Weyl current, scale invariance and Planck Scale generation

G. Ross, CORFU Summer Institute, Sept 2017

Based on:

No fifth force in a scale invariant universe
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1612.03157 [gr-qc], to appear Phys. Rev .D.

Weyl Current, Scale-Invariant Inflation and Planck Scale Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1610.09243 [hep-th]. Phys.Rev. D95 (2017) no.4, 043507.

Scale-Independent Inflation and Hierarchy Generation
By Pedro G. Ferreira, Christopher T. Hill, Graham G. Ross.
arXiv:1603.05983 [hep-th]. Phys.Lett. B763 (2016) 174-178.

See also:

M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 162 (2009).
M. Shaposhnikov and D. Zenhausern, Phys. Lett. B **671**, 187 (2009)
D. Blas, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 044001 (2011)
J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D **84**, 123504 (2011).
R. Kallosh and A. Linde, JCAP 1310 (2013), 033; J. J. M. Carrasco, R. Kallosh and A. Linde [arXiv:1506.00936[hep-th]]
K. Allison, C. T. Hill and G. G. Ross, Nucl. Phys. B **891**, 613 (2015) Phys. Lett. B **738**, 191 (2014)
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K. Kannike, G. Htsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio and A. Strumia, JHEP **1505**, 065 (2015)
G. K. Karananas and J. Rubio, Phys. Lett. B **761**, 223 (2016)
G. K. Karananas and M. Shaposhnikov, Phys. Rev. D **93**, no. 8, 084052 (2016)
I. Quiros, arXiv:1405.6668 [gr-qc]; arXiv:1401.2643 [gr-qc].

Motivation: Hierarchies

- $\Lambda \equiv \rho_{vac} \sim 10^{-122} M_P^4$

- $m_{Higgs} \sim 10^{-16} M_p$

- $m_{\text{inflaton}} \ll H_I$ (slow roll)

Symmetry origin?

$\Rightarrow$ SUSY

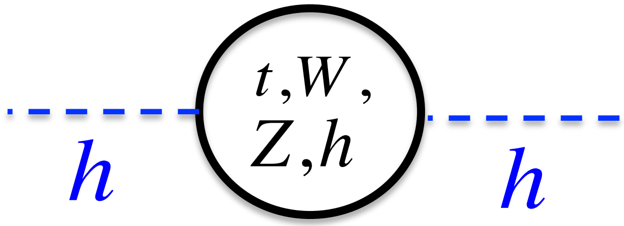
$$\Delta = \Delta_{SUSY} \sim TeV\,(?)$$
$$\Lambda \sim 10^{-60} M_P^4$$
$$m_{\text{inflaton}} \sim H_I \;(\eta - problem)$$

$\Rightarrow$ Pseudo-Goldstone scalars

$$\phi \rightarrow \phi + c \Rightarrow m_\phi = 0$$

$\Rightarrow$ **Scale invariance**

Scale independence?



$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right)\Lambda^2 = \left(\frac{\Lambda}{500GeV}\right)^2$$

Field theory: δm^2 not measureable

…only $m^2 = m_0^2 + \delta m^2$ "physical"

Only $m^2 = 0$ special ("classical" (quantum?) scale invariance)

$$\Rightarrow \quad \frac{d\, m_H^2}{d\, \ln \mu} = \frac{3m_H^2}{8\pi^2}\left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20}\right)$$

Gravity and scale invariance??

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\upsilon}\partial_\mu\sigma\partial_\nu\sigma - \Lambda + \frac{1}{2}M_P^2 R\right)$$

Gravity and scale invariance

$$S=\int\sqrt{-g}\left(\frac{1}{2}g^{\mu\upsilon}\partial_\mu\sigma\partial_\nu\sigma-\Lambda+\frac{1}{2}M_P^2R\right)$$

equivalent to

$$S=\int\sqrt{-g}\left(\frac{1}{2}g^{\mu\upsilon}\partial_\mu\phi\partial_\nu\phi-\frac{\lambda}{4}\phi^4-\frac{1}{12}\alpha\phi^2R\right)$$

$$\Lambda=\frac{\lambda}{4}\langle\phi\rangle^4,\quad M_P^2=-\frac{\alpha}{6}\langle\phi\rangle^2$$

Brans-Dicke

Weyl (scale) invariance:

Coordinates are scale free numbers.
Length is defined by the covariant metric.
Fields have canonical mass (length)$^{-1}$ dimension

$$g_{\mu\nu}(x) \to e^{-2\varepsilon(x)} g_{\mu\nu}(x), \quad \det(-g(x)) \to e^{-4\varepsilon(x)} \det(-g(x))$$

$$\phi(x) \to e^{\varepsilon(x)} \phi(x)$$

BD form is globally Weyl invariant:

$$S = \int \sqrt{-g} \left(\frac{1}{2} g^{\mu\upsilon} \partial_\mu \phi \partial_\nu \phi - \frac{\lambda}{4} \phi^4 - \frac{1}{12} \alpha \phi^2 R \right)$$

Noether current:

$$K_\mu = \frac{1}{\sqrt{\det(-g)}} \frac{\delta S}{\delta \partial^\mu \epsilon} = (1-\alpha) \phi \partial_\mu \phi$$

Noether current:

$$K^{\mu} = (1-\alpha)\phi\,\partial^{\mu}\phi, \quad \nabla_{\mu}K^{\mu} = 0$$

Integrable

$$K^{\mu} = \nabla^{\mu}K, \quad \text{Kernel } K = \frac{1}{2}(1-\alpha)\phi^{2}$$

Integrating:

$$\phi^{2} = \phi_{0}^{2} + c\int\frac{dt}{a(t)^{3}} \quad \Rightarrow \quad K \to \text{constant}$$

Spontaneously broken Weyl symmetry

Goldstone mode- dilaton, σ

$$\phi = \phi_0 \exp(\sigma/f)$$

…redefine metric $\quad g_{\mu\nu} = \exp(-2\sigma/f)\tilde{g}_{\mu\nu}$

$$S = \int \sqrt{-g}\left(\frac{1}{2}g^{\mu\upsilon}\partial_\mu\sigma\partial_\nu\sigma - \Lambda + \frac{1}{2}M_P^2 R\right)$$

$$\Lambda = \frac{\lambda}{4}\phi_0^4, \quad M_P^2 = -\frac{\alpha}{6}\phi_0^2$$

$$f = \sqrt{2K_0} \text{ where } K_0 = \frac{1}{2}(1-\alpha)\phi_0^2$$

Weyl symmetry is hidden !

Can our world have a hidden Weyl invariance?

… need to generate variety of mass scales

- $H_I << M_{P,} \quad m_{\text{inflaton}} \ll H_I$ (slow roll)

- $m_{Higgs} \sim 10^{-16} M_p$

- $\Lambda \equiv \rho_{vac} \sim 10^{-122} M_P^4$

To investigate this consider two scalar fields:

$$\phi \rightarrow Dilaton$$
$$\chi \rightarrow \text{Inflaton / "Higgs"}$$

Weyl invariant Brans-Dicke theory:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)]$$

"dilaton" "matter"

$$W(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2$$

$$\lambda \ll \delta \ll \xi$$

Weyl invariant Brans-Dicke theory:

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$$\lambda \ll \delta \ll \xi$$

Effective Planck mass

$$M_P^2 = M_\phi^2 + M_\chi^2 = -\tfrac{1}{6}\alpha\phi^2 - \tfrac{1}{6}\beta\chi^2$$

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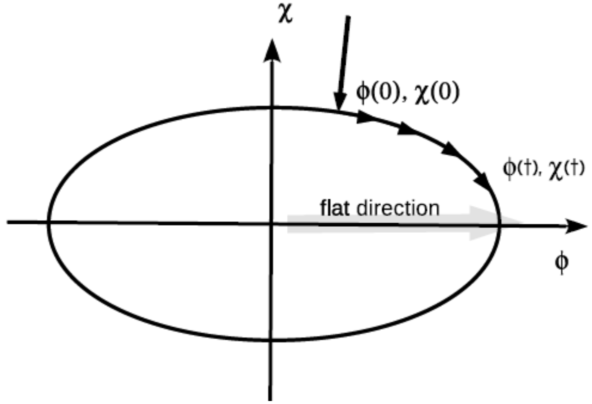
Effective Planck mass

$$M_P^2 = M_\phi^2 + M_\chi^2 = -\tfrac{1}{6}\alpha\phi^2 - \tfrac{1}{6}\beta\chi^2$$

Noether current

$$K^\mu = (1-\alpha)\phi\partial^\mu\phi + (1-\beta)\chi\partial^\mu\chi \equiv \partial^\mu K$$

$$K = \frac{1}{2}(1-\alpha)\phi^2 + \frac{1}{2}(1-\beta)\chi^2 \rightarrow \text{Constant}$$



Weyl invariant Brans-Dicke theory:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)]$$

Field equations

$$M^2 G_{\alpha\beta} = T^{\phi}_{\alpha\beta} + T^{\chi}_{\alpha\beta} - g_{\alpha\beta}W(\phi,\chi)$$

$$\begin{aligned} T^{\phi}_{\alpha\beta} &= \left(1-\frac{\alpha}{3}\right)\nabla_\alpha\phi\nabla_\beta\phi + \left(\frac{\alpha}{3}-\frac{1}{2}\right)g_{\alpha\beta}\nabla_\mu\phi\nabla^\mu\phi \\ &\quad -\frac{\alpha}{3}\phi\nabla_\alpha\nabla_\beta\phi + \frac{\alpha}{3}g_{\alpha\beta}\phi\Box\phi \\ T^{\chi}_{\alpha\beta} &= \left(1-\frac{\beta}{3}\right)\nabla_\alpha\chi\nabla_\beta\chi + \left(\frac{\beta}{3}-\frac{1}{2}\right)g_{\alpha\beta}\nabla_\mu\chi\nabla^\mu\chi \\ &\quad -\frac{\beta}{3}\chi\nabla_\alpha\nabla_\beta\chi + \frac{\beta}{3}g_{\alpha\beta}\chi\Box\chi \end{aligned}$$

$$\Box\phi - \frac{\alpha}{6}\phi R - \frac{\partial W}{\partial\phi} = 0, \quad \Box\chi - \frac{\beta}{6}\chi R - \frac{\partial W}{\partial\chi} = 0$$

Weyl invariant Brans-Dicke theory:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)]$$

Inflation?

$FRW:$

$$H^2 - \frac{D}{3M^2}H - \frac{\rho_T}{3M^2} = 0 \qquad D = \alpha\phi\dot{\phi} + \beta\chi\dot{\chi}$$

$$\begin{pmatrix} \Box\phi \\ \Box\chi \end{pmatrix} = \frac{1}{\mathrm{K}} \begin{pmatrix} 1 + \frac{\beta^2\chi^2}{6M^2} & -\frac{\alpha\beta\phi\chi}{6M^2} \\ -\frac{\alpha\beta\phi\chi}{6M^2} & 1 + \frac{\alpha^2\phi^2}{6M^2} \end{pmatrix} \begin{pmatrix} \mathcal{S}_\phi \\ \mathcal{S}_\chi \end{pmatrix}$$

$$\mathcal{S}_\phi = \alpha(\alpha-1)\frac{\phi\dot{\phi}^2}{6M^2} + \alpha(\beta-1)\frac{\phi\dot{\chi}^2}{6M^2} + \frac{4\alpha\phi}{6M^2}W + \frac{\partial W}{\partial\phi}$$

$$\mathcal{S}_\chi = \beta(\beta-1)\frac{\chi\dot{\chi}^2}{6M^2} + \beta(\alpha-1)\frac{\chi\dot{\phi}^2}{6M^2} + \frac{4\beta\chi}{6M^2}W + \frac{\partial W}{\partial\chi}$$

$$6M^2 = -(\alpha\phi^2 + \beta\chi^2)$$

Jordan Frame

Weyl invariant Brans-Dicke theory:

$$S = -\int d^4x\sqrt{-g}[\frac{1}{12}\alpha\phi^2 R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{1}{12}\beta\chi^2 R + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + W(\phi,\chi)]$$

Inflation?

$W(\phi,\chi) \simeq \xi\,\chi^4$

Slow roll:

$\beta \gg \alpha$

$\beta\chi^2 \gg \alpha\phi^2$

$M^2 \simeq -\frac{1}{6}\beta\chi^2$

$$H^2 - \frac{D}{3M^2}H - \frac{\rho_T}{3M^2} = 0 \qquad D = \alpha\phi\dot{\phi} + \beta\chi\dot{\chi}$$

$$\begin{pmatrix} \Box\phi \\ \Box\chi \end{pmatrix} = \frac{1}{\mathrm{K}}\begin{pmatrix} 1 + \frac{\beta^2\chi^2}{6M^2} & -\frac{\alpha\beta\phi\chi}{6M^2} \\ -\frac{\alpha\beta\phi\chi}{6M^2} & 1 + \frac{\alpha^2\phi^2}{6M^2} \end{pmatrix}\begin{pmatrix} \mathcal{S}_\phi \\ \mathcal{S}_\chi \end{pmatrix}$$

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$$\mathcal{S}_\chi = \beta(\beta-1)\frac{\chi\dot{\chi}^2}{6M^2} + \beta(\alpha-1)\frac{\chi\dot{\phi}^2}{6M^2} + \underbrace{\frac{4\beta\chi}{6M^2}W + \frac{\partial W}{\partial\chi}}_{O(\alpha)}$$

$$6M^2 = -(\alpha\phi^2 + \beta\chi^2)$$

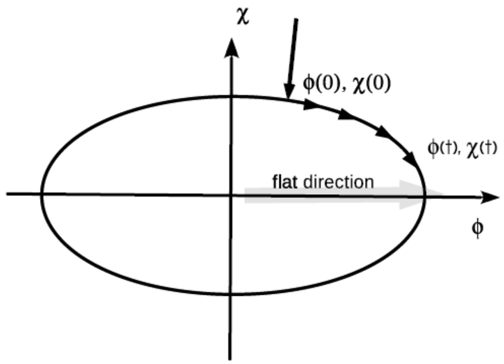
scale invariance

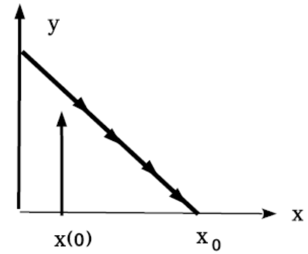
Slow roll

$$x = (1-\alpha)\phi^2, \quad y = (1-\beta)\chi^2$$

$$x + y = 1$$

$$\partial_N x \simeq -\frac{4}{3}\alpha x(x - x_0) \qquad N = \ln(a(t))$$





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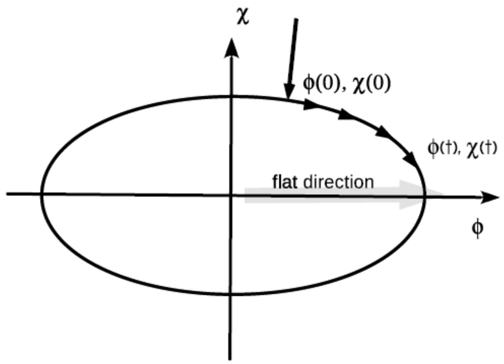

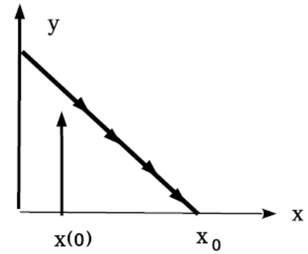

End of inflation:

$$x = 1 - O(\alpha) \qquad \varepsilon = \frac{4}{3}\alpha\frac{x}{1-x} \simeq 1$$

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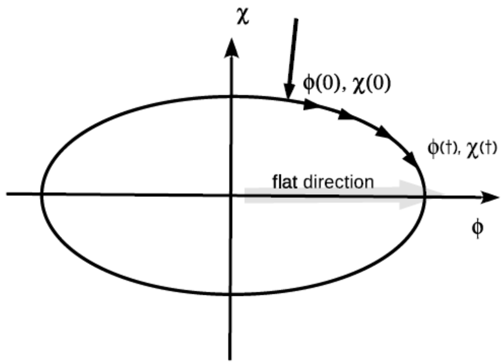
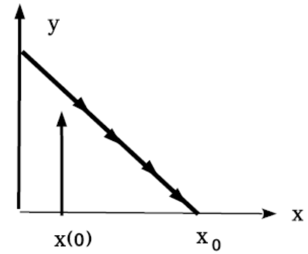




End of inflation:

$$x = 1 - O(\alpha) \qquad \varepsilon = \frac{4}{3}\alpha\frac{x}{1-x} \simeq 1$$

IR fixed point:

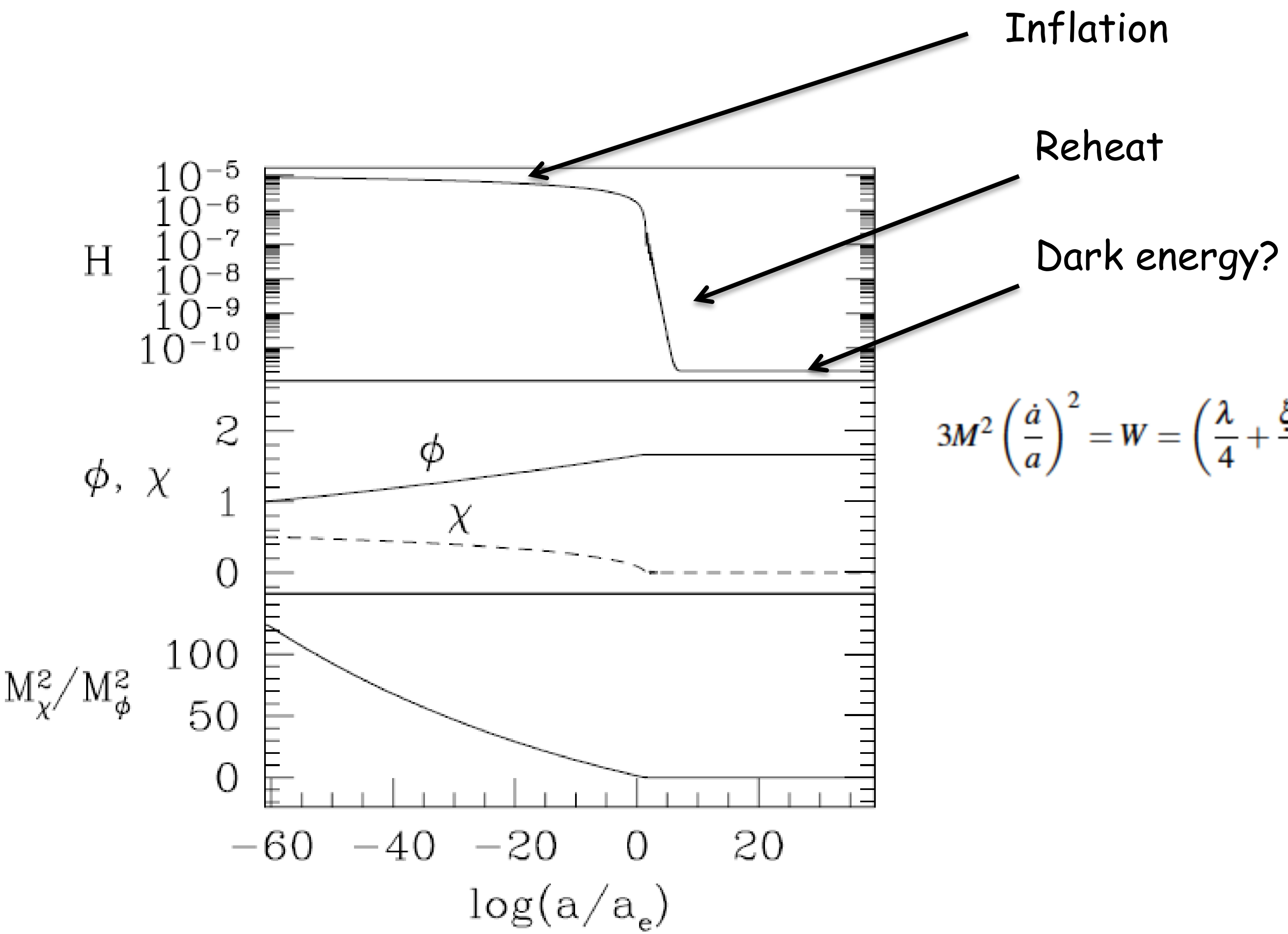
$$x(t \to \infty) = x_0$$

$$\frac{\chi_0^2}{\phi_0^2} = \frac{\beta\lambda - \alpha\delta}{\alpha\xi - \beta\delta}$$

$$W(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{\xi}{4}\chi^4 + \frac{\delta}{2}\phi^2\chi^2$$

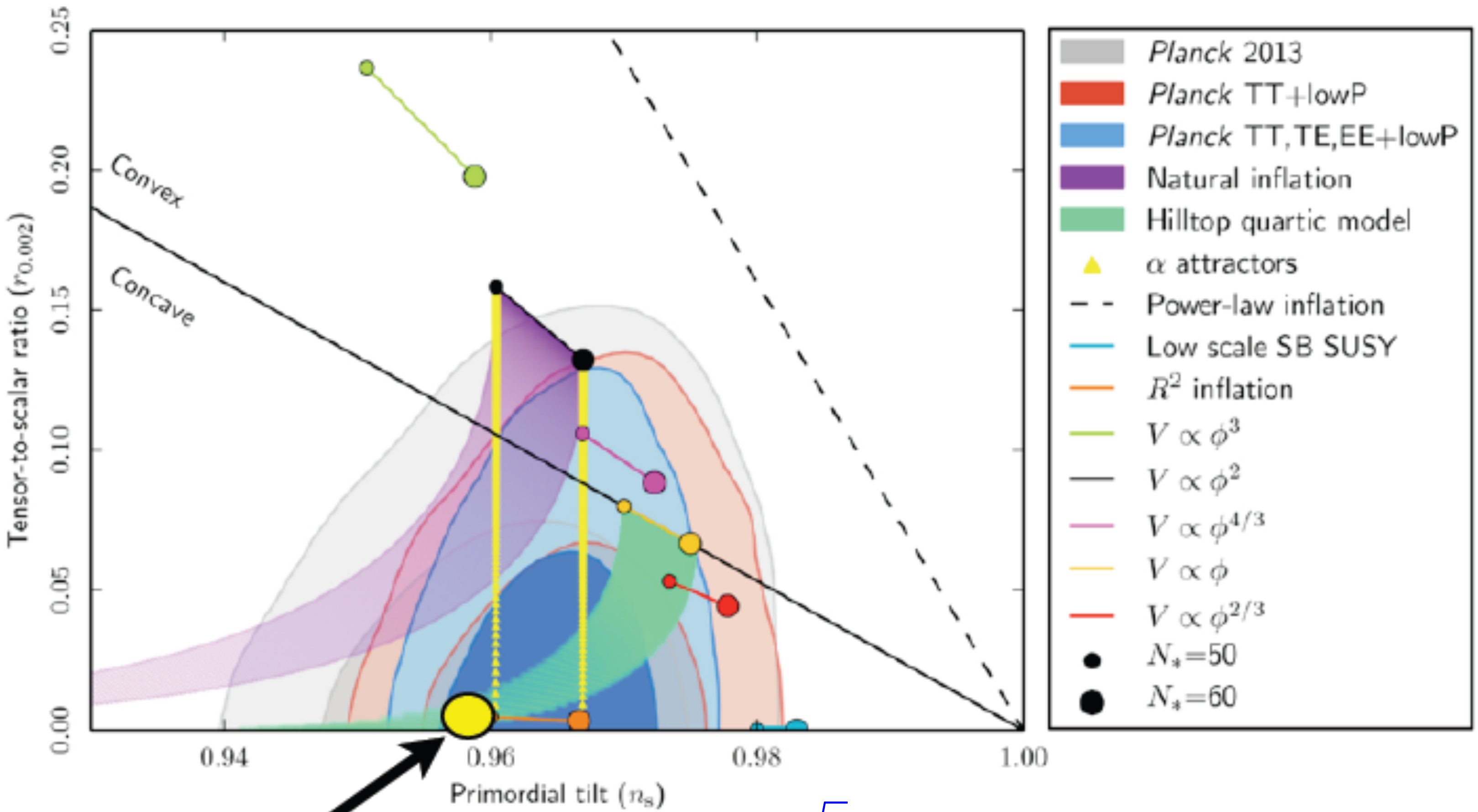
$$M_P^2 \approx -\frac{1}{6}\alpha\phi^2$$

Cosmology of symmetry broken phase



$$3M^2\left(\frac{\dot{a}}{a}\right)^2 = W = \left(\frac{\lambda}{4} + \frac{\xi\mu^4}{4} + \frac{\delta\mu^2}{2}\right)\phi_0^4$$

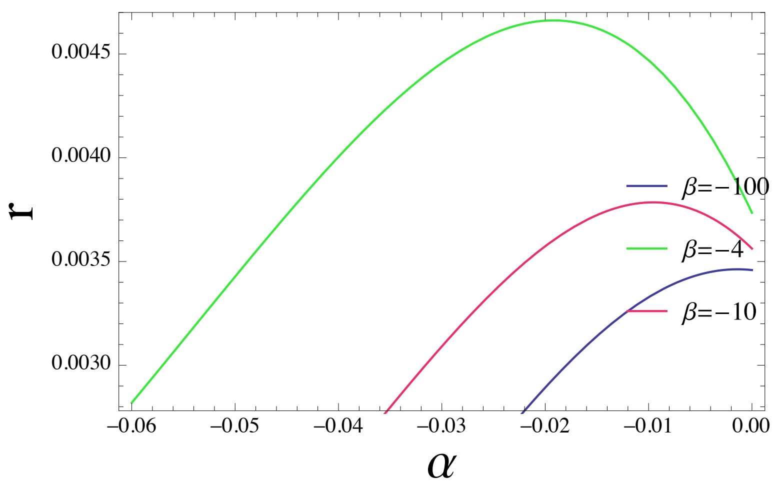
Cosmology of symmetry broken phase

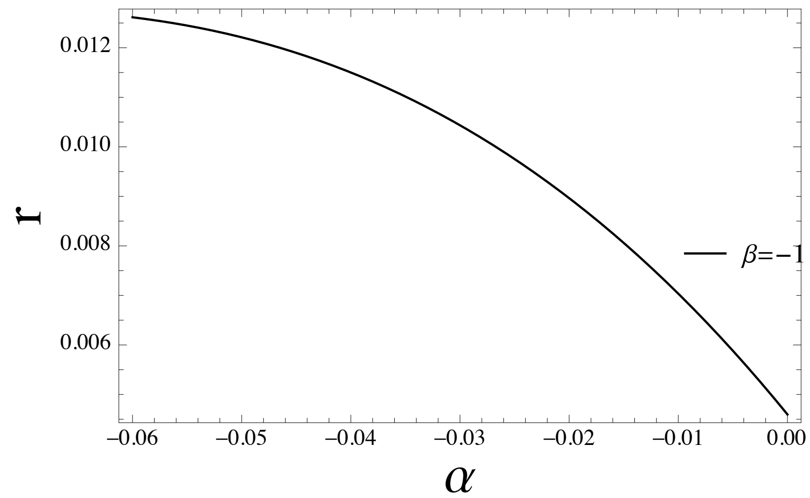


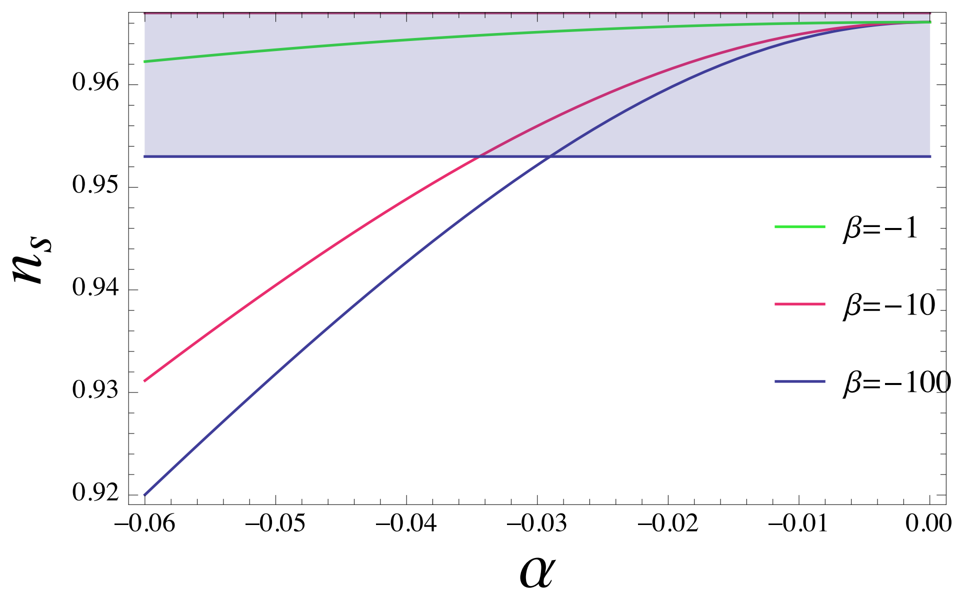
We are here

$$\frac{\sqrt{\xi}}{\beta} = 10^{-5}, \quad \alpha < 10^{-1}$$

Inflationary post-dictions







Summary

- Spontaneously broken scale-invariant "SM"+gravity
  - only dimensionless ratios meaningful

Small or zero – c.c. fine tuned

$$\frac{H_I^2}{M_P^2} \propto \frac{\xi}{\beta^2}, \quad \frac{m_{"h"}^2}{M_P^2} \propto \frac{\delta}{\alpha}, \quad \frac{H_0^2}{M_P^2} \propto \frac{1}{\alpha^2}\left(\frac{\lambda}{4} + \frac{\xi\mu^4}{4} + \frac{\delta\mu^2}{2}\right) \qquad \mu^2 = \frac{\delta}{\xi}$$

Hierarchy related to hierarchy of couplings (input but technically natural)

(No heavy states)

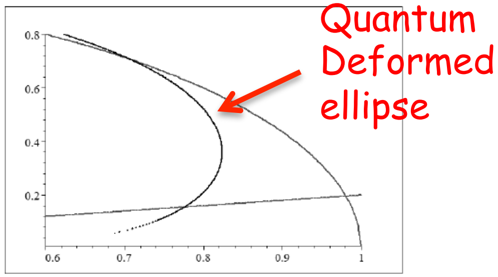
- Slow roll inflation with acceptable properties possible

$r < 0.01, n_s < 0.967$

- Massless dilaton – decouples …no 5th force avoiding BD bounds

- Classical ➔ Quantum scale invariance?

$$V(\phi) = \lambda\phi^4 + \frac{\beta_\lambda}{4}\phi^4 \ln\left(\frac{\phi}{M}\right); \quad M \to M(\phi, \chi)$$



c.f. D. Ghilencea's talk -tomorrow

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DOMI NVS ILLV MINA TIO MEA

Multiscalars

Conformal transformation with $\phi_i = e^{-\frac{\sigma}{f}}\hat{\phi}_i$ and $g_{\mu\nu} = e^{2\frac{\sigma}{f}}\hat{g}_{\mu\nu}$

σ is the dilaton and $\hat{\phi}_i$ satisfies $\bar{K} = \frac{1}{2}\sum_{i=1}^{N}(1-\alpha_i)\hat{\phi}_i^2 = f^2$

Transformed action:

$$S = -\int d^4x\sqrt{-\hat{g}}\left[-\frac{1}{12}\sum_i^N \alpha_i\hat{\phi}_i^2\hat{R} + \frac{1}{2}\sum_i^N \partial_\mu\hat{\phi}_i\partial^\mu\hat{\phi}_i \right.$$
$$\left. + \frac{1}{f^2}\bar{K}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{f}\underbrace{\partial_\mu\sigma\partial^\mu\bar{K}}_{=0} - W(\vec{\hat{\phi}}) + \lambda_{\mathrm{L}}\mathcal{C}(\vec{\hat{\phi}})\right]$$

(i.e. no coupling $\longrightarrow$ no fifth force!)

Ellipse is deformed

All dimensionless parameters run with RG

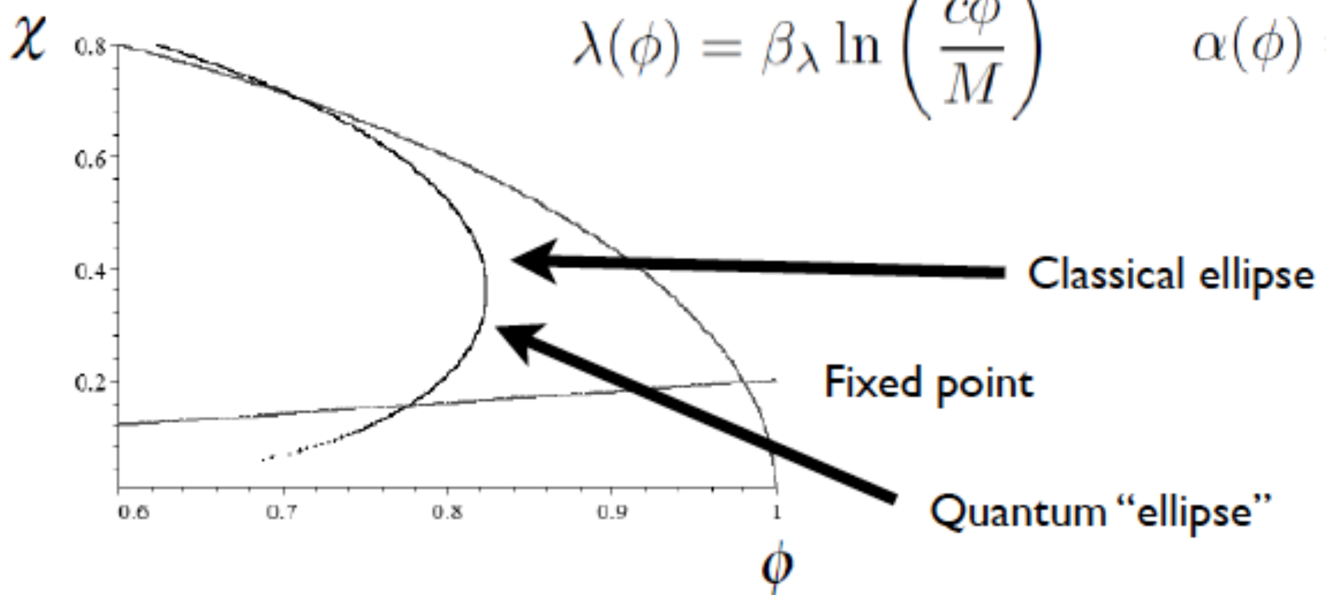
$$\phi\frac{\partial\lambda}{\partial\phi} = \beta_\lambda \left(=\frac{9\lambda^2}{8\pi^2}\right)$$

$$\phi\frac{\partial\alpha}{\partial\phi} = \beta_\alpha = (\alpha-1)\gamma_\alpha \qquad \left(\gamma_\alpha = \frac{3\lambda}{8\pi^2}\right)$$

One loop solution with internal renormalization

$$\lambda(\phi) = \beta_\lambda \ln\left(\frac{c\phi}{M}\right) \qquad \alpha(\phi) = 1 + (\alpha_0 - 1)\left(\frac{\phi}{M}\right)^{\gamma_\alpha}$$

$e.g.\quad M = K(\phi,\chi)$



Inflationary post-dictions

$$H^2(N) = \frac{18\xi}{\beta^2}\frac{M_\chi^4}{M^2}$$

Einstein frame:

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu} \qquad \Omega^2 = \frac{M_P^2}{M_\phi^2 + M_\chi^2}$$

$$\tilde{H} = \Omega(1 - \frac{d\ln\Omega}{dN})H$$

Observables:

$$r = 16\varepsilon \;\; = -16\frac{d\ln\tilde{H}}{d\tilde{N}}, \quad \eta = \varepsilon - \frac{\varepsilon'}{2\varepsilon}, \quad n_s = 1 + 2\eta - 4\varepsilon$$

$$\delta_H^2 = \frac{V_H}{150\pi^2 M_P^4\varepsilon} = \frac{36}{150\pi^2\varepsilon}\frac{\xi}{\beta^2} = 4.10^{-10} \quad \Rightarrow \quad \frac{\xi}{\beta^2} \sim 10^{-10}$$

Inflationary post-dictions