Fluxes and Warping for Gauge Couplings in F-Theory

Based on: T.W. Grimm, D. Klevers, M. P.: arXiv:1202.0285[hep-th]

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Aim of this talk

Type IIB perspective:

Gauge coupling function of D7 brane stack given as:

$$S = \text{Re} f \int F \wedge *F + \text{Im} f \int F \wedge F$$

$$f = \text{complexified volume of four-cycle} + \text{flux corrections}$$

F-theory perspective:

Sevenbranes get part of geometry.

Extract gauge coupling function from geometry!

Strong coupling problems of effective actions

Type II orientifold compactifications

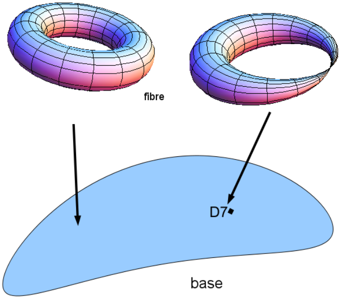
- are well understood
- are important for phenomenology as well as for conceptual reasons
- however assume a small string coupling constant (**inconsistent**, if D7 branes are present.)

Questions:

- Do the weak coupling results change for strong coupling?
- Is it possible to enter the strongly coupled regime?

F-theory as door opener to strong coupling regime

- F-theory geometrizes SL(2,$\mathbb{Z}$)-symmetry of type IIB Vafa '96
- F-theory provides access to strongly coupled type IIB vacua



Elliptically fibred Calabi-Yau fourfold Y_4

Examples for strong coupling phenomena

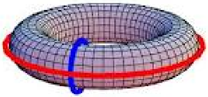
- O-Planes are bound states of (p,q)-sevenbranes
- only SU(N) on sevenbranes realized at strong coupling

$\Rightarrow$ Need for effective action of F-theory compactification on Y_4

Effective action of F-theory

F-theory has no

- low-energy effective action
- microscopic description



M-theory on Y_4

$\downarrow S^1$

IIA on $B_3 \times S^1_R$ $\xrightarrow{\text{T-dual}}$ IIB on $B_3 \times S^1_{1/R}$ $\xrightarrow{v \to 0}$ IIB on B_3

$\mathcal{N} = 1$ in $4d$ $\xleftrightarrow{\text{compare}}$ F-theory on Y_4

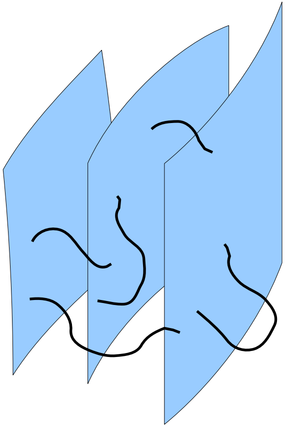
$\downarrow S^1$ $\qquad\qquad$ $\uparrow$ lift

$\mathcal{N} = 2$ in $3d$ $\xleftrightarrow{\text{compare}}$ M-theory on Y_4

Reminder on the Type IIB perspective

- U(N) gauge theory on the D7-branes
- gauge coupling function given as: Jockers, Louis '04

$$
\begin{aligned}
f &= f^{\mathrm{cl}} + f^{\mathrm{flux}} \\
f^{\mathrm{cl}} &= \int_S J \wedge J + i \int_S C_4 \\
f^{\mathrm{flux}} &= i\tau \int_S \mathcal{F} \wedge \mathcal{F}.
\end{aligned}
$$



Leading gauge coupling function from F-theory

Reduce the kinetic term of 11d supergravity

$$S_{kin} = \int_{\mathcal{M}_3 \times Y_4} G_4 \wedge * G_4$$

Kaluza-Klein ansatz for G_4

$$G_4 = F^i \wedge w_i + F^\alpha \wedge \omega_\alpha + F^0 \wedge \omega_0$$

dual to: w_i — blow-up divisors; ω_α — generic divisors; ω_0 — base

$$\int_{\mathcal{M}_3} F^i \wedge * F^j \int_{Y_4} w_i \wedge * w_j \longrightarrow \text{Real part of } f^{\text{cl}}$$

Grimm '10; Grimm, Kerstan, Palti, Weigand '11

$$\int_{\mathcal{M}_3} F^i \wedge * F^\alpha \int_{Y_4} w_i \wedge * w_\alpha \longrightarrow \text{Imaginary part of } f^{\text{cl}}$$

G_4-flux in M-theory

Background solution for M-theory compactification

$$ds_{11}^2 = e^{-A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-A/2}g_{a\bar{b}}dy^a d\bar{y}^{\bar{b}}$$

$$\Delta e^{3A/2} = *_{Y_4}\left(\mathcal{G}_4 \wedge \mathcal{G}_4 + X_8\right)$$

Becker, Becker '96; Haack, Louis '01

The presence of $\mathcal{G}_4$-flux

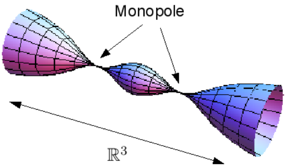
- back-reacts onto the geometry via $e^{3A/2} \longrightarrow$ Re f^{flux}
- modifies the Kaluza-Klein ansatz $\longrightarrow$ Im f^{flux}

Need a local model of the geometry $\Rightarrow$ Periodic Taub-NUT space

Ooguri, Vafa '96

Taub-Nut space

S^1_t-fibration over $\mathbb{R}^3$



$$ds^2 = \frac{1}{V}\big(dt + U\big)^2 + V(dx^2 + dy^2 + dz^2)$$

$$V_I = \frac{1}{|\vec{r} - \vec{r}_I|}, \qquad dV_I = *_3 dU_I$$

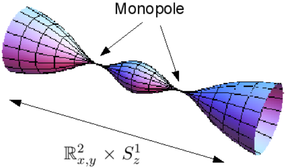
$$V = 1 + \sum V_I, \qquad U = \sum U_I$$

Dictionary: Geometry - Type II

M-theory	$\longrightarrow$	IIA	$\longrightarrow$	IIB
KK-monopole	$\longrightarrow$	D6	$\longrightarrow$	D7
Connection U	$\longrightarrow$	C_1	$\longrightarrow$	C_0
Radius of S^1_t	$\longrightarrow$	g_s^{IIA}	$\longrightarrow$	g_s^{IIB}

Periodic Taub-NUT space

S^1_t-fibration over $\mathbb{R}^2_{x,y} \times S^1_z$



$$ds^2 = \frac{1}{V}\big(dt + U\big)^2 + V(dx^2 + dy^2 + dz^2)$$

$$V = 1 + \sum V_I, \qquad U = \sum U_I$$

$$V_I \quad = \quad -\frac{r_{\rm A}}{2\pi r_{\rm B}}\left(\log\left(\frac{\hat\rho}{\Lambda}\right) - 2\sum_{\ell>0} K_0(2\pi\hat\rho\,\ell)\,\cos(2\pi\ell(\hat z - \hat z_I))\right)$$

$$U_I \quad = \quad -\frac{r_{\rm A}}{4\pi}\Big(1 + 2(\hat z - \hat z_I) + 4\hat\rho\sum_{\ell>0} K_1(2\pi\hat\rho\ell)\,\sin\big(2\pi\ell(\hat z - \hat z_I)\big)\Big)d\varphi$$

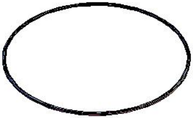
r_A: t-radius at infinity, r_B: z-radius, r_A/r_B: string coupling g_s^{IIB}.

(Leading) torus fibration

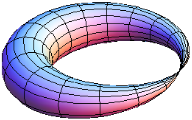
Far away from the singularity the geometry describes a torus fibration with modulus

$$\tau(u) = \tau_0 + \frac{k}{2\pi i} \log\left(\frac{u}{\Lambda}\right), \quad \tau_0 = C_0 + \frac{i}{g_s}$$

Vafa '96; Eyras, Lozano '99; Denef '08

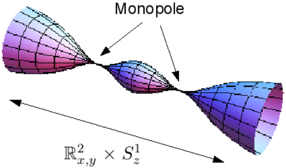


Near the singularity, the logarithmic singularity is smoothed out (i.e. there are corrections to the axiodilaton)



Exceptional divisors

This space is a local model of a resolved A_N-singularity. Blow-up divisors and their Poincaré duals can be explicitly constructed



$$\Omega_I^\infty = d\eta_I = \frac{1}{r_{\rm A}} d\Big(\frac{V_I}{V}(dt+U) - U_I\Big)$$

$$\int_{TN_k^\infty} \Omega_I^\infty \wedge \Omega_J^\infty = -\delta_{IJ} \, , \qquad *_4\Omega_I^\infty = -\Omega_I^\infty$$

Ruback '86; Ooguri, Vafa '96; Grimm, Klevers, MP '12

$\mathcal{G}_4$-flux descends to $\mathcal{F}_2$-flux in IIB via the reduction ansatz

$$\mathcal{G}_4 = \mathcal{F}^i \wedge \Omega_i,$$

$$\int_{Y_4} \mathcal{G}_4 \wedge *\mathcal{G}_4 \to \int_S \mathcal{F}^i \wedge \mathcal{F}^j \int_{TN} \underbrace{\omega_i \wedge \omega_j}_{\overset{g_s \to 0}{\longrightarrow} \delta(u)},$$

i.e. flux localization on the brane for small coupling

Denef 08; Grimm, Klevers, MP '12

Corrections to the real part

Use the geometry to explicitly solve the warp-factor equation

$$e^{3A/2} = 1 - \frac{n^I}{2r_{\rm A}^2 \mathcal{V}_{S_{\rm b}}} \Big(\frac{V_I^2}{V} - V_I \Big) , \quad n^I = \int_S \mathcal{F}^I \wedge \mathcal{F}^I$$

Remark: Dependence on the z-coordinate is crucial!

$$\int_{\mathcal{M}_3} F^i \wedge * F^j \int_{Y_4} \omega_i \wedge \omega_j \longrightarrow \int_{\mathcal{M}_3} F^i \wedge * F^j \int_{Y_4} e^{3A/2} \omega_i \wedge \omega_j$$

In the weak coupling limes one obtains:

$$\text{Re} f_{IJ}^{\text{flux}} = \tfrac{1}{8} g_s^{-1} \delta_{IJ} \big[n_{IIB}^I + \underbrace{\sum_{K \neq I} (n_{IIB}^I + n_{IIB}^K) V_K |_{\hat{Z}_I} }_{\propto g_s} \big] \, ,$$

Grimm, Klevers, MP ’12

Corrections to the imaginary part

Modify Kaluza-Klein ansatz

Dasgupta, Rajesh, Sethi '99; Grimm, Klevers, MP '12

$$C_3 = A^{\mathcal{A}} \wedge \omega_{\mathcal{A}} + \beta(M^{\Sigma}), \quad G_4 = F^{\mathcal{A}} \wedge \omega_{\mathcal{A}} + dM^{\Sigma} \wedge \beta_{\Sigma} + \mathcal{G}_4$$

C_3 now also has moduli(M^{Σ}) dependent three-forms

$$\beta(M^{\Sigma}) = \mathcal{F}^I \wedge \eta_I$$

This yields a coupling

$$d_{IC_0K} = -\tfrac{1}{4} \int_{\hat{Y}_4} \Omega_I^{\infty} \wedge \frac{\partial \beta}{\partial C_0} \wedge \frac{\partial \beta}{\partial \hat{z}^K}$$

This yields correction

$$\text{Im} f_{IJ}^{\text{flux}} = -\tfrac{1}{8} C_0 \Big[\delta_{IJ} \big(n^I_{IIB} + \underbrace{2 \sum_{K \neq I} n^I_{IIB} V_K|_{\hat{Z}_I} \big) + 2 n^I_{IIB} V_J|_{\hat{Z}_I} (\delta_{IJ} - 1) \Big]}_{\propto g_s}$$

Grimm, Klevers, MP '12

Conclusions and Outlook

- We found an explicit description of local D7-brane geometry
- We obtained the flux corrections to the gauge coupling function as well as new (loop) corrections in F-theory
- Explicit example for corrections obtained by backreaction onto geometry
- This backreaction is genuine F-theoretic, i.e. no lift of IIB warp factor.
- Possible to calculate other corrections, e.g. curvature corrections