TECHNISCHE UNIVERSITÄT WIEN
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Discrete gauge symmetries and Open strings

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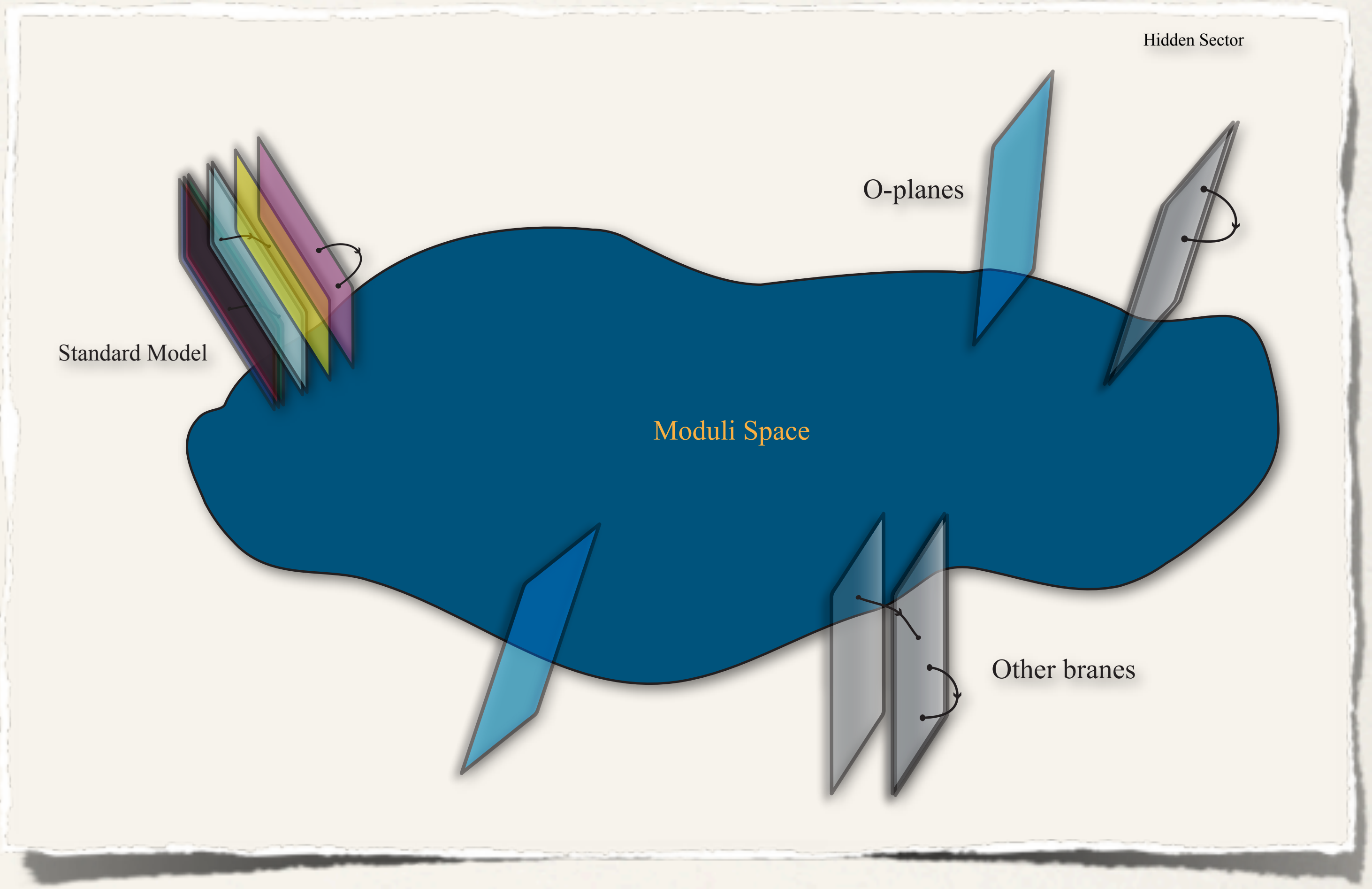
work in progress

with M. Cvetic, J. Halverson, R. Richter and P. Vaudrevange.

Corfu - 20/09/2012

Plan of the talk

- Motivation
- Discrete gauge symmetries
- D-Branes and Standard Model
- Discrete gauge symmetries in D-brane Standard Models
- Some specific examples
- Conclusions



Introduction and motivation

Motivation

- Discrete symmetries are often invoked in order to forbid unwanted or dangerous couplings in particle physics model building.

- However, these symmetries are poorly motivated in a more fundamental level.

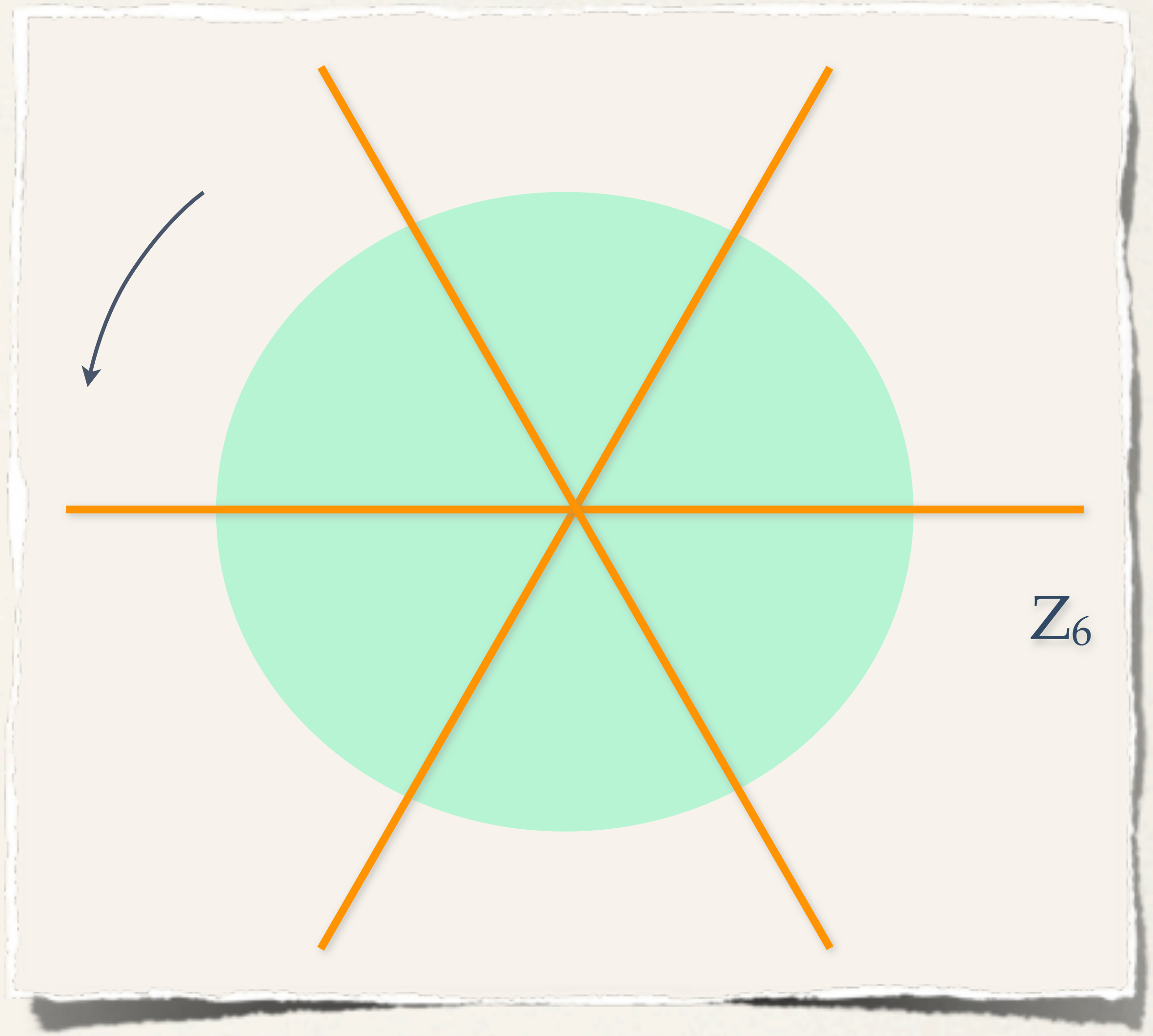
- Global discrete symmetries are expected to be violated in consistent theory of quantum gravity.

- Thus, discrete symmetries should have a gauge symmetry origin, and are called discrete gauge symmetries.

- Those discrete gauge symmetries are subject to discrete anomaly cancellation conditions, just as normal gauge symmetries are.

- The potential discrete gauge symmetries of the MSSM were classified. Ibanez Ross
  Dreiner Luhn Thormeier

- In this talk we will investigate the presence and the role of discrete gauge symmetries in D-brane realizations of the Standard Model.



Discrete gauge symmetries

Discrete gauge symmetries

- Consider the basic Lagrangian for a discrete symmetry:

$$-\frac{1}{4g^2}F^2 + (\partial_\mu a - kA_\mu)^2 \qquad\qquad a \rightarrow a+1$$

- This Lagrangian is invariant also under the gauge transformation:

$$A_\mu \rightarrow A_\mu + \partial\lambda \qquad\qquad a \rightarrow a + k\lambda$$

- A ***Z**$_k$* discrete gauge symmetry basically combines the two above, allowing for

$$\lambda = 1/k$$

fractional $U(1)$ gauge transformations.

- Such symmetries can remove dangerous couplings from the effective field theory.

Ibanez Ross

Discrete gauge symmetries in Standard Model

- Any family independent discrete gauge symmetry of the MSSM can be expressed as:

$$g_N = R_N^m \times A_N^n \times L_N^p$$

where $m, n, p = 0, 1, \ldots N-1$.

Dreiner Luhn Thormeier

- The MSSM particles are charged under these independent $\mathbf{Z}_N$ gauge symmetries:

	Q^i	U^i	D^i	L^i	E^i	N^i	H_u	H_d
R	0	-1	1	0	1	-1	1	-1
A	0	0	0	-1	1	1	0	0
L	0	0	-1	-1	0	1	0	0
$Q_{discrete}$	0	-m	m-n	-n-p	m+p	n+p-m	m	-m+n

- Given a discrete symmetry it is of utmost interest to investigate its phenomenological consequences on the various couplings.

Discrete gauge symmetries in Standard Model

- All possible family independent discrete gauge symmetries within the MSSM with:
  - Cancellation of all mixed anomalies:

$$\mathcal{A}_{SU(3)\times SU(3)\times Z_N}\,,\quad \mathcal{A}_{SU(2)\times SU(2)\times Z_N}\,,\quad \mathcal{A}_{Z_N\times Z_N\times Z_N}\,,\quad \mathcal{A}_{G\times G\times Z_N}$$

  - Allowed Yukawa couplings: $Q_L H_u U\,,\quad Q_L H_d D\,,\quad L H_d E$

  have been classified and belong to:

$$\mathbf{Z}_2\,,\quad \mathbf{Z}_3\,,\quad \mathbf{Z}_6\,,\quad \mathbf{Z}_9\,,\quad \mathbf{Z}_{18}$$

Dreiner Luhn Thormeier

- The $\mathbf{Z}_2$ is the usual matter parity.

- The class of $\mathbf{Z}_6$ solutions contains a proton hexality $L_6^2 R_6^5$ that

$$\text{allows:}\begin{cases}\text{the } \mu\text{-term,}\\ \text{the Weinberg operator.}\end{cases}\qquad \text{forbids:}\begin{cases}\text{R-parity violating,}\\ \text{dangerous dim 5 proton decay ops.}\end{cases}$$

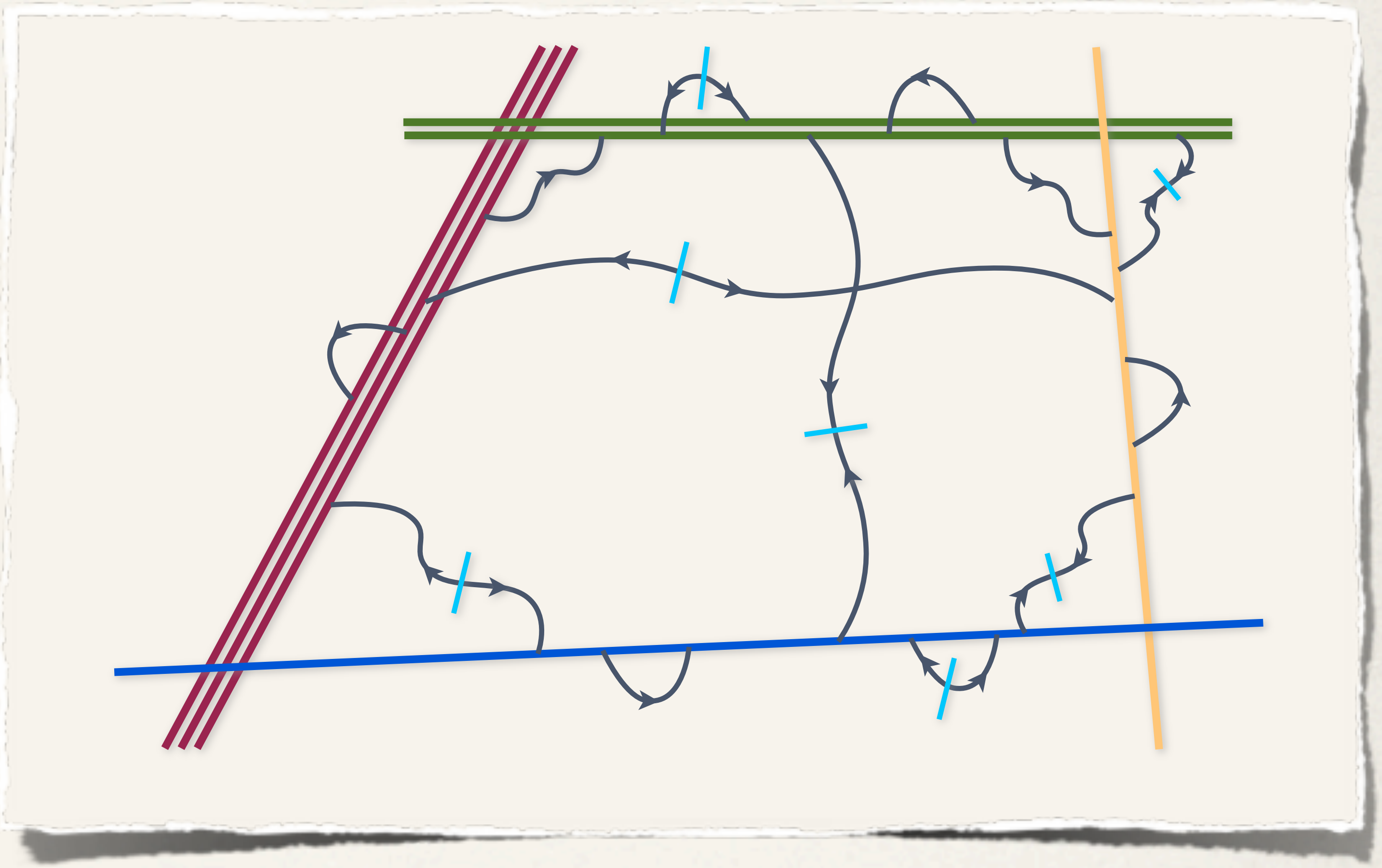
Discrete gauge symmetries vs coupings

- Physical consequences of the discrete gauge symmetries:

	R_2	R_3L_3	R_3	L_3	$R_3^2L_3$	$R_6^5L_6^2$	R_6	$R_6^3L_6^2$	$R_6L_6^2$	all $\mathbf{Z_9}$ & $\mathbf{Z_{18}}$
H_uH_d	✓	✓	✓	✓	✓	✓	✓	✓	✓	
H_uL		✓								
$LL\bar{E}$		✓								
$LQ\bar{D}$		✓								
$\bar{U}\bar{D}\bar{D}$				✓						
$QQQL$	✓		✓				✓			
$\bar{U}\bar{U}\bar{D}\bar{E}$	✓		✓				✓			
$QQQH_d$				✓						
$Q\bar{U}\bar{E}H_d$		✓								
LH_uLH_u	✓	✓				✓				

- The Yukawas couplings Q_LH_uU, Q_LH_dD, LH_dE are allowed for each of the above.

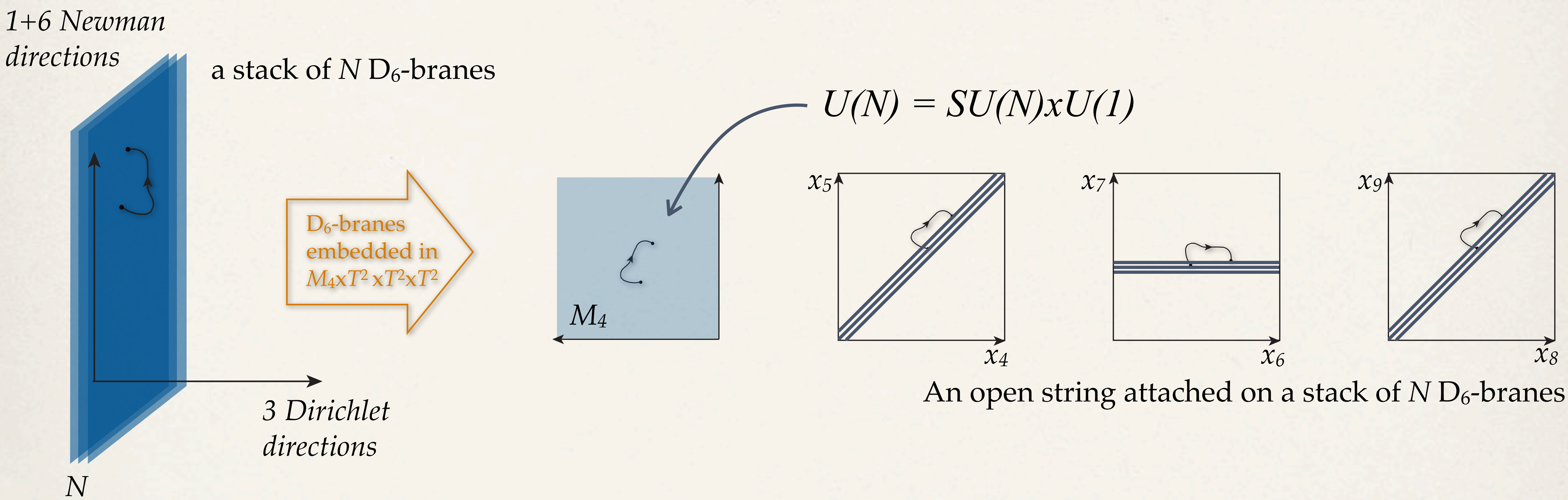
Dreiner Luhn Thormeier



D-branes and the Standard Model

D-branes and strings

- We focus on type IIA constructions with intersecting D6 branes:



- Strings with both ends on a stack of branes give rise to $U(\mathrm{N}) = SU(\mathrm{N}) \mathrm{x} U(1)$ group.

D-branes and strings

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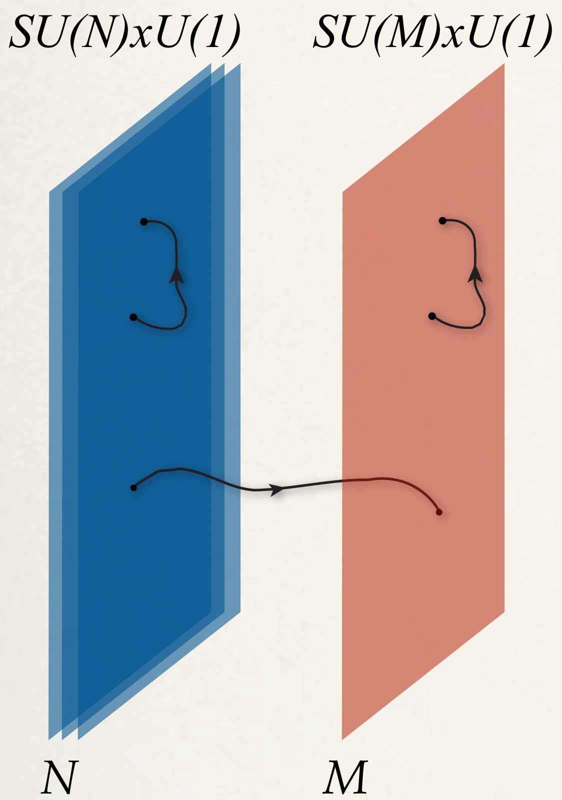

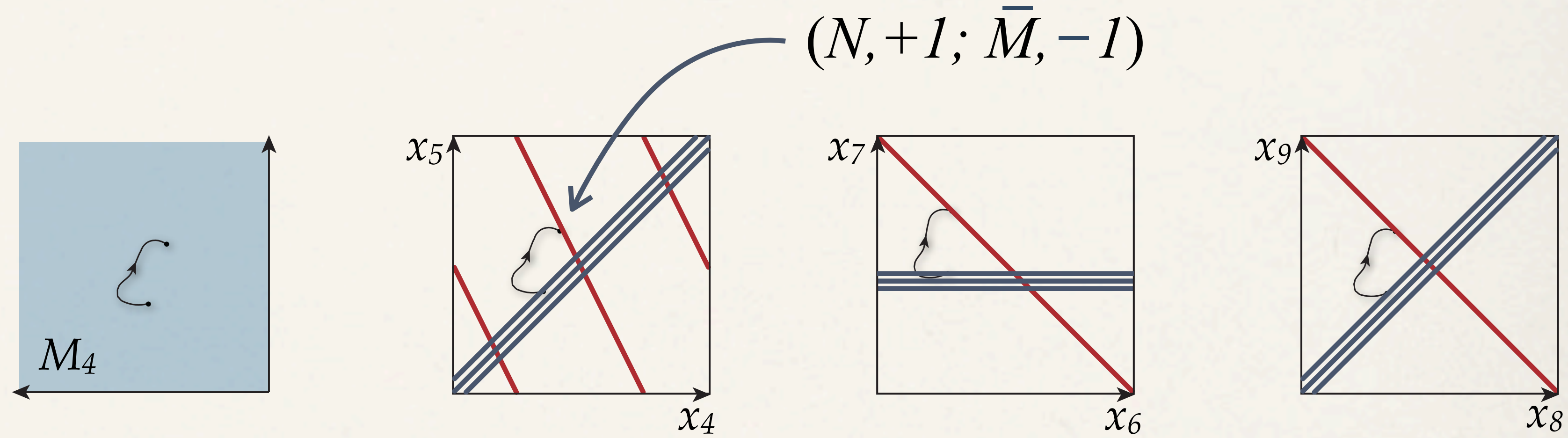

An open string stretched between branes.

- Strings with both ends on a stack of branes give rise to $U(\mathrm{N}) = SU(\mathrm{N})\mathrm{x}U(1)$ group.

- Strings stretched between different stacks transform as bifundamentals.

D-branes and strings

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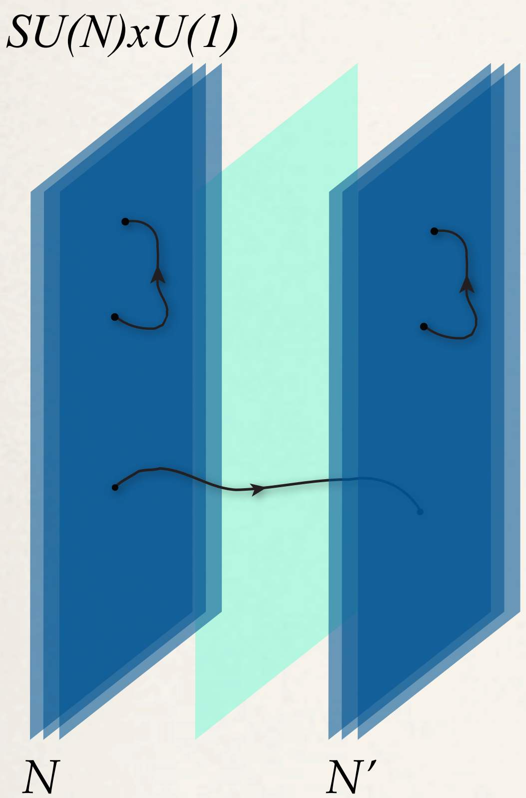

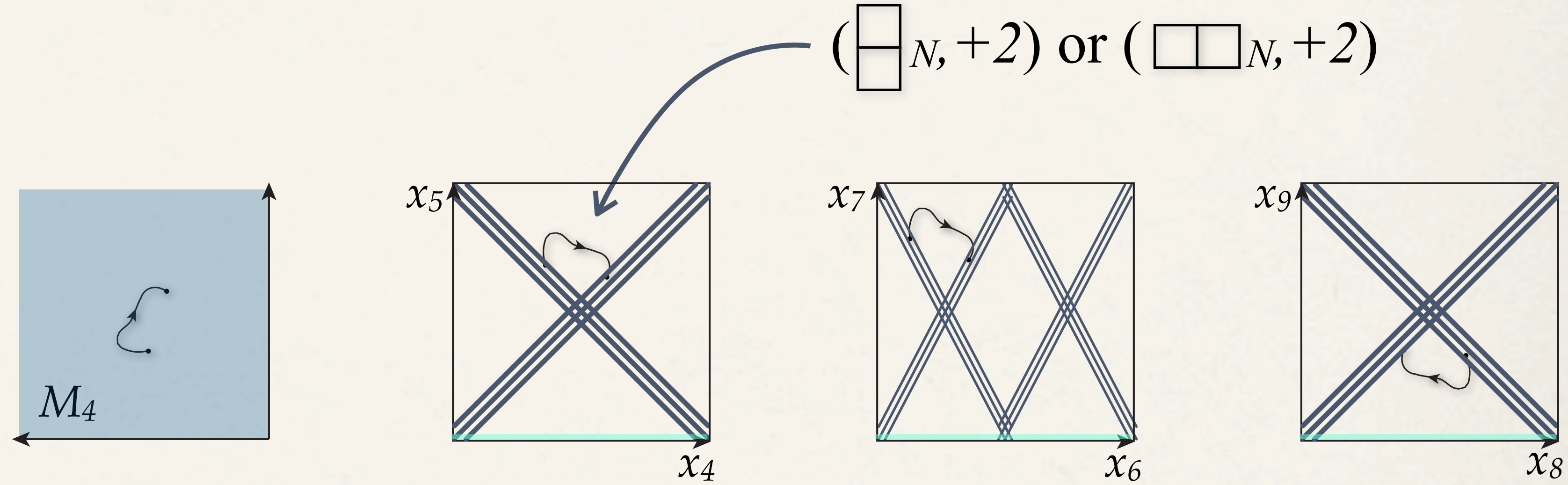

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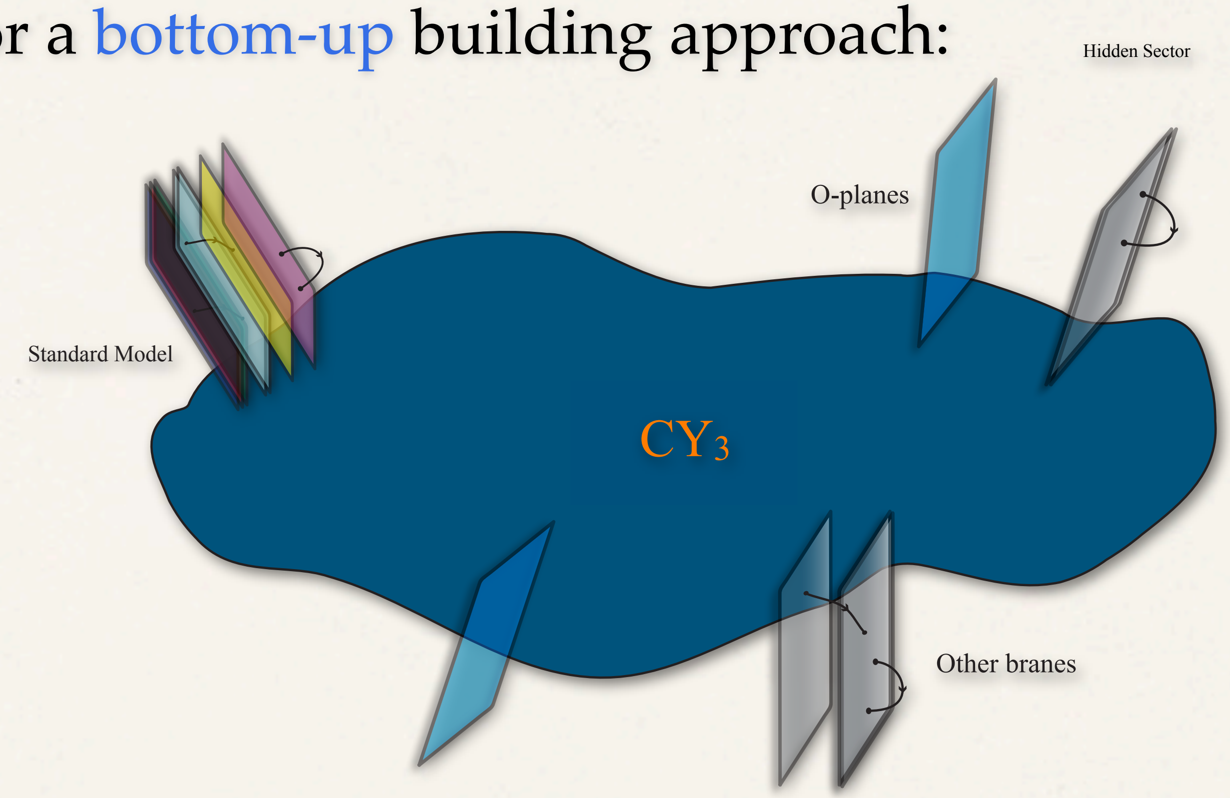
- Strings with both ends on a stack of branes give rise to $U(\mathrm{N}) = SU(\mathrm{N})\mathrm{x}U(1)$ group.

- Strings stretched between different stacks transform as bifundamentals.

- Strings stretched between a brane and its image transform as (anti)symmetric reps.

Local and Global models

- D-branes allow for a bottom-up building approach:



  - Consider a local set of D-branes at some region of a CY_3.
  - This set could eventually describe the Standard Model. Antoniadis Kiritsis Tomaras
Aldazabal Ibanez Quevedo Uranga
  - A hidden sector is typically necessary for the tadpole cancellation.

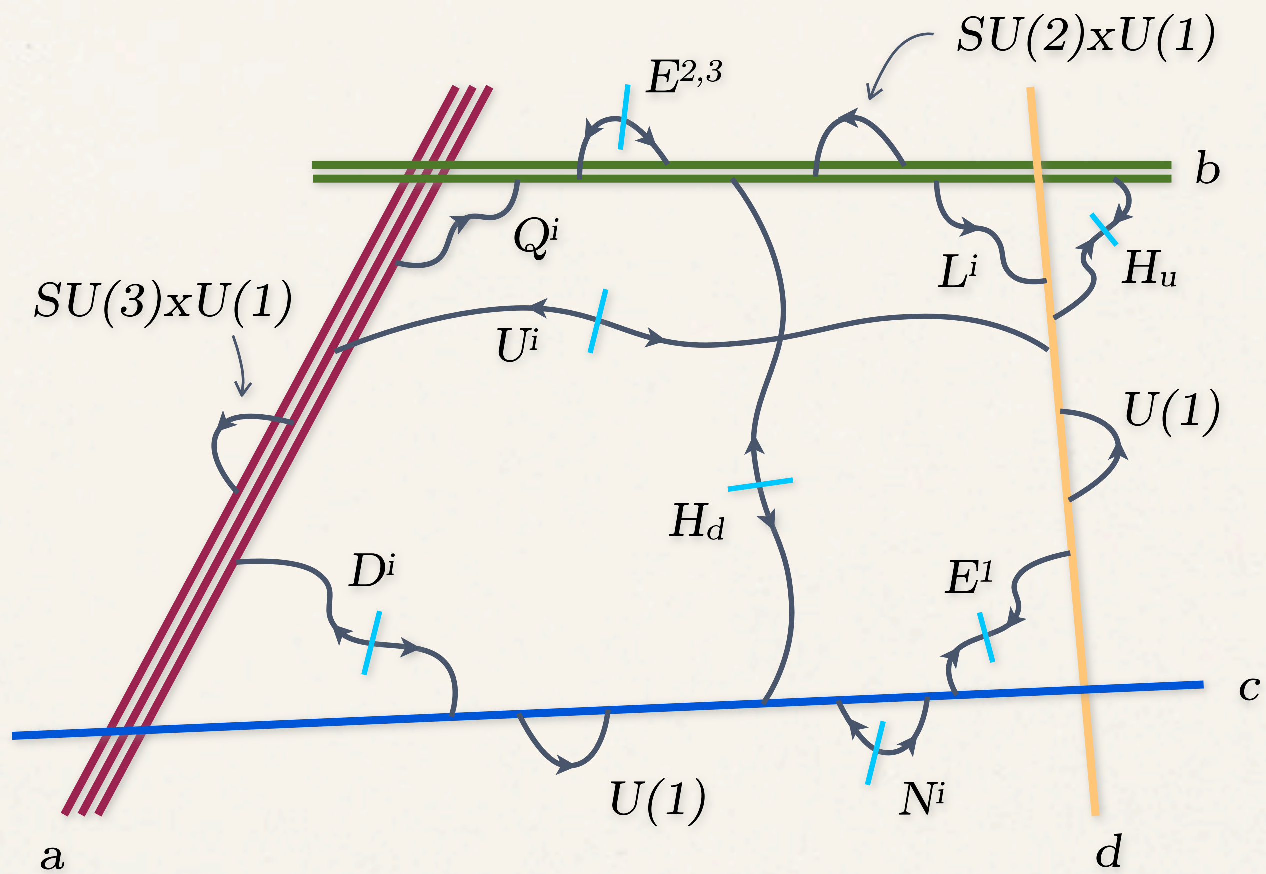
- We want to analyze in that bottom-up fashion, what kind of discrete gauge symmetries do appear in semi-realistic D-brane configurations.

A D-brane Standard Model

- Lets consider a specific D-brane Standard Model example:

$Q^i\,(\mathbf{3},\mathbf{2})_{1/6}$	$U^i\,(\bar{\mathbf{3}},1)_{-2/3}$	$D^i\,(\bar{\mathbf{3}},1)_{1/3}$	$L^i\,(1,\mathbf{2})_{-1/2}$	$E^i\,(1,1)_1$	$N^i\,(1,1)_0$	$H_u\,(1,\mathbf{2})_{+1/2}$	$H_d\,(1,\mathbf{2})_{-1/2}$
$3(a,\ \bar{b})$	$3(\bar{a},\ \bar{d})$	$3(\bar{a},\ \bar{c})$	$3(b,\ \bar{c})$	$(c,\ d), 2\,\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}_b$	$3\,\square\!\square_c$	$(b,\ d)$	$(b,\ c)$

Cvetic Halverson Richter



- The hypercharge is given by the linear combination: $Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$

- We will focus on this semi-realistic configuration and study which discrete symmetries survive and their effect on the superpotential couplings.

Tadpole conditions

- Consistency and stability of D-brane models require the tadpoles conditions:

$$\sum_x N_x(\pi_x + \pi_x') = 4\pi_O$$

- It is easy to transform the formula from the cycle- to the representation-language by:

$$\#\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a\right) = \frac{1}{2}\left(\pi_a \circ \pi_a' + \pi_a \circ \pi_{O6}\right) \qquad \#(\Box_a, \overline{\Box}_b) = \pi_a \circ \pi_b$$

$$\#(\Box\!\Box_a) = \frac{1}{2}\left(\pi_a \circ \pi_a' - \pi_a \circ \pi_{O6}\right) \qquad \#(\Box_a, \Box_b) = \pi_a \circ \pi_b'$$

which finally becomes for $U(N)$ branes:

$$\sum_{x \neq a} N_x \left(\#(\Box_a, \overline{\Box}_x) + \#(\Box_a, \Box_x)\right) + (N_a - 4)\#\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a\right) + (N_a + 4)\#(\Box\!\Box_a) = 0$$

For the $U(1)$ case:

$$\sum_{x \neq a} \left(\#(\Box_a, \overline{\Box}_x) + \#(\Box_a, \Box_x)\right) + 5\#(\Box\!\Box_a) = 0 \quad \mathrm{mod} \;\; 3$$

U(1) Masslessness conditions

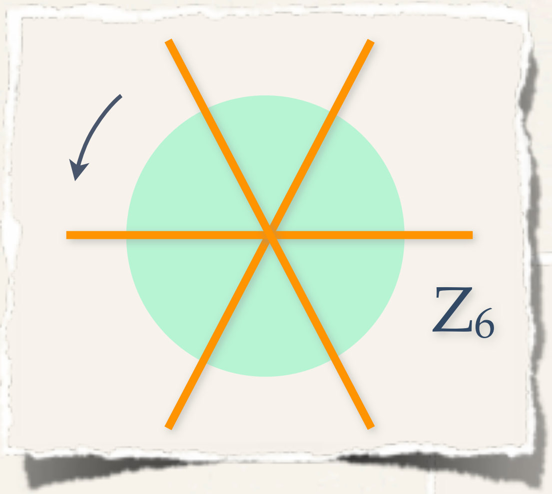
- Each D-brane carries a $U(1)$ which typically appears to be anomalous.
- The linear combination $U(1) = \sum_x q_x U(1)_x$ remains massless (no coupling to axions) if:

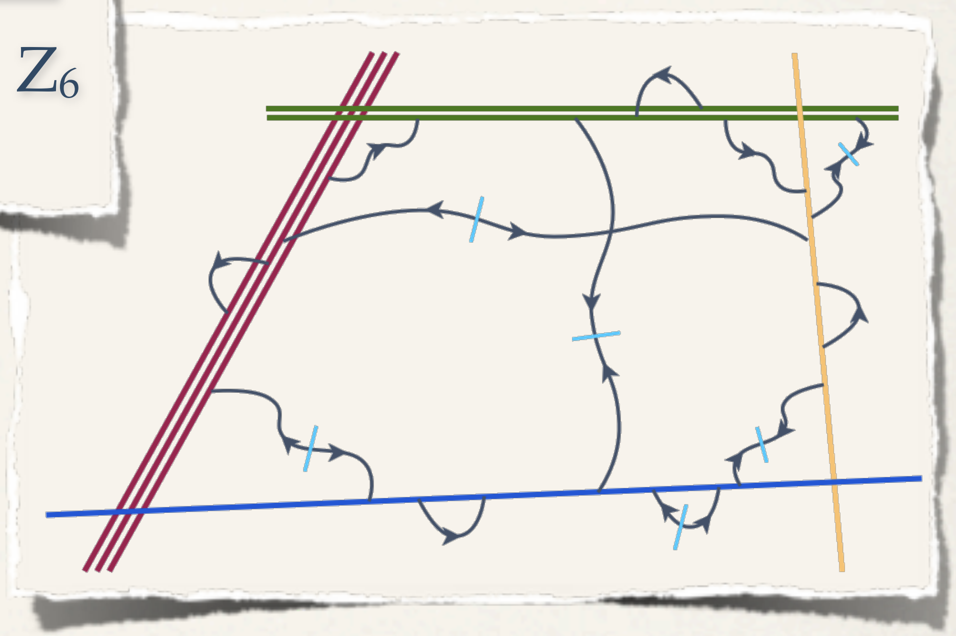
$$\frac{1}{2}\sum_x q_x N_x(\pi_x - \pi'_x) = 0$$

- Using again the cycle- to representation-dictionary we get:

$$\frac{1}{2}\sum_{x\neq a} q_x\, N_x\#(\square_a, \overline{\square}_x) - \frac{1}{2}\sum_{x\neq a} q_x\, N_x\#(\square_a, \square_x) \\ - \frac{q_a N_a}{2(4-N_a)}\left(\sum_{x\neq a} N_x\left(\#(\square_a, \overline{\square}_x) + \#(\square_a, \square_x)\right) + 8\#(\,\square\!\square_a)\right) = 0$$

- Here, we have used tadpole condition to substitute the confusing antisymmetric reps.
- The masslessness condition is a necessary condition for the hypercharge.





Discrete gauge symmetries in D-brane SM

Discrete symmetry conditions

- Abelian discrete gauge symmetries in D-brane compactifications are remnants of the anomalous $U(1)$ gauge symmetries living on D-branes.

- Those anomalous $U(1)$'s become massive via the Green-Schwarz mechanism and survive as global symmetries which are satisfied on the perturbative level.

- D-instanton effects can break those global symmetries inducing sometimes desired, but perturbatively forbidden, couplings (Majorana mass terms, Yukawa couplings etc).

- However, we have to ensure that other instantons do not induce dangerous couplings.

- Discrete gauge symmetries are an efficient way to guarantee it.

- Our aim is to do an analysis over semi-realistic D-brane Standard Model configurations by the effect of all allowed discrete gauge symmetries.

Discrete gauge symmetries in String theory

- Consider an abelian gauge field A_μ and some axions a_m which couple like:

$$\sum_m \left(\partial_\mu a_m - \sum_n k_n (R_{nm} - R_{n^c m}) A_\mu \right)^2 \qquad\qquad a_m \to a_m + 1$$

- Gauge invariance requires:

$$A_\mu \to A_\mu + \partial\lambda \qquad\qquad a_m \to a_m + \sum_n k_n (R_{nm} - R_{n^c m})\lambda$$

- Combining again the above, we have a $\mathbf{Z}_N$ discrete gauge symmetry if:

$$\sum_n k_n (R_{nm} - R_{n^c m}) = 0 \mod N$$

and using the homology classes of the branes it becomes:

$$\frac{1}{2}\sum_n k_n N_n (\pi_n - \pi'_n) = 0 \mod N$$

- Gauged discrete subgroups are preserved by any non-perturbative effect.

Berasaluce-Gonzalez Ibanez Soler Uranga

Discrete gauge symmetries

- Consider a discrete gauge symmetry $\mathbf{Z}_N = \sum_x k_x U(1)_x$
- This symmetry survives in the low energy effective action if:

$$\frac{1}{2}\sum_x k_x N_x(\pi_x - \pi'_x) = 0 \quad \mathrm{mod}\ N$$

- This condition becomes:

$$\frac{1}{2}\left(\sum_{x\neq a} k_x\, N_x \#(\square_a, \overline{\square}_x) - \sum_{x\neq a} k_x\, N_x \#(\square_a, \square_x) - \#(\,\square\!\square_a) - \#(\begin{smallmatrix}\square\\\square\end{smallmatrix}_a)\right) = 0 \quad \mathrm{mod}\ N$$

- Using tadpole conditions, we can substitute again the antisymmetrics and we get

$$\boxed{\begin{aligned}&\frac{1}{2}\sum_{x\neq a} k_x\, N_x \#(\square_a, \overline{\square}_x) - \frac{1}{2}\sum_{x\neq a} k_x\, N_x \#(\square_a, \square_x)\\ &\qquad - \frac{k_a N_a}{2(4-N_a)}\left(\sum_{x\neq a} N_x\left(\#(\square_a, \overline{\square}_x) + \#(\square_a, \square_x)\right) + 8\#(\,\square\!\square_a)\right) = 0 \quad \mathrm{mod}\ N\end{aligned}}$$

An additional discrete symmetry condition

- The fact that discrete symmetries require $0 \mod N$ instead of 0 brings troubles...

- One can compensate that by requiring an additional constraint:

$$\sum_a k_a N_a \left(\#(\genfrac{}{}{0pt}{}{\Box}{\Box}{}_a) - \#(\Box\!\Box_a)\right) = 0 \mod N$$

  arising from multiplying the homology class of the orientifold with the discrete symmetry constraint.

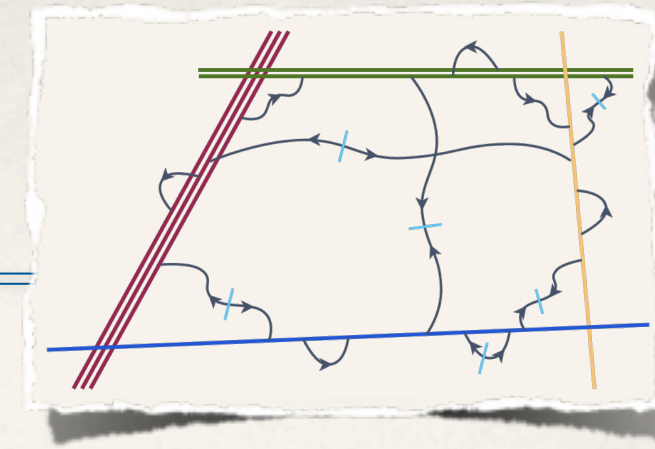
- After replacing again the antisymmetrics we get:

$$\boxed{\sum_a \frac{k_a N_a}{4 - N_a} \left(\sum_{x \neq a} N_x \left(\#(\Box_a, \overline{\Box}_x) + \#(\Box_a, \Box_x)\right) + 2N_a(\Box\!\Box_a) \right) = 0 \mod N}$$

- Combining the two conditions we can prove the absence of various mixed anomalies:

$$SU(N) \times SU(N) \times \mathbf{Z}_N \qquad\qquad G \times G \times \mathbf{Z}_N$$

D-brane Standard Models



- Consider again the previous embedding:

	$Q^i\,(\mathbf{3},\mathbf{2})_{1/6}$	$U^i\,(\bar{\mathbf{3}},1)_{-2/3}$	$D^i\,(\bar{\mathbf{3}},1)_{1/3}$	$L^i\,(1,\mathbf{2})_{-1/2}$	$E^i\,(1,1)_1$	$N^i\,(1,1)_0$	$H_u\,(1,\mathbf{2})_{+1/2}$	$H_d\,(1,\mathbf{2})_{-1/2}$
	$3(a,\ \bar{b})$	$3(\bar{a},\ \bar{d})$	$3(\bar{a},\ \bar{c})$	$3(b,\ \bar{c})$	$(c,\ d), 2\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}_b$	$3\,\Box\!\Box_c$	$(b,\ d)$	$(\bar{b},\ \bar{c})$

- With the discrete charges: $Q_{discrete} = k_a Q_a + k_b Q_b + k_c Q_c + k_d Q_d$

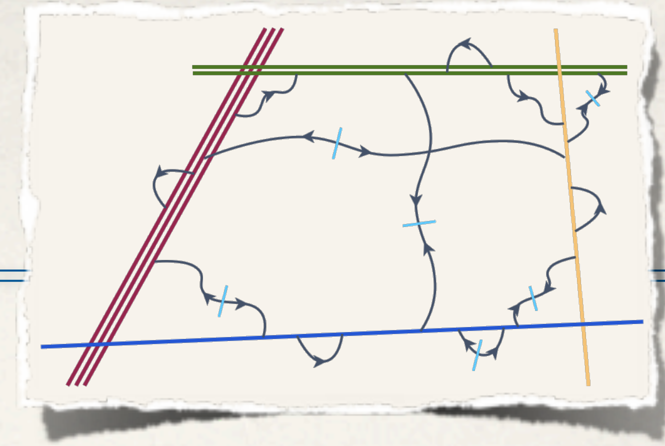
$Q_{discrete}$	k_a-k_b	-k_a-k_d	-k_a-k_c	k_c-k_c	k_c+k_d, $2k_b$	-$2k_c$	k_b+k_d	-k_b-k_c

- We want to find all $(k_a,\ k_b,\ k_c,\ k_d)$ that for various $\mathbf{Z}_N$ satisfy:

$$\frac{1}{2}\sum_x k_x N_x(\pi_x - \pi'_x) = 0 \mod N$$

- Each k take values from 0, 1, ... $2N$ (due to the 1/2 overall factor).

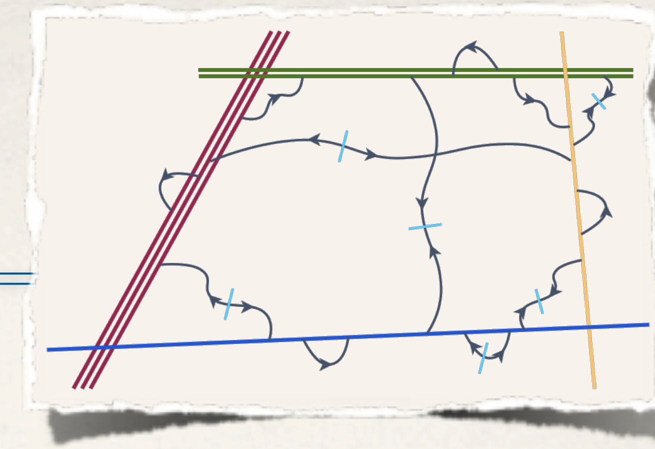
Search for discrete symmetries



- Not all sets of (k_a, k_b, k_c, k_d) are independent.
- To avoid overcounting , we have to remember that one solution gives others:
  - by a Hypercharge shift: $(k_a, k_b, k_c, k_d) + m(q_a, q_b, q_c, q_d) \mod N$
  - there is an overall freedom so we fix the discrete charge of Q_L to zero by: $k_a = k_b$.
- Within independent vectors (k_a, k_b, k_c, k_d) we check which of them satisfy:
  - the discrete symmetry condition
  - the Symmetric-Antisymmetric condition

  } Anomalies
  - allow Yukawa terms.
- For all $\mathbf{Z}_N$ with $N \in 2, 3, 4, \ldots 20$.

Results



- Discrete gauge symmetries (k_a, k_b, k_c, k_d) found:
  - The $\mathbf{Z}_2$: $R_2 = U(1)_a + U(1)_b + U(1)_c + U(1)_d$ is the usual matter parity.
  - The $\mathbf{Z}_3$: $L_3 R_3 = U(1)_a + U(1)_b + U(1)_d$ is the baryon triality.
  - The $\mathbf{Z}_6$: $L_6^2 R_6^5 = U(1)_a + U(1)_b + 9\,U(1)_d + 13\,U(1)_d$ is the proton hexality.
- Therefore, the above discrete gauge symmetries ensure for:
  - all desired Yukawa couplings,
  - allowed μ-term, Weinberg operator,
  - No bad terms (like R-violating, no proton decay operators).
- We have extended this analysis over all semi-realistic 4 stack D-brane models.

More results

- From the systematic search over all realistic 4 stack quivers (40) we find:
  - Only in few realizations proton hexality realized (3/40).
  - Baryon triality only rarely realized.
  - On the other hand matter parity may be oftentimes realized (25/40).
  - Other $\mathbf{Z}_3$ discrete symmetries appear fairly often.
  - No family dependence.
  - No $\mathbf{Z}_9$ and $\mathbf{Z}_{18}$ realizations (so far).

Conclusions

- We have analyzed all four D-brane Standard Model configurations with interesting phenomenology (around 40 local configurations).
- A few of them (3) may exhibit the discrete $\mathbf{Z}_6$ symmetry proton hexality.
- Matter parity is fairly often realized (around 25 out of 40).
- Baryon triality rarely realized, but other $\mathbf{Z}_3$ appear frequently.
- No family dependence, not in the quark sector which is somewhat expected and also desired, but also not in the lepton sector.
- Would be interesting to see whether the family independence holds true also for 5 stack realizations.
- Search in 5 stack quivers to make more substantial systematic comments that it would be interesting to see whether the patter on 4 stacks holds true also for 5 stacks.