Physical processes in strong external fields at linear colliders

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Based on a project with A.F. Hartin and G.A. Moortgat-Pick

Outline

1. Future linear colliders
2. Strong field effects at the Interaction Point (IP)
3. Furry Picture (FP)
4. Some results of FP calculations
5. Conclusions

Future e^+e^- linear colliders

ILC and **CLIC**:

- ILC 90 GeV - 1.5 TeV
  CLIC 500 GeV - 3 TeV
- $\mathcal{L} \sim 10^{34} - 10^{35}$ cm^{-2}s^{-1}
- clean

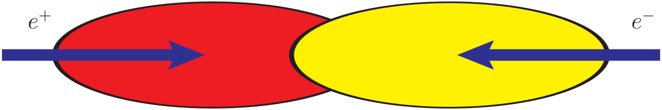
Natural steps after LHC:

(Good news from Kraków?!)

- For precision physics: Higgs, top, gauge bosons.
- Discovery of new physics BSM.

Future colliders: the Interaction Point (IP)

intense charge bunches



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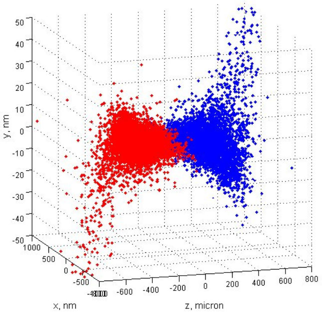

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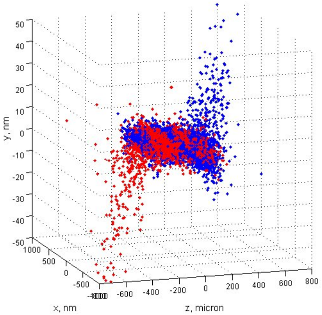
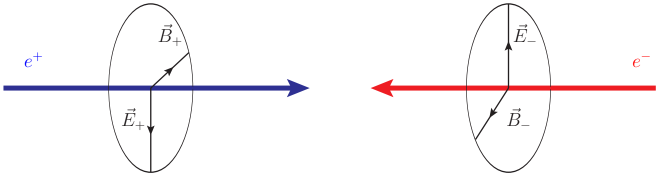


Walker'03

Future colliders: Strong fields at the IP

intense charge bunches $\longrightarrow$ strong field associated

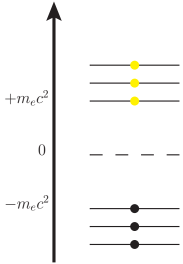


To a good approximation e^+ and e^- see 2 *almost* anticollinear **constant crossed fields**.

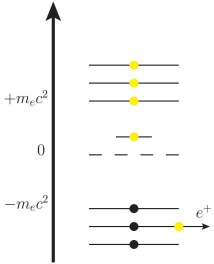
$$|\mathbf{E}| = |\mathbf{B}| \qquad \mathbf{E} \cdot \mathbf{B} = 0$$

static: em wave with infinite period of oscillations $\Rightarrow$ approximated as a classical field.

Schwinger critical field: $E_c = 1,3 \cdot 10^{18}$ V/m.



$\Rightarrow$ Turn on strong A^{ext} $\Rightarrow$



Vacuum is polarized.

Υ parameter: $\Upsilon \equiv \gamma \frac{B}{B_c} = \frac{e|\vec{a}|}{m_e E_c}(k \cdot p)$

CLIC-3TeV: $E = 10^{12}$ V/m, $\Upsilon_{av} = 3.34$, $\Upsilon_{max} = 10.9$.

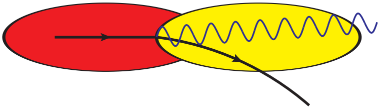
Quantum effects at the beam IP

Strong external fields

$\Downarrow$

quantum effects:

- Beamstrahlung
- Coherent Pair Production: int. with the collective field $\longrightarrow$ dominant at **CLIC**
- Incoherent Pair Production: int. with individual particles $\longrightarrow$ dominant at **ILC**



Usual treatment of strong field quantum effects

The previous quantum effects are presently estimated with approximations:

- Baĭer-Katkov approximation $\longrightarrow$ beamstrahlung & coherent pair production

The electron orbit is treated classically BUT the emission of a photon is a quantum process

- Equivalent photon approximation (EPA) $\longrightarrow$ incoherent pair production

Virtual photons are considered real.

In both the approximations e^+ and e^- only see one external field, the incoming one.

The next Linear Colliders programme requires highly precise knowledge of the processes at the IP:

- "2 *almost* anticollinear" external fields, due to deflection angle and other effects.
- EPA and Baĭer-Katkov approximations effective only if there is not significant transverse momentum.
- Then, analytically exact treatment of the external fields would be needed, even if time consuming.

Coming back,

Vacuum polarized by the strong "classical" external field $\Longrightarrow$ **Natural application for Furry Picture**

Furry Picture

[Furry51], [Moortgat-Pick09]

Interaction Picture: $\mathcal{H}_I = \mathcal{H}_0 + \mathcal{V}$

Furry Picture (FP): $\mathcal{H}_F = \mathcal{H}_0 + \mathcal{H}_{ext} + \mathcal{V} = \mathcal{H}_B + \mathcal{V}$

The external field is treated **classically**, not included in the interaction potential $\mathcal{V}$

Eigenstates in FP $\Rightarrow$ bound states of the electron in the external field.

Furry Picture

[Furry51], [Moortgat-Pick09]

Interaction Picture: $\mathcal{H}_I = \mathcal{H}_0 + \mathcal{V}$

Furry Picture (FP): $\mathcal{H}_F = \mathcal{H}_0 + \mathcal{H}_{ext} + \mathcal{V} = \mathcal{H}_B + \mathcal{V}$

related by a canonical transformation:

$$\Psi_F(x) = M^{-1}\Psi_I(x)M \qquad \Psi_F^\dagger(x) = M^{-1}\Psi_I^\dagger(x)M$$

Different basis system: $\lim_{A_\mu^{\text{ext}} \to 0}\{\Psi_F, \Psi_F^\dagger\} = \{\Psi_I, \Psi_I^\dagger\}$

Furry Picture

In the F.P. the QED Lagrangian:

$$\mathcal{L}_F = \bar{\psi}(i\not{\partial} - e\not{A}^{\text{ext}} - m)\psi - \frac{1}{4}FF - e\bar{\psi}\not{A}\psi$$

modified Dirac equation:

$$(i\not{\partial} - e\not{A}^{\text{ext}} - m)\psi = 0$$

Furry Picture

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Volkov solution [Volkov35]:

$$\Psi_p^V(k\cdot x) = \frac{1}{\sqrt{(2\pi)^3 2\epsilon_p}} E_p(k\cdot x) u_p$$

with

$$E_p(k\cdot x) \equiv \left(1 - \frac{eA\!\!\!/^{\text{ext}} k\!\!\!/}{2(k\cdot p)}\right) \exp\Big[- i\,(p)\cdot x - i\int_0^{(k\cdot x)} \left[\frac{e(A^{\text{ext}}(\phi)\cdot p)}{(k\cdot p)} - \frac{e^2 A^{\text{ext}}(\phi)^2}{2(k\cdot p)} \right] d\phi \Big]$$

Furry Picture

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They constitue an orthogonal and complete system [Ritus72].

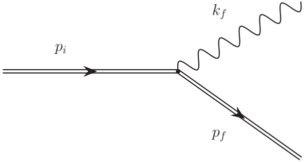
Furry Picture: QED Feynman Rules

- Fermion Green function:

$$G(x,x') = \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty} d^4p\, E_p(k\cdot x)\frac{\not p + m}{p^2-m^2}\overline{E}_p(k\cdot x')e^{ip\cdot(x'-x)}$$

- Photon propagator unchanged.

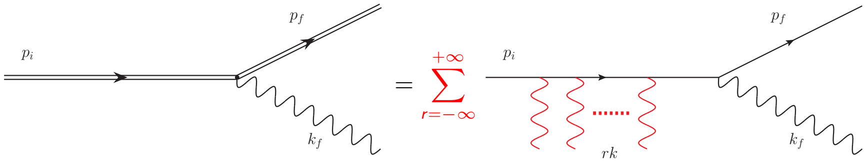
- QED vertex in momentum space:



$$\Gamma^e_\mu = (2\pi)^4 \sum_{r=-\infty}^{+\infty} \overline{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\delta^4(p_f+k_f-p_i-r\,k)$$

Interpretation of FP 1-vertex process

Momentum conservation encoded in $\delta^4(p_f + k_f - p_i - rk)$ allows 1-vertex processes, not permitted in absence of an external field:



Each term of the sum over r can be seen as the absorption or the emission of r photons of the external field [Nikishov64].

Strong field quantum effects with Furry Picture

Beamstrahlung and coherent pair production are 1^{st} order Furry Picture processes:

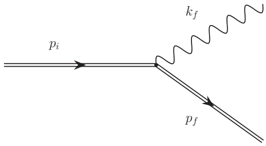


Figure: Beamstrahlung.

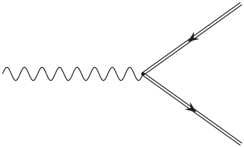


Figure: Coherent pair production.

The incoherent pair production instead is a 2^{nd} order (2-vertices) process:

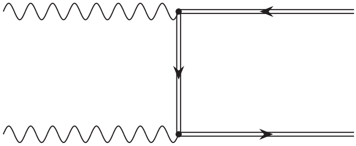


Figure: Incoherent pair production, Breit-Wheeler process.

Results: photon emission by an electron and pair production

The photon emission by an electron and pair production were studied:

- in a polarized plane electromagnetic wave and in a constant field [Nikishov64].

$$W_{\text{cost}} = -\frac{e^2 m_e}{2} \int_0^{+\infty} \frac{du}{(1+u)^2} \Big[\int dz + \frac{1+(1+u)^2}{z(1+u)} \frac{d}{dz} \Big] \text{Ai}(z) \quad \text{with } z \equiv \left(\frac{u m_e^2}{\nu (k \cdot p_i)} \right)^{2/3}$$

- in two collinear, linearly and orthogonally polarized waves [Lyul'ka74].
- in N collinear fields [Hartin11].
- in two constant crossed fields of any orientation [Hartin12].

Observed a dependence of the energy of the radiated photon on the intensity of external field ($\nu = ea/m_e$).

Results: W leptonic decays

Volkov solution for W_μ boson:

$$W_\mu(x) = E^{W}_{p\,\mu\nu}\, e^{-ip\cdot x}\, \epsilon^{W\,\nu}_{p}$$

with:

$$E^{W}_{p\,\mu\nu} = \Big(g_{\mu\nu} + \frac{e}{k\cdot p}\int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{\mathrm{ext}\,2} k_{\mu\nu}\Big)\exp\Big[-\frac{i}{2(k\cdot p)}\Big(2e\Big(A^{\mathrm{ext}}\cdot p\Big) - e^2 A^{\mathrm{ext}\,2}\Big)\Big]$$

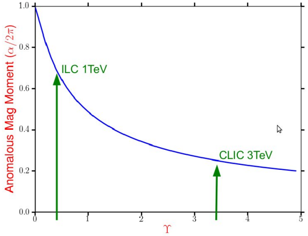
- The partial decay width $\Gamma(W^\pm \to l^\pm \nu_l)$ in strong external fields has been considered [Kurilin03], revealing important correction $\mathcal{O}(10)$ for $\Upsilon \gg 1$.

One-loop

A few one-loop effects in a costant field at the IP regions have been studied [Ritus70], [Ritus72]:

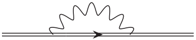
- anomalous magnetic moment:

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^{+\infty}\frac{dx\,2\pi}{(1+x)^3}\left(\frac{x}{\Upsilon}\right)^{\frac{1}{3}}\mathrm{Gi}\left(\frac{x}{\Upsilon}\right)^{\frac{1}{3}}$$



Hartin'11

- Photon mass. 

- Shift in the electron mass. 

We want to study systematically one-loop effects and renormalization.

Conclusions and outlook

- In the next e^+e^- linear colliders, the em fields associated to the charge bunches are so strong that their effect at the IP are not negligible.
- Precision physics and search for BSM require these processes to be known as precisely as possible.
- The **Furry Picture** take entirely into account of the effects of the strong external field at the IP.

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Work in progress:

- Analytically calculate 2-vertices Furry Picture processes, ex. $e^+e^- \to W^+W^-$.
- Extend these calculation to the field of two bunches; extend most general shape of external fields, for applications in other contexts, ex. laser or plasma physics.
- Study loop corrections and development of a coherent treatement of renormalization.

Thank you for your attention!

References

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Backup: ILC and CLIC parameters

	ILC (1 TeV)	CLIC (3 TeV)
$\mathcal{L}$	$4 \cdot 10^{34}$	$3.6 \cdot 10^{34}$
N_{coh}	0	$6.8 \cdot 10^{8}$
N_{inc}	$3.9 \cdot 10^{5}$	$3.8 \cdot 10^{5}$
Υ_{av}	0.27	3.34
Υ_{max}	0.94	10.9

Υ estimated taking into account small bunch dimensions (with `CAIN`).

$$\Upsilon \equiv \frac{2}{3}\frac{\hbar\omega_c}{\epsilon_e} = \gamma\frac{B}{B_c} = \frac{e|\vec{a}|}{m_e E_c}(k \cdot p) = \frac{e}{m_e^3}\sqrt{|(F_{\mu\nu}p^{\nu})^2|}$$

$$\Upsilon_{av} \approx \frac{2Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x + 1.85\sigma_y)} \qquad \Upsilon_{max} \approx \frac{5}{6}\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x + \sigma_y)}$$

$$E_c = m_e^2/e = 1.32 \cdot 10^{18}\ \text{V/m}.$$

Backup: Furry Picture

Furry Picture is related to the Dirac Picture by a canonical transformation:

$$\Psi_F(x) = M^{-1}\Psi_I(x)M \qquad \Psi_F^\dagger(x) = M^{-1}\Psi_I^\dagger(x)M$$

Ψ_F and Ψ_I spanned by a different basis system so that

$$\{\Psi_F, \Psi_F^\dagger\} \neq \{\Psi_I, \Psi_I^\dagger\}$$

The usual commutation relations are recovered in the limit $A_\mu^{\text{ext}} \to 0$.

Gauge transformation:

$$A^\mu(x) \to A^\mu(x) - \frac{\delta\Lambda(x)}{\delta x^\mu}, \qquad A^\mu_{ext}(x) \to A^\mu_{ext}(x) - \frac{\delta\Lambda_{ext}(x)}{\delta x^\mu}, \qquad \Psi(x) \to e^{-ie\Lambda(x) - ie\Lambda_{ext}(x)}\Psi(x)$$

Backup: dependence on $\nu = ea/m_e$

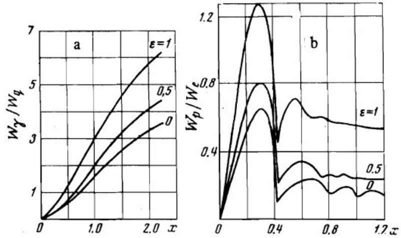

FIG. 1. The probability W_γ of emission of a photon, (a), and the probability W_p of pair production, (b); $\chi = 1$.

Lyul'ka'74