

Photons in massive and non-linear theories

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Highlights of the talk

- Motivations
- Non-linear theories (Born-Infeld, Heisenberg-Euler...): Magnetar.
- Massive theories (de Broglie-Proca, Stueckelberg, Podolsky).
- Experimental state of affairs of photon mass. Solar wind and FRBs.
- LOFAR, NenuFAR, OLFAR: a novel window, the (sub-)MHz region.
- Massive photons, energy dissipation from SuSy and LoSy breaking.
- Applications to cosmology: Supernovae and Red-Shift.

Since 2015 Non-Maxwellian EM (before GR)

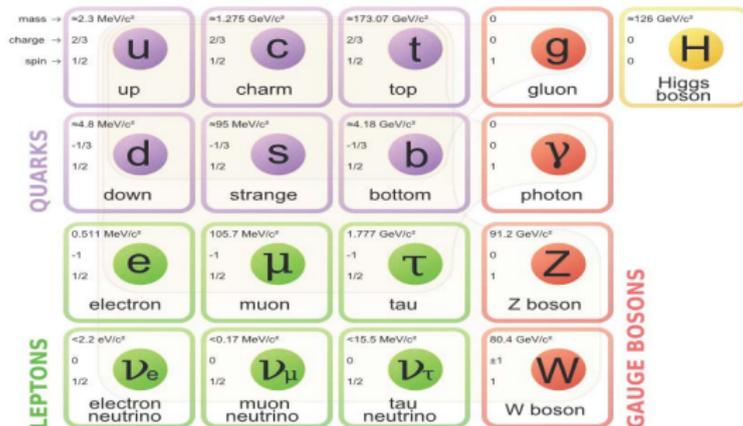
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Motivations: 1/4

- GW detection 2015, but universe understanding based on EM observations.
- As photons are the main messengers, fundamental physics has a concern in testing the foundations of electromagnetism.
- 96% universe dark (unknown), only part of 4% is known: yet precision cosmology.
- Striking contrast: complex and multi-parameterised cosmology - linear and non dissipative electromagnetism from the 19th century.
- Conversely to the graviton, photon mass is less frequently assumed.
- There is no theoretical prejudice against a photon small mass, technically natural, in that all radiative corrections are proportional to mass ('t Hooft).
- Electromagnetic radiation has zero rest mass to propagate at c . Since it carries momentum and energy, it has non-zero inertial mass. Hence, for EP, it has non-zero gravitational mass: \rightarrow light must be heavy ('t Hooft).
- The Einstein demonstration of the equivalence of mass and energy (wagon at rest on frictionless rails, photon shot *inside* end to end) implies a massive photon.
- Regularisation of the singularities of point particles, e.g. Born-Infeld.

Motivations: 2/4

- The photon is the only free massless particle of the Standard Model.
- The SM successful but shortcomings: Higgs is too light, neutrinos are massive, no gravitons...



- Physics at the end of the XIX century:
 - Laws of physics are valid anywhere and anytime.
 - Galilei transformations hold.
 - Michelson-Morley: light speed constancy.
- Conclusion: æther does not exist and light has to be reinterpreted.

- Physics at the end of the XX century:
 - Expansion is accelerating (questioned) and rotation curves.
 - GR holds.
 - No detection of dark ingredients.
- Two options: search more and better the dark or extend GR.
- Third **complementary** option to previous options: light has to be reinterpreted.

- non-Maxwellian theories are non-linear (initiated by Born and Infeld; Heisenberg and Euler) or massive photon based (de Broglie-Proca).
- Massive photon and yet gauge invariant theories include: Bopp, Laudé, Podolsky, Stueckelberg, Chern-Simons, Carroll-Field-Jackiw.
- Impact on relativity? Difficult answer: variety of the theories above; removal of ordinary landmarks and rising of interwoven implications.
- Massive photons evoked for dark matter, inflation, charge conservation, magnetic monopoles, Higgs boson, redshifts; in applied physics, superconductors and "light shining through walls" experiments. The mass can be considered effective, if depending on given parameters.

- The Born-Infeld Lagrangian

$$\mathcal{L} = \sqrt{1 + F} - 1 + j^\mu A_\mu \quad (1)$$

- The equations are

$$\partial_\mu \left(\frac{F^{\mu\nu} (1 + F)^{-\frac{1}{2}}}{2} \right) = j^\nu \quad (2)$$

- Electromagnetic field gives origin to the mass of the charge.
- Avoidance of infinities out of self-energy $\phi(0) = 1.8541 \frac{e}{r_0}$.
- The parameter r_0 is computed out of analytic expressions.

- The Heisenberg-Euler Lagrangian

$$\mathcal{L} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{e^2}{\hbar c} \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \cdot \left\{ i\frac{\eta^2}{2} F^{\mu\nu} F_{\mu\nu}^* \cdot \frac{\cos\left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F_{\mu\nu}^*}\right] + \cos\left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F_{\mu\nu}^*}\right]}{\cos\left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F_{\mu\nu}^*}\right] - \cos\left[\frac{\eta}{\mathfrak{E}_k} \sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F_{\mu\nu}^*}\right]} + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6} \cdot F_{\mu\nu}F^{\mu\nu} \right\} \quad (3)$$

$$F_{\mu\nu}^* = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \quad (4)$$

- Photon-Photon interaction and Photon splitting since HE theory relates to second order QED.
- Vacuum polarisation occurs for $E_c > 1.3 \times 10^{18}$ V/m or $B_c > 4.4 \times 10^{13}$ G.

Non-linear theories: Magnetar 3/3

Heisenberg-Euler on magnetars overcritical magnetic field. Blue or red shift depending on polarisation for a photon emitted up to similar values to the gravitational redshift.

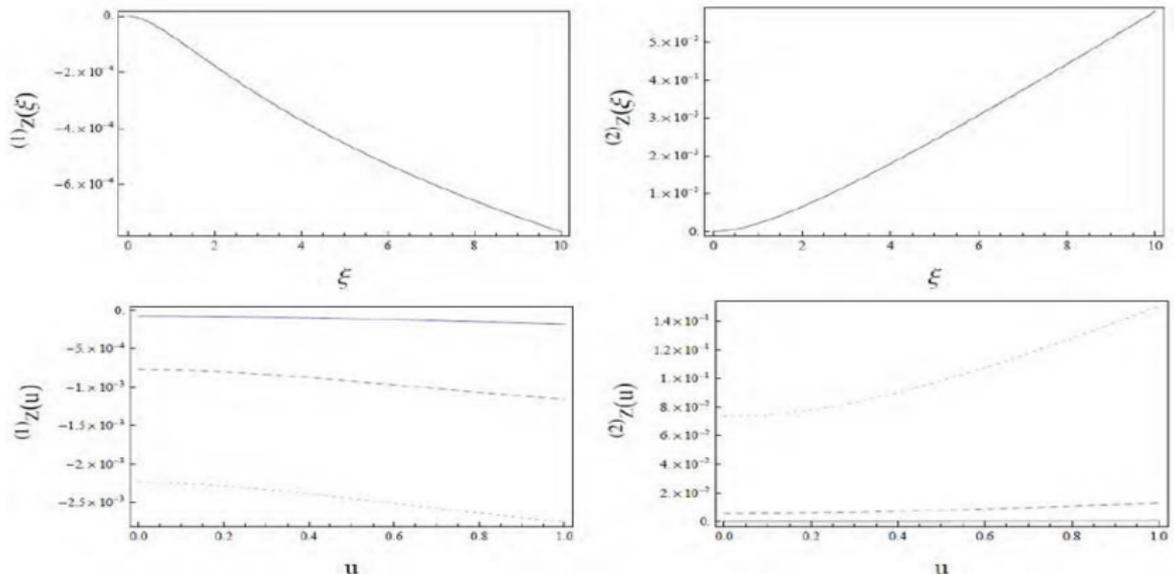


Fig.1. EMS (Electromagnetic shift) of the two photon polarisations versus the ratio of the magnetic/overcritical fields (upper panel), and the azimuthal angle (lower panel). The EMS can reach comparable values to the gravitational Einstein shift. The figure is taken from [Bonetti, Perez Bergliaffa, Spallicci, 2016].

Massive theories: de Broglie-Proca 1/5

- The concept of a massive photon has been vigorously pursued by Louis de Broglie from 1922 throughout his life. He defines the value of the mass to be lower than 10^{-53} kg. A comprehensive work of 1940 contains the modified Maxwells equations and the related Lagrangian.
- Insted, the original aim of Alexandru Proca, de Broglie's student, was the description of electrons and positrons. Despite Proca's several assertions on the photons being massless, his work has been used.

Massive theories: de Broglie-Proca 2/5

$$\mathcal{L} = -\frac{1}{4\mu} F_{\alpha\beta} F^{\alpha\beta} - \frac{\mathcal{M}^2}{2\mu} A_\alpha A^\alpha - j^\alpha A_\alpha \quad (5)$$

$F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$. Minimal action (Euler-Lagrange) \rightarrow inhomogeneous eqs.

Ricci Curbastro-Bianchi identity $\partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} + \partial^\mu F^{\nu\lambda} = 0 \rightarrow$ homogeneous eqs.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2 \phi, \quad (6)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A}, \quad (7)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (8)$$

$$\nabla \cdot \vec{B} = 0, \quad (9)$$

ϵ_0 permittivity, μ_0 permeability, ρ charge density, \vec{j} current, ϕ and \vec{A} potential.

$\mathcal{M} = m_\gamma c / \hbar = 2\pi / \lambda$, \hbar reduced Planck (or Dirac) constant, c speed of light, λ Compton wavelength, m_γ photon mass.

Eqs. (6, 7) are Lorentz-Poincaré transformation but not Lorenz gauge invariant, though in static regime they are not coupled through the potential.

Massive theories: de Broglie/Proca 3/5

From the Lagrangian we get $\partial_\alpha F^{\alpha\beta} + \mathcal{M}^2 A^\beta = \mu j^\beta$. With the Lorentz subsidiary condition $\partial_\gamma A^\gamma = 0$,

$$[\partial_\mu \partial^\mu + \mathcal{M}^2] A^\nu = 0 \quad (10)$$

Through Fourier transform, at high frequencies (photon rest energy $<$ the total energy; $\nu \gg 1$ Hz), the positive difference in velocity for two different frequencies ($\nu_2 > \nu_1$) is

$$\Delta v_g = v_{g2} - v_{g1} = \frac{c^3 \mathcal{M}^2}{8\pi^2} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right), \quad (11)$$

being v_g the group velocity. For a single source at distance d , the difference in the time of arrival of the two photons is

$$\begin{aligned} \Delta t &= \frac{d}{v_{g1}} - \frac{d}{v_{g2}} \simeq \frac{\Delta v_g d}{c^2} = \frac{dc \mathcal{M}^2}{8\pi^2} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \\ &\simeq \frac{d}{c} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) 10^{100} m_\gamma^2. \end{aligned} \quad (12)$$

- The Stueckelberg Lagrangian

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + m^2 \left(A_\mu - \frac{\partial_\mu B}{m} \right)^2 - (\partial^\mu A_\mu + mB)^2 \quad (13)$$

where B is a scalar field to render the dBP *manifestly* gauge invariant.

- We have two fields and two equations of motion. The wave equations are

$$\partial_\mu \partial^\mu A^\nu + m^2 A^\nu = 0 \quad (14)$$

$$\partial_\mu \partial^\mu B + m^2 B = 0 \quad (15)$$

- First massive photon theory, gauge invariant

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda \quad B \rightarrow B + m\Lambda \quad (\partial^2 + m^2)\Lambda = 0$$

- Used as alternative to dark energy, Akarsu et al., 2017, Foundations of Physics on Cosmology (Capozziello, Prokopec, Spallicci Eds.)

- The Podolsky Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{b^2}{4}(\partial^\nu F^{\mu\nu})\partial_\nu F_{\mu\nu} + j^\mu A_\mu \quad (16)$$

where b has the dimension of m^{-1} .

- The equations are

$$-b^2\partial_\mu\partial^\mu(\vec{\nabla}\cdot\vec{E}) + \vec{\nabla}\cdot\vec{E} - \rho = 0 \quad (17)$$

$$-b^2\partial_\mu\partial^\mu\left[\frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B}\right] + \frac{\partial\vec{E}}{\partial t} - \vec{\nabla}\times\vec{B} + \vec{j} = 0 \quad (18)$$

- Gauge invariant $A_\mu \rightarrow A_\mu + \partial_\mu\Lambda$
- Magnetic monopoles? and massive photons.
- Cut-off for short distances $\phi = \frac{e}{4e\pi}(1 - e^{-\frac{r}{b}})$

Experimental mass limits: PDG 1/8

γ MASS

Results prior to 2008 are critiqued in GOLDHABER 10. All experimental results published prior to 2005 are summarized in detail by TU 05.

The following conversions are useful: $1 \text{ eV} = 1.783 \times 10^{-33} \text{ g} = 1.957 \times 10^{-6} m_{\oplus}$; $\lambda_C = (1.973 \times 10^{-7} \text{ m}) \times (1 \text{ eV}/m\gamma)$.

VALUE (eV)	CL%	DOCUMENT ID	TECN	COMMENT
<1 $\times 10^{-10}$		1 RYUTOV	07	MHD of solar wind
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
<1.8 $\times 10^{-14}$		2 BONETTI	16	Fast Radio Bursts, FRB 150418
<1.9 $\times 10^{-15}$		3 RETINO	16	Ampere's Law in solar wind
<2.3 $\times 10^{-9}$	95	4 EGOROV	14	COSM Lensed quasar position
		5 ACCIOLY	10	Anomalous mag. mom.
<1 $\times 10^{-26}$		6 ADELBERGER 07A		Proca galactic field
no limit feasible		6 ADELBERGER 07A		γ as Higgs particle
<1 $\times 10^{-19}$		7 TU	06	Torque on rotating magnetized toroid
<1.4 $\times 10^{-7}$		ACCIOLY	04	Dispersion of GHz radio waves by sun
<2 $\times 10^{-16}$		8 FULLEKRUG	04	Speed of 5-50 Hz radiation in atmosphere
<7 $\times 10^{-19}$		9 LUO	03	Torque on rotating magnetized toroid
<1 $\times 10^{-17}$		10 LAKES	98	Torque on toroid balance
<6 $\times 10^{-17}$		11 RYUTOV	97	MHD of solar wind
<8 $\times 10^{-16}$	90	12 FISCHBACH	94	Earth magnetic field
<5 $\times 10^{-13}$		13 CHERNIKOV	92	SQID Ampere's Law null test
<1.5 $\times 10^{-9}$	90	14 RYAN	85	Coulomb's Law null test
<3 $\times 10^{-27}$		15 CHIBISOV	76	Galactic magnetic field
<6 $\times 10^{-16}$	99.7	16 DAVIS	75	Jupiter's magnetic field
<7.3 $\times 10^{-16}$		HOLLWEG	74	Alfven waves
<6 $\times 10^{-17}$		17 FRANKEN	71	Low freq. res. circuit
<2.4 $\times 10^{-13}$		18 KROLL	71A	Dispersion in atmosphere
<1 $\times 10^{-14}$		19 WILLIAMS	71	CNTR Tests Coulomb's Law
<2.3 $\times 10^{-15}$		GOLDHABER	68	Satellite data

Experimental mass limits: the graviton 2/8

- LIGO upper limit 2×10^{-58} kg
- Often determination of graviton mass upper limit supposes massless photons

3. Graviton mass limits:

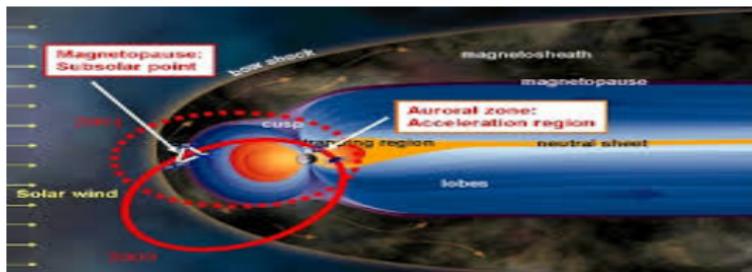
Gravitation wave dispersion (Finn and Sutton, 2002)	3×10^{12}	8×10^{-20}	10^{-55}	Question mark for scalar graviton
Pulsar timing (Baskaran <i>et al.</i> , 2008)	2×10^{16}	9×10^{-24}	2×10^{-59}	Fluctuations due to graviton phase velocity
Gravity over cluster sizes (Goldhaber and Nieto, 1974)	2×10^{22}	10^{-29}	2×10^{-65}	
Near field constraints (Gruzinov, 2005)	3×10^{24} (10^8 pc)	6×10^{-32}	10^{-67}	For DGP model
Far field constraints (Dvali, Gruzinov, and Zaldarriaga, 2003)	3×10^{26} (10^{10} pc)	6×10^{-34}	10^{-69}	For DGP model

Experimental mass limits: 3/8

- Laboratory experiment (Coulomb's law) 2×10^{-50} kg.
- Dispersion-based limit 3×10^{-49} kg (lower energy photons travel at lower speed). Note: quantum gravity affects high frequencies (GRB, Amelino-Camelia).
- Ryutov finds $m_\gamma < 10^{-52}$ kg in the solar wind at 1 AU, and $m_\gamma < 1.5 \times 10^{-54}$ kg at 40 AU (PDG value). These values come partly from *ad hoc* models. Limits:
 - (i) the magnetic field is assumed exactly always and everywhere a Parker's spiral;
 - (ii) the accuracy of particle data measurements (from e.g. Pioneer or Voyager) has not been discussed; (iii) there is no error analysis, nor data presentation.
- Speculative lower limits from modelling the galactic magnetic field: 3×10^{-63} kg include differences of ten orders of magnitude on same data.
- New theoretical limits from black holes stability, gravitational light bending, CPT violation.

- Quote "Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterisations. This is perhaps due to the temptation to assert too strongly something one knows to be true. A look at the summary of the Particle Data Group (Amsler et al., 2008) hints at this. In such a spirit, we give here our understanding of both secure and speculative mass limits."
Goldhaber and Nieto, Rev. Mod. Phys., 2000
- The lowest theoretical limit on the measurement of any mass is dictated by the Heisenberg's principle $m \geq \hbar \Delta t c^2$, and gives 3.8×10^{-69} kg, where Δt is the supposed age of the Universe.

Experimental mass limits: Cluster 5/8



- Highly elliptical evolving orbits in tetrahedron: perigee $4 R_{\oplus}$ apogee $19.6 R_{\oplus}$, visited a wide set of magnetospheric regions. Inter-spacecraft separation ranging from 10^2 to 10^4 km.
- Small mass \rightarrow precise experiment or very large apparatus (Compton wavelength). The largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind.

Experimental mass limits: Cluster 6/8

- $j_P = 1.86 \cdot 10^{-7} \pm 3 \cdot 10^{-8} \text{ A m}^{-2}$, while $j_B = |\nabla \times \vec{B}|/\mu_0$ is $3.5 \pm 4.7 \cdot 10^{-11} \text{ A m}^{-2}$. A_H is an estimate, not a measurement.

$$A_H^{\frac{1}{2}} (m_\gamma + \Delta m_\gamma) = A_H^{\frac{1}{2}} \left(m_\gamma + \left| \frac{\partial m_\gamma}{\partial j_P} \right| \Delta j_P + \left| \frac{\partial m_\gamma}{\partial j_B} \right| \Delta j_B \right) = k \left[(j_P - j_B)^{\frac{1}{2}} + \frac{\Delta j_P + \Delta j_B}{2(j_P - j_B)^{\frac{1}{2}}} \right]. \quad (19)$$

Considering j_P and Δj_P of the same order, $j_P = 0.62 \Delta j_P$, and both much larger than j_B and Δj_B , Eq. (19), after squaring, leads to

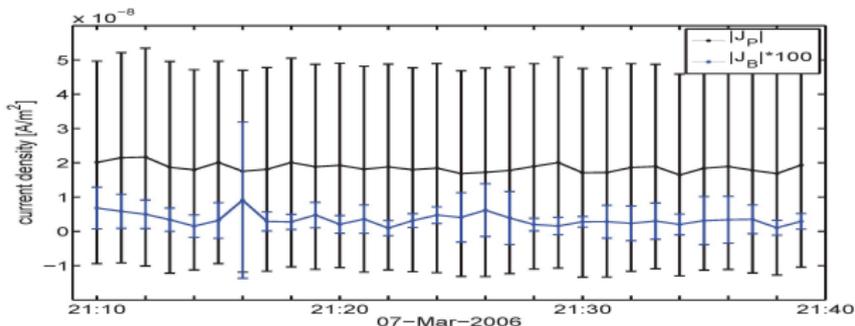
$$A_H^{\frac{1}{2}} (m_\gamma + \Delta m_\gamma) \sim k (j_P + \Delta j_P)^{1/2}. \quad (20)$$

Table: The values of m_γ (according to the estimate on A_H).

A_H [T m]	0.4	29 (Z)	637
m_γ [kg]	1.4×10^{-49}	1.6×10^{-50}	3.4×10^{-51}

Experimental mass limits: Cluster 7/8

- The particle current density $\vec{j} = \vec{j}_P = ne(\vec{v}_i - \vec{v}_e)$ from ion and electron currents; n is the number density, e the electron charge and \vec{v}_i , \vec{v}_e the velocity of the ions and electrons, respectively.
- An accurate assessment of the particle current density in the solar wind is difficult due to inherent instrument limitations.
- $j_P \gg j_B$ (up to four orders of magnitude), mostly due to the differences in the i, e velocities, while the estimate of density is reasonable. While we can't exclude that this difference is due to the dBP massive photon, the large uncertainties related to particle measurements hint to instrumental limits.



Experimental mass limits: dispersion 8/8

- Photon mass reproduces plasma dispersion, the frequency f^{-2} dependence of the group velocity of the pulsar or FRB radiation through the ionised components of the interstellar medium. Again, pulses at lower radio frequencies arrive later than those at higher frequencies.
- In absence of an alternative way to measure plasma dispersion, there is no way to disentangle plasma effects from a dBP photon

$$\frac{m_\gamma}{\sqrt{n}} \left[\text{kg m}^{3/2} \right] = 6.62 \times 10^{-50}, \quad (21)$$

implies that for this ratio, a massive photon and the average electron density along the line of sight determine the same dispersion.

- Data on FRB 150418 indicate $m_\gamma \lesssim 1.8 \times 10^{-14} \text{ eV c}^{-2}$ ($3.2 \times 10^{-50} \text{ kg}$), for a redshift $z = 0.492(?)$, while for FRB 121102 $m_\gamma \lesssim 2.2 \times 10^{-14} \text{ eV c}^{-2}$ ($3.9 \times 10^{-50} \text{ kg}$). The different redshift dependences of the plasma and photon mass contributions to DM can be used to improve the sensitivity to m_γ .

SuSy and LoSy breaking: 1/4

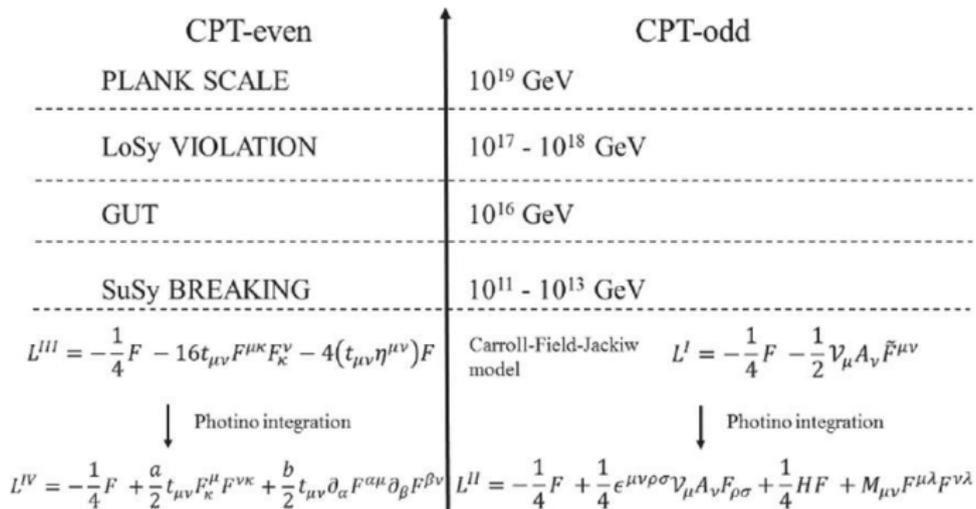


FIG. 1: Breaking energy values and the Lagrangians. A different hierarchy of LoSy, SuSy breaking and Grand Unification Theories (GUT) does not interfere with the dispersion laws of the photonic sector at low energies.

- Standard Model extensions (SMEs) address issues like the Higgs boson mass lightness, the dark universe, neutrino oscillations and their mass.
- Four models involving Super and Lorentz symmetries breaking and analysis in the photon sector. Dispersion relations show a non-Maxwellian behaviour for CPT even and odd sectors.
- In the latter, a massive photon behaviour, f^{-2} in the group velocities emerges.
- A massive and gauge invariant Carroll-Field-Jackiw term in the Lagrangian is extracted and shown to be proportional to the background vector.
- The mass is lower than 10^{-18} eV or 10^{-55} kg.

SuSy and LoSy breaking: OLFAR nanosatellites 3/4

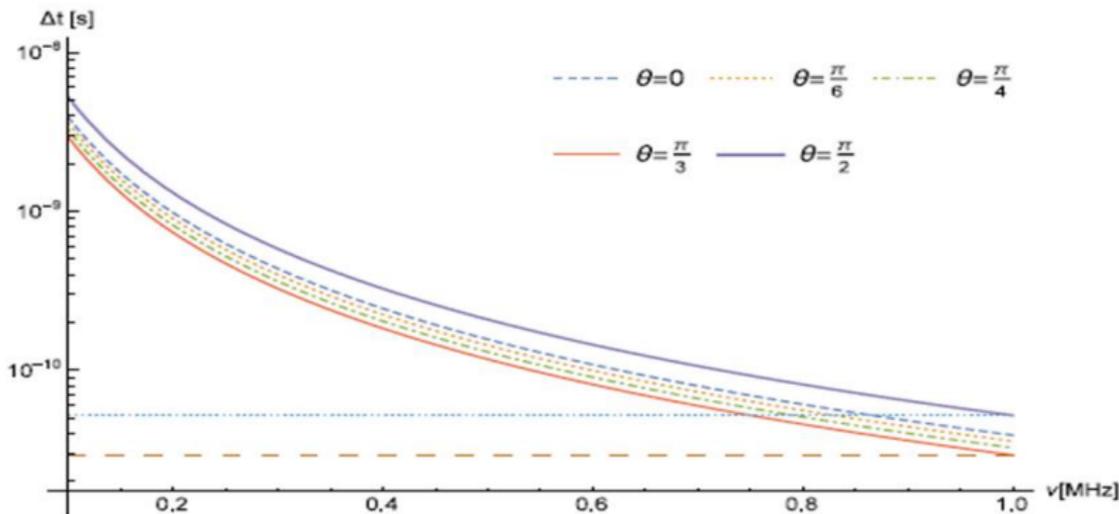


FIG. 2: For Class I, we plot the delays [s], Eq. (16), for different angles, Eqs. (12,13), using $|\vec{V}| = 10^{-19}$ eV [40], versus frequency. We have supposed the source to be at a distance of 4 kpc. The frequency range 0.1 - 1 MHz has been chosen since it is targeted by recently proposed low radio frequency space detectors, composed by a swarm of nano-satellites; see [41] and references therein. There is a feeble dependence of the delays on θ . The delay is of about 50 ps at 1 MHz for $\theta = \pi/2$, Eq. (13), and around half of this value for θ approaching $\pi/2$, Eq. (12).

SuSy and LoSy breaking: wave dissipation 4/4

Photon energy-momentum tensor conservation

$$\begin{aligned} \partial_\mu \theta^\mu_\rho = & j^\nu f_{\nu\rho} - (\partial_\mu F_B^{\mu\nu}) f_{\nu\rho} - V_\mu * F_B^{\mu\nu} f_{\nu\rho} - \frac{1}{2} (\partial_\mu V_\rho) * f^{\mu\nu} a_\nu + \frac{1}{4} \left(\partial_\rho k_F^{\mu\nu\kappa\lambda} \right) f_{\mu\nu} f_{\kappa\lambda} \\ & - \left(\partial_\mu k_F^{\mu\nu\kappa\lambda} \right) F_{B\kappa\lambda} f_{\nu\rho} - k_F^{\mu\nu\kappa\lambda} (\partial_\mu F_{B\kappa\lambda}) f_{\nu\rho} - \zeta n_\mu \left[(n \cdot \partial)^2 * F_B^{\mu\nu} \right] f_{\nu\rho}, \end{aligned} \quad (22)$$

with

$$\begin{aligned} \theta^\mu_\rho = & f^{\mu\nu} f_{\nu\rho} + \frac{1}{4} \delta_\rho^\mu f^2 - \frac{1}{2} V_\rho * f^{\mu\nu} a_\nu + k_F^{\mu\nu\kappa\lambda} f_{\kappa\lambda} f_{\nu\rho} + \frac{1}{4} \delta_\rho^\mu k_F^{\kappa\lambda\alpha\beta} f_{\kappa\lambda} f_{\alpha\beta} \\ & + \zeta n^\mu n_\nu f_{\kappa\rho} (n \cdot \partial) * f^{\nu\kappa} + \frac{\zeta}{2} n_\rho (n \cdot \partial a_\nu) (n \cdot \partial * f^{\mu\nu}) \end{aligned} \quad (23)$$

The right hand-side of Eq.(22) displays all types of terms that describe the exchange of energy between the photon $f^{\mu\nu}$, the LSV background (V_ρ CPT odd, $k_F^{\kappa\lambda\alpha\beta}$ CPT even, ζ Myers-Pospelov) and the external field $F_B^{\mu\nu}$, taking into account an x^μ dependence of the LSV background and the external field.

Does wave energy loss translate into frequency damping for a photon, 'tired light' ?

- Do non-linear theories produce dissipation?
- How wave dissipation is translated into photon language?
- Could non-linear redshifts complement or be part of the cosmological (accelerated) expansion? Uncertainties: 1. Hubble constant range 65-77 km/s per Mpc (Jackson, 2015, Liv. Rev. Rel.) 2. data SN1 consistent with constant expansion (Nielsen, Guffanti, Sarkar, 2016, Scient. Rep.).
- If SN spectrum shifts towards lower frequencies, massive photon may mimic time dilation, even if the source is not moving. Relevant corrections?
- For alternative cosmologies see 2017 Lopez-Corredoira on Foundations of Physics (Capozziello, Prokopec, Spallicci, Eds.)
- Experiment on local expansion and/or non-Maxwellian redshift? 10^{-18} m/s per m, compatible with state of the art technology.

Grazie per la vostra attenzione

