Aspects of String Cosmology

Nicolaos Toumbas
Work in collaboration with
C. Angelantonj, I. Florakis, C. Kounnas, H. Partouche and J. Troost

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The hot Big-Bang model gives an accurate description of the evolution of the Universe

from the time of Nucleosynthesis until present (at least)

Key cosmological questions concerning its structure and makeup

- flatness and homogeneity puzzles
- horizon and large entropy puzzles
- monopole problem . . .

have inspired inflationary cosmology,

where a phase of superluminal expansion occurs in the very early cosmological history,
eventually settling into a radiation-dominated evolution.

The model can be adapted so as to take into account the current accelerating expansion of the Universe,
observed in the light from distant galaxies and supernovae

as well as gravitational effects observed in very large scale structures,

by introducing dark energy and cold dark matter.

$\longrightarrow$ ΛCDM model.

So the result is:
a very rich but highly involved phenomenological model

based on classical GR + QFT

period of rapid inflation, high temperature eras, symmetry breaking phase transitions, and

proportionally large amounts of dark matter and dark energy, dominating the very late time evolution ...

$\longrightarrow$ Our vacuum is a very non-trivial one with lots of stuff.

The standard cosmological model, presents some of the greatest challenges to fundamental physics.

Two in particular:

1.) If we extrapolate the cosmological evolution back in time, using the equations of GR and QFT $\rightarrow$

we are driven to an initial singularity,

where the Universe collapses to zero volume

and the description breaks down.

The second concerns the nature of dark energy.

The simplest explanation for it is a positive,

however unnaturally small, cosmological constant $\rightarrow$

$$\Lambda \sim 10^{-120} \; M_p^4$$

(many orders of magnitude smaller than the Planck and elementary particle physics scales)

To date no symmetry principle or mechanism is known to explain its value.

If dark energy persists arbitrarily long, it would imply that the Universe approaches de Sitter space in the far future $\rightarrow$

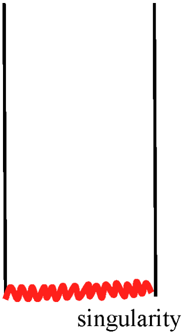
$\rightarrow$ cosmic event horizon

so portions of space will remain unobservable, forever.

The observable part of the universe is in a highly mixed state.

So within the context of General Relativity and the Standard Model

we lack a coherent framework to analyze the cosmology of our Universe, from beginning to end.



If string theory is a complete theory of quantum gravity, it should eventually provide a consistent cosmological framework

The hope is that by incorporating fundamental duality symmetries and stringy degrees of freedom in time-dependent settings $\longrightarrow$

able to obtain complete cosmological histories, free of any essential singularities

new tools for model building.

Indeed, string dualities have given us profound insights into the nature of *Space*,

with many surprising phenomena arising at short distances of order the string scale $l_s = \sqrt{\alpha'}$ or the Planck length l_p.

These include:

- UV finiteness

- T-duality $R \leftrightarrow l_s^2/R$, small/large volume duality
  $\longrightarrow$ shrinking radii past the string scale does not produce a theory at shorter distances.

- Stringy spacetime uncertainty principles: $\Delta x \Delta t \sim l_s^2$, $\Delta x \Delta t \sim l_p^2 \longrightarrow$ minimal length, UV/IR connection.

- Non-perturbative strong/weak coupling dualities
- Singularity resolution and topology change. E.g. orbifold and conifold singularities.
  The appearance of extra massless states *localized at the singularity*, make it fuzzy and smooth.
- Non-conventional thermodynamics (Hagedorn phases + black holes)$\rightarrow$ pointing at a maximal temperature and non-trivial phase transitions.
- Holographic gauge/gravity dualities.

$\rightarrow$ illustrating how String Theory can provide concrete answers to many of the puzzles one has to face in trying to quantize Einstein's theory of general relativity.

Lessons:

• locality, geometry and even topology are approximate concepts, well-defined at low enough energies, $E \leq 1/l_s$.

• From Holography: Gravity and *Space* can emerge from special quantum mechanical systems.

E.g. a large N maximally supersymmetric gauge theory in 4d gives rise to a 10d gravitational theory, a string theory in $AdS_5 \times S_5$.

• Apparently singular regimes and/or geometries $\rightarrow$ mapped via string dualities into non-singular ones, with well defined effective descriptions.

Much of this insight has been obtained from studies of static, equilibrium configurations of superstrings

given the principle of relativity $\longrightarrow$ seems inevitable that similar results hold for the time-dependent cases.

Progress in String Cosmology can be made if

$\rightarrow$ extend the web of
string dualities to time-dependent, cosmological settings.

Technically and conceptually challenging:

In phases with broken susy $+$ geometric variation, string theory can become an inflexible tool.

• With the moduli acquiring time dependence $\rightarrow$ may wander through cross-over regions of moduli space, where we have no control over the quantum corrections, and the effective field theory approach breaks down.

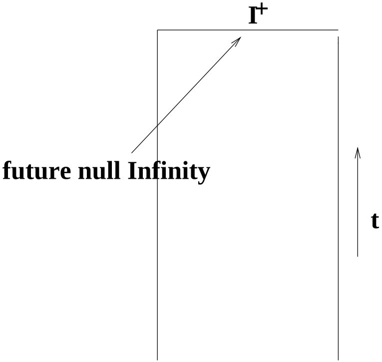
• At a more fundamental level: it is hard to identify (and compute) the correct, precise observables.

2nd quantized version to probe the theory directly off-shell ??

Holography: the construction of the holographic theory is very sensitive to the global structure of spacetime

with the dual variables living at the boundary of spacetime.

For asymptotically de Sitter cosmologies (like our own) $\longrightarrow$ the natural boundaries lie to the infinite future



suggesting a form of space-like holography. E.g. the dS_4/Euclidean CFT_3 correspondence.

central charge:

$$c \sim 1/[H(t \to \infty)]^2 G$$

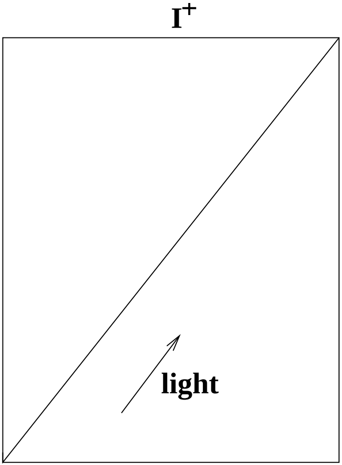
$H_\infty \to$ asymptotic value of the Hubble parameter. $H(t)$ decreases monotonically with time.

Such a boundary CFT may be non-unitary; no explicit, microscopic construction in string theory

but if realized, it would be a holographic example where *Time* and the cosmological history emerge, perhaps as RG flow in the CFT.
RG flow $\leftrightarrow$ time reversed evolution.

• Reconstructing the very early Universe, would amount to the difficult task of decoding the hologram in the deep infrared of the boundary CFT.

• More importantly, no observer can measure the boundary CFT correlators.



Bouncing String Cosmologies

In these lectures, we will explore the possibility of realizing

eternal string cosmologies

where an initially contracting phase bounces/emerges into an expanding thermal phase.

Before focusing on stringy examples, review the situation in GR.

Within classical GR, the singularity theorems [Penrose, Hawking]

show that such a reversal is impossible unless an energy condition of the form

$$T_{\mu\nu} v^\mu v^\nu \geq 0$$

(for a suitable class of vectors v^μ), is violated.

Let us see how this applies for homogeneous and isotropic $4D$ FRW cosmologies

$$ds^2 = -dt^2 + [a(t)]^2 d\Omega_k^2, \qquad H = \frac{\dot{a}}{a} \rightarrow \text{Hubble parameter}$$

supported by sources with total energy density ρ and pressure P.
(sources comprise together a perfect fluid)

The relevant equations are:

$$H^2 = \frac{8\pi}{3} G\rho - \frac{k}{a^2} \rightarrow \text{Friedmann/Hubble equation}$$

and the 1st law for adiabatic evolution

$$\dot{\rho} + 3H(\rho + P) = 0$$

implying $\rightarrow$

$$\dot{H} = -4\pi G(\rho + P) + \frac{k}{a^2}$$

So if the null energy condition is satisfied $\rho + P \geq 0$,

for flat ($k = 0$) and open ($k = -1$) Universes, the Hubble parameter decreases monotonically with time: $\dot{H} \leq 0$

and reversal from contraction ($H < 0$) to expansion ($H > 0$) is not possible.

They are separated by a singularity or the expanding phase is past geodesically incomplete.

The known macroscopic sources of energy and matter in the Universe satisfy the NEC.

For closed Universes ($k = 1$) we get

$$\frac{d^2 a}{dt^2} = -\frac{4\pi}{3} G a(\rho + 3P),$$

so for the bounce to occur at a singularity the strong energy condition, $\rho \geq -3P$, must be satisfied.

There are field theoretic sources for which the NEC holds but the SEC is violated, e.g. positive vacuum energy density or a positive cosmological constant $\rightarrow$

and global de Sitter space is an example of a closed FRW cosmology, where (at the classical level) a contracting phase smoothly reverses to an expanding inflationary phase.

The problem with such an initially, exponentially contracting phase, is that it requires the Universe to be *sufficiently empty*

for an infinite amount of time.

During exponential contraction, perturbations grow large $\longrightarrow$ the Universe is very likely to thermalize before expansion sets in, and within the field theoretic context

$\longrightarrow$ collapse to a singularity.

Another possibility is to consider a Universe which is eternally inflating, or expanding sufficiently fast forever.

Even without requiring an energy condition, Borde, Guth and Vilenkin show that such a Universe is not past geodesically complete

($H_{\mathrm{av}} > 0$, throughout the cosmological history)

and must have a beginning, presumably an initial singularity.

Scalar field-driven inflation cannot be the ultimate theory of the very early Universe.

There are many attempts to overcome the reversal problem within the string theoretic set up.

1.) various incarnations of the pre big bang scenario [Veneziano *et al.*]

Typically in the pre big bang phase the dilaton runs from weak to strong coupling; new terms in the effective action such as higher derivative interactions and potentials can become relevant

invalidating some of the assumptions of the singularity theorems, and thus facilitating the bounce.

difficulty: lack of analytical control over the strongly coupled dynamics.

2.) Various null/spacelike orbifold models of the singularity [see e.g. Liu, Moore, Seiberg]

$\rightarrow$ large backreaction from blue-shifted modes as they pass through the orbifold singularity.

3.) String gas cosmology [Brandenberger, Vafa; Tseytlin, Vafa]

Here the idea is that the Universe starts as a compact space, e.g. a 9D torus with all radii close to the string scale,

and temperature close to (but below) the Hagedorn temperature: $T \sim T_H$.

A gas of winding states keeps the Universe from expanding, while a gas of momentum modes keeps it from shrinking $\rightarrow$

$\rightarrow$ until thermal fluctuations cause the winding states to annihilate, and some dimensions to expand.

At the level of the two derivative dilaton/gravity effective action, there is a singularity a finite time in the past, where also the dilaton evolves to strong coupling.

4.) Modular cosmology, Brane collisions ...

I will focus on another class of bouncing thermal superstring cosmologies, exploiting stringy phase transitions/phenomena that can occur at temperatures close to the Hagedorn temperature. [Florakis, Kounnas, Partouche, NT]

The non-singular cosmological solutions are based on a mechanism that resolves the Hagedorn instabilities of finite temperature strings,

realizable in a large class of initially $\mathcal{N}=(4,0)$ type II superstring models [Angelantonj, Florakis, Kounnas, Partouche, NT].

I will also review several aspects of strings at finite temperature, and (partial) spontaneous supersymmetry breaking via geometrical fluxes.

Consider weakly coupled type II $(4,0)$ vacua, on initially flat backgrounds:

$$R^d \times T^{10-d}$$

These are vacua with 16 real supersymmetries arising from the left moving sector of the worldsheet.
The internal toroidal radii are taken close to the string scale.

Right-moving susy broken spontaneously by twisting some of the internal cycles with F_R, the right-moving spacetime fermion number.

Under the Z_2 symmetry $(-1)^{F_R} \rightarrow$ the right moving R sector changes sign.

Pattern of susy breaking $\rightarrow$ extended symmetry points,

when internal radii are at the fermionic point.

At finite T, such points in moduli space are preferred,

with the moduli participating in the breaking of the right-moving supersymmetries $\rightarrow$ stabilized at the extended symmetry point values.

The odd-F_R sector is massive, $m^2 \geq 1/(2\alpha')$.

As an illustrative example, consider the $d = 2$ Hybrid Vacua, on $R^2 \times T^8$,

where the internal radii of T^8 are taken at the fermionic point: $R = 1/\sqrt{2}$ (work in units where $\alpha' = 1$.)

At this point, the 8 compact supercoordinates can be replaced with 24 left-moving and 24 right-moving worldsheet fermions.

24 left-moving fermions are split in two groups of 8 and 16.

The one-loop partition function is:

$$Z = \frac{V_2}{(2\pi)^2} \int_{\mathcal{F}} \frac{d^2\tau}{4(\mathrm{Im}\tau)^2} \, \frac{1}{\eta^8} \, \Gamma_{E_8}(\tau) \, (V_8 - S_8) \, \left(\bar{V}_{24} - \bar{S}_{24}\right) .$$

left-moving sector: 8 fermions are described in terms of the $SO(8)$ characters, as in the conventional superstring models.

$SO(8)$ characters:

$$O_8 = \frac{\theta_3^4 + \theta_4^4}{2\,\eta^4}\,, \qquad V_8 = \frac{\theta_3^4 - \theta_4^4}{2\,\eta^4}\,,$$

$$S_8 = \frac{\theta_2^4 - \theta_1^4}{2\,\eta^4}\,, \qquad C_8 = \frac{\theta_2^4 + \theta_1^4}{2\,\eta^4}\,.$$

The other 16 left-moving fermions are described by the chiral E_8 lattice: Γ_{E_8}.

right-moving sector: in terms of the MSDS $SO(24)$ characters

$$\bar{V}_{24} - \bar{S}_{24} = \frac{1}{2\eta^{12}} \left(\theta_3^{12} - \theta_4^{12} - \theta_2^{12}\right) = 24.$$

Despite the breaking of right-moving supersymmetry, this sector exhibits *Massive Spectrum Degeneracy Symmetry* [Kounnas, Florakis].

Massless sector: 24×8 bosons and 24×8 fermions, arising in the $V_8 \bar{V}_{24}$ and $S_8 \bar{V}_{24}$ sectors.

The right-moving R sector is massive.

local gauge group: $U(1)_L^8 \times [SU(2)_R]_{k=2}^8$.

The model can be exhibited as a freely acting, asymmetric orbifold compactification of the type II superstring to 2 dimensions.

The relevant half-shifted $(8,8)$ lattice is:

$$\Gamma_{(8,8)}\begin{bmatrix}\bar{a}\\ \bar{b}\end{bmatrix} = \Gamma_{E_8} \times \bar{\theta}\begin{bmatrix}\bar{a}\\ \bar{b}\end{bmatrix}^8 \rightarrow$$

$$\frac{\sqrt{\det G_{IJ}}}{(\sqrt{\tau_2})^8}\sum_{\tilde{m}^I, n^J} e^{-\frac{\pi}{\tau_2}(G+B)_{IJ}(\tilde{m}+\tau n)^I(\tilde{m}+\bar{\tau} n)^J} \times e^{i\pi(\tilde{m}^1\bar{a}+n^1\bar{b}+\tilde{m}^1 n^1)}$$

- The modular covariant cocycle describes the coupling of the lattice to F_R.
- At the MSDS point the G_{IJ} and B_{IJ} tensors take special values $\rightarrow$ holomorphic/anti-holomorphic factorization, enhanced gauged symmetry.

$(4,0)$ models can be constructed in various dimensions.

Next consider the models at finite temperature.

To avoid strong Jeans instabilities (gravitational collapse into black holes) $\rightarrow$ compactify R^{d-1} on a large torus, with each cycle having radius $R >> 1$,
and take the string coupling to be sufficiently weak.

E.g. in 4d, Schwarzschild radius: $R_S \sim GM = G\rho R^3$, where the energy density ρ is set by the temperature.

Requiring this to be much smaller than the size of the system $\rightarrow$

$$1 \ll R \ll \frac{1}{\sqrt{G\rho}} \sim \frac{1}{g_s} \quad \text{for T close to the string scale}$$

String theory: New instabilities at $T \sim 1/l_s$
$\rightarrow$ the Hagedorn instabilities

$\rightarrow$ which signal non-trivial phase transitions.

Due to the exponential rise in the density of states as a function of mass

$$n(m) \sim e^{\beta_H m}, \quad \text{asymptotically for large } m$$

the (single) string partition function

$$Z \sim \int^{\infty} dm n(m) e^{-\beta m} = \int^{\infty} dm e^{-(\beta - \beta_H)m}$$

diverges for temperatures above Hagedorn: $T > T_H = 1/\beta_H$.

$$\beta_H = 2\pi\sqrt{2\alpha'} \quad \text{in Type II superstrings}$$

The critical behavior as $\beta \to \beta_H$ is governed by string states of large mass $m \gg 1/l_s$, or high level N (recall that $m^2 \sim N/\alpha'$.)

At weak coupling, the typical size of such a string state is large, of order $l \sim N^{1/4} l_s \leftarrow$ random walk of $N^{1/2}$ bits.

So close to the Hagedorn temperature, we have percolation phenomena, where multiple strings coalesce into fluctuations of single long strings.

The critical behavior can be also described by an effective field theory of a complex field becoming massless, in one fewer spacetime dimension $\longrightarrow$

manifesting a UV/IR connection.

The (Euclidean) thermal theory is obtained $\longrightarrow$

by compactifying Euclidean time on a circle with period $2\pi R_0 = 1/T$.

Anti-periodic boundary conditions for fermions.
$\longrightarrow$ implemented by twisting the Euclidean time circle with F, the spacetime fermion number.

For type II superstrings this amounts to coupling the Euclidean time $\Gamma_{1,1}$ lattice with the following co-cycle:

$$e^{i\pi(\tilde{m}^0(a+\bar{a})+n^0(b+\bar{b}))}$$

In this picture, the Hagedorn instabilities appear at a critical compactification radius;

certain string winding modes ($n_0 \neq 0$) become massless

precisely when $R_0 = R_H = 1/(2\pi T_H)$.

Tachyonic, when $R_0 < R_H$.

Thermal scalar mass:

$$m^2(R_0) = R_0^2 - R_H^2$$

Critical behavior of the partition function is captured by the thermal scalar path integral:

$$\mathcal{Z} \sim \int [d\phi] e^{-S[\phi]}$$

$$S[\phi] \sim \int d^{d-1}x (\partial_i \phi^* \partial^i \phi + m^2(R_0)\phi^* \phi)$$

To see this, let us recover the asymptotic density of states $n(m)$.

$$Z_c = \ln \mathcal{Z} = -\sum_a \ln \lambda_a$$

λ_a eigenvalues of $-\nabla^2 + m^2(R_0)$.

When the sizes of the spatial dimensions are small compared to the inverse thermal winding mass

the cut-off sum is dominated by lowest eigenvalue: $\lambda_0 = m^2(R_0) \rightarrow$

$$Z_c \sim -\ln \lambda_0 \sim -\ln(R - R_H) \sim \int^{\infty} dm e^{-\beta m} n(m),$$

for $n(m) = e^{\beta_H m}/m$. When $d-1$ spatial dimensions are non-compact, $n(m) \sim V_{d-1} e^{\beta_H m}/m^{(d+1)/2}$.

$\rightarrow$ Hagedorn divergence for $R_0 < R_H$ can be interpreted as an IR instability of the underlying Euclidean thermal background.

Condensing the winding tachyon leads to large backreaction, and presumably brings the thermal ensemble to a speedy end ($F \sim 1/g_s^2$) [Attick, Witten].

In type II $(4,0)$ models, perturbatively stable configurations can be produced, if in addition to temperature

turn on vacuum potentials associated to the graviphoton G_{I0} and B_{I0} fields. $U(1)_L$ combination.

(I along an internal direction twisted by F_R.)
At finite T, these cannot be gauged away $\rightarrow$ topological vacuum parameters.

These *gravitomagnetic* fluxes modify the thermal masses of all states charged under the graviphoton fields $\rightarrow$ tachyonic instabilities can be lifted.

Equivalently the contribution to the free energy of the massive oscillator states gets regulated (refined),
reducing the effective density of thermally excited states $\rightarrow$ Asymptotic Supersymmetry.

Hagedorn free models $\leftrightarrow$ freely acting asymmetric orbifolds $(-1)^{F_L}\delta_0$ [Florakis, Kounnas, NT].

$$\frac{Z_{\rm Hyb}}{V_1} = \int_{\mathcal{F}} \frac{d^2\tau}{8\pi(\mathrm{Im}\tau)^{3/2}} \, (\bar{V}_{24} - \bar{S}_{24}) \, \frac{\Gamma_{E_8}}{\eta^8}$$
$$\times \sum_{m,n} \left(V_8 \, \Gamma_{m,2n} + O_8 \, \Gamma_{m+\frac{1}{2},2n+1} - S_8 \, \Gamma_{m+\frac{1}{2},2n} - C_8 \, \Gamma_{m,2n+1} \right) .$$

Z is finite for all values of R_0.

Models exhibit a number of universal properties.

Fluxes lead to a restoration of the stringy T-duality symmetry along the thermal circle:

$$R_0 \to R_c^2/R_0; \qquad S_8 \leftrightarrow C_8.$$

- In all models: self-dual point $R_c = 1/\sqrt{2} \leftarrow$ fermionic point.
- Z is duality invariant, but not a smooth function of R_0:

At R_c additional thermal states become massless leading to
1.) enhanced gauge symmetry: $U(1)_L \to [SU(2)_L]_{k=2}$

2.) a conical structure in Z as a function of $R_0 \to$ stringy phase transition.

E.g. in the Hybrid model 2×24 states (from the $O_8 \bar{V}_{24}$ sector) become massless:

$$m^2 = \left(\frac{1}{2R_0} - R_0\right)^2 .$$

Vertex operators:

$$p_L = \pm 1, \quad p_R = 0, \quad O_\pm = \psi_L^0 \, e^{\pm i X_L^0} \, \mathcal{O}_R$$

$\rightarrow$ non-trivial momentum and winding charges.

$$\frac{Z_{\text{Hyb}}}{V_1} = 24 \times \left(R_0 + \frac{1}{2R_0}\right) - 24 \times \left| R_0 - \frac{1}{2R_0} \right| .$$

Exact result thanks to r.m. MSDS symmetry.

Complete suppression of the massive oscillator contributions, but stringy behavior survives inducing conical structure.

In $d > 2$ the partition function acquires a higher order conical structure:

$$\sim \left| R_0 - \frac{1}{2R_0} \right|^{d-1}$$

implying a higher order transition as a function of R_0.

Recall however that we must keep all but at least one of the large spatial dimensions compact.

Thermal duality $\rightarrow$ the existence of two dual asymptotic regimes dominated by

1.) the light thermal momenta, $R_0 \gg R_c$

2.) the light thermal windings, $R_0 \ll R_c$.

In the regime of light momenta

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{(2\pi R_c)^{d-1}} \left(\frac{R_c}{R_0} \right)^{d-1} + \mathcal{O}\left(e^{-R_0/R_c} \right)$$

$\rightarrow$ characteristic behavior of massless thermal radiation in d dimensions, $T = 1/2\pi R_0$.

($n^* \rightarrow$ number of effectively massless degrees of freedom.)

In the regime of light windings ($R_0 \ll R_c$)

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{(2\pi R_c)^{d-1}} \left(\frac{R_0}{R_c}\right)^{d-1} + \mathcal{O}\left(e^{-R_c/R_0}\right)$$

$Z \to 0$ as $R_0 \to 0$.

Standard thermodynamics: Z decreases monotonically as T decreases.
So the correct definition of temperature cannot be $T = 1/2\pi R_0$ in this regime.

Winding excitations are non-local in X^0, local in the T-dual of X^0

By T-duality, interpret as ordinary thermal excitations associated with the large T-dual circle; radius $\tilde{R}_0 = R_c^2/R_0$.

$$T = 1/2\pi\tilde{R}_0 = R_0/(2\pi R_c^2)$$

Thus the system at small radii is again effectively cold.

Two dual asymptotically cold phases.
Thermal phases arise via spontaneous symmetry breaking, as we deform away from the intermediate extended symmetry point.

distinguished by the light thermally excited spinors:
At $R > R_c$: S_8-Spinor of $SO(8)$
At $R < R_c$: C_8-Spinor.

Extended symmetry point: stringy, no precise thermal interpretation, but T-duality invariant.

Operators associated with the extra massless states induce transitions between the purely momentum and winding modes

O_+, O_- raise and lower p_L by one unit, leave p_R unchanged $\longrightarrow$

$$< C_8 | O_- | \, S_8 > \neq 0.$$

Extra massless states give rise to genus-0 backgrounds, localized at $R_0 = R_c$, *"gluing together" the momentum and winding regimes.*

$\nabla_\perp \varphi^I \neq 0 \longrightarrow$ localized negative pressure contributions.

Thus thermal duality $\rightarrow$ a maximal critical temperature $T \leq T_c$.

Stringy system "tries to hide" its short distance behavior.

Set $R_0/R_c = e^\sigma$; T-duality: $\sigma \rightarrow -\sigma$.

$$T = T_c \ e^{-|\sigma|}, \quad T_c = \frac{1}{2\pi R_c} = \frac{1}{\sqrt{2}\pi}$$

T is duality invariant, expression valid in both asymptotically cold regimes

$\rho \leq \rho_c$, $P \leq P_c$ $\rightarrow$ bounded, never exceeding certain maximal values.
crucial difference from thermal field theory models.

Hybrid Model:

$$\frac{Z}{V_1} = (24\sqrt{2})\ e^{-|\sigma|} = \Lambda\ T, \qquad \Lambda = \frac{24\sqrt{2}}{T_c}.$$

$$P = \rho = \Lambda\ T^2, \qquad \rho_c = \Lambda\ T_c^2 = \frac{24}{\pi}.$$

So in each phase the equation of state is effectively that of thermal massless radiation in two dimensions $\leftarrow$ right moving MSDS symmetry.

In the higher d cases, r.m. MSDS structure gets replaced by *right moving asymptotic supersymmetry.*

$\rightarrow$ up to the critical point, $Z \sim T^{d-1}$ $\rightarrow$ dominated by the contributions of the thermally excited massless states.

The backreaction on the initially flat metric and dilaton background $\rightarrow$ will induce a cosmological evolution.

We consider the situation where the thermal modulus σ is a monotonic function in time $\rightarrow$ scanning all three regimes of the string thermal system.

Each regime admits a local effective theory description, associated with *a distinct* α'*-expansion:*

- $R_c/R_0 \gg 1$, ($\sigma \ll 0$); regime of light thermal windings: $\{\mathcal{W}(\sigma < 0)\}$.
- $|R_0/R_c - R_c/R_0| \ll 1$, ($\sigma \sim 0$); $SU(2)_L$ extended symmetry point: $\{\mathcal{B}(\sigma = 0)\}$. $\rightarrow$ exact CFT description (in the Euclidean).

- $R_0/R_c \gg 1$, $(\sigma \gg 0)$; regime of light thermal momenta: $\{\mathcal{M}(\sigma > 0)\}$.

• So a stringy transition occurs at T_c $(\sigma = 0)$, connecting two asymptotically cold phases.

• Near T_c condensates associated with the extra massless scalars can form and decay $\rightarrow$ S-brane interpretation.

At T_c, the extra massless scalars give rise to non-trivial genus-0 contributions such that [Kounnas, Partouche, NT]

$$G_{IJ}\nabla_{\hat{\mu}}\varphi^I\nabla_{\hat{\nu}}\varphi^J = \frac{\kappa}{(d-1)}g_{\hat{\mu}\hat{\nu}}$$

($G_{IJ} \to$ metric in field configuration space. $g_{\hat{\mu}\hat{\nu}}$ metric on spatial slice at constant $\sigma = 0$.)

Effectively, this results in *negative pressure contributions, localized in time,* in the Lorentzian.

Intermediate regime $\to$ "Brane regime."

In the limit of "thin" S-brane: $S_{\mathrm{brane}} = -\kappa \int d\sigma \sqrt{g_\perp} e^{-2\phi} \delta(\sigma) \to$

$-\kappa \int d\tau \sqrt{g_\perp} e^{-2\phi} \delta(\tau - \tau_c)$. $(\sigma(\tau_c) = 0.)$

Since all thermodynamical quantities are duality invariant $\to$ functions of $|\sigma|$.

Their 2nd time derivatives, and hence those of the back-reacted fields $\longrightarrow$ may give rise to (apparently) singular terms, proportional to $\delta(\sigma)$.

These singularities are resolved in the presence of the S-brane, *gluing together the smooth geometrical phases associated with the "Winding" and "Momentum" regimes.*

1st class of non-singular string cosmologies consists of transitions from a "Winding regime" to a "Momentum regime" via a thin S-brane:

$$\{\mathcal{C}_{\rm String}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c)\} \oplus \{\mathcal{B}(\tau = \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}$$

Since in each thermal phase the Universe is asymptotically cold, this must be a bouncing cosmology:

Asymptotically, the string frame scale factor and temperature satisfy: $aT = constant \leftarrow$ adiabatic evolution.

Entropy conservation (irrespectively of the running dilaton)

$$S = \frac{a^{d-1}}{T}(\rho + P) \sim (aT)^{d-1}$$

$T \leq T_c$, all thermodynamical quantities are bounded from above: $a \geq a_c$, bounded from below.

a_c large in string units if the entropy S is large.

The singular $a \to 0$ regime of classical GR is absent.

The bounce is facilitated by the extra negative pressure at the critical point $\leftarrow$ S-brane.
Including, as we will see that of the dilaton.

The exact spread in time will be governed by the CFT at the critical point, including interactions.

Effective Lorentzian Action (in string frame):

Let $\tau_c = 0$. Then $S = S_0 + S_1 + S_{\mathrm{brane}}$,

$$S_0 = \int d^d x\ e^{-2\phi}\ \sqrt{-g}\left(\frac{1}{2}\ R + 2(\nabla\phi)^2\right)\ \ (\mathrm{dilaton-gravity\ action})$$

$$S_1 = \int d^d x\ \sqrt{-g}\ P \quad (\mathrm{Thermal\ effective\ potential,\ at\ genus\ 1})$$

$$S_{\mathrm{brane}} = -\kappa \int d^d x \sqrt{g_\perp} e^{-2\phi}\delta(\tau)$$

We look for homogeneous and isotropic solutions:

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\Omega_k^2$$

Ω_k is a $(d-1)$-dimensional Einstein space with curvature $k \leq 0$.

There is also a running dilaton.

(Concentrate on the spatially flat cases: k=0.)

$$(N): \quad \frac{1}{2}(d-1)(d-2)H^2 = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi}N^2\rho$$

$$P = T_c e^{-|\sigma|}\frac{Z}{V_{d-1}}, \qquad \rho = -P - \frac{\partial P}{\partial|\sigma|} \sim (d-1)P.$$

$\rho = P$ exact in the 2d Hybrid models.

$$(a): \quad (d-2)\left(\frac{\ddot{a}}{a} - H\frac{\dot{N}}{N}\right) + \frac{1}{2}(d-2)(d-3)H^2$$

$$= 2\ddot{\phi} + 2(d-2)H\dot{\phi} - 2\dot{\phi}^2 - 2\frac{\dot{N}}{N}\dot{\phi} - e^{2\phi}N^2 P + \kappa N\delta(\tau).$$

For the dilaton:

$$\ddot{\phi} + (d-1)H\dot{\phi} - \dot{\phi}^2 - \frac{\dot{N}}{N}\dot{\phi} - \frac{d-1}{2}\left(\frac{\ddot{a}}{a} - H\frac{\dot{N}}{N}\right) - \frac{1}{4}(d-1)(d-2)H^2$$

$$= -\frac{1}{2}\kappa N\delta(\tau).$$

Effect of the brane in any d is such that:

$$\ddot{\phi} = -\frac{1}{2}\kappa N_c \delta(\tau) + \dots$$

$\ddot{a} \to$ smooth.

- Induces a discontinuity in $\dot{\phi}$: $2(\dot{\phi}_+ - \dot{\phi}_-) = -N_c \kappa$
$\dot{a}$ is continuous.

- Criticality in temperature $\to \dot{a} = 0$.

- Then from the N-equation $\to \dot{\phi}_+ = -\dot{\phi}_-$.

The dilaton bounces elastically across the brane.

$$\dot{\phi}_+ = -\dot{\phi}_- = -N_c \kappa / 4$$

Since the brane tension κ is positive, the dilaton is initially increasing, crosses the brane and then decreases.

$\rightarrow$ Entropy conservation everywhere and across the brane:
$\dot{\rho} + (d-1)H(\rho + P) = 0.$

Assuming massless thermal radiation throughout: $P = n^* \Sigma_d T^d$

$$n^* = n^B + n^F \frac{2^{d-1}-1}{2^{d-1}}, \quad \Sigma_d \rightarrow \text{Stefan/Boltzmann constant}$$

entropy in a co-moving cell of unit coordinate volume (physical volume a^{d-1}) $\rightarrow$ $S = d(aT)^{d-1} n^* \Sigma_d = \text{constant}$ $\rightarrow$

$$a_c T_c = aT = \left(\frac{S}{d n^* \Sigma_d} \right)^{1/(d-1)}$$

Cosmological solutions in the conformal gauge: $N = a$

$$\ln\left(\frac{a}{a_c}\right) = \frac{1}{d-2}\ \left[\eta_+ \ln\left(1 + \frac{\omega a_c|\tau|}{\eta_+}\right) - \eta_- \ln\left(1 + \frac{\omega a_c|\tau|}{\eta_-}\right)\right]$$

$$\phi = \phi_c + \frac{\sqrt{d-1}}{2}\ \left[\ln\left(1 + \frac{\omega a_c|\tau|}{\eta_+}\right) - \ln\left(1 + \frac{\omega a_c|\tau|}{\eta_-}\right)\right]$$

ω is proportional to the brane tension κ:

$$\omega = \kappa \frac{d-2}{4\sqrt{d-1}}$$

$$\eta_\pm = \sqrt{d-1} \pm 1, \quad \kappa = 2\sqrt{2(d-1)}\ \sqrt{n^* \Sigma_d}\ \ T_c^{d/2}\ e^{\phi_c}$$

In the neighborhood of the brane $|\kappa a_c \tau| \ll 1$,

the metric is regular while the dilaton exhibits a conical singularity:

$$\ln\left(\frac{a}{a_c}\right) = \frac{1}{16(d-1)}(\kappa a_c \tau)^2 + \mathcal{O}(|\kappa a_c \tau|^3)$$

$$\phi = \phi_c - \frac{|\kappa a_c \tau|}{4} + \mathcal{O}((\kappa a_c \tau)^2)$$

Far from the brane $|\kappa a_c \tau| \gg 1$, the dilaton asymptotes to a constant,

the temperature drops and the scale factor tends to infinity.

The maximal value of the dilaton is given by ϕ_c, and hence $g_s \leq g_c = e^{\phi_c}$.

Our perturbative approach is valid if g_c is sufficiently small.

Ricci scalar attains its maximal value at the brane:
$\mathcal{R} = \kappa^2/4 = \mathcal{O}(g_c^2) \rightarrow$ *no essential singularity.*

• *Both g_s and α' corrections remain under control,* provided that g_c is small.

• The $d = 2$ Hybrid cases can be recovered by taking the formal limit $d \rightarrow 2$ in the solutions above.

We transform to the Einstein frame via

$$(N_E, a_E, 1/T_E) = e^{-\frac{2\phi}{d-2}} (N, a, 1/T)$$

Thus the geometrical quantities in the Einstein frame develop conical singularities, inherited from the dilaton factor.

The conical singularities are resolved by the brane at T_c, supporting the extra massless thermal scalars.

Notice that at the brane, $\dot{T} = 0 \longrightarrow$ with the temperature freezing at its maximal, critical value T_c.

We examine now the possibility of "spreading" the "Brane regime", for an arbitrarily long period of time in the past.

The requirement for establishing a long "Brane regime" is keeping the string frame temperature constant, frozen at its critical value.

It follows via the entropy conservation law that the string frame scale factor must also be constant: $a = a_c$.

Modulo the running dilaton, this model shares features with the "Emergent Cosmological Scenario,"

where the Universe is in a long quasi-static phase, with $H = 0$, before it exits into an expanding phase.

Various condensates associated with the extra massless fields $\rightarrow$ dilaton effective potential $V(\phi)$.

In order for the temperature and string frame scale factor to remain constant $\rightarrow$

$$V(\phi) = B + Ce^{-2\phi}, \quad B = P_c$$

It turns out that the parameters B and C can be obtained in terms of fluxes in the underlying effective gauged supergravity description of the thermal system at the extended symmetry point: $\sigma = 0$.

Euclidean time circle $\rightarrow$ $SU(2)_L$ gauge symmetry.

Right moving susy breaking circle $\rightarrow$ $SU(2)_R$ gauge symmetry.

So at the fermionic extended symmetry point symmetries reorganize and the S^1 cycle gets extended to an $SU(2)$-manifold.

At this point the target space can be described by an $n = d + 2$-dimensional space.

Now consider the $d + 2$-dimensional gauged supergravity.

Once fluxes and gradients along the 2 compact directions are turned on, the kinetic terms of the internal massless scalars, graviphotons and matter gauge bosons give rise to the following dilaton effective potential:

$$Ae^{-2\phi} + B(\mathcal{U}) + \tilde{C}e^{-2\phi},$$

$\mathcal{U}$ stands for the volume of the $10 - d$ internal manifold.

So supporting a brane regime requires:

- $B(\mathcal{U}) = P_c$
- $A + \tilde{C} = C$, arbitrary

This form persists when the theory is reduced to d-dimensions, since the extra compact $SU(2)_{k=2}/U(1)$ manifold has fixed volume at the string scale.

Thus the only non-trivial evolution is that of the dilaton, which depends on the flux parameter C of the effective potential.

$$2\dot{\phi}^2 = a_c^2 \left[(\rho_c + P_c) e^{2\phi} + C \right]$$

$$C = 0 \ : \qquad e^{-\phi} = a_c \sqrt{\frac{\rho_c + P_c}{2}}\,(-\tau), \qquad \forall\, \tau \leq \tau_+ < 0$$

$$C > 0 \ : \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{C}}\, \sinh\Big[\, a_c \sqrt{\frac{C}{2}}(-\tau)\Big], \quad \forall\, \tau \leq \tau_+ < 0$$

- In both cases the "Brane regime" starts at $\tau = -\infty$.

- At $\tau_+ < 0$, this regime exits via a thin S-brane to the "Momentum, radiation regime."

The choice of τ_+ determines the string coupling at the transition towards the Momentum phase: $\phi_+ = \phi(\tau = \tau_+)$.

$$\rightarrow \{\mathcal{C}_{\rm String}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}$$

- Initially the Universe has constant σ-model temperature and scale factor, $T = T_c$ and $a = a_c$.

- The string coupling grows from very weak values in the very early past reaching a maximal value $g^*_{\rm str}$ at τ_c.

- For $(\tau > \tau_c)$, the Universe exits into the radiation dominated "Momentum regime".

- g_s corrections are under control provided that $g^*_{\rm str}$ is small enough.

Finally the case $C < 0$ allows us to obtain a "Brane regime" of finite time duration.

$$C < 0: \; e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{|C|}} \sin\left[a_c \sqrt{\frac{|C|}{2}}(-\tau)\right] \quad \tau_- \leq \tau \leq \tau_+ < 0$$

Eternal, bouncing cosmologies open new perspectives to address the cosmological puzzles.

Most problems of standard cosmology are based on the assumption that the Universe starts out very small and hot (Planck temperature.)

In our set up:

- The horizon problem is essentially nullified. Causal contact over large scales is assured.

- The large entropy problem does not arise either. If the Universe begins cold and large (larger than the present-day Universe), it will by dimensional analysis reach a state of large entropy.

Homogeneity problem? Growth of cosmological fluctuations? Scale invariant spectrum?

Work under progress: [Brandenberger, Kounnas, Partouche, Patil, NT].

Parafermionic Cosmologies

Exact, cosmological solutions to classical superstring theory exist
$\longrightarrow$

described by a worldsheet SCFT of the form

$$SL(2,R)_{-|k|}/U(1) \;\times\; K$$

first factor: gauged WZW model; K is an internal, compact CFT.

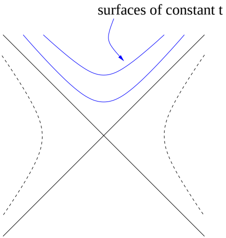
The sigma-model metric and dilaton are given by

$$(\alpha')^{-1} ds^2 = (|k|+2)\frac{-dT^2+dX^2}{1+T^2-X^2}$$

$$e^{2\Phi} = \frac{e^{2\Phi_0}}{1+T^2-X^2}$$

Geometry: singularity-free light-cone region, time-like curvature singularities in the regions outside the light-cone horizons.

Singularities at $X = \pm\sqrt{1 + T^2}$, where the dilaton field is also singular. Proper distance: $L = \pi\sqrt{(|k| + 2)\alpha'} \to$ finite.



If we perform a double analytic continuation, we obtain Witten's 2d black hole.

Equivalent to changing the sign of the level k.

At the singularities the sigma-model geometrical description breaks down. The CFT gives a prescription to describe them.

The cosmological region of interest is the future part of the lightcone region.

It is an expanding, asymptotically flat geometry with the string coupling vanishing at late times:

$$(\alpha')^{-1} ds^2 = (|k| + 2)\frac{-dt^2 + t^2 dx^2}{1 + t^2}, \qquad e^{2\Phi} = \frac{e^{2\Phi_0}}{1 + t^2}$$

Asymptotically we get a timelike linear dilaton background.

The cosmological observer never encounters the singularities $\rightarrow$ behind the horizons at $T = \pm X$.

But signals from the singularities can propagate into the lightcone region, at $t = 0$, and influence its future evolution.
$\rightarrow$ Similar to a Big-Bang cosmology.

The central charge of the superconformal $SL(2,R)/U(1)$ model at negative level k, is given by

$$c = 3 - \frac{6}{|k|+2}, \quad \hat{c} = 2 - \frac{4}{|k|+2}$$

4d model: add a large (super) T^2 plus a compact, SCFT of central charge $\delta\hat{c} = 6 + 4/(|k|+2)$.

E.g. at $|k| = 2$, $K \equiv T^2 \times T^7 \rightarrow$ super-critical model.

The metric in Einstein frame is given by

$$ds_E^2 = (|k| + 2)\alpha'(-dt^2 + t^2 dx^2) + R^2\ (1 + t^2)(dy^2 + dz^2).$$

Anisotropic cosmology. At late times however, and for large R, we get an isotropic flat Friedmann cosmology.

Scale factor: $a \sim t$

$$\rho_{\rm eff} = -3P_{\rm eff}$$

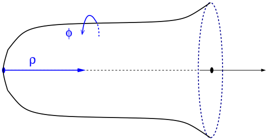
At the cross-point between accelerating and decelerating Universes.

Rotating to Euclidean signature, we obtain a disk.

$$(\alpha')^{-1} ds^2 = (|k| + 2)\frac{d\rho^2 + \rho^2 d\phi^2}{1 - \rho^2}, \qquad e^{2\Phi} = \frac{e^{2\Phi_0}}{1 - \rho^2}$$

The singularity now occurs at the boundary circle $\rho = 1$.

The radial distance of the center to the boundary is finite, but the circumference of the boundary circle at $\rho = 1$ is infinite.
Geometrically the space looks like a bell.



Euclidean background $\leftrightarrow$ well defined CFT based on the $SU(2)/U(1)$ coset model at level $|k|$.

The interesting feature is that the Euclidean CFT is compact and unitary.

$N = 2$ minimal model: the level $|k|$ must be an integer.

The worldsheet CFT is perfectly well behaved at $\rho = 1$.

Near this region the gauged WZW action is given by

$$S_{WZW} = -\frac{|k| + 2}{2\pi} \int d^2 z \phi F_{z\bar{z}} + \dots$$

$\rightarrow$ topological theory.

Worldsheet instantons for which $\int F_{z\bar{z}} = 2\pi i n$ break the $U(1)$ symmetry shifting the angle ϕ to a discrete symmetry $Z_{|k|+2} \rightarrow$ description in terms of $Z_{|k|+2}$ parafermions.

We argue that the non-singular description of the theory is an almost-geometrical one, in terms of a "T-fold".

To obtain it, perform T-duality along the angular direction ϕ.

The resulting sigma model is based on the metric and dilaton

$$(\alpha')^{-1} ds'^2 = \frac{(|k|+2)}{1-\rho'^2}\left(d\rho'^2 + \frac{\rho'^2}{(|k|+2)^2} d\phi'^2\right)$$

$$e^{2\Phi'} = \frac{e^{2\Phi_0}}{(|k|+2)(1-\rho'^2)}, \qquad \rho' = (1-\rho^2)^{\frac{1}{2}}$$

The transformation on ρ exchanges the boundary of the disk and its center.

The T-dual description is weakly coupled near $\rho = 1$ or $\rho' = 0$.

The only singularity there is a benign orbifold singularity.

The T-dual $\leftrightarrow$ $Z_{|k|+2}$ orbifold of the original model.

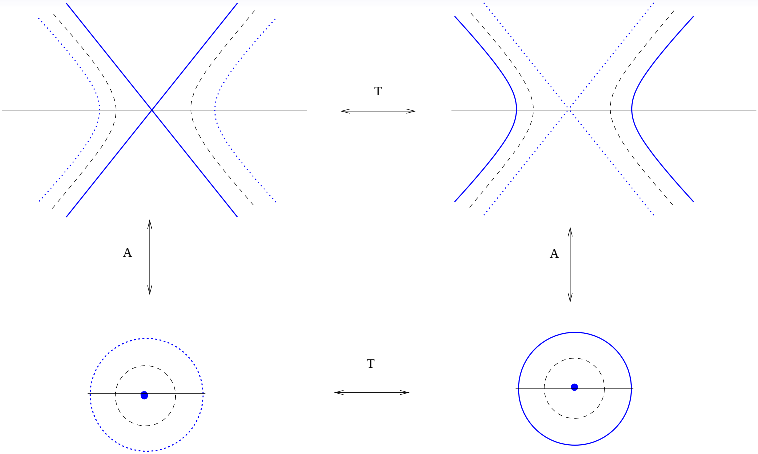
By gluing the two T-duals along a non-singular circle (e.g. at $\rho = \rho' = 1/\sqrt{2}$) we obtain a compact T-fold.

This has no boundaries or singularities.
The gluing is non-geometrical. It involves a T-duality transformation on the metric and other fields.

For the cosmology as well, we can obtain a regular T-fold description as the target space of the CFT.

T-duality interchanges the light-cone and the singularities.

We must glue the T-duals along a hyperbola in between the lightcone and the singularities.

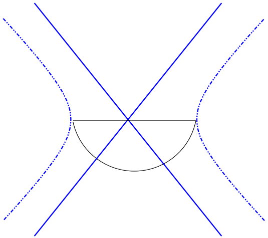


The resulting almost-geometrical description is very much like 2d de-Sitter space, which we can think of as a hyperboloid embedded in three-dimensional flat space.

We may think of the Euclidean T-fold (or the corresponding compact CFT) as describing a string field theory instanton.

The T-fold allows us to *formally* define a Hartle-Hawking wavefunction for the cosmology

a vector in the Hilbert space of the 2nd quantized string field theory. Observables: "S-Vector"



Perform a "half T-fold" Euclidean path integral over *fluctuations of all* target space fields with specified values on the boundary:

$$\Psi[h_\partial, \phi_\partial, \dots] = \int [dg][d\phi] \dots e^{-S(g,\phi,\dots)}$$

Hard to compute, but we can understand some of its global properties by computing its norm:

$$||\Psi||^2 = e^{Z_{string}}$$

Z_{string} is the connected string partition function. Calculable perturbatively in 1st quantized formalism if $e^{2\Phi_0}$ is small enough.

Finite if underlying SCFT is compact and leads to a tachyon free Euclidean model. (Via suitable GSO projection.)

As in the case of de Sitter space, the norm of the wavefunction can be interpreted as a thermal ensemble.

$$||\Psi_{dS}||^2 = e^{3/(8G^2\Lambda)}, \quad \frac{3}{8G^2\Lambda} \rightarrow \text{de Sitter Entropy}.$$

$\rightarrow Z_{string}$ corresponds to a thermal string amplitude. No genus zero contribution.

Effective temperature: $T = 1/2\pi R$, where

$$R = \sqrt{(|k|+2)\alpha'},$$

below Hagedorn $R_H = \sqrt{2\alpha'}$ for all $|k| > 0$ $\rightarrow \Psi$ is normalizable.

E.g. for $|k| = 2$, $\hat{c} = 1$, parafermionic factor is equivalent to a fermion + compact boson at radius $2\sqrt{\alpha'}$.
$R = R_H$ at $|k| = 0$, when the cosmology disappears from the target space.

$||\Psi||^2$ is a function of the moduli associated to the internal CFT K.

It would be nice to use it to define relative probabilities for different string compactifications.

Conclusions

• Cosmological consequences of stringy gluing mechanisms between different string effective field theories.

• The gluing mechanism is triggered by extra massless thermal states when the temperature reaches a maximal critical value.

• The region $T = T_c$, admits a "brane interpretation;" brane tension given in terms of non-trivial gradients associated with the extra massless scalars.

• In the Einstein cosmological frame, all solutions correspond to bouncing Universes.

• The cosmological solutions remain perturbative, provided that the critical value of the string coupling at the brane is sufficiently small.

• This class of bouncing cosmologies provides the first examples, where

both the Hagedorn instability as well as the classical Big Bang singularity are successfully resolved, remaining in a perturbative regime throughout the evolution.