X$  level is



$N_G$  allowed geometries

$$|g^{(a)}\rangle, \quad a = 0, 1, \dots, N_G - 1$$

so  $|\psi\rangle = |g^{(a)}, \phi\rangle$

The distribution of the  $X$  between geometry and the fields in general changes during the unitary evolution.

**Assume**

$$\mathbf{H} = \bigoplus_{i=1}^{N_T} T_{(i)}, \quad \dim \mathbf{H} = N_T N$$

**where**

$$T_{(i)} = \mathbb{H}_G^{p_i} \otimes \mathbb{H}_F^{q_i}, \quad p_i q_i = N$$

**A general state  $|\psi\rangle \in \mathbf{H}$  admits then the expansion**

$$|\psi\rangle = \bigoplus_{i=1}^{N_T} \sum_{I=1}^{p_i} \sum_{n=0}^{q_i-1} c_{In}^{(i)} |I_i\rangle \otimes |n_i\rangle$$

$|I_i\rangle$ s and  $|n_i\rangle$ s bases of  $\mathbb{H}_G^{p_i}$  and  $\mathbb{H}_F^{q_i}$ , resp.

Denote by  $P_{(i)} : \mathbb{H} \mapsto T_{(i)}$  a projector onto  $T_{(i)}$ . Then

$$p_{(i)} = \|P_{(i)}|\psi\rangle\|^2$$

is the probability of finding the system in the state with the topology  $T_{(i)}$ .

In general, a state in  $T_{(i)}$  has entanglement between geometry and field

$$P_{(i)}|\psi\rangle = \sum_{I,n} c_{In}^{(i)} |I_i\rangle \otimes |n_i\rangle$$

The associated density matrix representing the state of the field is

$$\rho_{(i)} = \text{Tr}_{\mathbb{H}_G^{p_i}} |\psi\rangle_i \langle\psi|_i$$

where  $|\psi\rangle_i = p_{(i)}^{-1/2} P_{(i)}|\psi\rangle$

The corresponding entanglement entropy

$$S_{(i)} = -\text{Tr}_{\mathbb{H}_F^{q_i}} \rho_{(i)} \ln \rho_{(i)}$$

is the entanglement entropy between the geometry and the fields for a given topology

Since the observer does not distinguish between different topologies, the expected value of the entanglement between the fields and the geometrical dof is

$$\langle S \rangle = \sum_i p_{(i)} S_{(i)}$$

Assume  $N_T = 2$ ,  $N_G = 30$ ,  $P_G(T_{(1)}) = P_G(T_{(2)}) = H_G$  and  $N = 1500$  and let us set ( $\dim H = 3000$ )

$$\begin{aligned} T_{(1)} &= H_G^{30} \otimes H_F^{50}, & p_1 \times q_1 &= 30 \times 50 \\ T_{(2)} &= H_G^{60} \otimes H_F^{25}, & p_2 \times q_2 &= 60 \times 25 \end{aligned}$$

Define the “mass operator”  $M$

$$M|g^{(a)}\rangle = M^{(a)}|g^{(a)}\rangle \equiv \varepsilon a |g^{(a)}\rangle$$

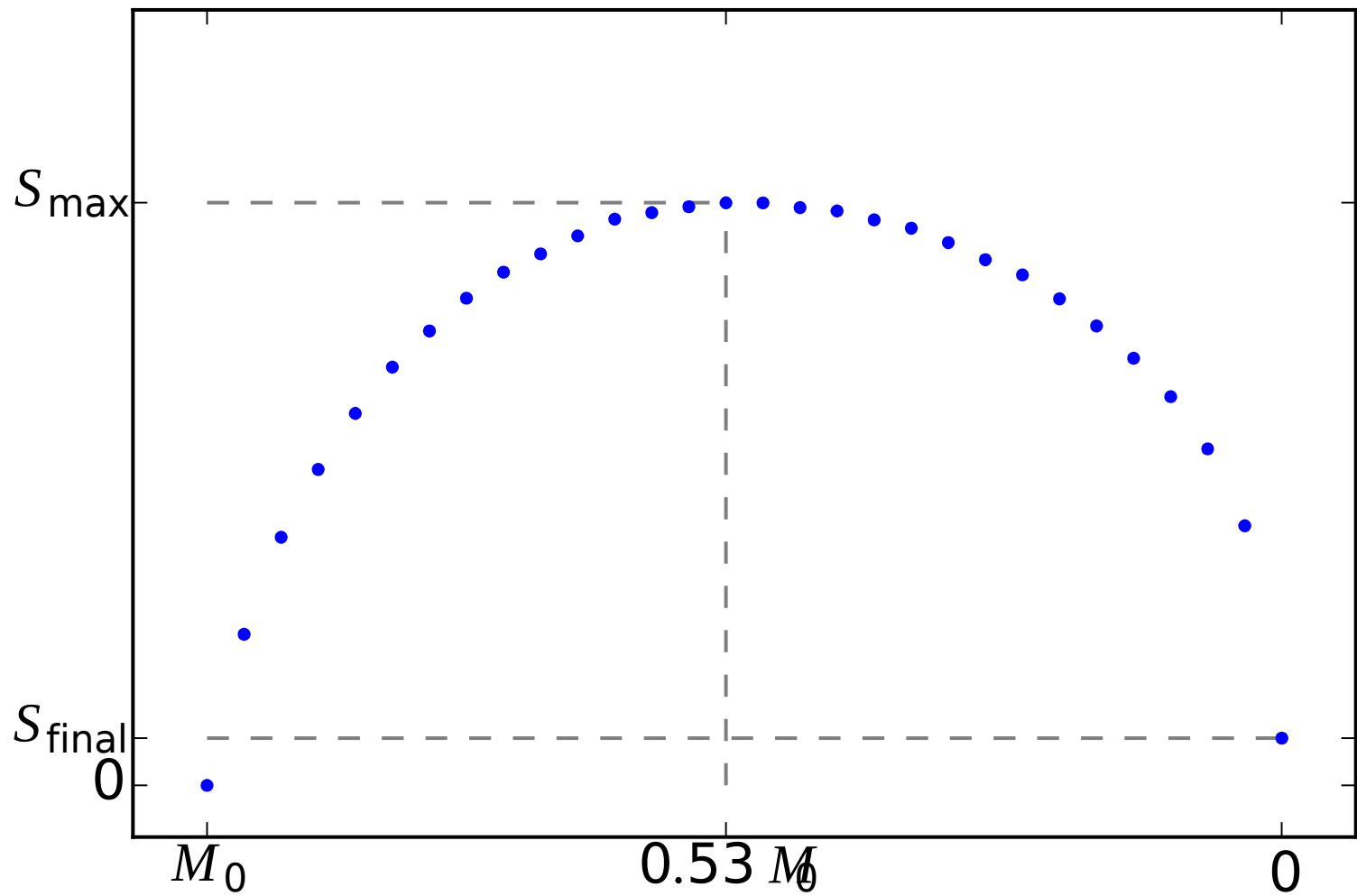
For a BH of  $M^{(a)} = a \varepsilon$  one state in  $H_G^{30}$  mapped to one  $|g^{(a)}\rangle$  by  $P_G$ , while in  $H_G^{60}$  to two such states

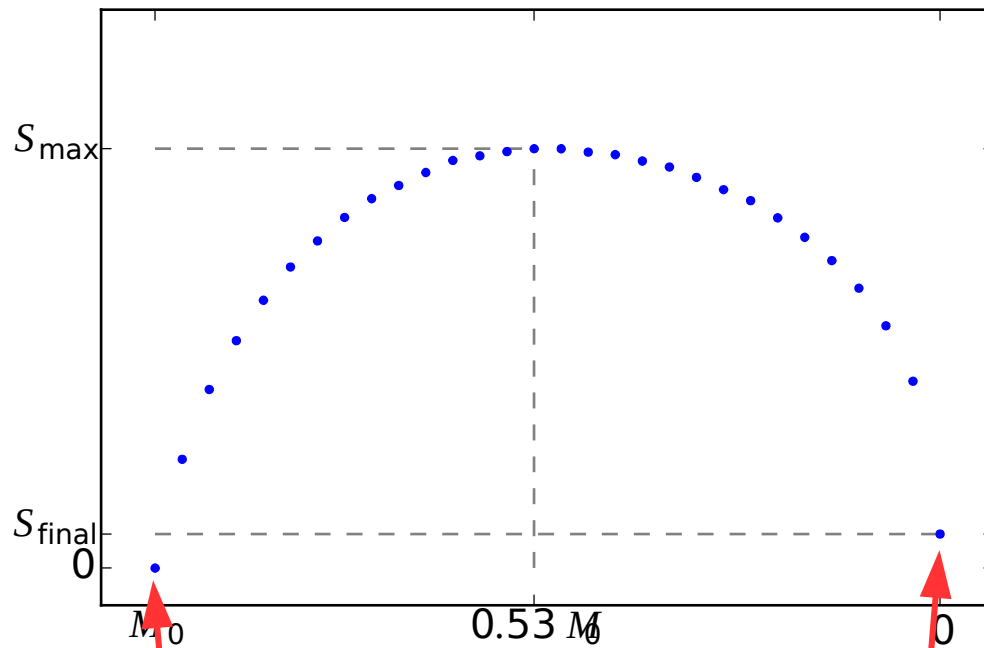
“Snapshots” of the continuous and unitary evolution in  $H$

$$\langle M \rangle = (N_G - 1 - k) \quad \text{and} \quad \langle n \rangle = k,$$

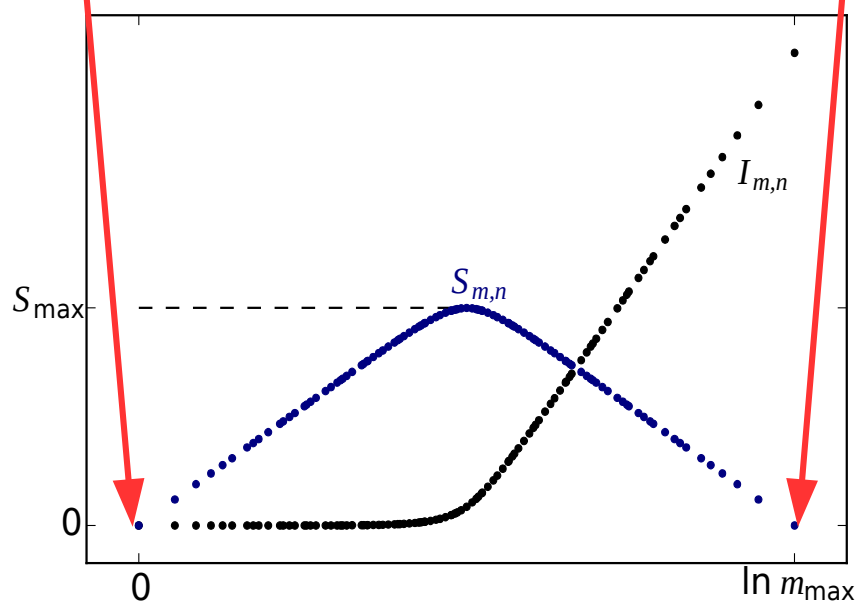
where  $k = 0, 1, \dots, N_G - 1$ .

Making the long story of the estimate of the expected entanglement entropy for a random state with prescribed expected  $\langle n \rangle$  and  $\langle M \rangle$  short (a story of Hopf coordinates parametrization on  $S^3$ , of solving constraints, of generation of sequences and their random phases... done 5000 times)



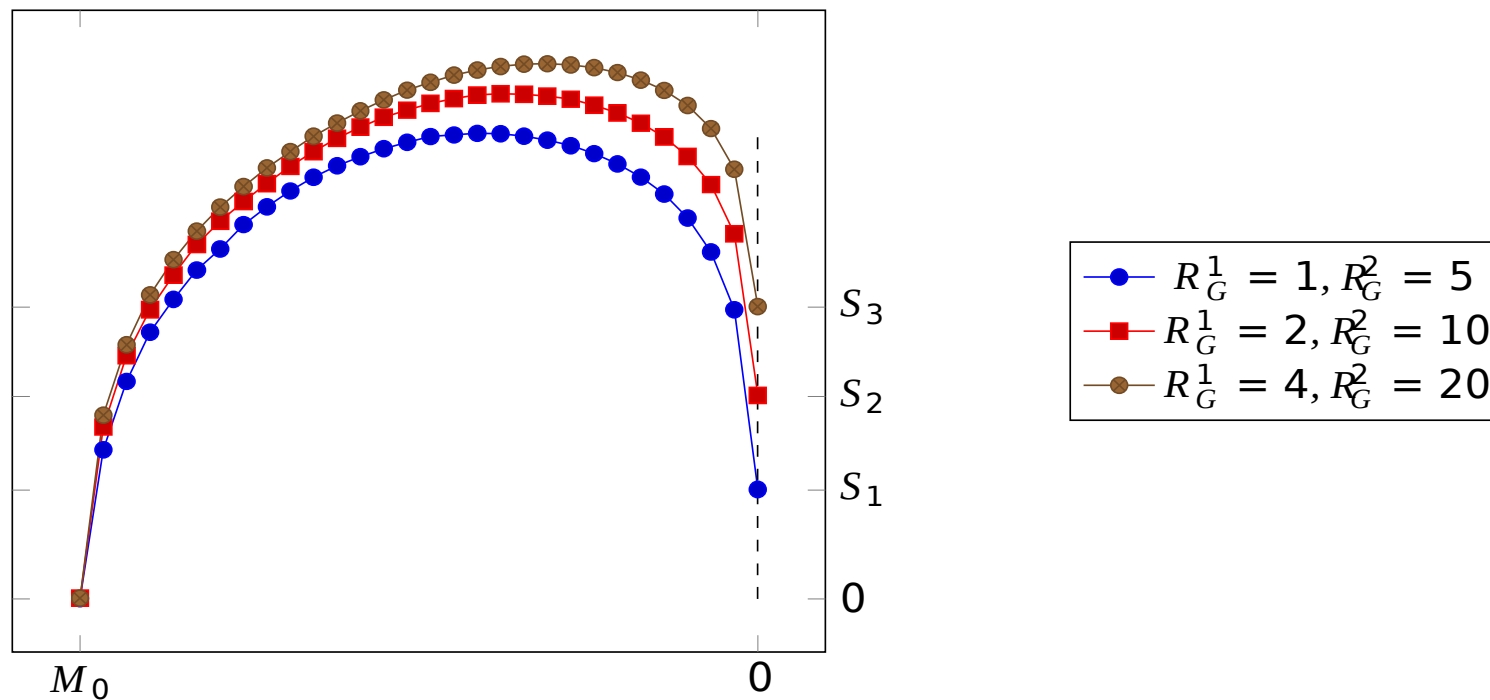


**NONZERO!**



**ZERO**





Here  $p_i = N_G R_G^i$  and  $q_i = N_F^i R_F^i$

We choose  $N_G = 30$ ,  $N_T = 2$ , and  $R_F^i = 1$  for each topology

We plot three different cases:

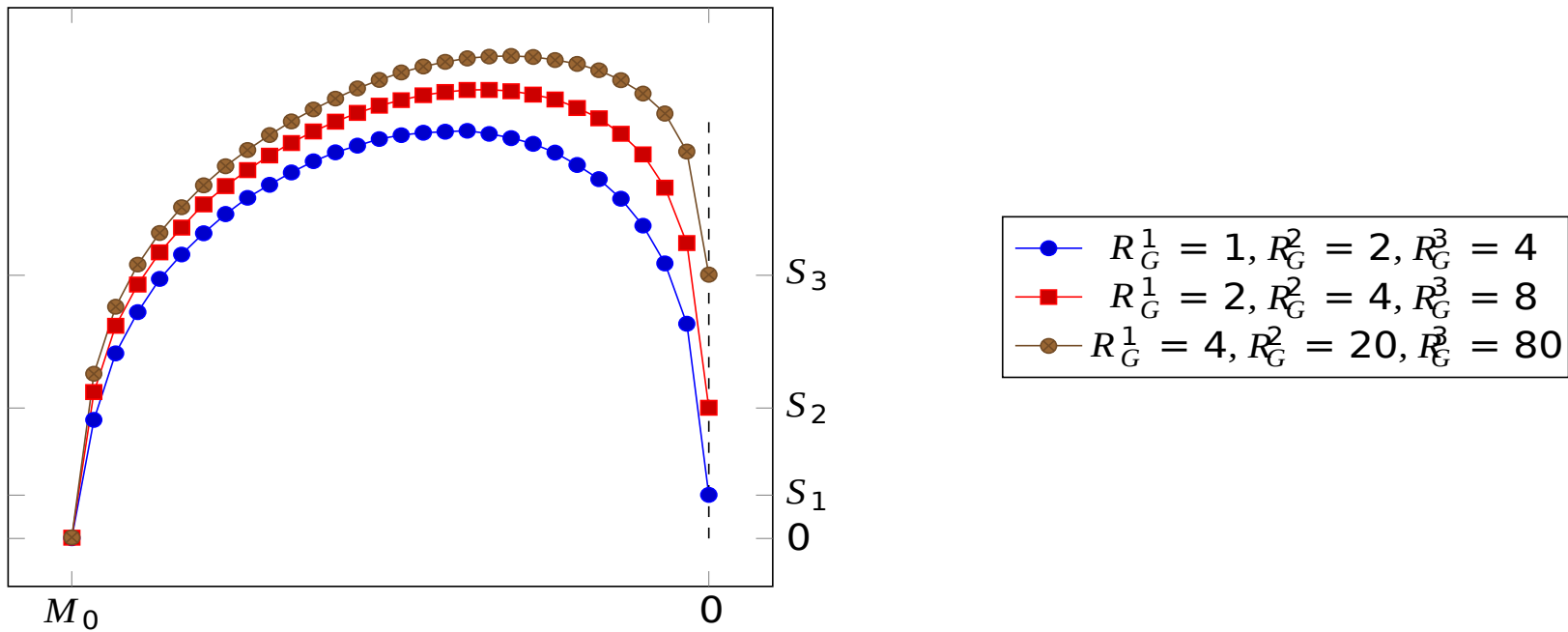
$$N_F^1 = 200, R_G^1 = 1, N_F^2 = 40, R_G^2 = 5$$

$$N_F^1 = 200, R_G^1 = 2, N_F^2 = 40, R_G^2 = 10$$

$$N_F^1 = 200, R_G^1 = 4, N_F^2 = 40, R_G^2 = 20$$

The residual entropies are

$$S_1 = 0.77, \quad S_2 = 1.43, \quad S_3 = 2.06$$



Again  $p_i = N_G R_G^i$  and  $q_i = N_F^i R_F^i$

As before  $N_G = 30$ , but  $N_T = 3$ , and

$$N_F^1 = 120, N_F^2 = 60, N_F^3 = 30 \quad \text{and} \quad R_G^1 = 1, R_G^2 = 2, R_G^3 = 4,$$

$$N_F^1 = 120, N_F^2 = 60, N_F^3 = 30 \quad \text{and} \quad R_G^1 = 2, R_G^2 = 4, R_G^3 = 8,$$

$$N_F^1 = 200, N_F^2 = 40, N_F^3 = 10 \quad \text{and} \quad R_G^1 = 4, R_G^2 = 20, R_G^3 = 80,$$

In this case, the residual entropies are

$$S_1 = 0.34, \quad S_2 = 1.02, \quad S_3 = 2.06$$

# What to do with this?

Our quasiparticle picture makes a lot of sense (to us!)

Plenty of further theoretical research (= we still don't understand most of what have done!):

dynamical realizations; more realistic BH evaporation; exact computations of  $S_{ent}$ ; realistic estimate of the degeneracy; the classical limit; coherent states; new Stone-von Neumann thrm; dark matter; the fundamental nature of oscillating particles; etc.

We shall probably follow that road, but this will not (and cannot) stop the info-loss-yes-or-not story to keep going forever...

Perhaps, one should try to look for something to measure? Is there anything around that resembles this?

First we need quasi-particles

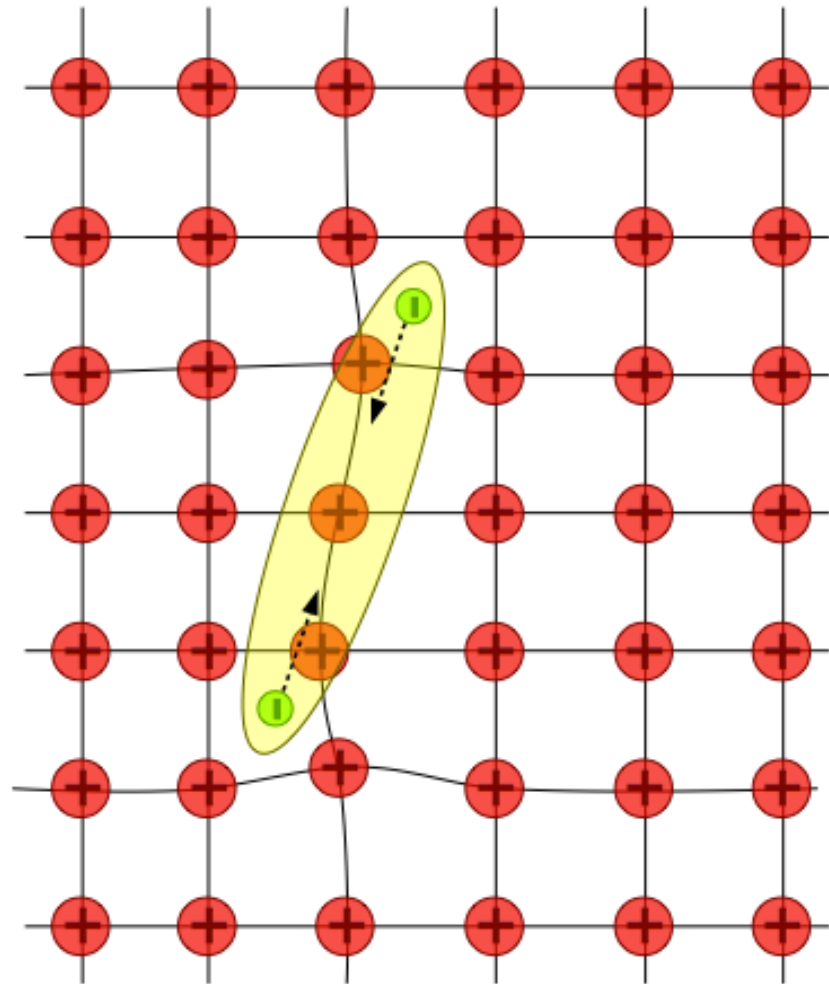
BCS pairs

Plasmons

Phonons

Magnons

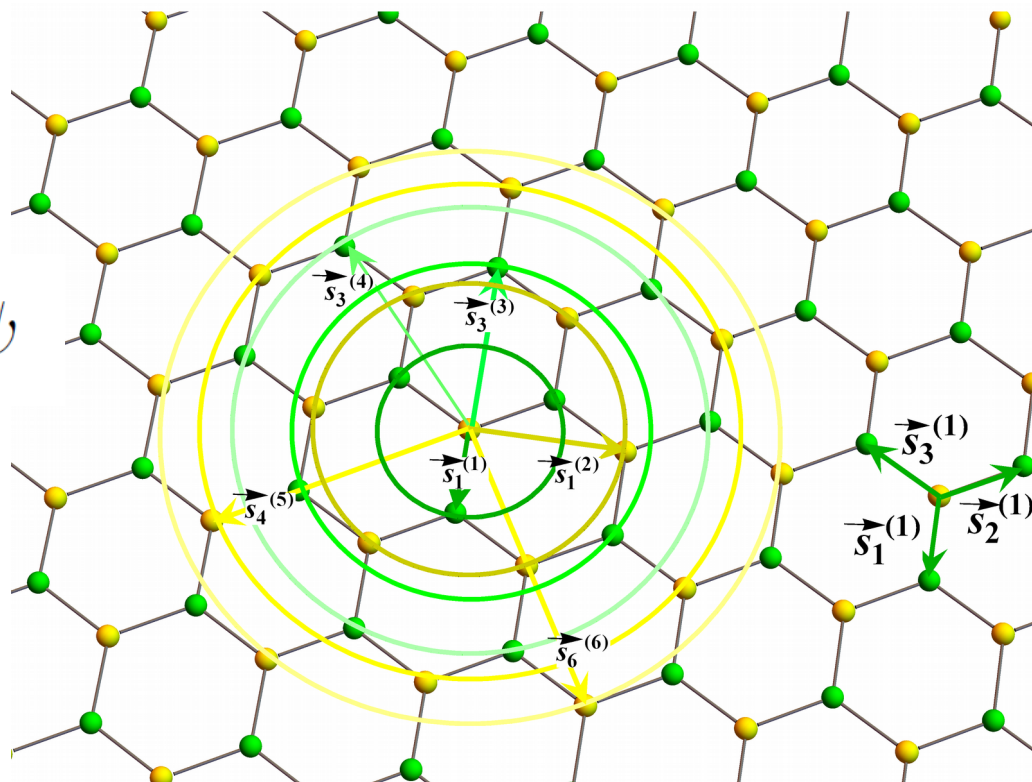
...



Rep of one Cooper pair from  
Chernodub, M.N. Lect. Notes  
Phys. 871 (2013) 143

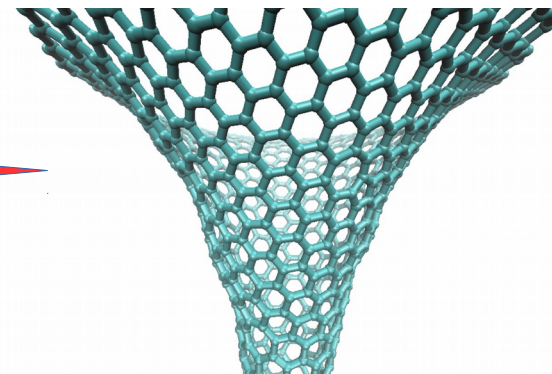
# Graphene is unique

$$i\hbar v_F \int d^3x \bar{\psi} \not{\partial} \psi$$



$$ds_{\text{graphene}}^2 = \frac{r^2}{\tilde{y}^2} \left[ \frac{\tilde{y}^2}{r^2} dt^2 - d\tilde{y}^2 - d\tilde{x}^2 \right]$$

$(\eta, \psi)$



## Dispersion relations

$$E_{\pm} = V_F \left( \pm |\vec{P}| - A |\vec{P}|^2 \right)$$

with  $\vec{P} \equiv (\hbar/\ell)(\text{Re}\mathcal{F}_1, \text{Im}\mathcal{F}_1)$ ,  $V_F \equiv \eta_1 \ell / \hbar$ ,  $A \equiv (\ell/\hbar)\epsilon(\eta_2)/\eta_1$  and

$$\mathcal{F}_1 = \sum_{i=1}^3 e^{i\vec{k} \cdot \vec{s}_i^{(1)}} = e^{-i\ell k_y} \left[ 1 + 2e^{i\frac{3}{2}\ell k_y} \cos\left(\frac{\sqrt{3}}{2}\ell k_x\right) \right]$$

and  $\mathcal{F}_2 = |\mathcal{F}_1|^2 - 3$

Henceforth *deformed* Dirac Hamiltonian

$$H(P) = V_F \sum_{\vec{k}} \psi_{\vec{k}}^{\dagger} (\not{P} - A \not{P} \not{P}) \psi_{\vec{k}}$$

with *standard* commutation relations,  $[X_i^P, P_j] = i\hbar \delta_{ij}$

Or *standard* Dirac Hamiltonian,  $\vec{Q} \equiv \vec{P}(1 - A|\vec{P}|)$

$$H(Q) = V_F \sum_{\vec{k}} \psi_{\vec{k}}^{\dagger} \not{Q} \psi_{\vec{k}}$$

with *deformed* commutation relations

$$[X_i^P, Q_j] = i\hbar \left[ \delta_{ij} - A \left( Q \delta_{ij} + \frac{Q_i Q_j}{Q} \right) \right]$$

To have nonzero intrinsic curvature  $\mathcal{K}$  on an hexagonal lattice we need disclination defects

$$\sum_p (6 - p)n_p = 6\chi_M \quad (\clubsuit)$$

and

$$\int_M \mathcal{K}(x) \equiv \mathcal{K}_{tot} = 2\pi\chi_M \quad (\spadesuit)$$

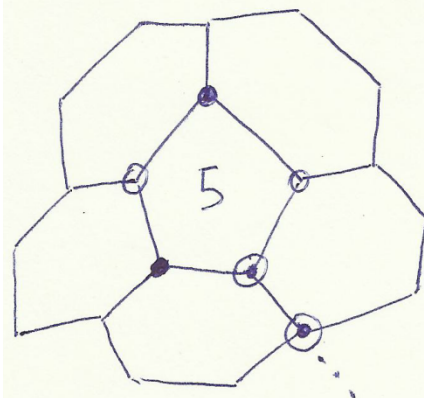
E.g.,  $M = S^2$  ( $\chi_{S^2} = 2$ )

$$(6 - 7)n_7 + (6 - 6)n_6 + (6 - 5)n_5 = 12$$



Thus, ( $\clubsuit$ ) and ( $\spadesuit$ ) together give

$$\mathcal{K}_5 = +\left(\frac{3}{\pi}\right) \frac{\mathcal{K}_{tot}}{12}$$

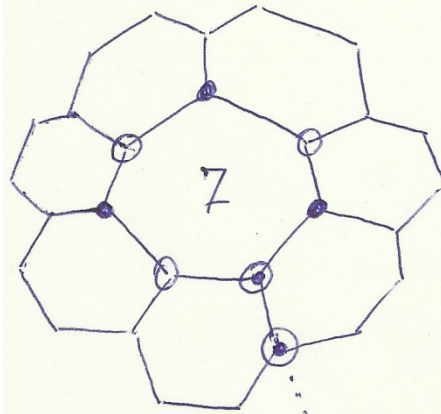


→ 1 unit of  
positive  
curvature

(126)

and

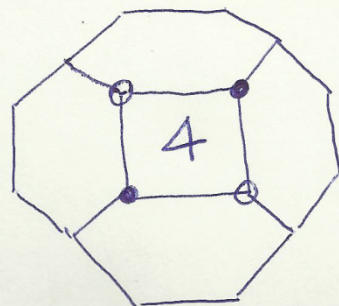
$$\mathcal{K}_7 = -\left(\frac{3}{\pi}\right) \frac{\mathcal{K}_{tot}}{12}$$



→ 1 unit of  
negative  
curvature

(127)

and so on

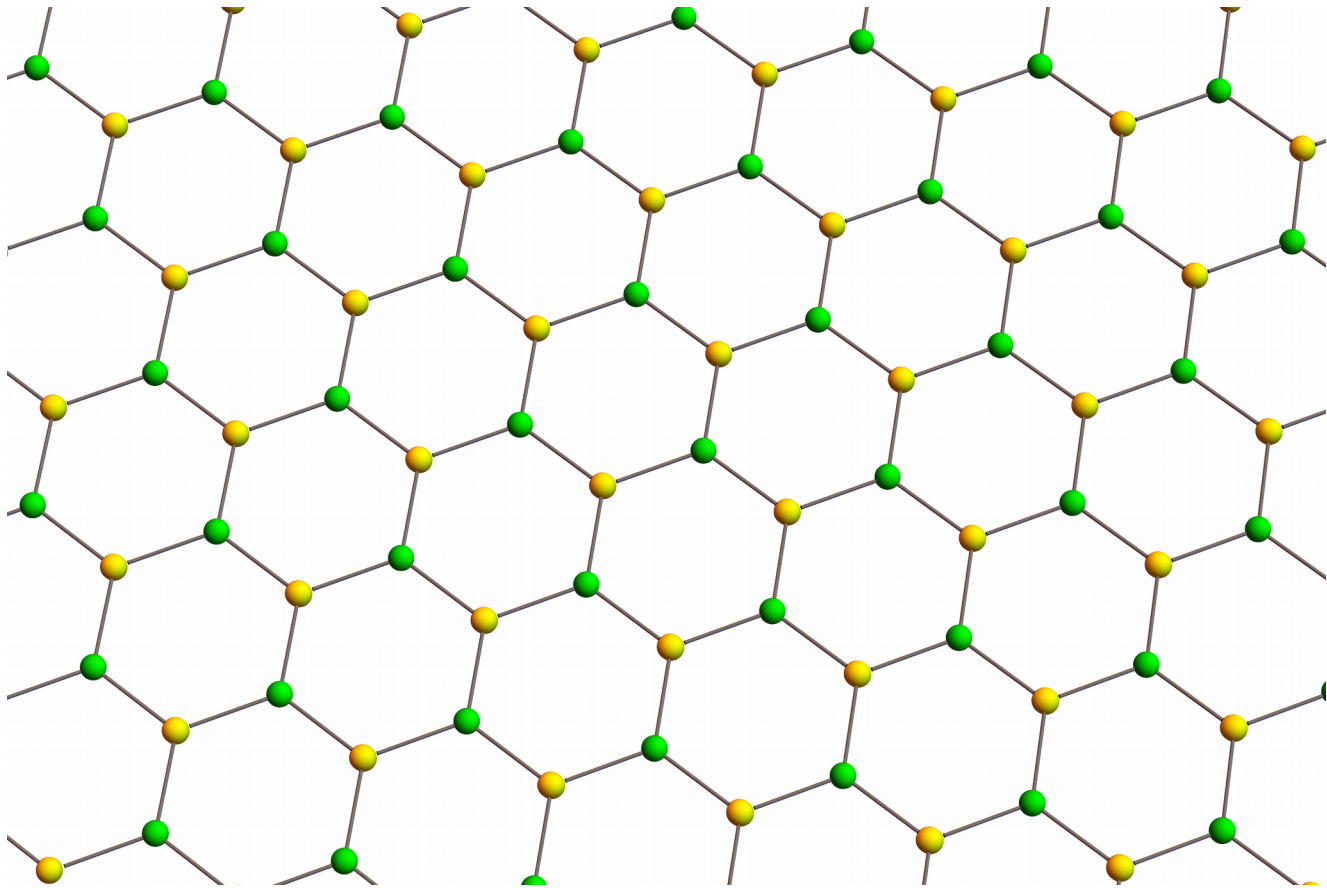


→ 2 units of  
positive  
curvature

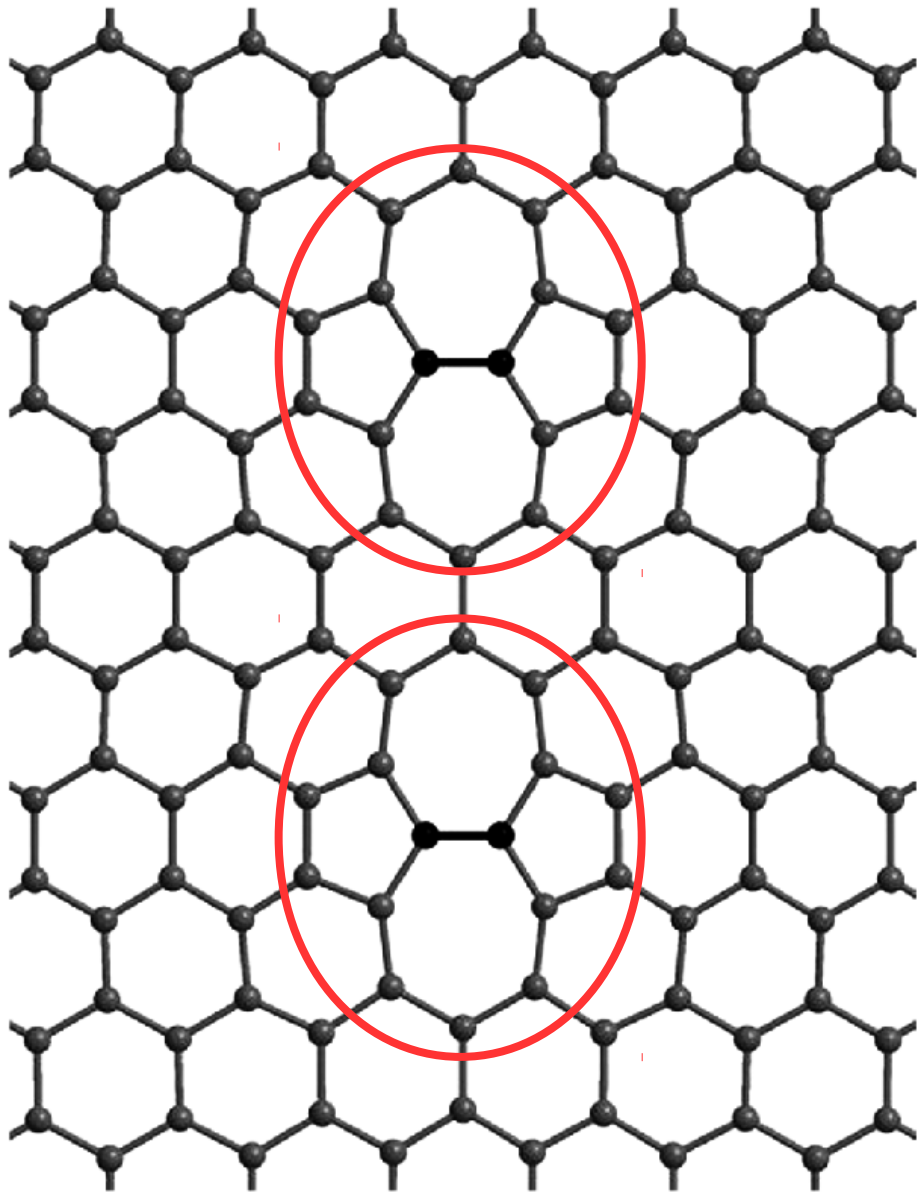
(128)

This is behind  $\omega_{\mu}^a$

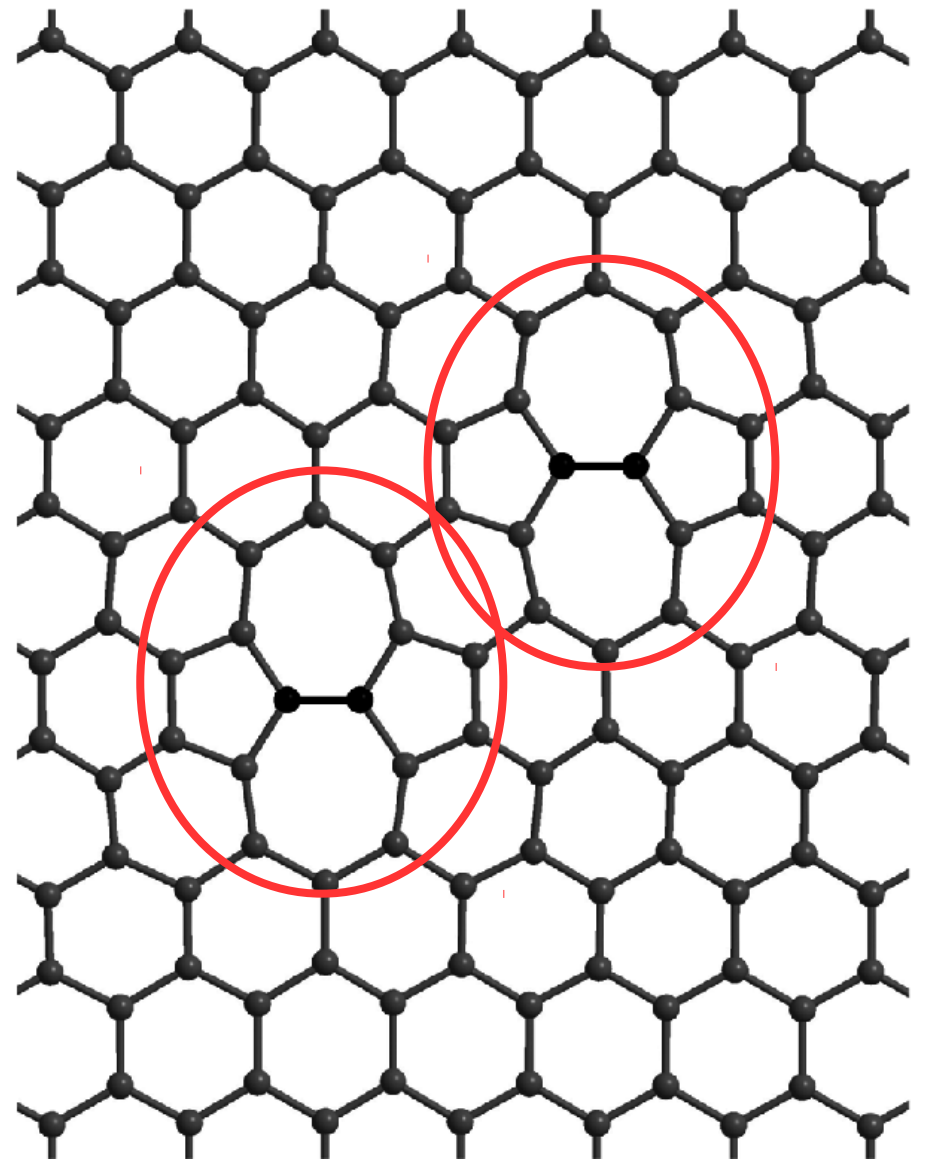




$$(\eta_{\mu\nu}, \psi)$$



$$(\eta_{\mu\nu}, \psi')$$



$$(\eta_{\mu\nu}, \psi'')$$

# *Conclusions and credits*

- The Bekenstein bound may imply the existence of the “ $X$  level”
- Both fields and geometry need be made of the same “ $X$  material”
- Even assuming unitary evolution at the  $X$  level, such unitarity is unaccessible
- We (= me) do not understand Page curve
- Should we try to test that on... graphene?

G. Acquaviva, AI, M. Scholtz, (AoP *tbp*), arXiv:1704.00345;  
and letter (sbmtd PLB)

AI, P. Pais, I.A. Elmashad, A.F. Ali, et al, arXiv:1706.01332  
(sbmtd PRD)

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J Phys: Cond Mat 28 (2016) 13LT01

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AI, G. Lambiase, PLB 716 (2012) 334; PRD 90 (2014)  
025006

AI, Ann Phys, 326 (2011) 1334

UNDERSTANDING GRAVITY:  
SPACE-TIME IS LIKE A  
RUBBER SHEET. MASSIVE  
OBJECTS DISTORT THE  
SHEET, AND—



WAIT.

THEY DISTORT IT  
BECAUSE THEY'RE  
PULLED DOWN  
BY... WHAT?



SIGH



SPACE-TIME IS LIKE THIS  
SET OF EQUATIONS, FOR  
WHICH ANY ANALOGY MUST  
BE AN APPROXIMATION.







**Giovanni  
Acquaviva**



**Martin  
Scholtz**



**Georgios  
Lukes-Gerakopoulos**



**Pablo  
Pais**



**Adamantia  
Zampeli**