Non-Abelian T-duality

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Disclaimers

1) I will not be talking about String theory.

2) “T-duality” is a bit of a misnomer.

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D4-D8 near-horizon

$AdS_6 \times S^4$ in massive IIA

Ferrara, Kehagias, Partouche, Zaffaroni (1998); Brandhuber, Oz (1999)

Recently...

Orbifold theories $\Leftrightarrow AdS_6 \times S^4/\mathbb{Z}_n$ Bergman & Rodriguez-Gómez

Localization SCFT_5 Jafferis & Pufu; Kim^2 & Lee

Uniqueness in massive IIA Passias

"In fact this is the only known class of supersymmetric AdS_6 solutions."

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Punchline

I KNOW ONE MORE*.

*We all probably know one more. I am saying that I know one more than that!

with D. Rodriguez-Gómez, K. Sfetsos

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NEW SUSY AdS_6 solution

$$d\hat{s}^2 = \frac{1}{4}W^2\left[9ds^2(AdS_6) + 4d\theta^2\right] + e^{-2A}dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}}ds^2(S^2),$$

$$\hat{B} = \frac{r^3}{r^2 + e^{4A}}\text{vol}(S^2),$$

$$e^{-2\hat{\phi}} = e^{-2\phi}e^{2A}(r^2 + e^{4A}),$$

$$F_1 = -G_1 - mrdr,$$

$$F_3 = \left[-\frac{r^3}{r^2 + e^{4A}}G_1 + \frac{mr^2 e^{4A}}{r^2 + e^{4A}}dr\right] \wedge \text{vol}(S^2),$$

where

$$W = (m\cos\theta)^{-1/6}, \quad e^{\phi} = \frac{2}{3(m\cos\theta)^{5/6}},$$

$$e^A = \frac{W\sin\theta}{2}, \quad G_1 = \frac{5}{8}(m\cos\theta)^{1/3}\sin^3\theta d\theta.$$

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Outline

- Abelian, non-Abelian dualities
- Review T-duality à la Buscher
- Status of non-Abelian T-duality
- Consistent KK reductions
- Return to D4-D8

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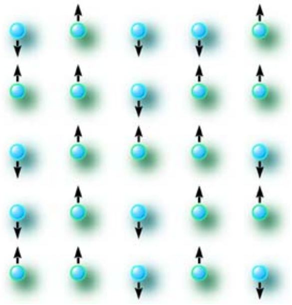
Traditional (Abelian) Dualities



- Kramers-Wannier (1941)
- $\mathcal{F}_{\text{low}T} \Leftrightarrow \mathcal{F}_{\text{high}T}$
- Phase transition

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non-Abelian duality problem

Well Motivated: Nature is often far from linear

QCD, Heisenberg ferromagnets, FLRW metric

't Hooft, Mandelstam, Englert (1978-1980)

Technical problems: Ex. $D = 2$ $O(3)$ Heisenberg ferromagnet

$$H = -J \sum \cos\left(\Delta_\mu \Omega_i\right), \quad Z = \int D\Omega \exp\left(\frac{J}{kT} \sum \cos\left(\Delta_\mu \Omega_i\right)\right)$$

Expand in $Y_{l,m}$, integral gives constraints on four l, m of $Y_{l,m}$

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T-duality

NLSM

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi [\sqrt{h}(h^{\mu\nu} g_{ij} + i\epsilon^{\mu\nu} b_{ij})\partial_\mu x^i \partial_\nu x^j]$$

Gauge isometry $x^0 \equiv \theta$

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi [\sqrt{h} h^{\mu\nu} g_{00} (\partial_\mu \theta + A_\mu)(\partial_\nu \theta + A_\nu) + \cdots 2i\epsilon^{\mu\nu} \tilde{\theta} F_{\mu\nu}]$$

Integrate out A, $\tilde{\theta}$ dual coordinate

$$A_\mu = -\frac{1}{g_{00}} (g_{0\alpha} \partial_\mu x^\alpha + i \frac{\epsilon_\mu^{\ \nu}}{\sqrt{h}} (b_{0\alpha} \partial_\nu x^\alpha + \partial_\nu \tilde{\theta}))$$

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T-duality

1. Compare SM with T-dual SM - Buscher (1988)

$$\begin{aligned} \tilde{g}_{00} &= g_{00}^{-1}, \quad \tilde{g}_{0\alpha} = -g_{00}^{-1} b_{0\alpha}, \quad \tilde{b}_{0\alpha} = -g_{00}^{-1} g_{0\alpha}, \\ 2\tilde{\phi} &= 2\phi - \ln g_{00}, + \cdots \end{aligned}$$

2. RR-flux: reduction to *unique* $\mathcal{N} = 2$ $D = 9$ supergravity
   Bergshoeff, Hull & Ortin (1995)

3. KS rotation: $\tilde{\epsilon}_- = \epsilon_-, \quad \tilde{\epsilon}_+ = \Omega \epsilon_+, \quad \Omega = \sqrt{g_{00}^{-1}} \Gamma_{11} \Gamma^0$

$$\begin{aligned} \delta \tilde{\lambda}_- &= \delta \lambda_- - g_{00}^{-1} \Gamma^0 \delta \psi_{-0}, \\ \delta \tilde{\lambda}_+ &= \Omega \left(\delta \lambda_+ - g_{00}^{-1} \Gamma^0 \delta \psi_{+0} \right). \end{aligned}$$

Rotation on RR sector Hassan (1999)

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Status of non-Abelian T-duality

Old story Buscher (1988, 1992), De la Ossa, Quevedo; Roček, Verlinde
Gasperini, Ricci, Veneziano (1993), Alvarez, Alvarez-Gaume, Lozano (1994)

Gauge isometry $\partial x^m \to Dx^m \equiv \partial x^m + A^\alpha (T_\alpha)^m{}_n x^n$

Symmetry of type II sugra with $SU(2) \times SU(2)$ PCM Sfetsos, Thompson (2010)

i) $AdS_3 \times S^3 \times T^4$ ii) $AdS_5 \times S^5 \Rightarrow$ Gaiotto-Maldacena

Coset G/H manifolds, $S^4 \simeq SO(4)/SO(3)$, $S^5 \simeq SO(6)/SO(5)$
chirality unchanged Lozano, Ó C, Sfetsos, Thompson (2011)

Not traditional duality - one way, isometries (SUSY) break, non-compact

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IIB reduction on S^3

Motivation: Can one emulate Bergshoeff, Hull, Ortin?

Natural ansatz with $SO(4)$ symmetry Itsios, Lozano, Ó C, Sfetsos (2012)

$$ds^2 = ds^2(M_7) + e^{2A} ds^2(S^3).$$

Fluxes

$$\begin{aligned} F_5 &= G_2 \wedge \mathrm{Vol}(S^3) - e^{-3A} \star_7 G_2 \,, \\ F_3 &= G_3 - m\mathrm{Vol}(S^3) \,, \\ F_1 &= G_1 \,. \end{aligned}$$

Background B, Φ

$SO(4)$ singlets $\Rightarrow$ *consistent* reduction to $D = 7$ theory (not SUGRA)

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Massive IIA on $\mathbb{R} \times S^2$

NS sector

$$\begin{aligned} ds^2 &= ds^2(M_7) + e^{-2A} dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}} ds^2(S^2), \\ \tilde{B} &= B + \frac{r^3}{r^2 + e^{4A}} \mathrm{vol}(S^2), \\ e^{-2\tilde{\Phi}} &= e^{-2\Phi} e^{2A} (r^2 + e^{4A}). \end{aligned}$$

Flux ansatz

$$\begin{aligned} \hat{F}_0 &= m\,, \\ \hat{F}_2 &= \frac{m r^3}{r^2 + e^{4A}} \mathrm{vol}(S^2) + r dr \wedge G_1 - G_2\,, \\ \hat{F}_4 &= \frac{r^2 e^{4A}}{r^2 + e^{4A}} G_1 \wedge dr \wedge \mathrm{vol}(S^2) - \frac{r^3}{r^2 + e^{4A}} G_2 \wedge \mathrm{vol}(S^2) + r dr \wedge G_3 + e^{3A} \star_7 G_3\,. \end{aligned}$$

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$D = 7$ Action

Field content: $G_1 = dC_0, \;\; G_2 = dC_1 - mB, \;\; G_3 = dC_2 - C_0 H$

Diagonalisation & rescaling: $\Phi = 5\tilde{\Phi} + \frac{3}{2}A$

$$\begin{aligned}\mathcal{L}_{\text{Einstein}} &= R - 3(\partial A)^2 - 20(\partial\Phi)^2 - \frac{1}{2}e^{10\Phi+3A}(\partial C_0)^2 - \frac{1}{12}e^{-8\Phi}H^2 \\ &- \frac{1}{2}\left(m^2 e^{14\Phi-3A} - 3e^{4\Phi-2A} + \frac{1}{2}e^{6\Phi-3A}G_2^2 + \frac{1}{6}e^{2\Phi+3A}G_3^2\right) \\ &+ G_2 \wedge C_2 \wedge H\end{aligned}$$

Any solution uplifts to both IIB and massive IIA: pp-wave, LLM, Lifshitz

cf reduction to $D = 5$ Romans Lu, Pope, Tran (1999); Gauntlett, Varela (2007)

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SUSY

Rotation on the Killing spinor generic

$$\epsilon = e^{X}\tilde{\epsilon} = \exp\left(-\frac{1}{2}\tan^{-1}\left(\frac{e^{2A}}{r}\right)\Gamma^{\theta\phi}\sigma^{3}\right)\tilde{\epsilon}$$

Variations between IIA and IIB match up to

$$\delta\psi_r = e^{X}\left[\frac{1}{2}\partial\!\!\!/ A\Gamma_r - \frac{e^{-A}}{4}\Gamma^{\theta\phi}\sigma^{3} + \frac{e^{\Phi}}{8}\left(me^{-3A}\Gamma^{\theta\phi}\Gamma_r(i\sigma^{2}) - G\!\!\!/_1(i\sigma^{2}) - \frac{e^{-3A}}{2}G\!\!\!/_2\Gamma^{\theta\phi}\Gamma_r\sigma^{1} - \frac{1}{6}G\!\!\!/_3\sigma^{1}\right)\right]\tilde{\epsilon}$$

Redefinitions $\tilde{\epsilon}_+ = \Gamma^{r}\epsilon_+, \ \tilde{\epsilon}_- = -\epsilon_-;$

$\delta\psi_r$ reproduces D3, D1-D5 projections of Sfetsos, Thompson

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D4-D8

$AdS_6 \times S^4$

$$\begin{aligned} ds^2 &= \frac{1}{4} W^2 \left[9 ds^2(AdS_6) + 4 \left(d\theta^2 + \sin^2\theta ds^2(S^3) \right) \right], \\ F_4 &= 5 (m \cos\theta)^{1/3} \sin^3\theta d\theta \wedge \text{vol}(S^3), \\ e^{\phi} &= \frac{2}{3 (m \cos\theta)^{5/6}}, \quad W = (m \cos\theta)^{-1/6}. \end{aligned}$$

Warping breaks $SO(5)$ to $SO(4) \simeq SU(2)_R \times SU(2)_G$

Can do Abelian Hopf-fibre T-duality Cvetic, Lu, Pope, Vazquez-Portiz (1999)

More generally, also have freedom for $SU(2)$.

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Future directions

Is there a dual CFT? Rodriguez-Gómez, Ó C, Sfetsos (sometime soon)

Classification of AdS_6 in IIB, more solutions?

NA T-duality turns on a B-field - applications?

Do thermodynamics change? - 1206.1367

non-Abelian pre Big Bang cosmology? Gasperini, Veneziano (1992)

SUSY breaking models?

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