Multi-field inflationary trajectories with a fast-turn feature

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Introduction

$H \approx \text{const}$	$H \propto a^{-n},\, n > 1$
Zero spatial curvature is an attractive fixed point of the evolution equations	Zero spatial curvature is an repulsive fixed point of the evolution equations
Comoving scales continuously leave the causally connected patch	Comoving scales continuously enter the causally connected patch
All particle densities quickly washed out	

Reasons to contemplate inflation; there is an extra bonus: generation of cosmological perturbations.

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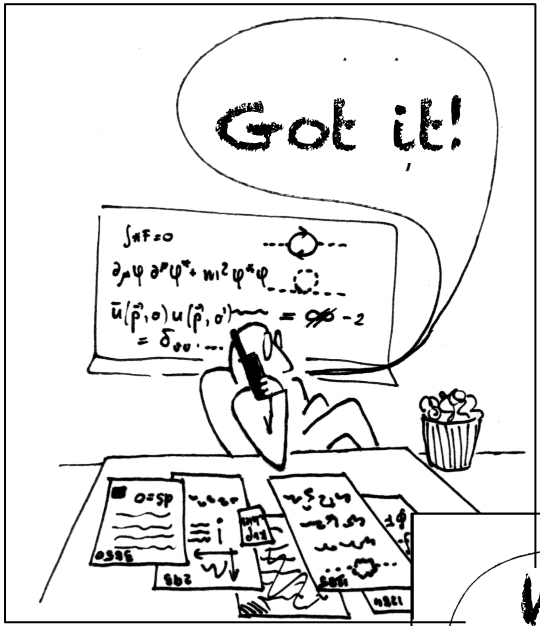

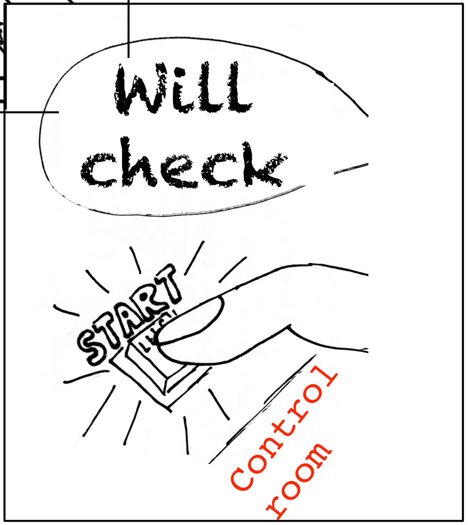

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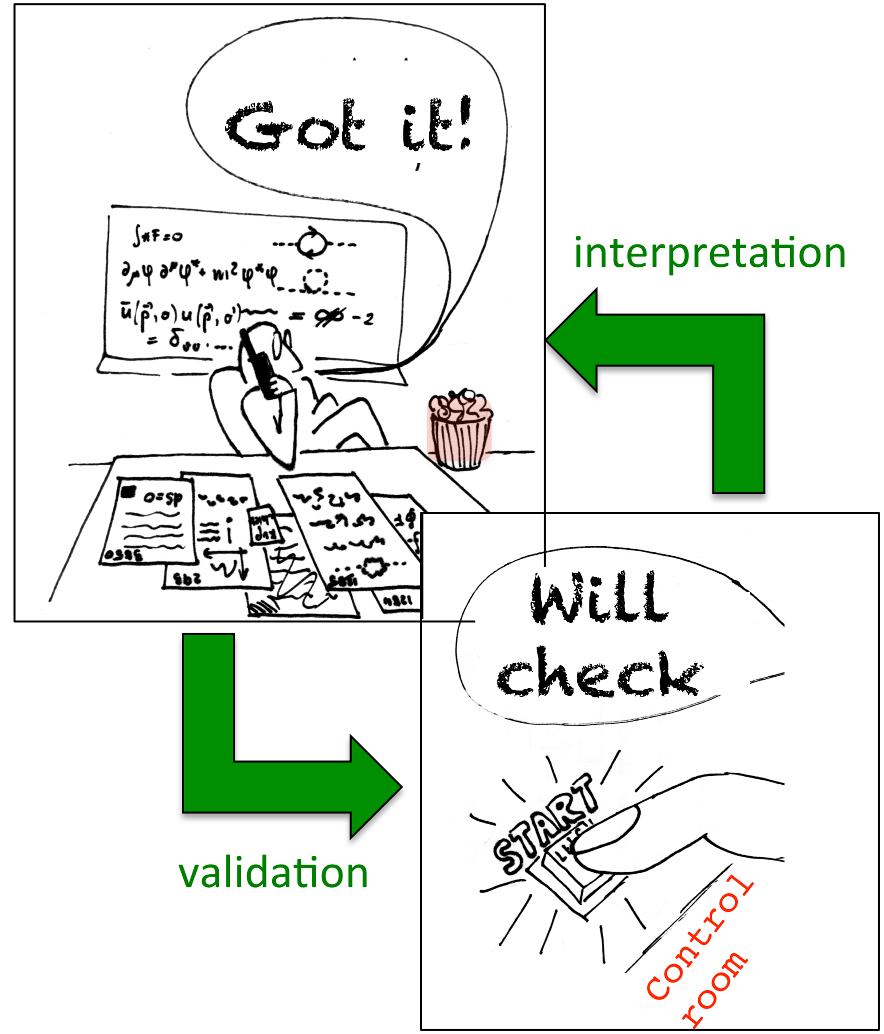
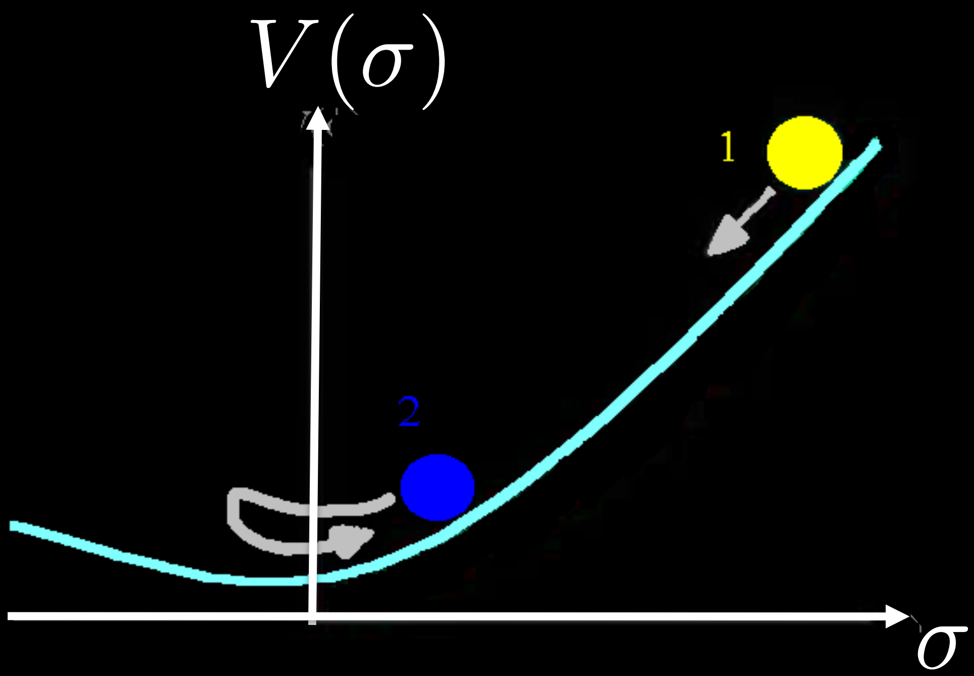


Reasons to contemplate inflation; there is an extra bonus: generation of cosmological perturbations.



1. Quasi-de-Sitter solution

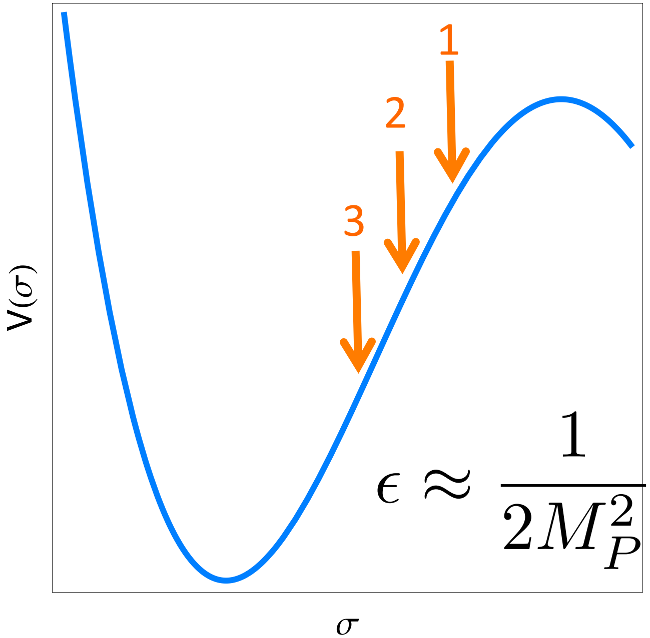
A scalar field slowly rolls down a potential

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$$

$$\epsilon \approx \frac{1}{2M_P^2}\left(\frac{V_\sigma}{V}\right)^2$$

$$\eta \approx \frac{1}{M_P^2}\frac{V_{\sigma\sigma}}{V}$$

$$\mathcal{P}_\mathcal{R} = \frac{V}{24\pi^2 M_P^4 \epsilon}$$



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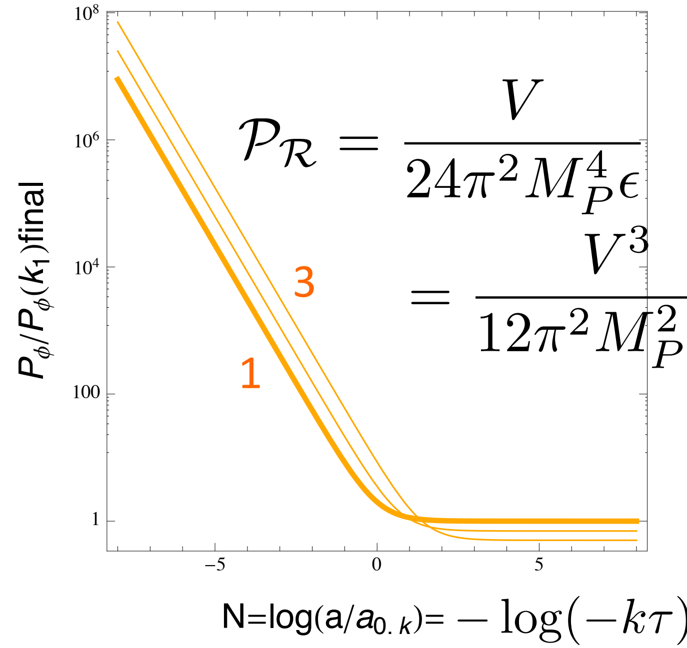


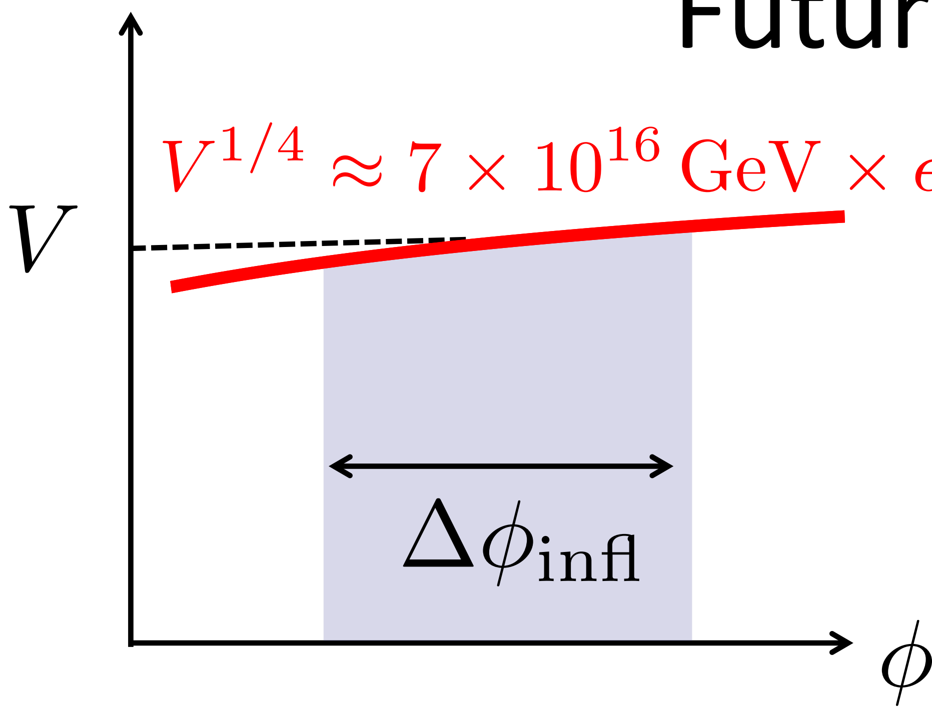
$$\mathcal{P}_\mathcal{R} = \frac{V}{24\pi^2 M_P^4 \epsilon} = \frac{V^3}{12\pi^2 M_P^2 V_\sigma^2}$$

Assume $\mathcal{P}_\mathcal{R} = A\,(k/k_0)^{n_s - 1}$

$$n_s - 1 = \frac{\mathrm{d}\,\log \mathcal{P}_\mathcal{R}}{\mathrm{d}\,\log k} = -6\epsilon + 2\eta$$

Future?

$$V^{1/4} \approx 7 \times 10^{16}\,\mathrm{GeV} \times \epsilon^{1/4}$$



$$\mathcal{P}_{\mathcal{R}} = \frac{V}{24\pi^2 M_P^4 \epsilon}$$

$$n_s = 1 - 6\epsilon + 2\eta$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon$$

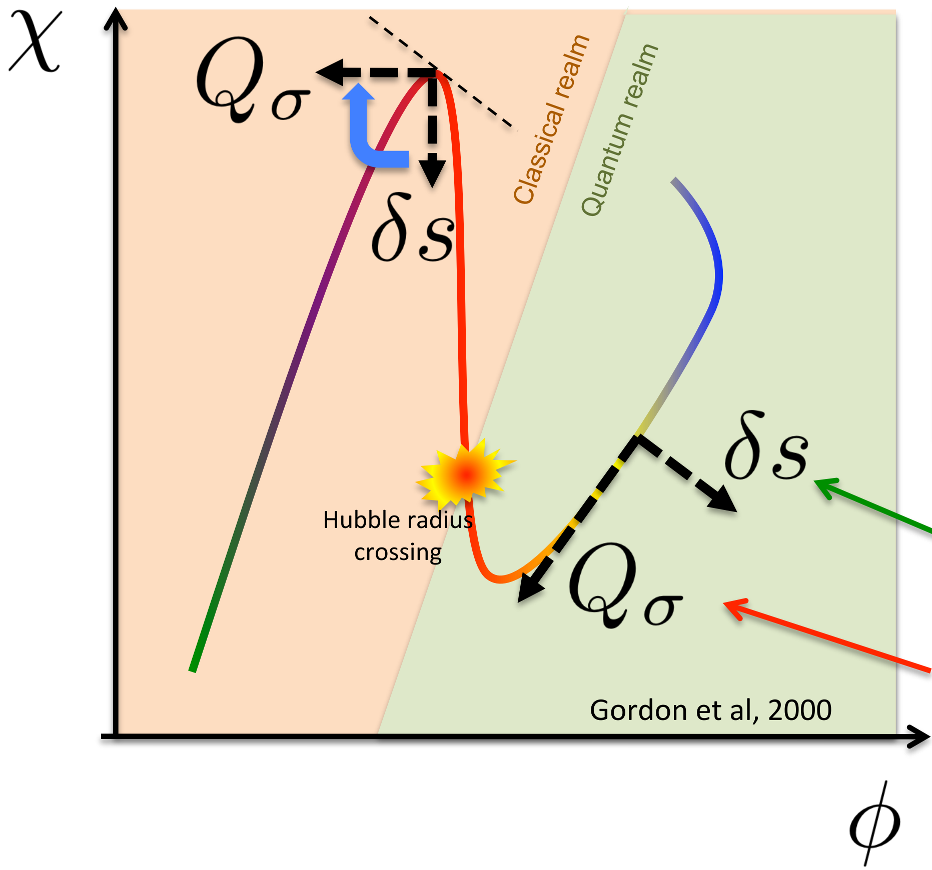
~~α_s f_{NL} g_{NL}~~

Is this interpretation SAFE?

Yes&No

Multi-field inflation

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi)$$



On super-Hubble scales, along a **turn** in the inflationary trajectory, there is a coupling between the **adiabatic** and **entropy** modes and the **latter** can source the **former**.

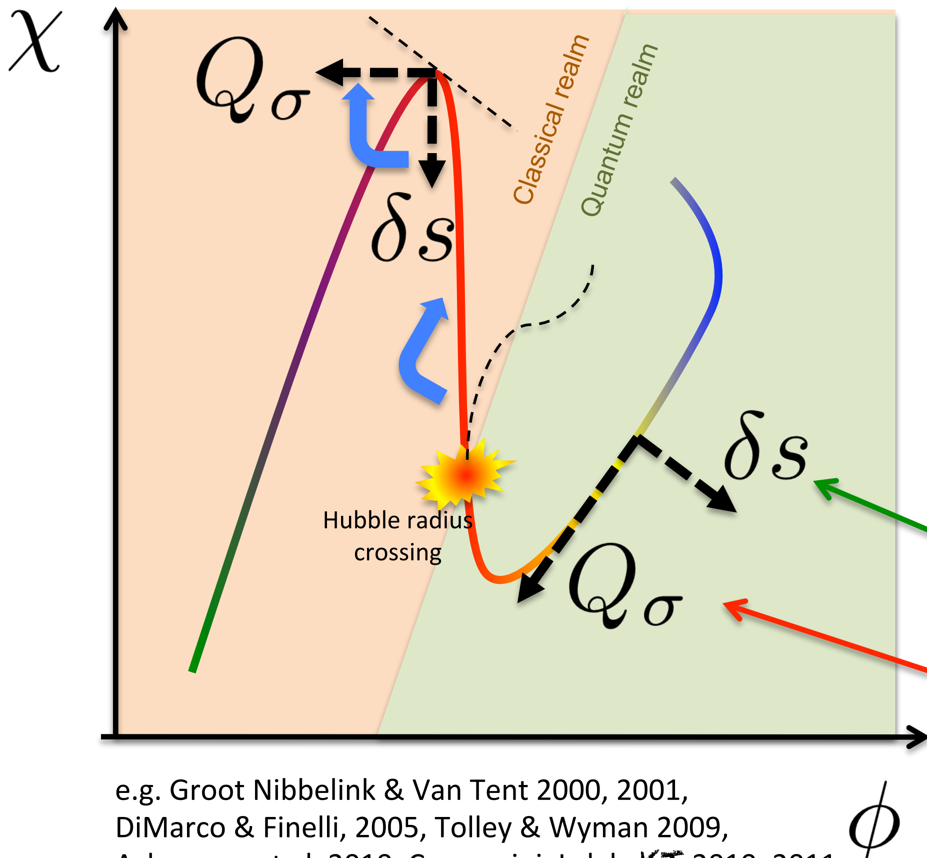
Curiously, sometimes ignored; see eg. account in Avgoustidis, Cremonini, Davis, Ribeiro, KT & Watson '11

instantaneous entropy (isocurvature) perturbation

instantaneous adiabatic (curvature) perturbation

Multi-field inflation

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{e^{2b(\phi)}}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi)$$



On super-Hubble scales, when the inflationary trajectory **turns away from geodesic lines** in the field space, there is a coupling between the **adiabatic** and **entropy** modes and the **latter** can source the **former**.

instantaneous entropy (isocurvature) perturbation

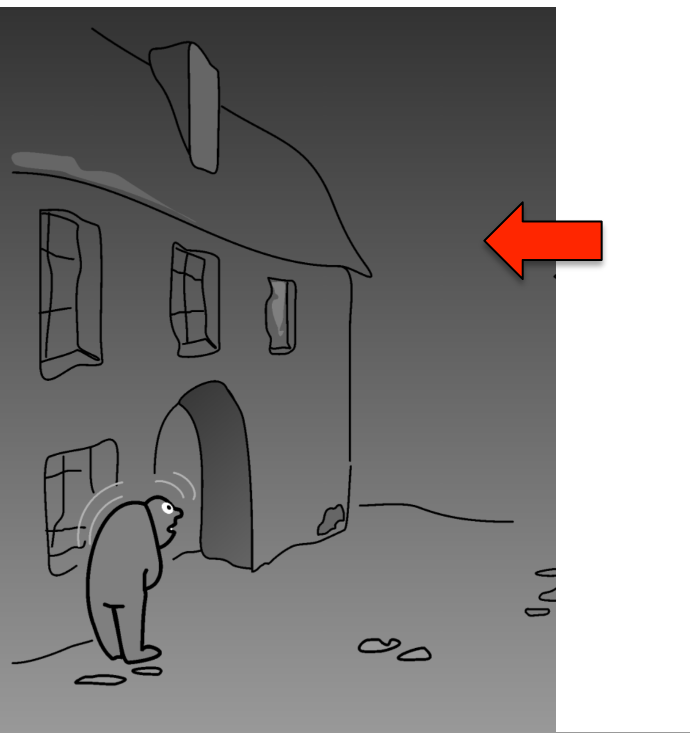
instantaneous adiabatic (curvature) perturbation

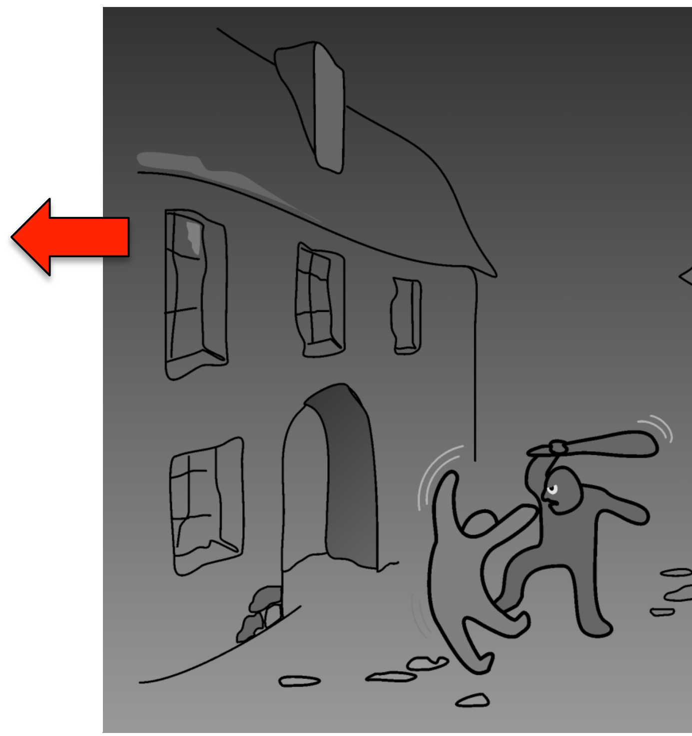
e.g. Groot Nibbelink & Van Tent 2000, 2001, DiMarco & Finelli, 2005, Tolley & Wyman 2009, Achucarro et al. 2010, Cremonini, Lalak, KT 2010, 2011…

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi)$$

Violent events in the past?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi)$$





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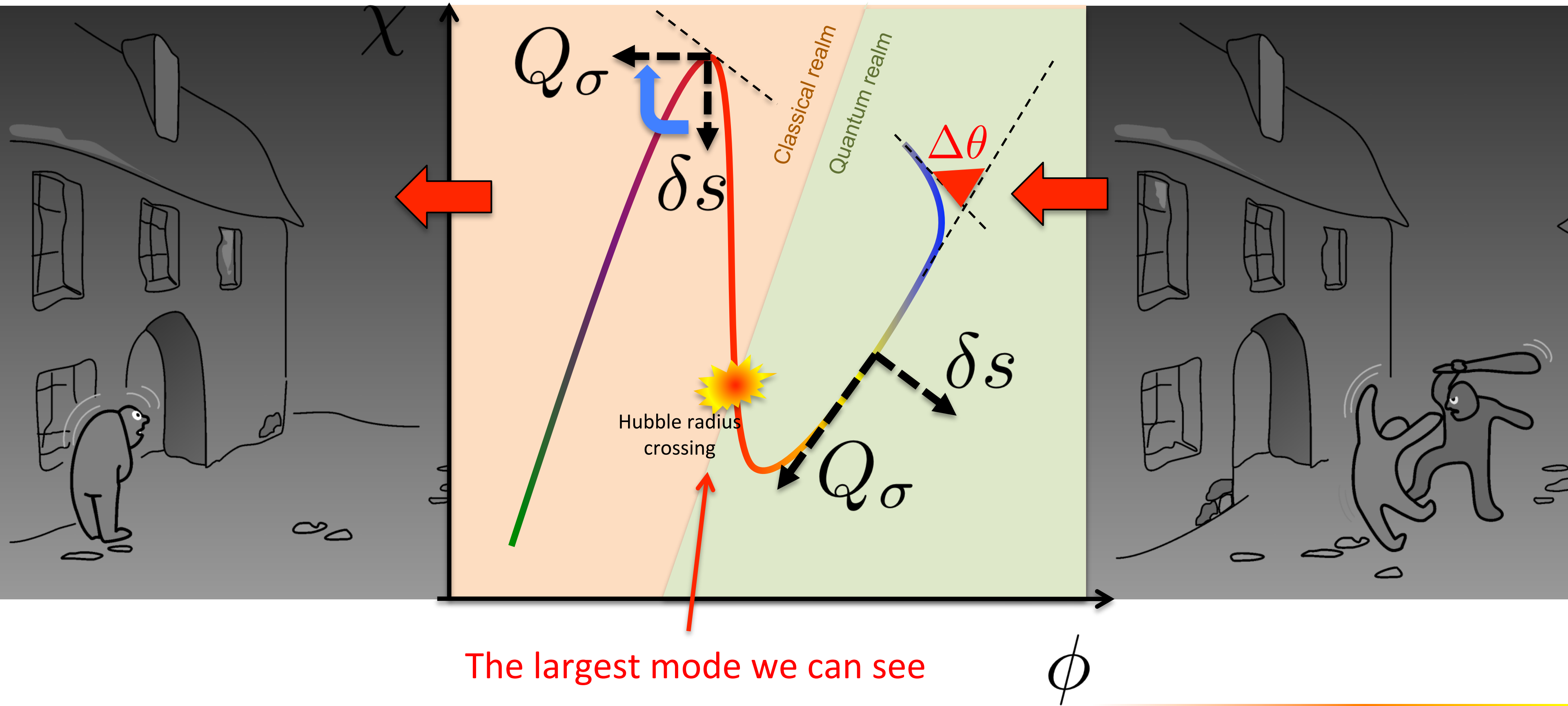

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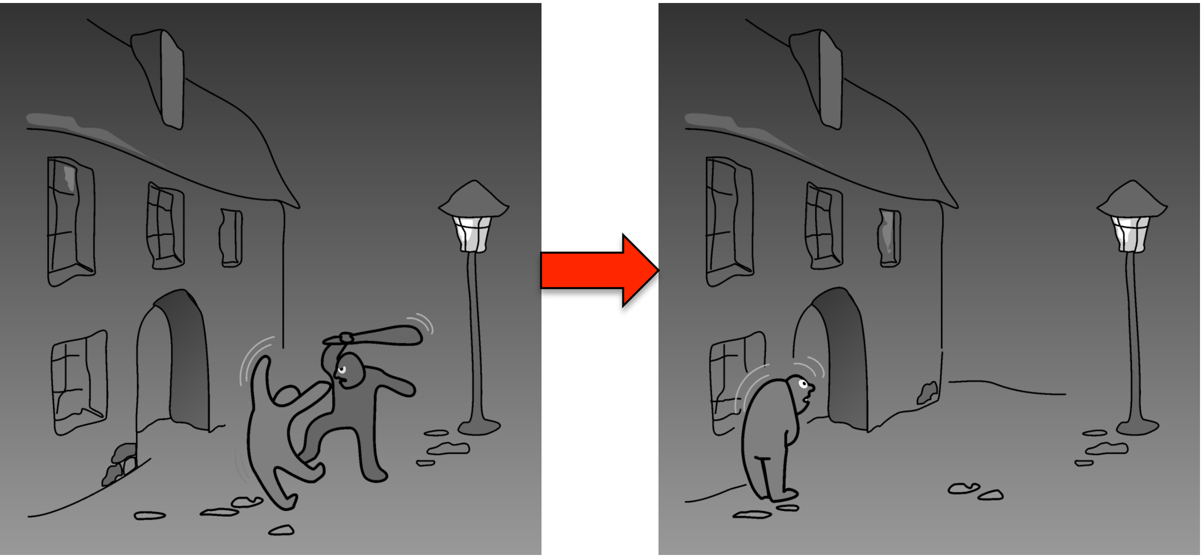
Violent events in the past?



$$\mathcal{P}_{\mathcal{R}}/\mathcal{P}_0 = 1 + 2\,\Delta\theta\ \sin(2k/k_0)$$

Shiu & Xu, 2011

$$\mathcal{P}_{\mathcal{R}}/\mathcal{P}_0 = 1 + e^{H^2\Delta t^2}\Delta\theta^2$$

Gao, Langlois & Mizuno, 2012

$$\mathcal{P}_{\mathcal{R}}/\mathcal{P}_0 = 1$$ EOM's unaffected for $k_{\mathrm{ph}} \gg \dot{\theta}$

Moderately fast turns

EOM's neglecting terms suppressed by the slow-roll parameter $\epsilon = -\dot{H}/H^2$

$$v_\sigma'' + \left(k^2 + \frac{\mu_\sigma^2 - \rho^2 - 2}{\eta^2}\right) v_\sigma + \left(\frac{2\rho}{\eta} v_s\right)' - \frac{2\rho}{\eta^2} v_s = 0$$

$$v_s'' + \left(k^2 + \frac{\mu_s^2 - \rho^2 - 2}{\eta^2}\right) v_s - \frac{2\rho}{\eta} v_\sigma' - \frac{2\rho}{\eta^2} v_\sigma = 0$$

$$(v_\sigma = a\, Q_\sigma\,,\ v_s = a\,\delta s\,,\ \rho = \dot{\theta}/H)$$

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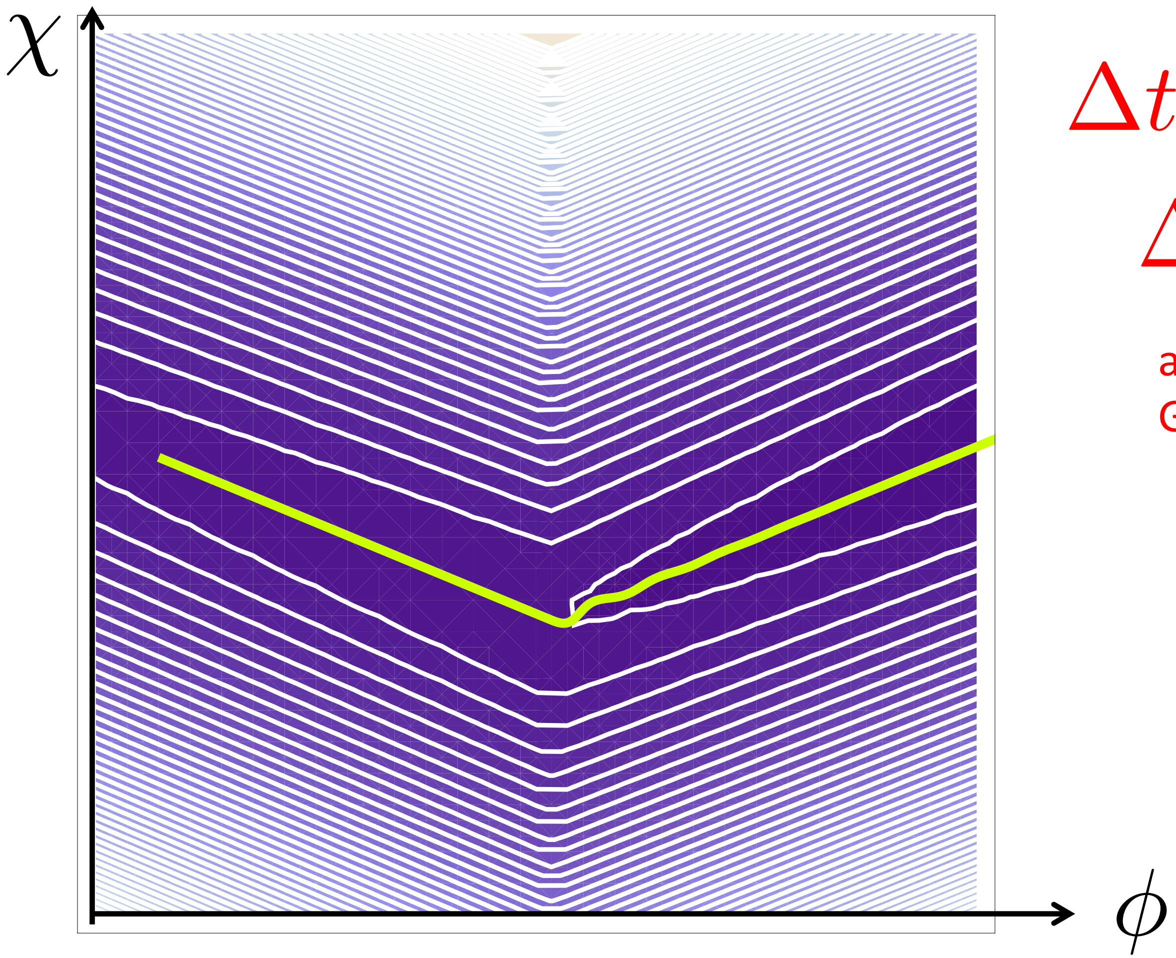
have the following solutions for $k \gg \rho \gg \mu_\sigma^2/k, \mu_s^2/k$

$$\begin{pmatrix} v_\sigma \\ v_s \end{pmatrix} = e^{-\imath k\eta} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix},\ e^{-\imath k\eta} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

Moderately fast turns

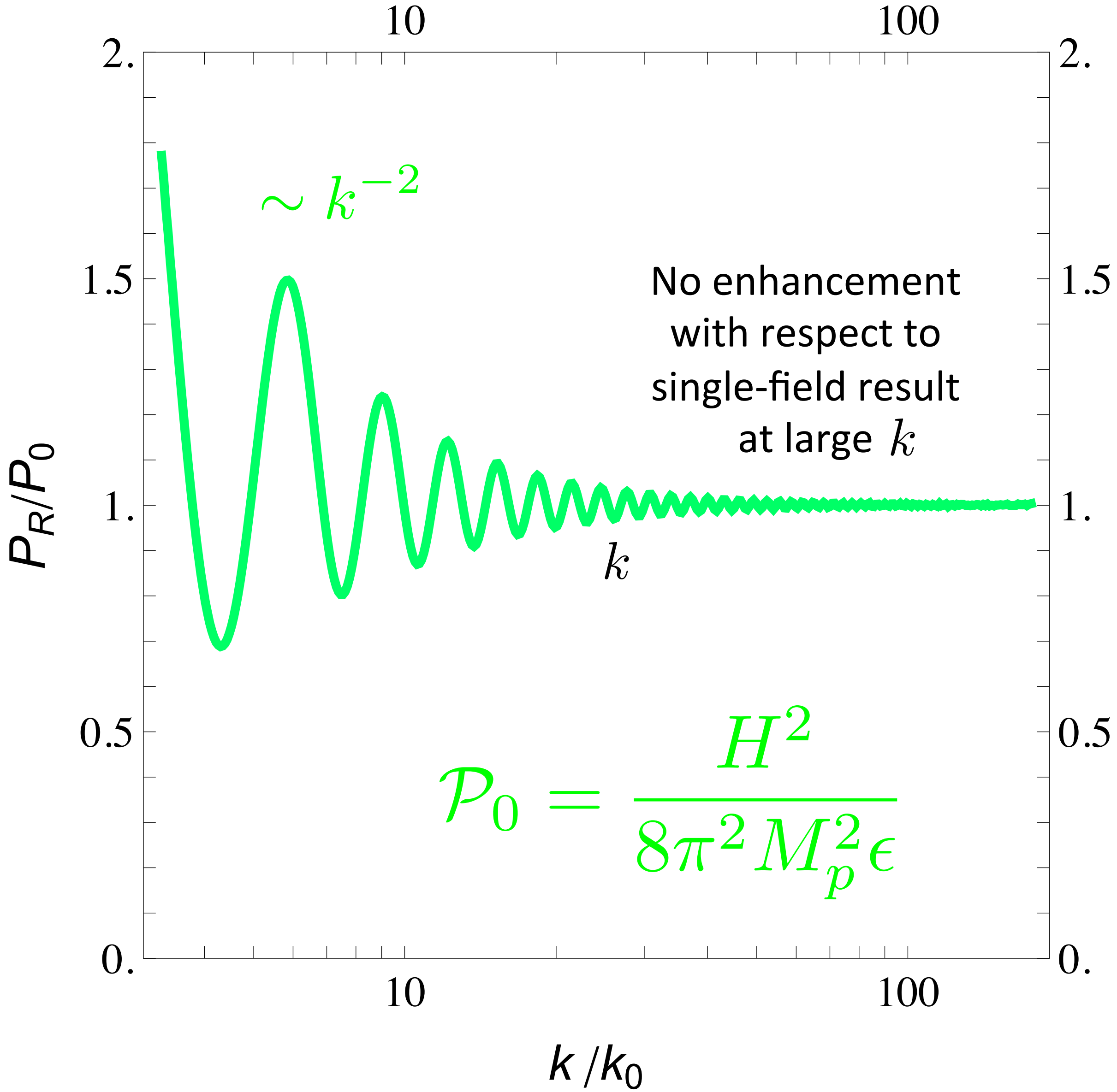


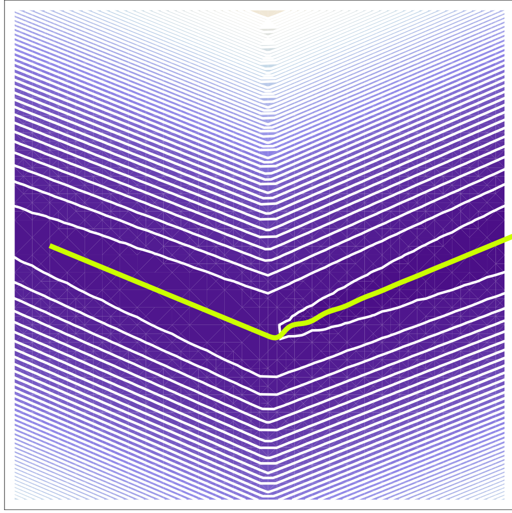
$$\Delta t = 0.0002 H^{-1}$$

$$\Delta\theta = \pi/4$$

analysis of the model by
Gao, Langlois & Mizuno, 2012

Moderately fast turns





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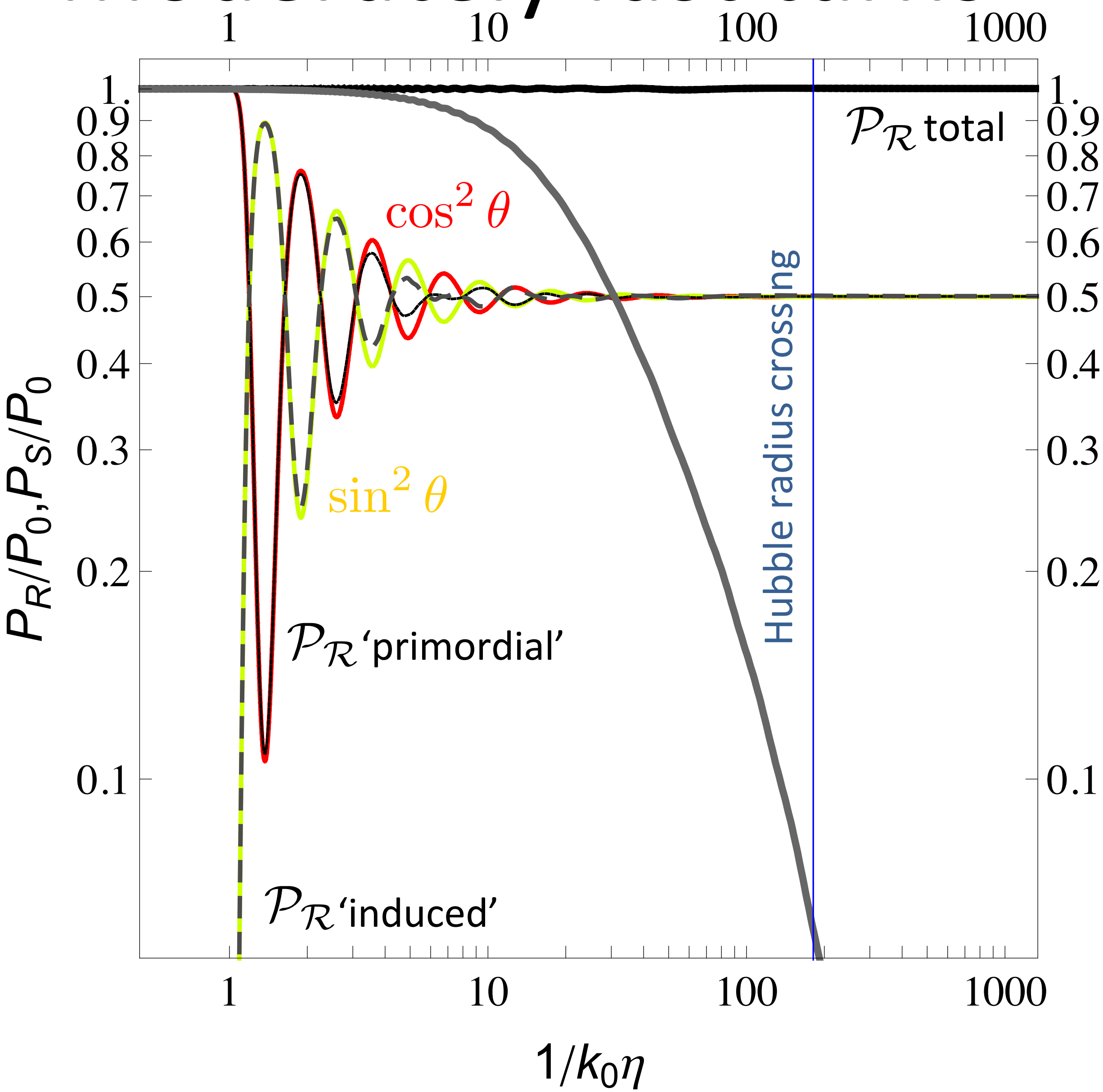

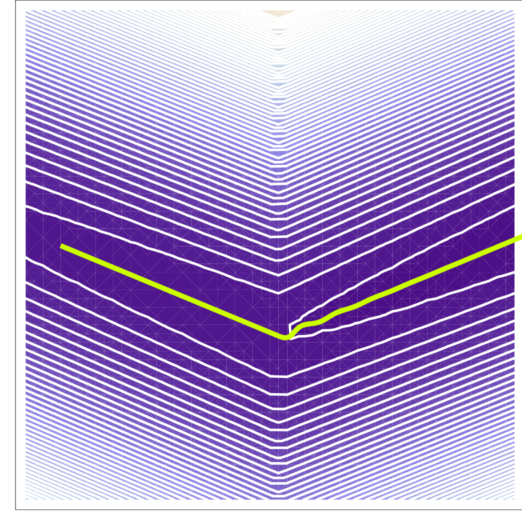

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Very fast turns

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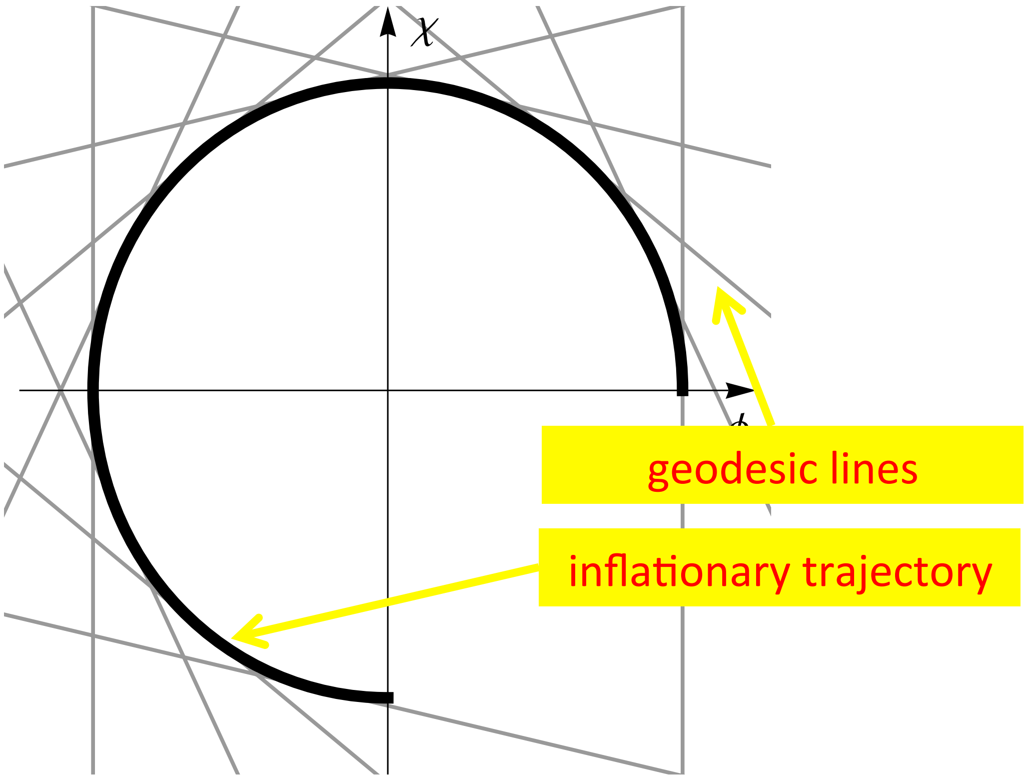
Assume $\rho/\eta = -\Delta\theta\,\delta(\eta - \eta_0)$ (Shiu & Xu 2001)

$$\rho = \dot{\theta}/H$$

No effect on $\mathcal{P}_\mathcal{R}$, either

Quasi-single field inflation



weak coupling $\dot{\theta}/H$

Chen & Wang, 2009

Quasi-single field inflation

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{e^{2b(\phi)}}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi)$$

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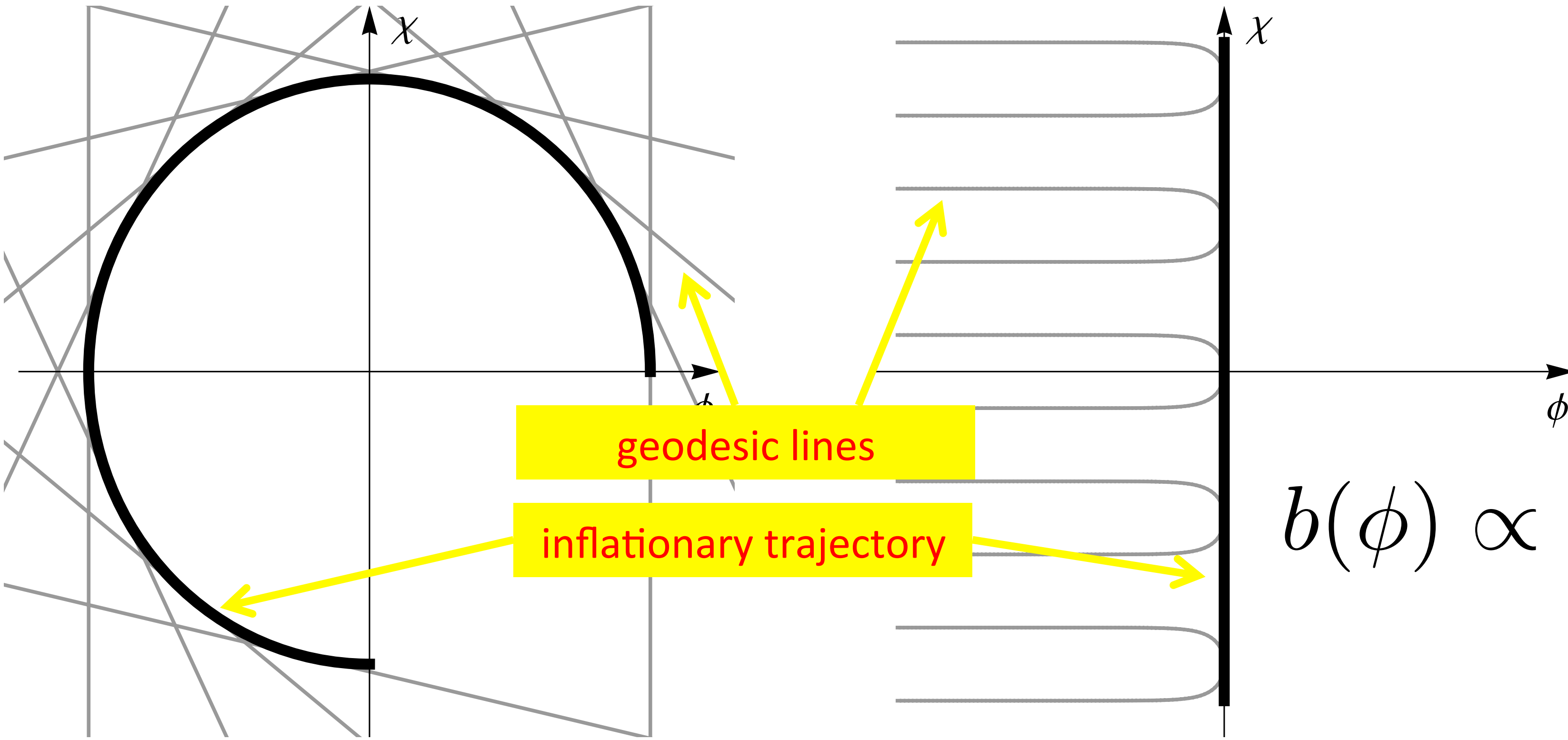


$$b(\phi) \propto -\phi$$

weak coupling $\dot{\theta}/H$

Chen & Wang, 2009

possibility of a strong coupling

$1 \ll M_P^2 b'^2$ e.g. roulette inflation, Bond et al. 2006

application with a weak coupling

Cremonini, Lalak & KT, 2010

Quasi-single field inflation

<u>Toy model</u>

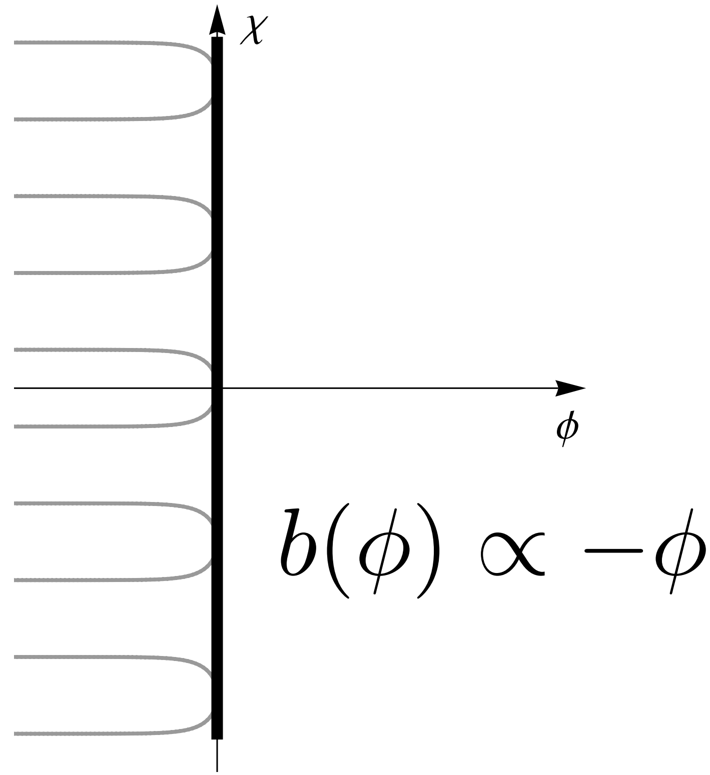
Cremonini, Lalak & KT, 2011

$$b(\phi) = -\phi/M, \;\; M \ll M_P$$

$$V(\phi,\chi) = V_0 \left(1 + \alpha \cdot \left(\frac{\phi-\phi_0}{M_P}\right)^2 + \beta \frac{\chi}{M_P}\right)$$

small, slow-roll $\epsilon = 2\beta^2$

large, $m_\perp \gg H$ or $\eta_{ss} = m_\perp^2/3H^2 \gg 1$



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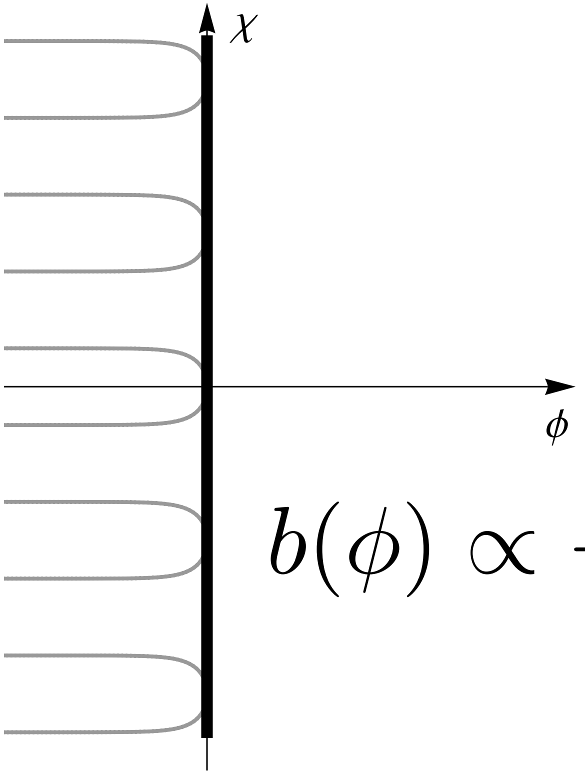

Homogeneous EOMs

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi - b' e^{2b}\dot{\chi}^2 = 0\,,$$

$$\ddot{\chi} + 3H\dot{\chi} + 2b'\dot{\phi}\,\dot{\chi} + e^{-2b}V_\chi = 0$$

$$b(\phi) \propto -\phi$$

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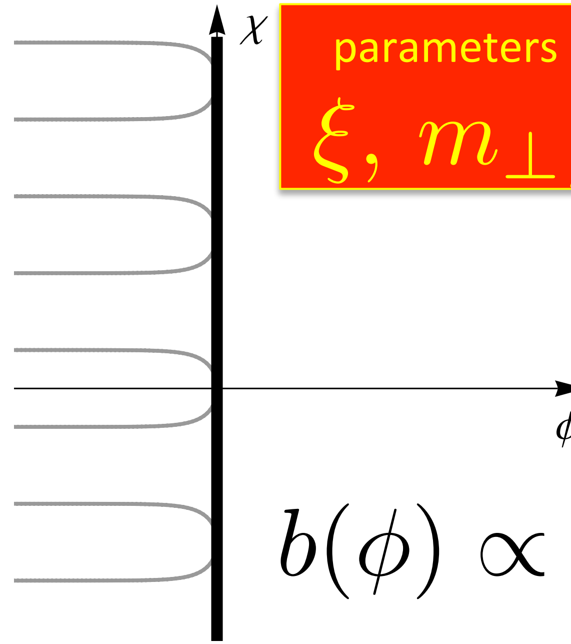

parameters

$$\xi,\, m_\perp/H$$

$$b(\phi) \propto -\phi$$

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EOMs for perturbations

$$\left[\left(\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + k^2 - \frac{2}{\tau^2}\right) + \begin{pmatrix} 0 & \frac{2\xi}{\tau} \\ -\frac{2\xi}{\tau} & 0 \end{pmatrix}\frac{\mathrm{d}}{\mathrm{d}\tau} + \begin{pmatrix} 0 & -\frac{4\xi}{\tau^2} \\ -\frac{2\xi}{\tau^2} & \frac{1}{\tau^2}\frac{m_\perp^2}{H^2} \end{pmatrix}\right]\begin{pmatrix} u_{\rm cur} \\ u_{\rm iso} \end{pmatrix} = 0$$

- solve semi-analytically (alas, no time)
- solve numerically

Lalak, Langlois, Pokorski & KT, 2007

possibility of a strong coupling

$$1 \ll M_P^2 b'^2$$ e.g. roulette inflation, Bond et al. 2006

$$1 \ll 2M_P^2 b'^2 \epsilon \equiv \xi^2$$

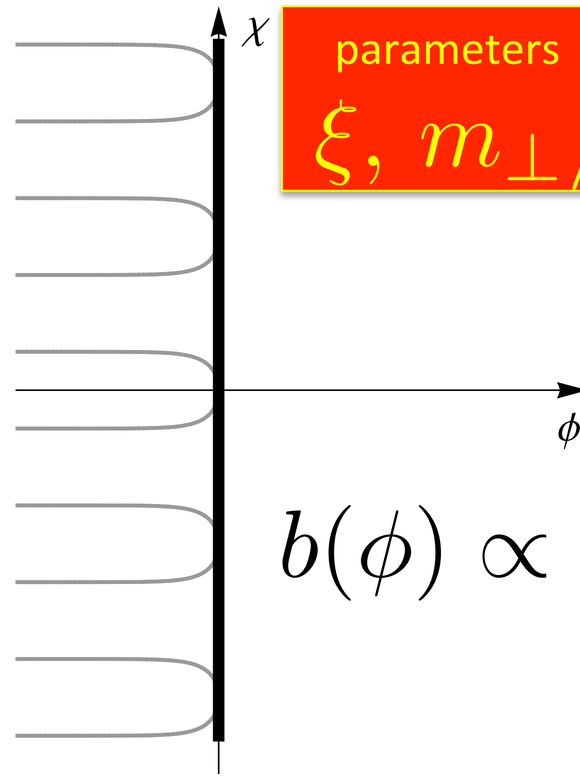
Quasi-single field inflation

Later development

For $k|\tau| < \xi$ and $m_\perp \ll \xi H$
curvature perturbations described
by an effective theory with a single degree
of freedom and a modified dispersion relation.

$$\omega^2 = \frac{m_\perp^2}{4\xi^2 H^2} k_{\mathrm{ph}}^2 + \frac{1}{4\xi^2 H^2} k_{\mathrm{ph}}^4$$

Baumann & Green, 2011;
first term: Tolley & Wyman, 2009, Achucarro et al, 2010
see also: Ashoorioon et al. 2011



parameters

$\xi,\ m_\perp / H$

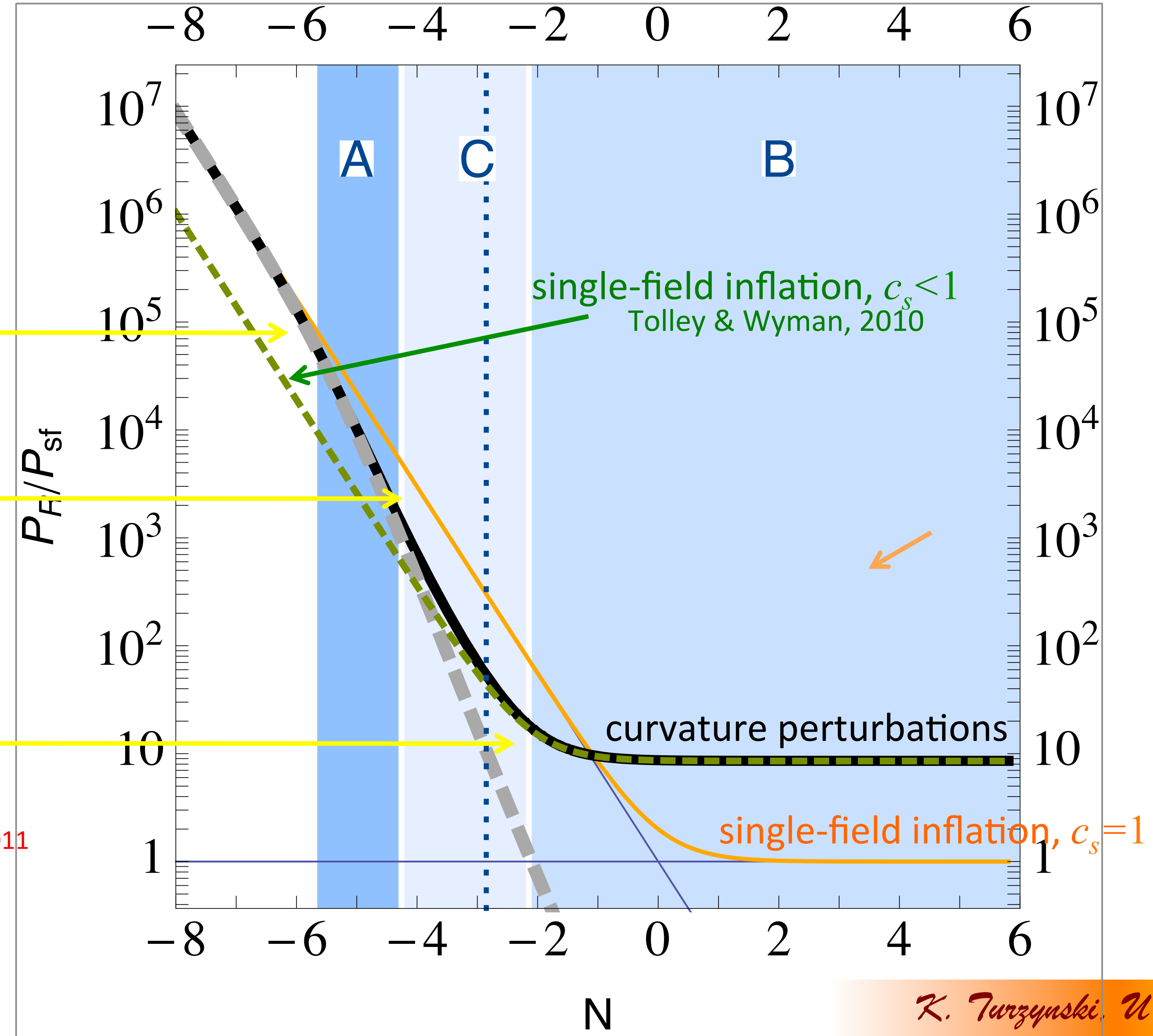
$$b(\phi) \propto -\phi$$

possibility of a strong coupling

$1 \ll M_P^2 b'^2$ e.g. roulette inflation, Bond et al. 2006

$$1 \ll 2 M_P^2 b'^2 \epsilon \equiv \xi^2$$

$$\xi = 300,\ m_\perp^2 = 5000H^2$$



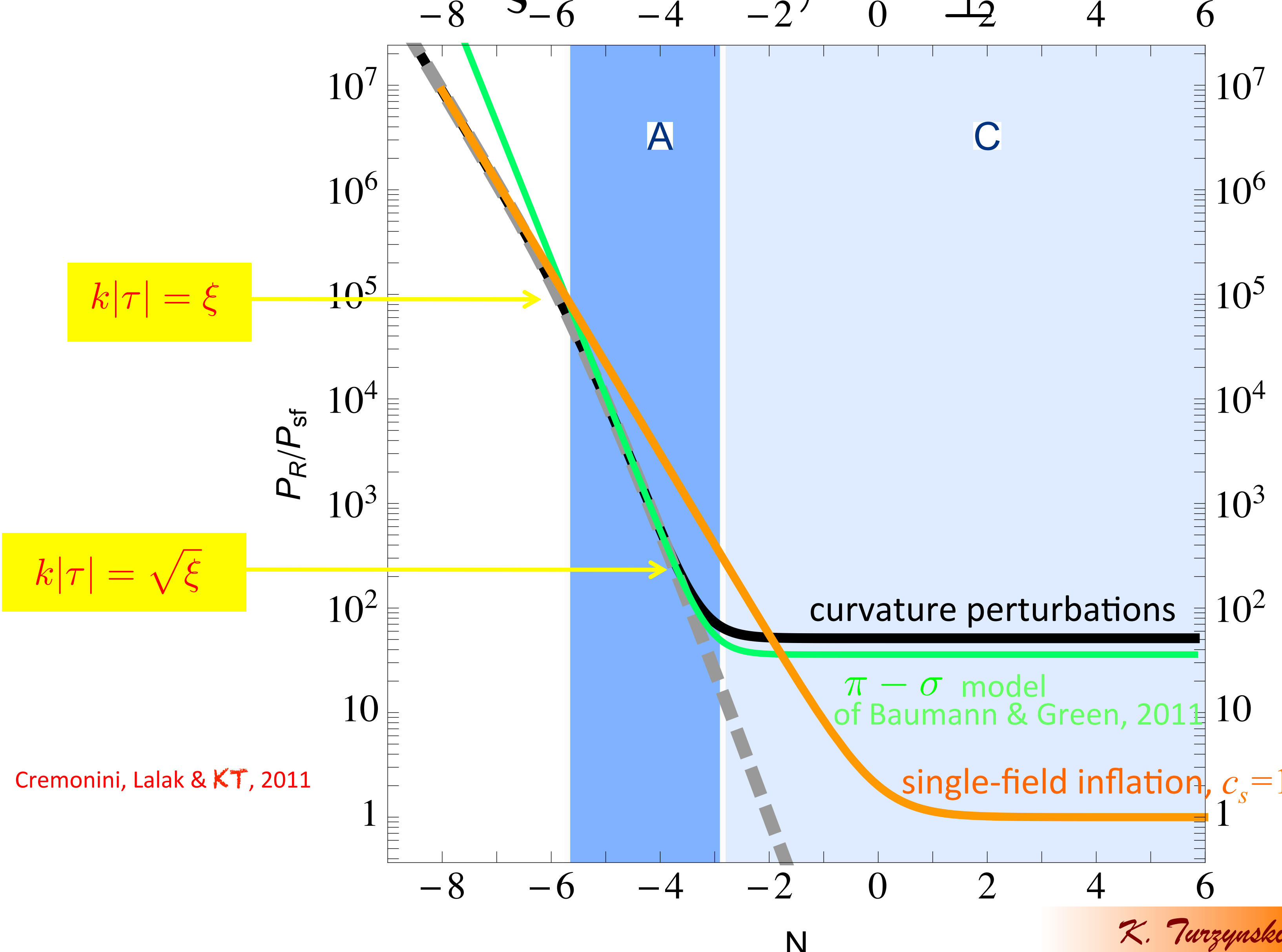
$k|\tau| = \xi$

$k|\tau| = m_\perp/H$

$k|\tau| = 1/c_s$

Cremonini, Lalak & KT, 2011

$\xi = 300,\ m_\perp^2 \to 0$



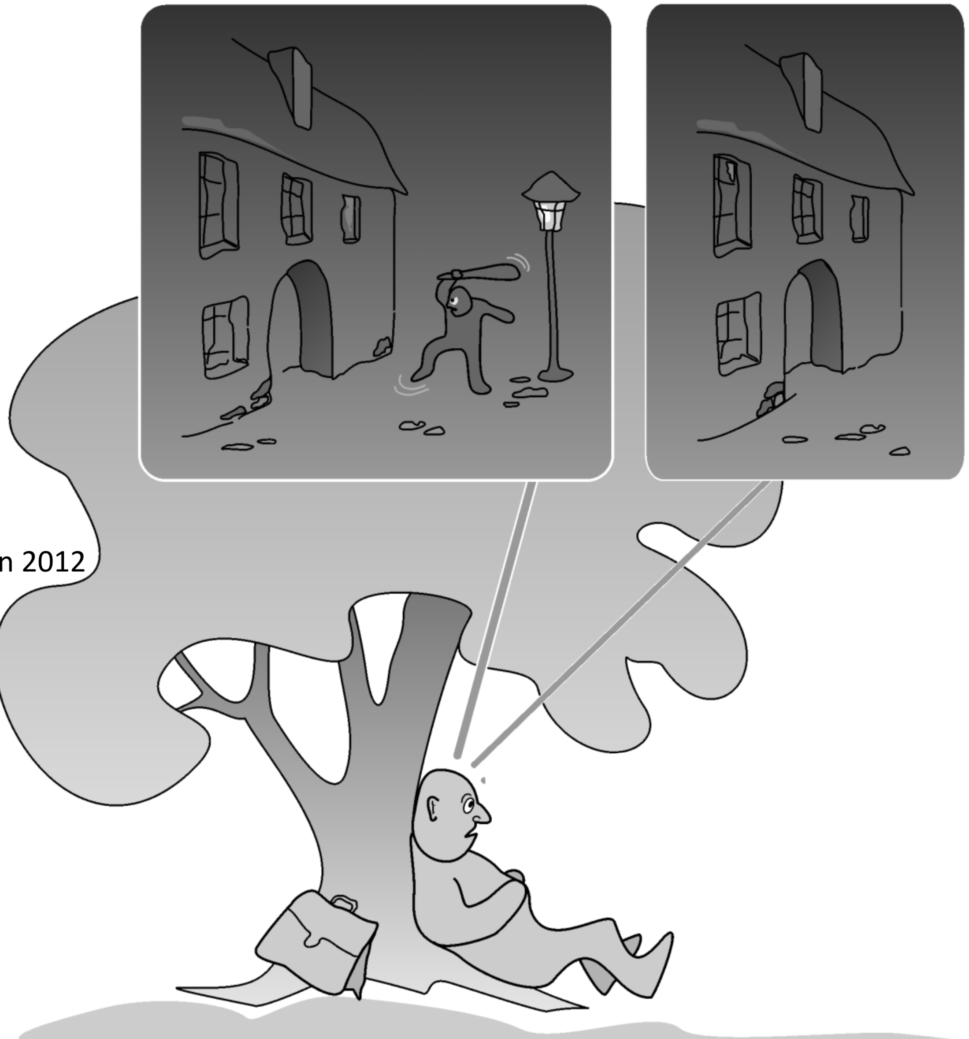
Cremonini, Lalak & KT, 2011

Summary

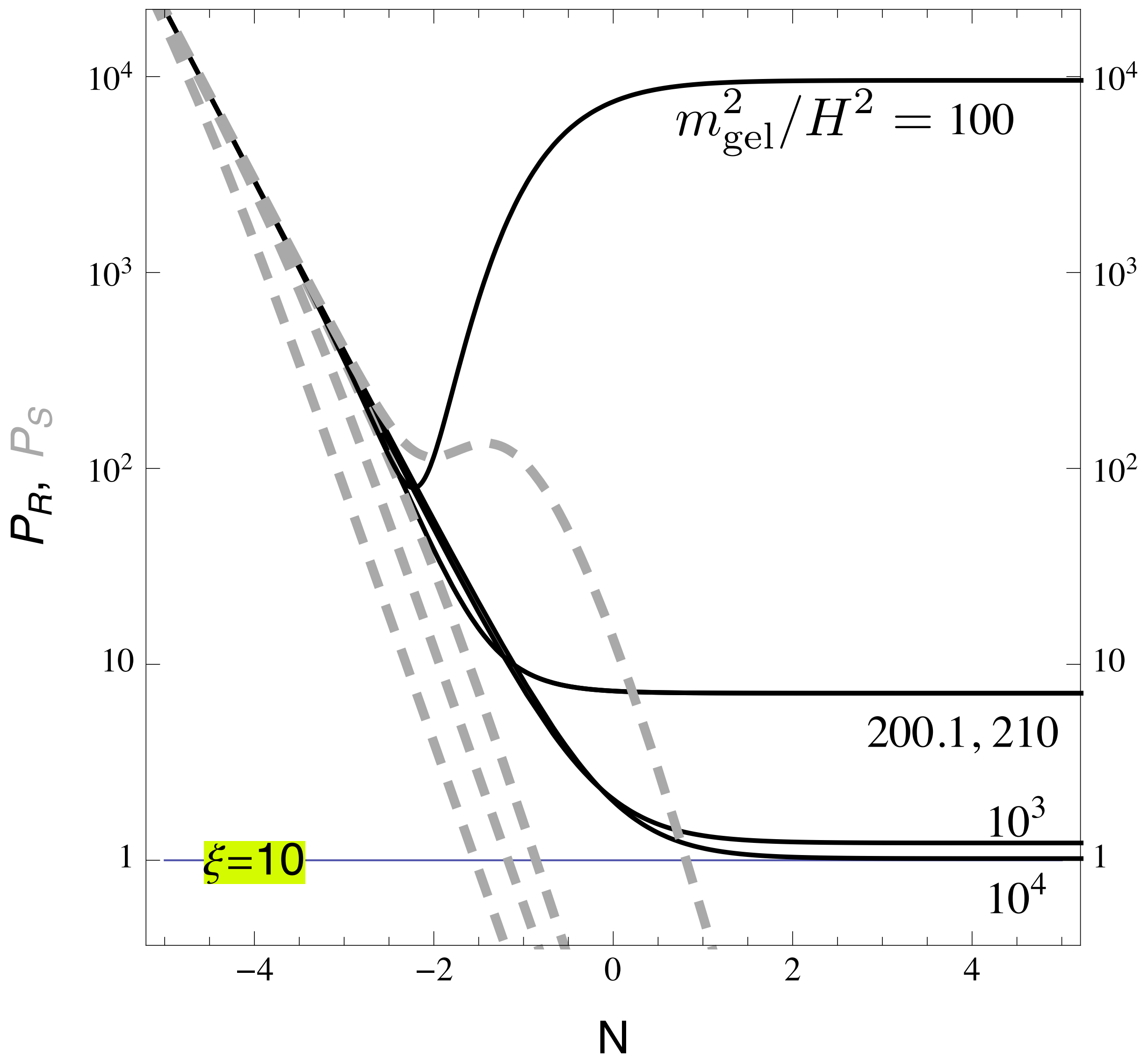
- Several examples show that the effects of sub-horizon fast turns decouple as k^{-2} (at least).

- Some new things possible with non-canonical kinetic terms
  Cremonini, Lalak, KT 2010, 2011
  Avgoustidis, Cremonini, Davis, Ribeiro, KT, Watson 2012

- Work in progress…



Just one backup slide



Temporary instability
at the Hubble radius crossing

Enhancement of the curvature perturbations by many orders of magnitude

Cremonini, Lalak, KT 2011