Scale-dependence of non-Gaussianities as a probe of the early Universe

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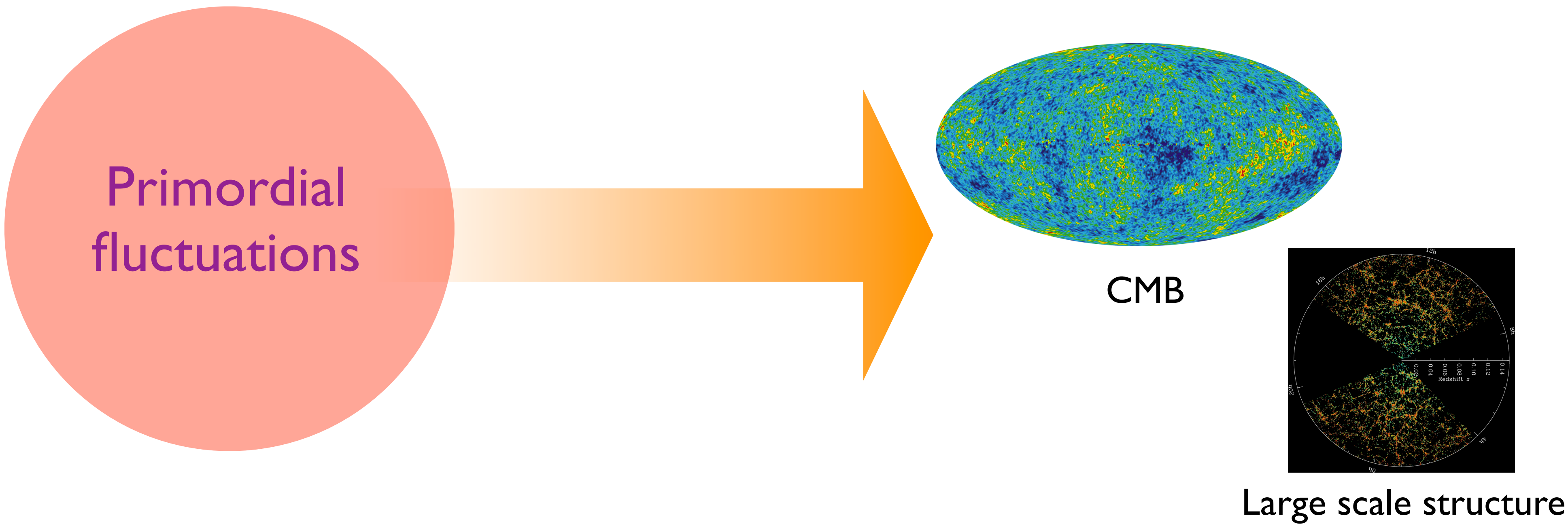
Primordial fluctuations as a probe of the early Universe

- Primordial density fluctuations (the origin of cosmic structure) are considered to be generated in the very early Universe.

➡ They should give the information of the early Universe.

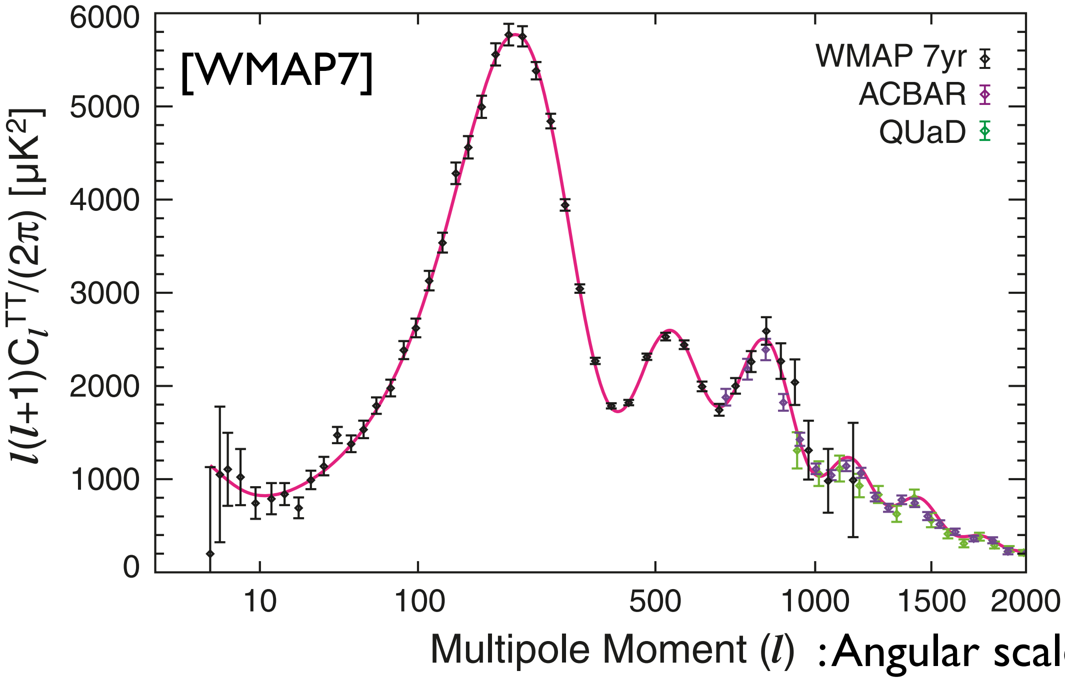


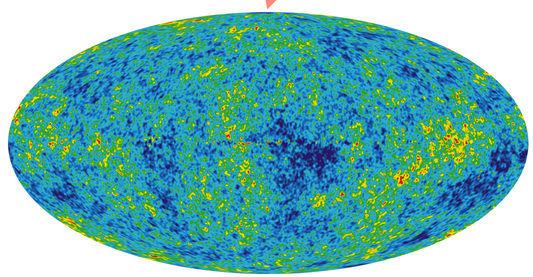
Power spectrum

Primordial curvature perturbation

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \right\rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1)$$

Initial condition





:Angular scales $l \sim 180^\circ/\theta \quad \left(l \sim 14000 \frac{k}{\mathrm{Mpc}^{-1}} \right)$

Primordial fluctuations

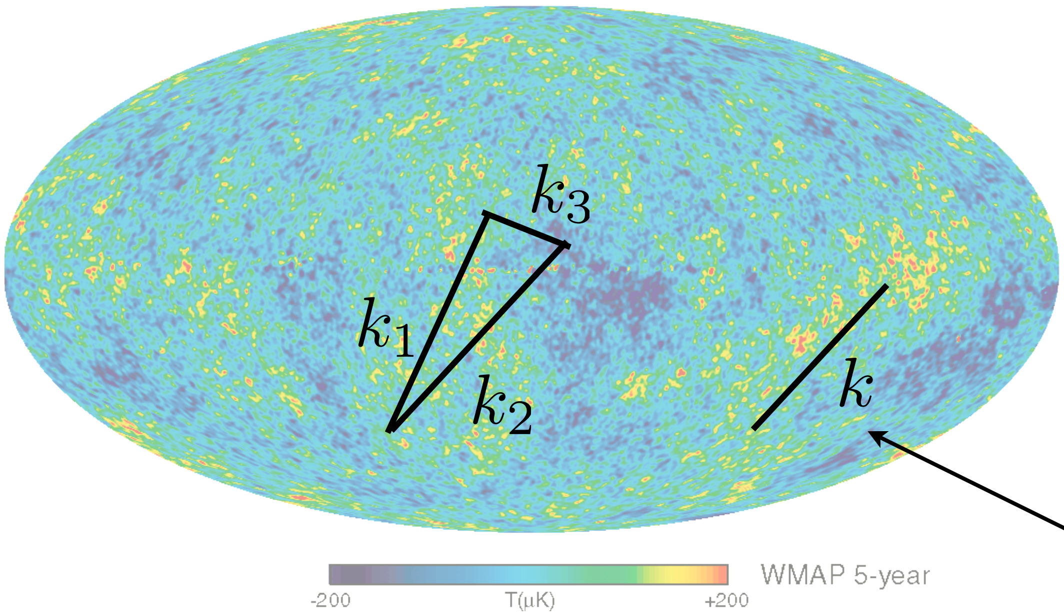
(almost) adiabatic

(almost) scale invariant

amplitude ~ 10^{-5}

3 point function: a measure of non-Gaussianity

$$\left\langle \frac{\Delta T}{T}(\vec{k_1})\frac{\Delta T}{T}(\vec{k_2})\frac{\Delta T}{T}(\vec{k_3}) \right\rangle$$



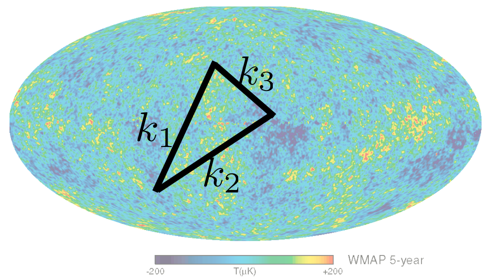
2-point function
(power spectrum)

3 point function: a measure of non-Gaussianity

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$$

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\rm NL} \left(P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right)$$

- Non-Gaussianity is usually characterized by fNL



  - Amplitude of the bispectrum (3-point function)
  - If fluctuations are Gaussian, $f_{\rm NL} = 0$
  - A critical test of inflation: For (most) inflation models, $f_{\rm NL} \ll \mathcal{O}(1)$

Observations of Primordial non-Gaussianity

- Current constraint on f_{NL}

$$f_{\rm NL}^{\rm local} = 32 \pm 21 \quad (68\% \ {\rm CL})$$ → may suggest $f_{\rm NL} \neq 0$?

[(local type) WMAP7, Komatsu et al 2010]

→ Detection of non-Gaussianity would rule out (many) inflation models (of a single field with a canonical kinetic term)

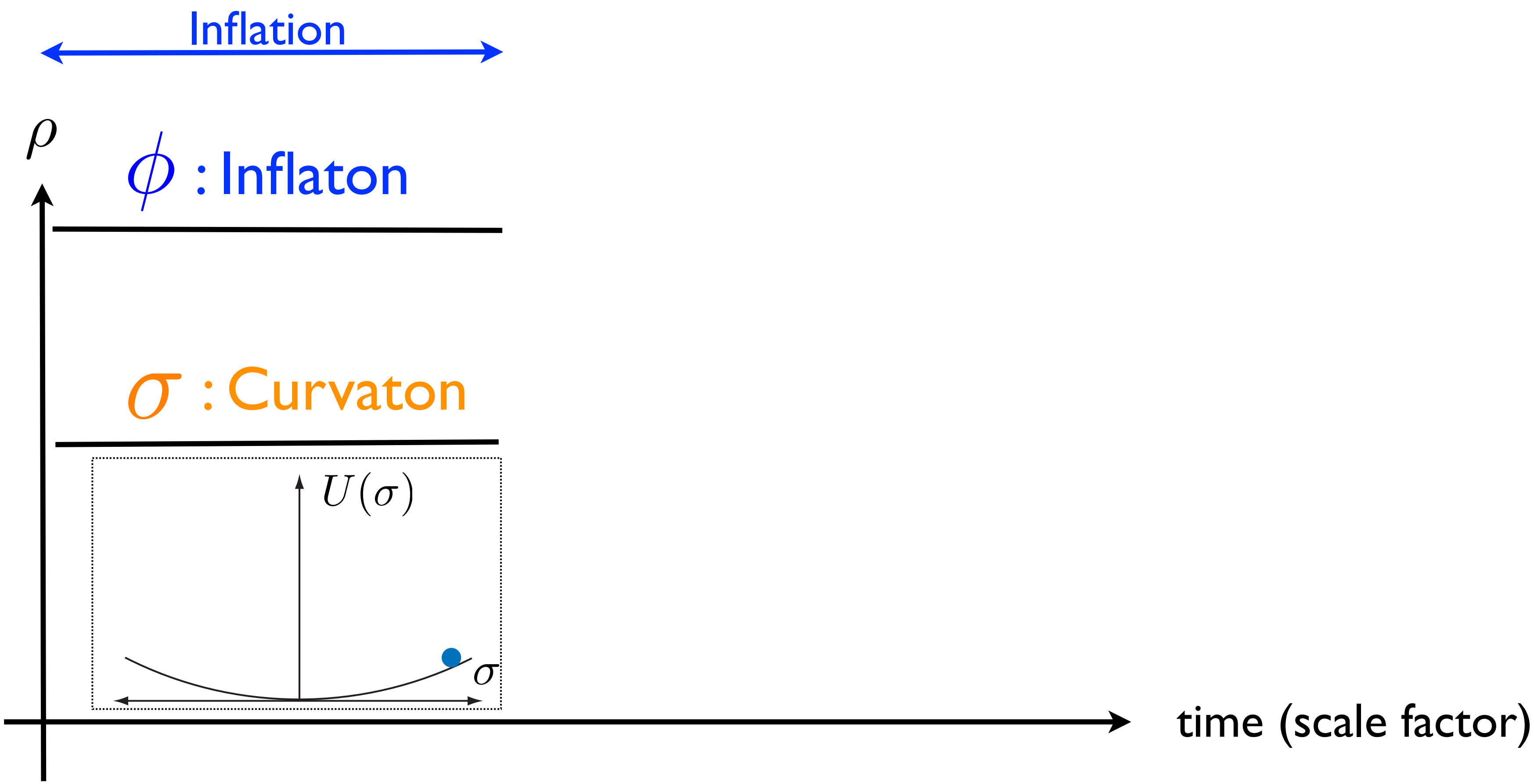
→ If non-Gaussianity is detected $(f_{\rm NL} \neq 0)$, we MUST need some mechanism of generating primordial density fluctuations!

Models generating primordial fluctuations

- Inflation (fluctuations of the inflaton)
- Curvaton model [Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]

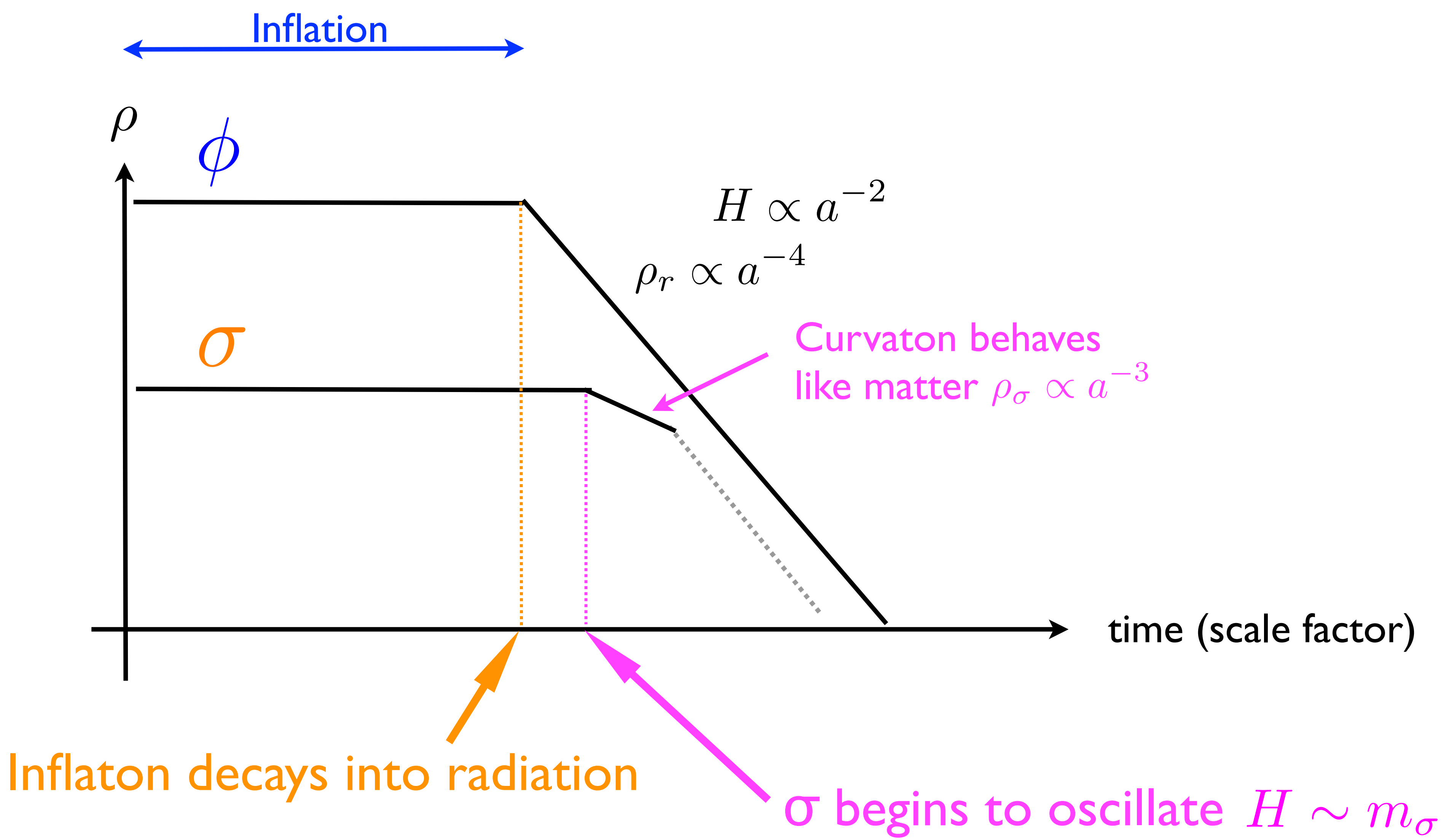
⋮

Thermal history with curvaton

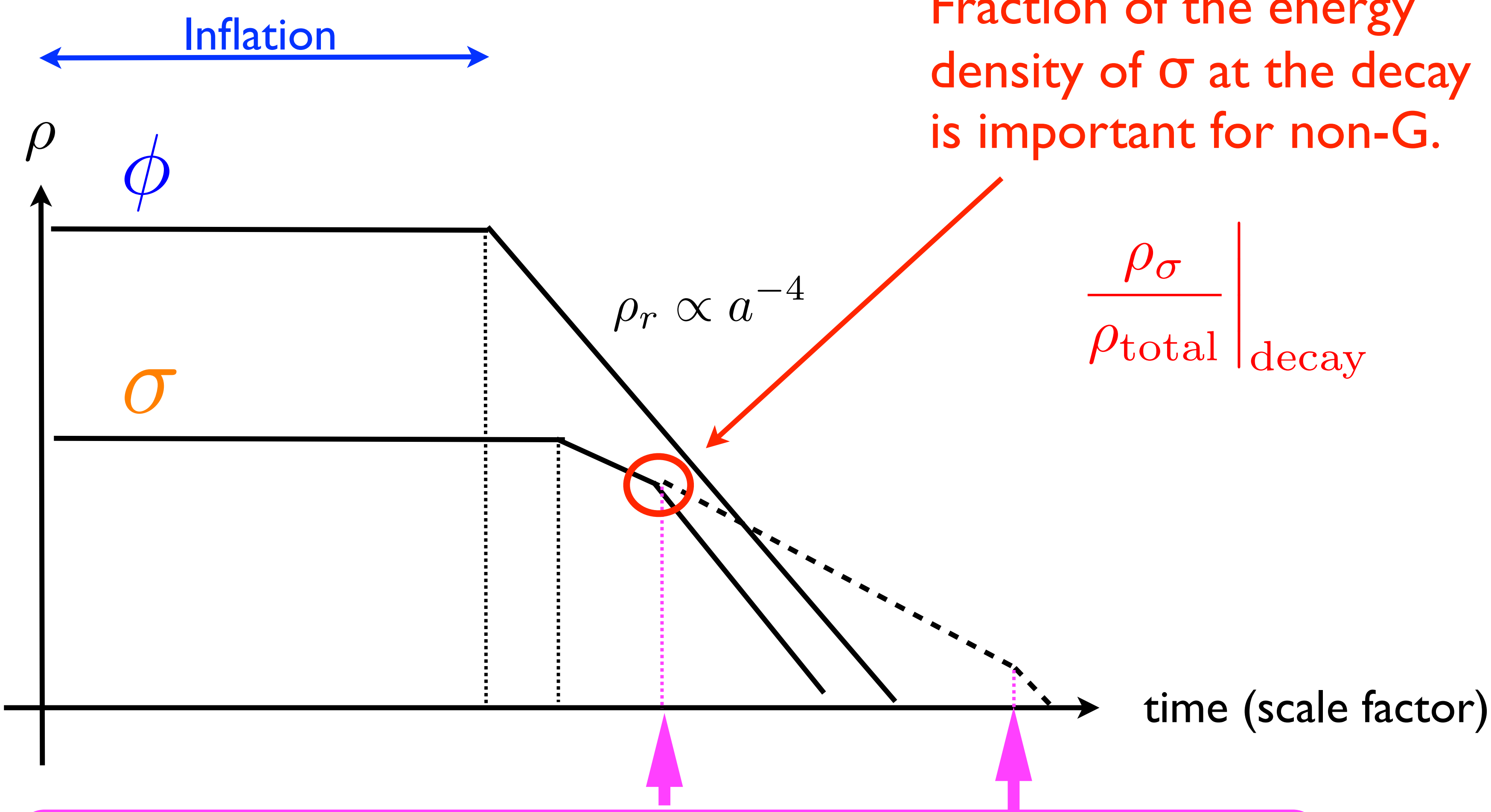


- Inflation is driven by the inflaton $(\rho_\phi \gg \rho_\sigma)$
- σ field can also acquire fluctuations

Thermal history with curvaton



Thermal history with curvaton



- σ decays (into radiation)
- Fluctuations of σ gives adiabatic perturbations

Example: Curvaton model

Non-Gaussianity: $\zeta = \zeta_{(1)} + \frac{3}{5} f_{\rm NL} \zeta_{(1)}^2$

- Curvature perturbation (~ total density fluctuations)

$$\zeta \sim \left.\frac{\rho_\sigma}{\rho_{\rm total}}\right|_{\rm decay} \frac{\delta\rho_\sigma}{\rho_\sigma} \sim 10^{-5}$$

fraction of energy density

density fluc

$$f_{\rm NL} \sim \left(\left.\frac{\rho_\sigma}{\rho_{\rm total}}\right|_{\rm decay} \right)^{-1}$$

$$\rho_\sigma \sim m_\sigma^2 \sigma^2$$
$$\delta\rho_\sigma \sim m_\sigma^2 \sigma \delta\sigma$$
$$\rightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$$

If $\left.\frac{\rho_\sigma}{\rho_{\rm total}}\right|_{\rm decay} \sim 0.01 \rightarrow \frac{\delta\rho_\sigma}{\rho_\sigma} \sim 10^{-3} \rightarrow f_{\rm NL} \sim 100$ (Local-type)

$(\zeta \sim 10^{-5})$

Models generating primordial fluctuations

- Inflation (fluctuations of the inflaton)
- Curvaton model [Enqvist & Sloth; Lyth & Wands; Moroi & TT, 2001]
- Inhomogeneous (modulated) reheating [Dvali, Gruzinov, Zaldarriaga 2003; Kofman 2003]
- Inhomogeneous end of hybrid inflation [Bernardeau, Uzan 2003, Bernardeau et al, 2004, Lyth 2005]
- Inhomogeneous phase transition (e.g., end of thermal inflation) [Matsuda,2009; Kawasaki, TT, Yokoyama, 2009]
- Modulated trapping [Langlois, Sorbo, 2009]

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⋮

f_{NL} can be large...

Even just for the Curvaton model.....

- (simple) Curvaton model with $V = \frac{1}{2}m^2\sigma^2$
- Curvaton with self-interaction [Enqvist, Nurmi 2005; Enqvist, TT, 2008; Enqvist, Nurmi, Taanila, TT, 2009]
- pseudo-Nambu-Goldstone curvaton [Dimopoulos et al 2003; Kawasaki et al 2008]
- Mixed inflaton + curvaton scenario [Langlois, Vernizzi, 2004; Moroi, TT, Toyoda, 2005; Ichikawa, Suyama, TT, Yamaguchi, 2008]
- Multi-Curvaton model (two curvatons) [Assadullahi, Valiviita, Wands, 2007]

→ All of these can give large f_{NL}. We need to differentiate ....

beyond f_{NL}

- There are many models giving large f_{NL}

→ f_{NL} is NOT enough to differentiate models

- We need something beyond f_{NL}:

  - Information of trispectrum (4-pt. function)

    [for a comprehensive discussion, see e.g., Suyama, TT, Yamaguchi, Yokoyama, 1009.1979].

  - Isocurvature fluctuations

    [for nonG, Kawasaki et al 2008; Langlois et al 2008; Hikage, Koyama, Matusbara, TT, Yamaguchi 2008; Kawakami et al 2009; Langlois, Lepidi 2010; Langlois, TT 2010].

  - Looking at scale-dependence of f_{NL}

n_{fNL} :Scale-dependence of $f_{\rm NL}$

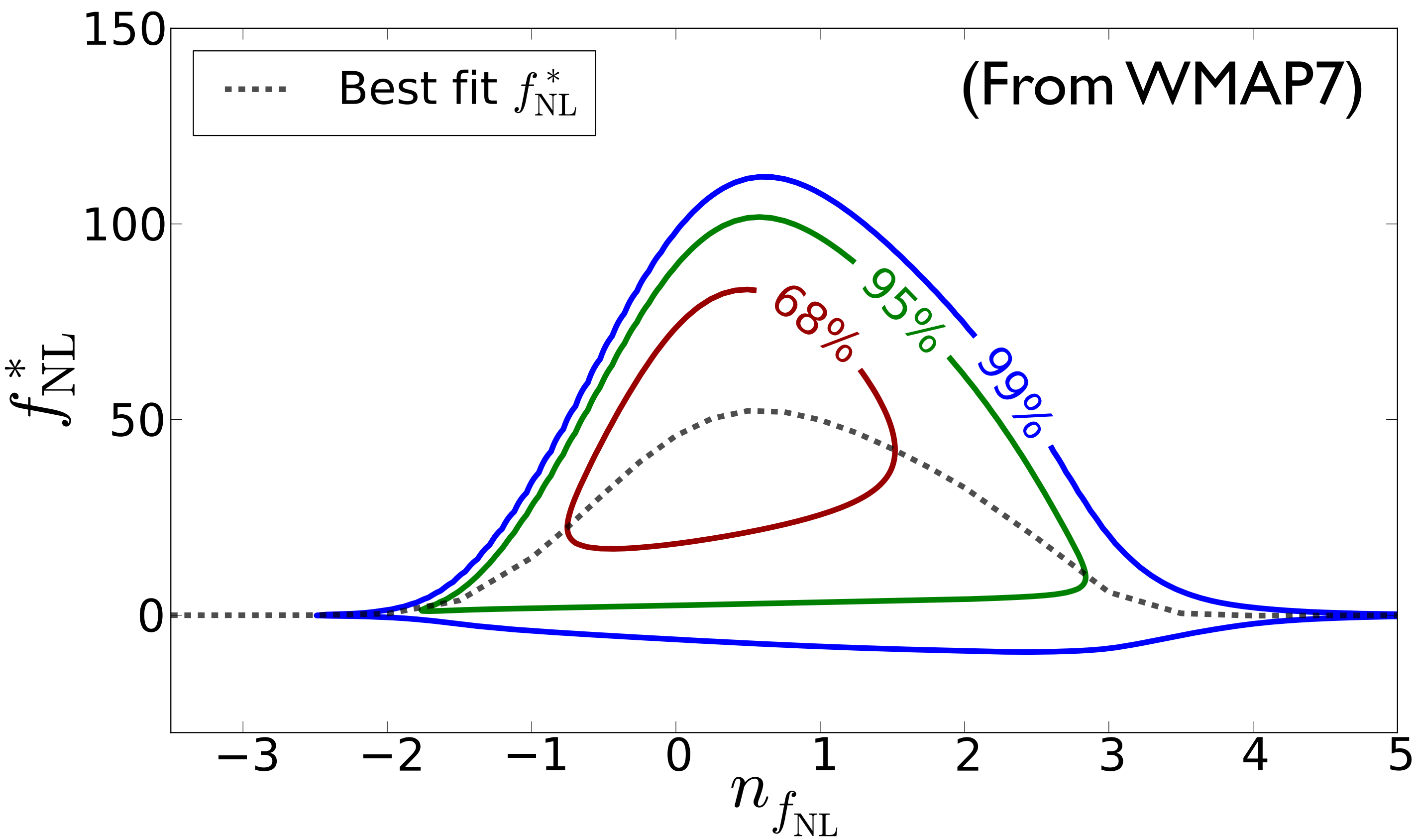
- Definition: $n_{f_{\rm NL}} \equiv \dfrac{d\ln|f_{\rm NL}|}{d\ln k}$

$$\longrightarrow \quad f_{\rm NL}(k) = f_{\rm NL}(k_{\rm ref})\left(\frac{k}{k_{\rm ref}}\right)^{n_{f_{\rm NL}}}$$

In the following, we consider "local type": $\zeta = \zeta_G + \frac{3}{5} f_{\rm NL} \zeta_G^2$

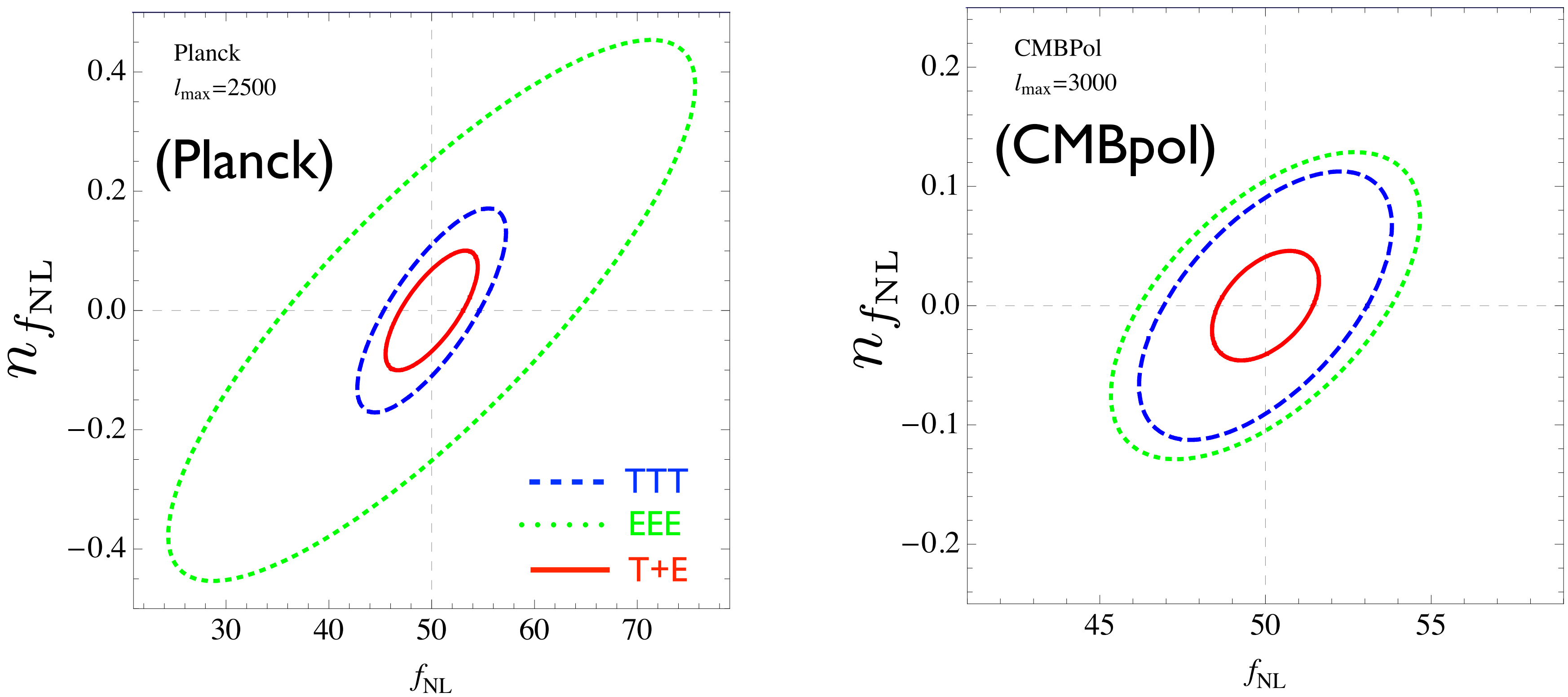
Current limit on n_{fNL}

$$f_{\rm NL}(k) = f^*_{\rm NL}\left(\frac{k}{k_*}\right)^{n_{f_{\rm NL}}} \qquad (k_* \simeq 0.064\ h{\rm Mpc}^{-1})$$



[Becker, Huterer 1207.5788]

Projected limit on n_{fNL}



[Sefusatti et al. 2009]

$$\Delta n_{f_{\rm NL}} = 0.05 \frac{50}{f_{\rm NL}} \frac{1}{\sqrt{f_{\rm sky}}} \quad \text{(CMBpol)}$$

$n_{f_{NL}}$ as a discriminator of models of large f_{NL}

[Byrnes et al, 2009, 2010]

- f_{NL} can be (strongly) scale-dependent when:

  - the potential deviates from the quadratic form.

  - multi-fields are responsible for the perturbations.

$n_{f_{\rm NL}}$ from non-quadratic potential

■ When the potential for a light field deviates from a quadratic form, f_{NL} can be scale dependent.

$$f_{\rm NL} n_{f_{\rm NL}} \sim \frac{V'''}{3H^2}$$

cf. for power spectrum

$$\left(n_s - 1 = -2\epsilon + \frac{2V''}{3H^2} \right)$$

- When the potential is quadratic, no scale-dependence

- Non-zero $n_{f_{\rm NL}}$ can give important information on the potential.

Example: Curvaton with non-quadratic potential

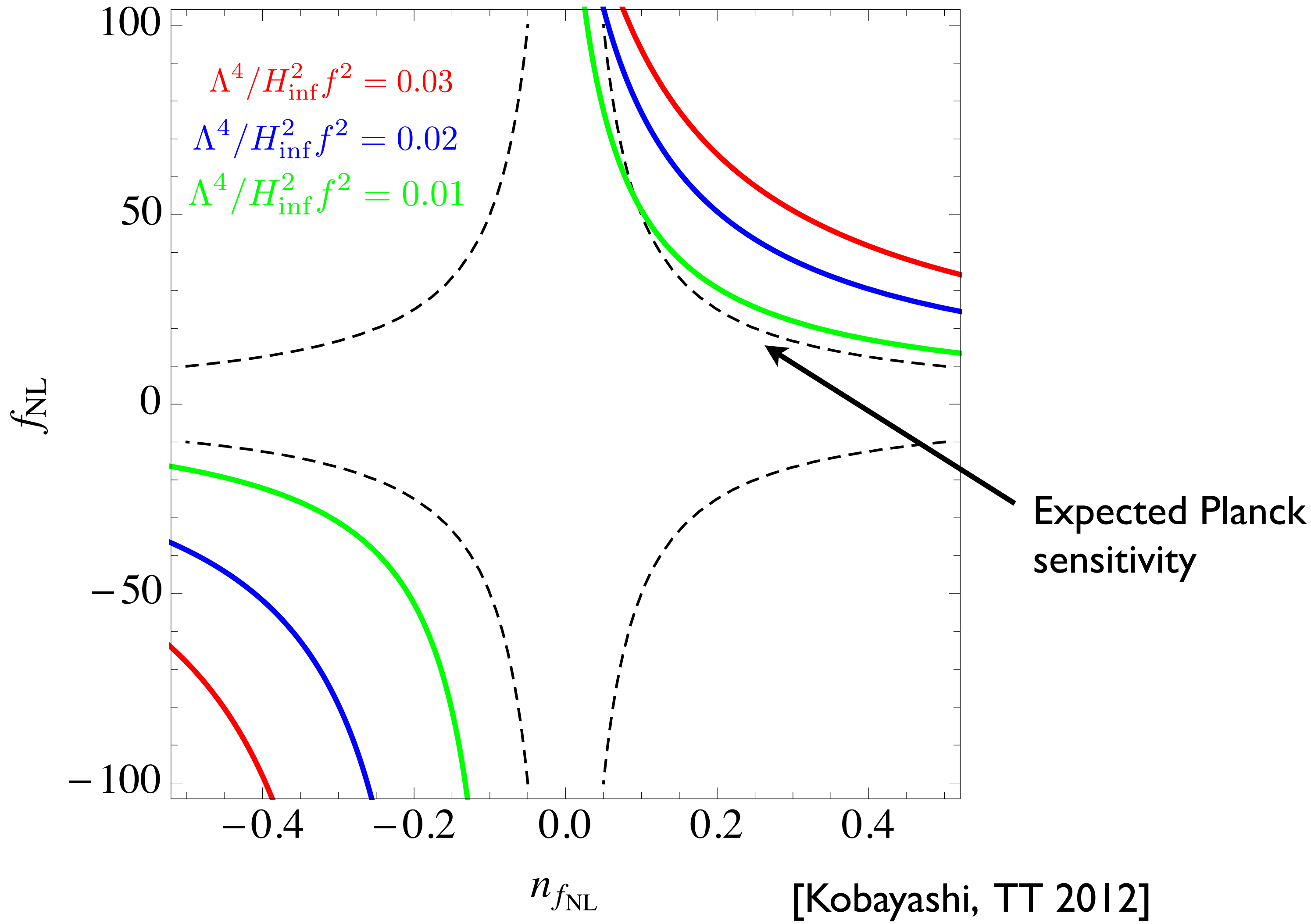
- Self-interacting curvaton [Byrnes, Enqvist, TT 2010; Byrnes, Enqvist, Nurmi, TT 2011; Kobayashi, TT 2012]

$$V(\sigma) = \Lambda^4 \left[\left(\frac{\sigma}{f} \right)^2 + \left(\frac{\sigma}{f} \right)^m \right]$$

- pseudo-Nambu-Goldstone (NG) curvaton [Huang 2010, Kobayashi, TT 2012]

$$V(\sigma) = \Lambda^4 \left[1 - \cos \left(\frac{\sigma}{f} \right) \right]$$

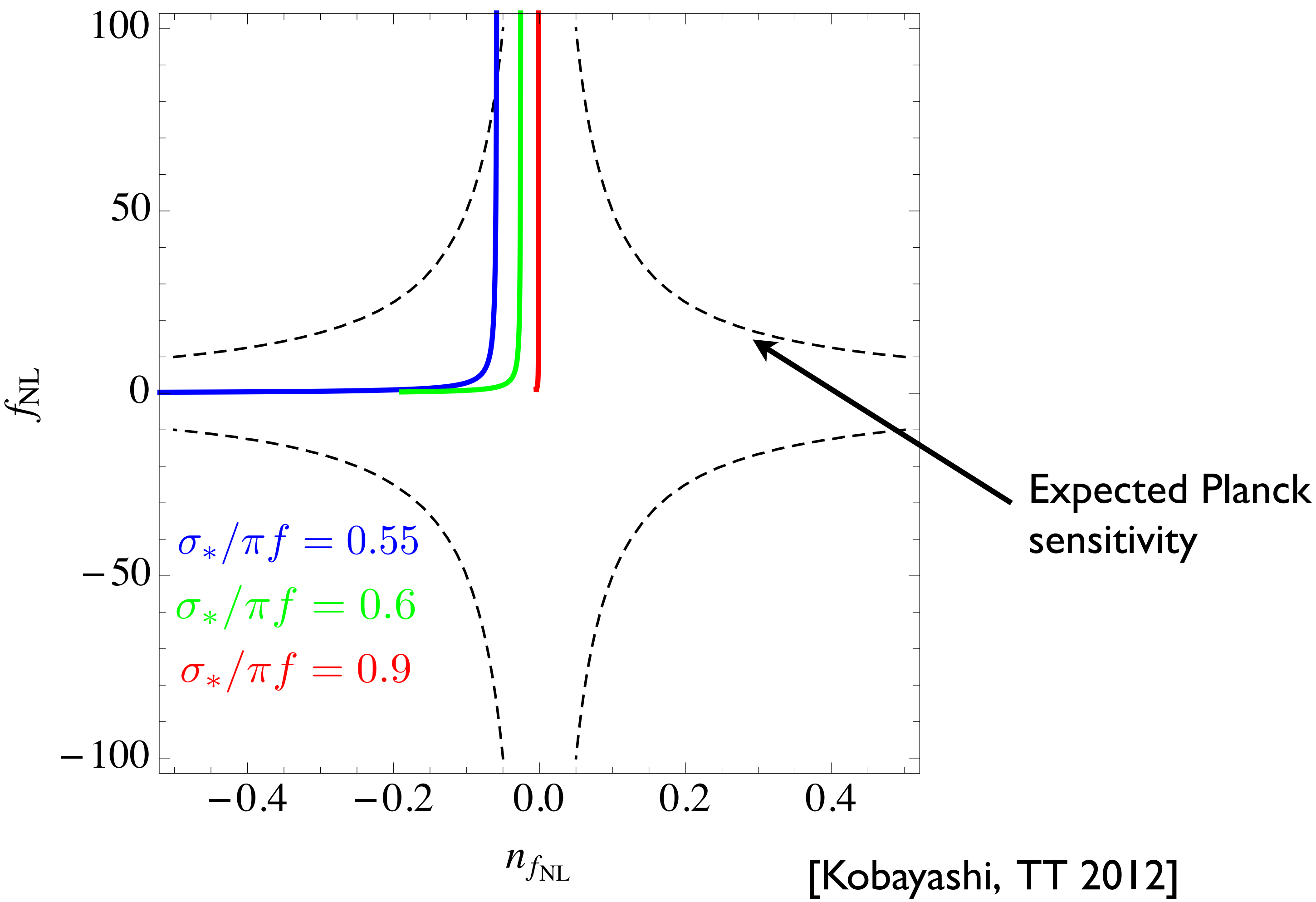
$n_{f_{\rm NL}}$ in the self-interacting curvaton



[Kobayashi, TT 2012]

For positive fNL, nfNL is positive.

$n_{f_{\rm NL}}$ in the psuedo-Nambu-Goldstone curvaton



[Kobayashi, TT 2012]

In this case, nfNL is negative. ➡ differentiate the potential

Multi-field (mixed source) model

- Multiple scalar fields can be simultaneously responsible for density perturbations.

(e.g., in the curvaton model, fluctuations of the inflaton can also exist.)

$$\zeta^{(\mathrm{total})} = \zeta^{(\phi)} + \zeta^{(\sigma)}$$

$$\Rightarrow \quad f_{\mathrm{NL}}^{(\mathrm{total})} = \left(\frac{P_\phi(k)}{P_{\mathrm{total}}(k)} \right)^2 f_{\mathrm{NL}}^{(\phi)} + \left(\frac{P_\sigma(k)}{P_{\mathrm{total}}(k)} \right)^2 f_{\mathrm{NL}}^{(\sigma)}$$

$$P_{\mathrm{total}}(k) = P_\phi(k) + P_\sigma(k)$$

Multi-field (mixed source) model

$$f_{\rm NL}^{(\rm total)} = \left(\frac{P_\phi(k)}{P_{\rm total}(k)}\right)^2 f_{\rm NL}^{(\phi)} + \left(\frac{P_\sigma(k)}{P_{\rm total}(k)}\right)^2 f_{\rm NL}^{(\sigma)}$$

$$P_{\rm total}(k) = P_\phi(k) + P_\sigma(k)$$

- If the scale-dependence of *P(k)* for two fields are different, "total" *fNL* can be scale-dependent.

- $n_{f_{\rm NL}} \equiv \dfrac{d\ln|f_{\rm NL}|}{d\ln k}$ comes from $\dfrac{d}{d\ln k}\left(\dfrac{P_\phi(k)}{P_{\rm total}(k)}\right)$ or $\dfrac{d}{d\ln k}\left(\dfrac{P_\sigma(k)}{P_{\rm total}(k)}\right)$

When $P_{\rm total}(k) \simeq P_\phi(k)$ or $P_{\rm total}(k) \simeq P_\sigma(k)$

no scale dependence in *fNL*

Mixed inflaton-curvaton model

ϕ → Inflaton σ → Curvaton

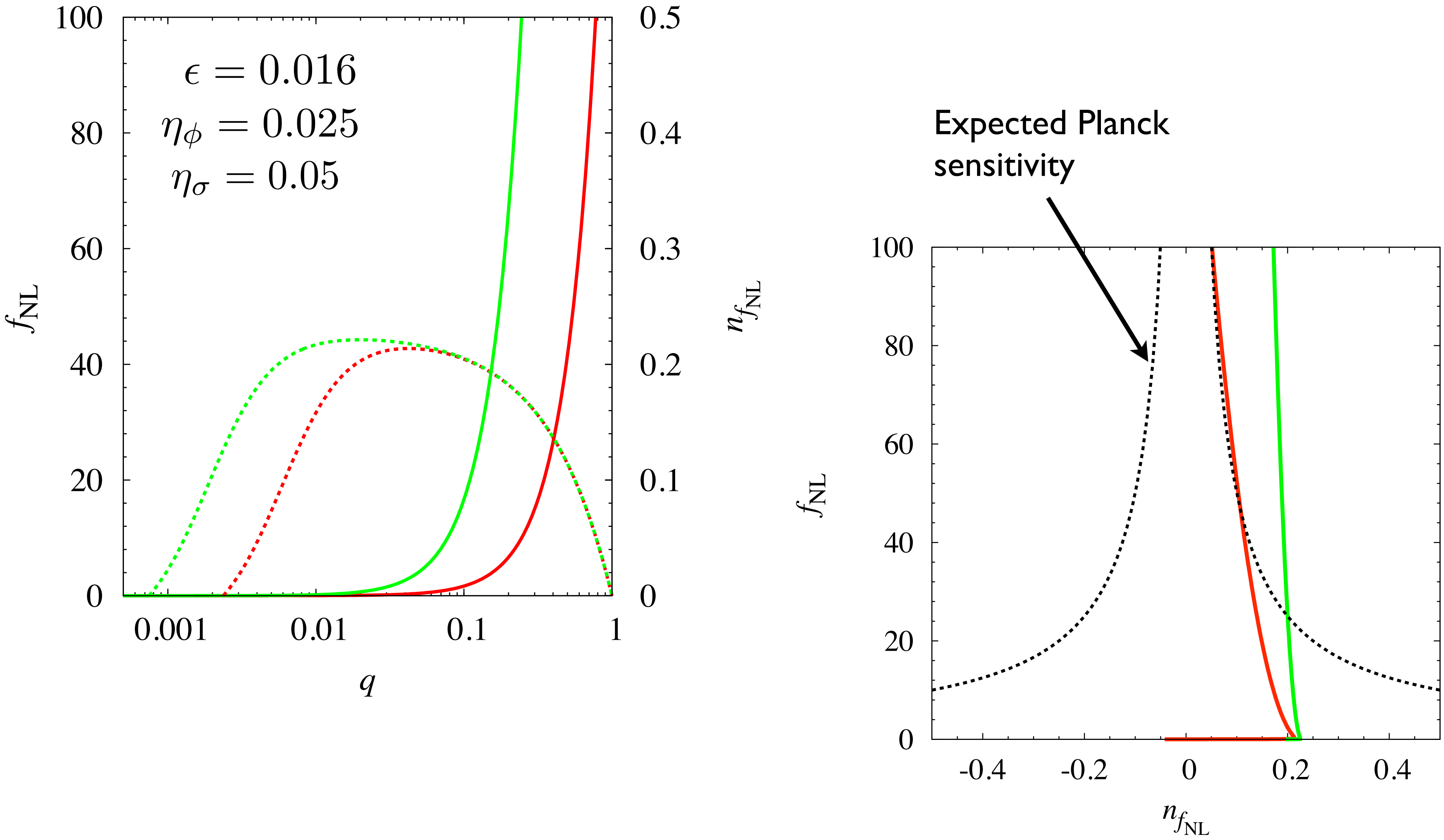
$$n_s^{(\phi)} - 1 = -6\epsilon + 2\eta_\phi \qquad n_s^{(\sigma)} - 1 = -2\epsilon + 2\eta_\sigma$$

where $\epsilon = -\frac{\dot{H}_*}{H_*^2} \quad \eta_\phi = \frac{U^{''}(\phi)}{3H_*^2} \quad \eta_\sigma = \frac{V^{''}(\sigma)}{3H_*^2} = \frac{m_\sigma^2}{3H_*^2}$

→ $$n_{f_{\rm NL}} \simeq 4\left(1 - \frac{P_\sigma(k)}{P_{\rm total}(k)}\right)(2\epsilon + \eta_\sigma - \eta_\phi)$$

(for $f_{NL} >> 1$)

$n_{f_{\rm NL}}$ in the mixed inflaton-curvaton model



[Kobayashi, TT 2012]

Two curvaton model

ϕ → Curvaton 1 σ → Curvaton 2

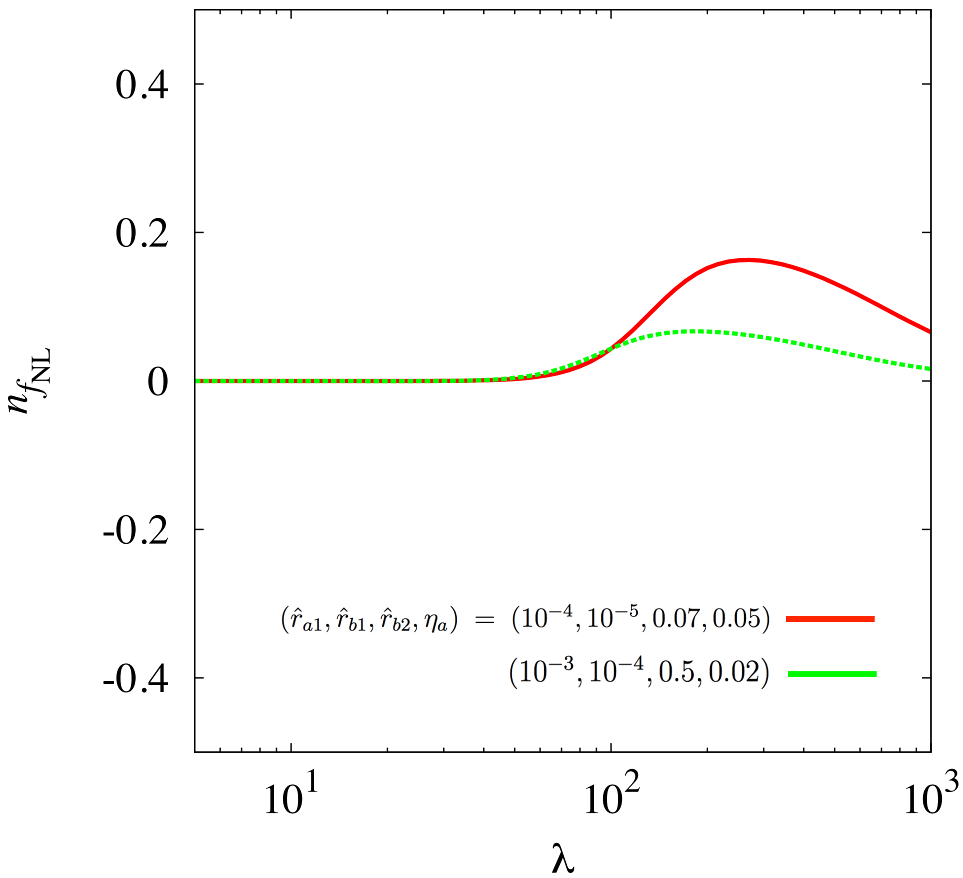
$$n_s^{(\sigma_1)} - 1 = -2\epsilon + 2\eta_{\sigma_1} \qquad n_s^{(\sigma_2)} - 1 = -2\epsilon + 2\eta_{\sigma_2}$$

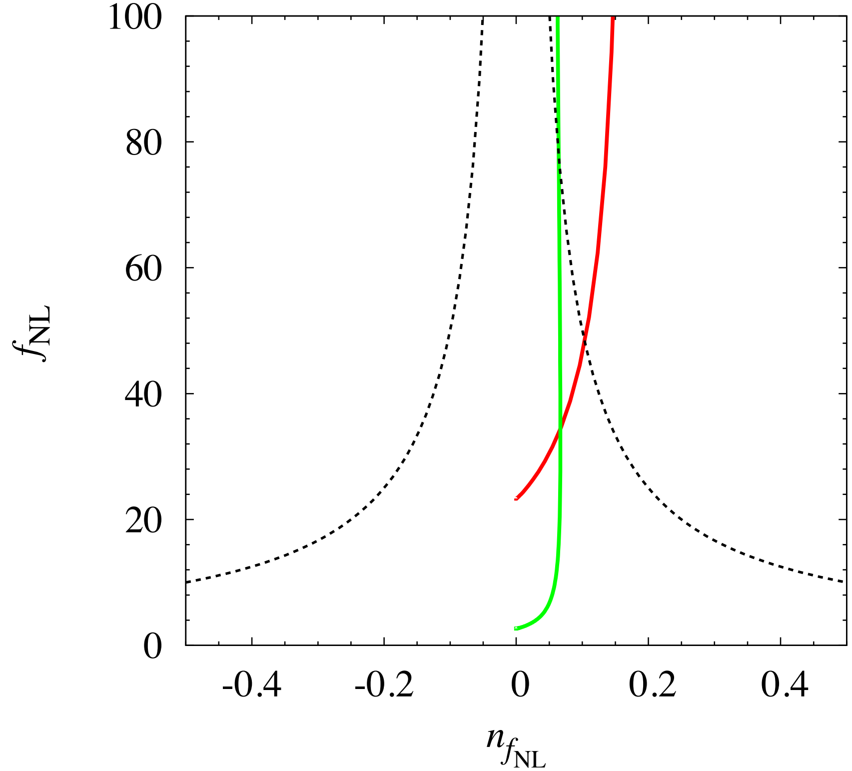
where $\epsilon = -\frac{\dot{H}_*}{H_*^2} \quad \eta_\phi = \frac{U''(\phi)}{3H_*^2} \quad \eta_\sigma = \frac{V''(\sigma)}{3H_*^2} = \frac{m_\sigma^2}{3H_*^2}$

→ $$n_{f_{\rm NL}} \simeq 4\left(1 - \frac{P_{\sigma_1}(k)}{P_{\rm total}(k)}\right)(\eta_{\sigma_1} - \eta_{\sigma_2})$$

(for $f_{NL} >> 1$)

Two curvaton model





[Kobayashi, TT 2012]

Summary

- Information on f_{NL} is NOT enough to differentiate models of primordial fluctuations.

- Scale-dependence of non-Gaussianity ($n_{f_{NL}}$) can be useful to discriminate models of large non-G.

- Some models predict large $n_{f_{NL}}$ which can be testable with Planck.

- Scale-dependence of non-G. can give important information on the physics of the early Universe.