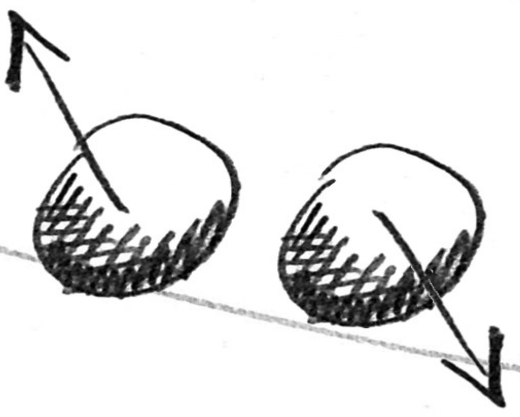
Unbalanced Holographic Superconductor & Spintronics

Daniele Musso

Université Libre de Bruxelles

XVIII European Workshop on String Theory

Bigazzi, Cotrone, Musso, Pinzani, Seminara JHEP 1202, 078 (2012), arXiv:1111.6601 [hep-th]



Outline

- Unbalanced Superconductor & Spintronics
- A Holographic Model
- Results (at & out of equilibrium)
- Future Perspectives

Outline

- Unbalanced Superconductor & Spintronics
- A Holographic Model
- Results (at & out of equilibrium)
- Future Perspectives

"Unbalanced Fermi mixtures"

Relevant for both Condensed Matter and QCD

(superconductors, spintronics, cold atoms, neutron stars,...)

General framework & context

- AdS/CFT Correspondence (gauge/gravity)
- Holography: *Classical gravity as an analytic tool to compute correlation functions in the dual, strongly coupled quantum field theory*
  (strongly coupled plasmas, quantum phase transitions,...)
- Holographic superconductor
  (Hartnoll-Horowitz- Herzog, 2008)

High Tc (i.e. non BCS) superconductors and superconducting mechanism at strong coupling ??

Specific problem

Unbalanced Fermi mixtures at strong coupling

Unbalance Fermi mixtures (standard lore)

- Fermions “pile up” because of the Pauli exclusion principle and give rise to Fermi surfaces.
- Different fermionic species in a system can have different Fermi surfaces: “imbalance”.

Unbalanced Superconductor

Different Fermi surfaces for spin “up” and “down” electrons. *(Magnetic impurities and doping, external fields,...)*

Effective chemical potential mismatch δμ

$$\mathcal{H}_I = H\,\mu_b\,\overline{\Psi}\gamma^3\Psi = \delta\mu\,\overline{\Psi}\gamma^3\Psi$$

Zeeman splitting of single electron energy levels

*

Relevant not only for condensed matter

→ QCD: *baryon* and *isospin* symmetries

Cooper Pairing

Cooper condensate:

$$\langle \mathcal{O} \rangle = \langle \psi^{\uparrow}(\vec{k})\ \psi^{\downarrow}(-\vec{k}) \rangle \longrightarrow \langle \mathcal{O}(\vec{x}) \rangle = \Delta$$

- The chemical potential imbalance hinders the Cooper pairs formation
- Large imbalance brings loss of homogeneous superconductivity *(Chandrasekar-Clogston 1962)*
- Inhomogeneous, finite momentum condensate (Larkin-Ovchinnikov 1964, Fulde-Ferrel 1964)

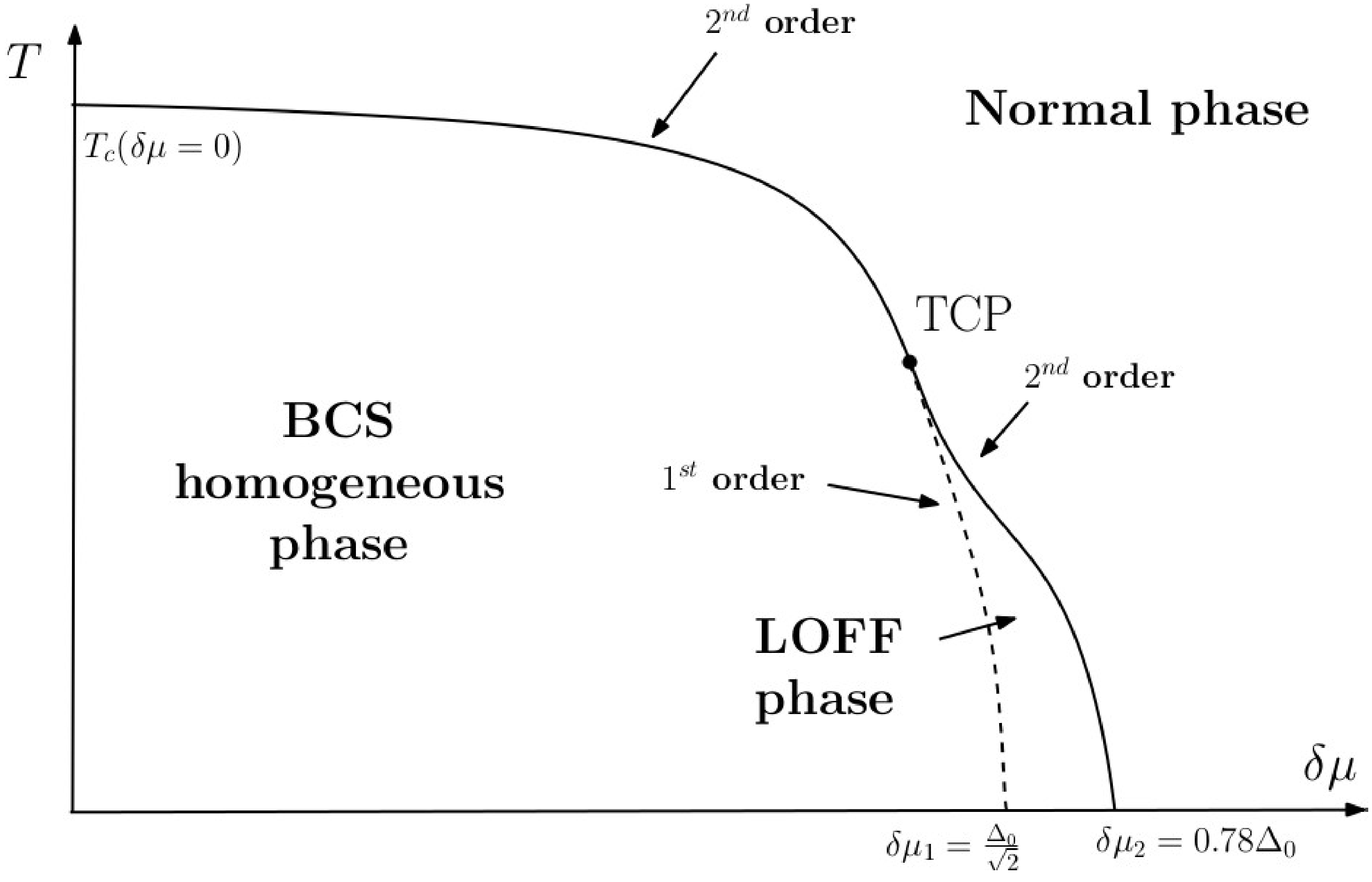
$$\langle \mathcal{O} \rangle = \langle \psi^{\uparrow}(\vec{k})\ \psi^{\downarrow}(-\vec{k}+\vec{q}) \rangle \longrightarrow \langle \mathcal{O}(\vec{x}) \rangle = \Delta\ e^{\mathrm{i}\vec{q}\cdot\vec{x}}$$

Why Now?

- BCS theory for balanced S.C. (1957)
- CC bound (1962)
- LOFF phase and unbalanced S.C. (1965)
-
- ...
-
- MIT & Rice University experiments (2006)

Stringent experimental conditions required
Low spin relaxation

Phase Diagram



1st PROBLEM: Phase diagram at strong coupling??

Spintronics

- Mixed spin-electric transport properties.
- Theoretical basis: *"Two-current model"* *(Mott, 1936)*.

Giant Magneto Resistance

(2007 Nobel Prize to Fert and Grünberg)

- GMR: great change in electric resistance depending on the applied external magnetic field.
- Usage: common hard disk read heads.

Macroscopic picture

- Superconductivity: zero DC resistivity arising directly from the spontaneous breaking of $U(1)_e$ *(Weinberg 1985)*. Condensation of electrically charged operator $\langle \mathcal{O} \rangle$.
- Unbalance: $\delta\mu$ chemical potential for $U(1)_s$ (decoupled from space-time symmetries in IR).

$$U(1)_e \times U(1)_s$$

with order parameter $\langle \mathcal{O} \rangle$
electrically charged under $U(1)_s$

Mott two-current model

At “small” T , “up” and “down” electron currents treated separately *(Mott 1936, Fert-Campbell 1968)*

$$J^{\mathrm{e}} \sim J^{\uparrow} + J^{\downarrow} \qquad J^{\mathrm{spin}} \sim J^{\uparrow} - J^{\downarrow}$$

$$J^{\mathrm{e}} = \sigma_{\mathrm{e}} E^{\mathrm{e}} \qquad J^{\mathrm{spin}} = \sigma_{\mathrm{spin}} E^{\mathrm{spin}}$$

$$E^{\mathrm{e}} \leftrightarrow \mathrm{U}(1)_e \qquad E^{\mathrm{spin}} \leftrightarrow \mathrm{U}(1)_{\mathrm{spin}}$$

EFFECTIVE

Dynamical magnetization, “spin motive field”:

$$E^{\mathrm{spin}} \sim \nabla \delta\mu$$

Spin-electric response

- In the presence of a net spin density (i.e. imbalance up/down), then an external electric field produces also a spin current *(Aronov 1976)*.

$$J^{\mathrm{spin}} = \gamma E^{\mathrm{e}}$$

- Opposite also true: an external “spin-motive field” induces also an electric current spin *(Berger 1986, van Son-van Kempen-Wyder 1987, Johnson-Silsbee 1987)*

$$J^{\mathrm{e}} = \gamma E^{\mathrm{spin}}$$

It is possible to induce spin transport with electric fields and electric transport with spin motive forces

Optical Conductivities

$$\begin{pmatrix} J^e \\ Q \\ J^{\mathrm{spin}} \end{pmatrix} = \begin{pmatrix} \sigma_{\mathrm{e}} & \alpha T & \gamma \\ \alpha T & \kappa T & \beta T \\ \gamma & \beta T & \sigma_{\mathrm{spin}} \end{pmatrix} \cdot \begin{pmatrix} E^{\mathrm{e}} \\ -\frac{\nabla T}{T} \\ E^{\mathrm{spin}} \end{pmatrix}$$

Electrical current J^e

Heat current Q

Spin current J^{spin}

Electric conductivity σ_{e}

Thermal conductivity κ

Thermo-electric conductivity α

...

2nd PROBLEM: Transport at strong coupling??

Statement

We build the simplest holographic model of s-wave unbalanced (2+1 dim → layered) "superconductor" and of Mott's "two-current model".

(Bigazzi-Cotrone-Musso-Pinzani-Fokeeva-Seminara 2011)

p-wave unbalanced holographic superconductor has been considered in the literature

(Erdmenger-Grass-Kerner-Hai Ngo 2011)

Holographic Model *(ingredients)*

Minimal holographic set of dual ingredients to describe a superconductor *(Gubser 2008, Hartnoll-Herzog-Horowitz 2008)*

Effective Q.F.T. describing the Su.Co. at "the bounday":

- U(1) symmetry *(global)*
- Charged scalar ψ

⟺

Dual gravitational model in AAdS "bulk":

- U(1) gauge field A
- Charged scalar ψ

IMBALANCE: Extra dual ingredient:

Another gravity gauge field B associated to the U(1)s

(Bigazzi-Cotrone-Musso-Pinzani-Fokeeva-Seminara 2011)

ψ *chargerd w.r.t.* A *and uncharged w.r.t.* B $\uparrow\downarrow$

Holographic Model

Dual gravitational action:

$$S = \frac{1}{2\kappa_4^2}\int dx^4\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{4}Y_{ab}Y^{ab} - V(|\psi|) - |\partial\psi - \mathrm{i}qA\psi|^2\right]$$

$$F = dA\ , \qquad Y = dB\ , \qquad V(|\psi|) = -\frac{m^2}{L^2}\psi^\dagger\psi$$

Above the BF bound for AdS4 $\quad m^2 = -2$

Gravitional background

Ansatz:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{r^2}{L^2}(dx^2 + dy^2) + \frac{dr^2}{g(r)}$$

$$\psi = \psi(r) \ , \qquad A_a dx^a = \phi(r)dt \ , \qquad B_a dx^a = v(r)dt$$

IR :

$$g(r_H) = \phi(r_H) = v(r_H) = 0 \ , \qquad \psi(r_H) \ , \chi(r_H) \ \text{const.}$$

UV :

$$\psi(r) = \frac{C_1}{r} + \frac{C_2}{r^2} + \ldots \ , \qquad \phi(r) = \mu - \frac{\rho}{r} + \ldots \ ,$$

$$v(r) = \delta\mu - \frac{\delta\rho}{r} + \ldots \ , \qquad g(r) = r^2 - \frac{\epsilon}{2r} + \ldots. \ ,$$

$$\chi(r) = 0 + \ldots$$

$C_1 = 0 \ \rightarrow$ **SPONTANEOUS**

Fluctuations

Vector Fluctuations $\longrightarrow$ A_x, B_x, g_{tx}

$$A_x(r) = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \ldots \qquad r \to \infty$$

$$B_x(r) = B_x^{(0)} + \frac{B_x^{(1)}}{r} + \ldots \qquad g_{tx}(r) = g_{tx}^{(0)} - \frac{g_{tx}^{(1)}}{r} + \ldots$$

$$S_{\text{quad}} = \int d^3x \left(\frac{1}{2} A_x^{(0)} A_x^{(1)} + \frac{1}{2} B_x^{(0)} B_x^{(1)} - 3 g_{tx}^{(0)} g_{tx}^{(1)} - \frac{\epsilon}{2} g_{tx}^{(0)} g_{tx}^{(0)} \right)$$

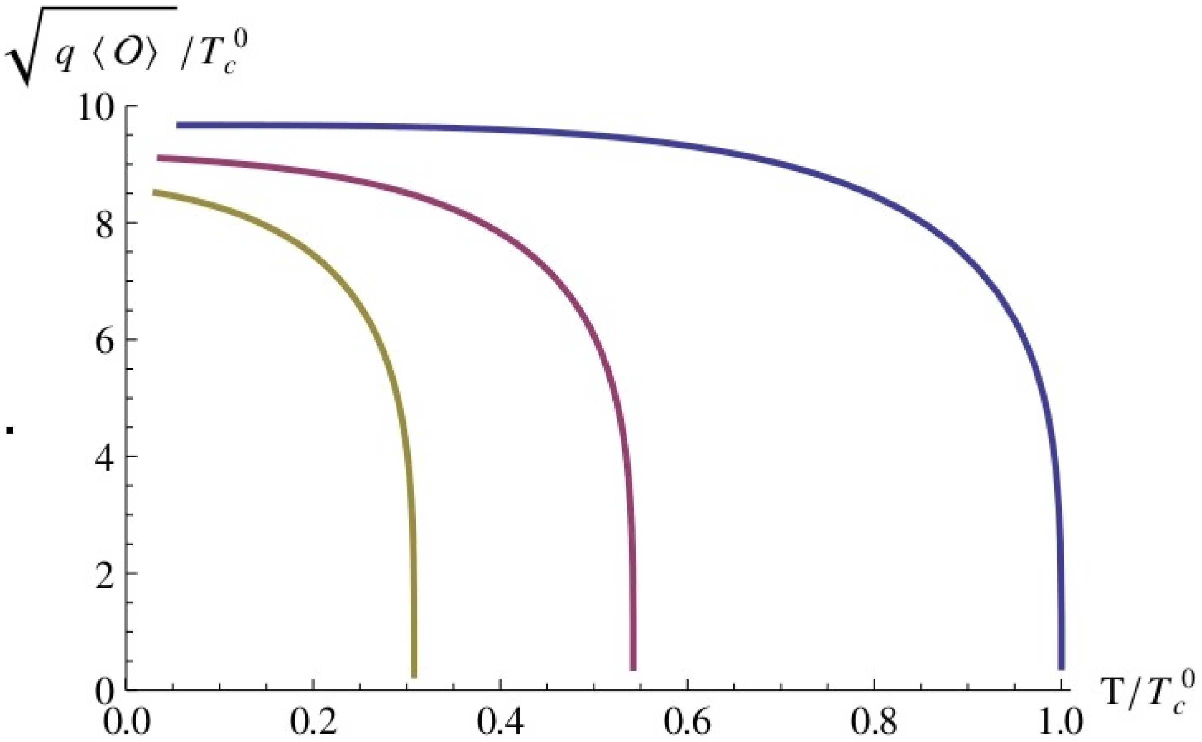
$$J^A = \frac{\delta S_{\text{quad}}}{\delta A_x^{(0)}} \qquad J^B = \frac{\delta S_{\text{quad}}}{\delta B_x^{(0)}} \qquad Q = \frac{\delta S_{\text{quad}}}{\delta g_{tx}^{(0)}}$$

Electric current — Spin current — Heat current

RESULTS

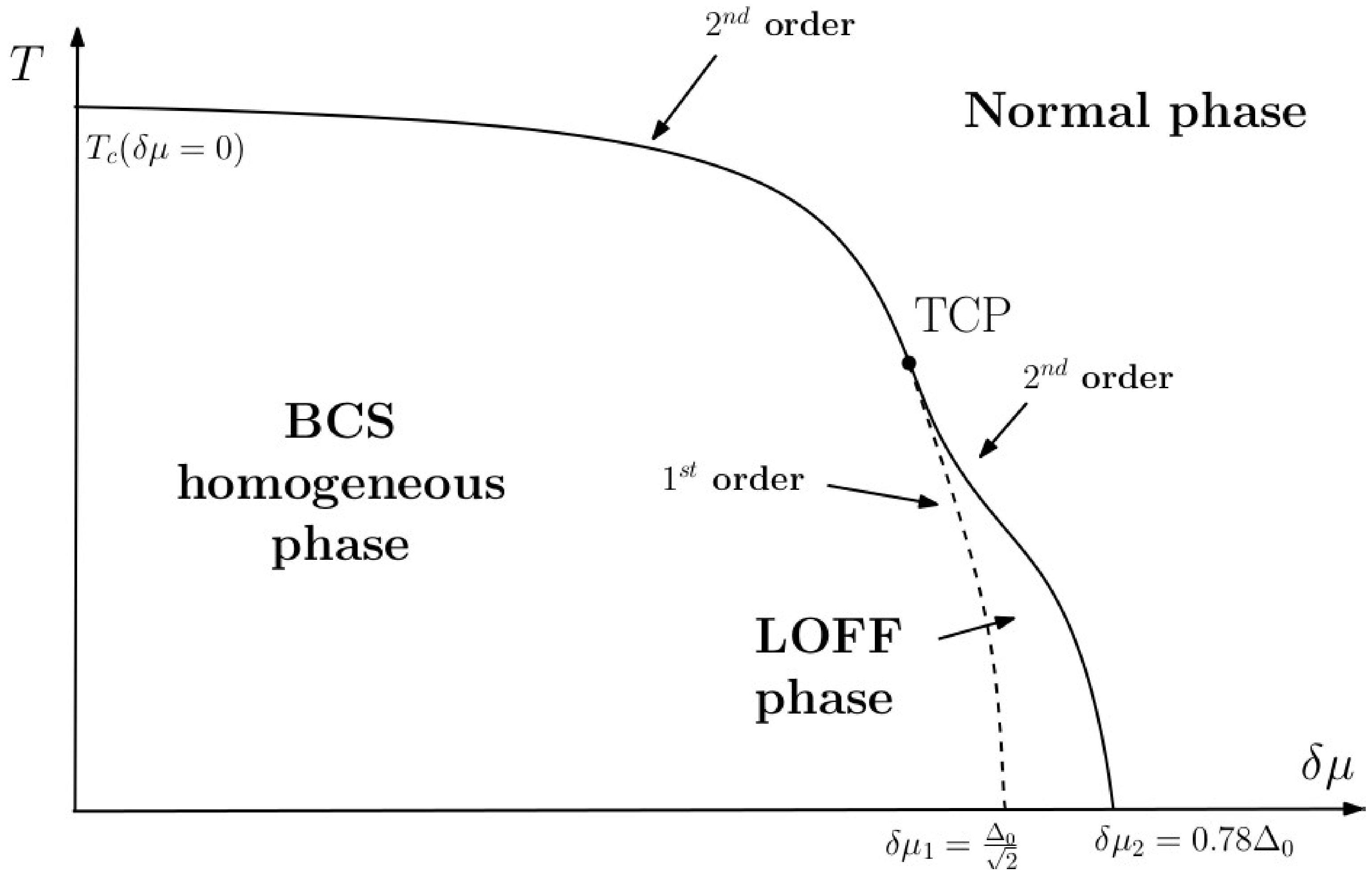
The BH instability leading to scalar hair formation corresponds to the dual Cooper condensation.

The condensate plotted for different values of $\delta\mu/\mu$: 0 (blue), 1 (purple) and 1.5 (green). *The imbalance hinders the condensation.*



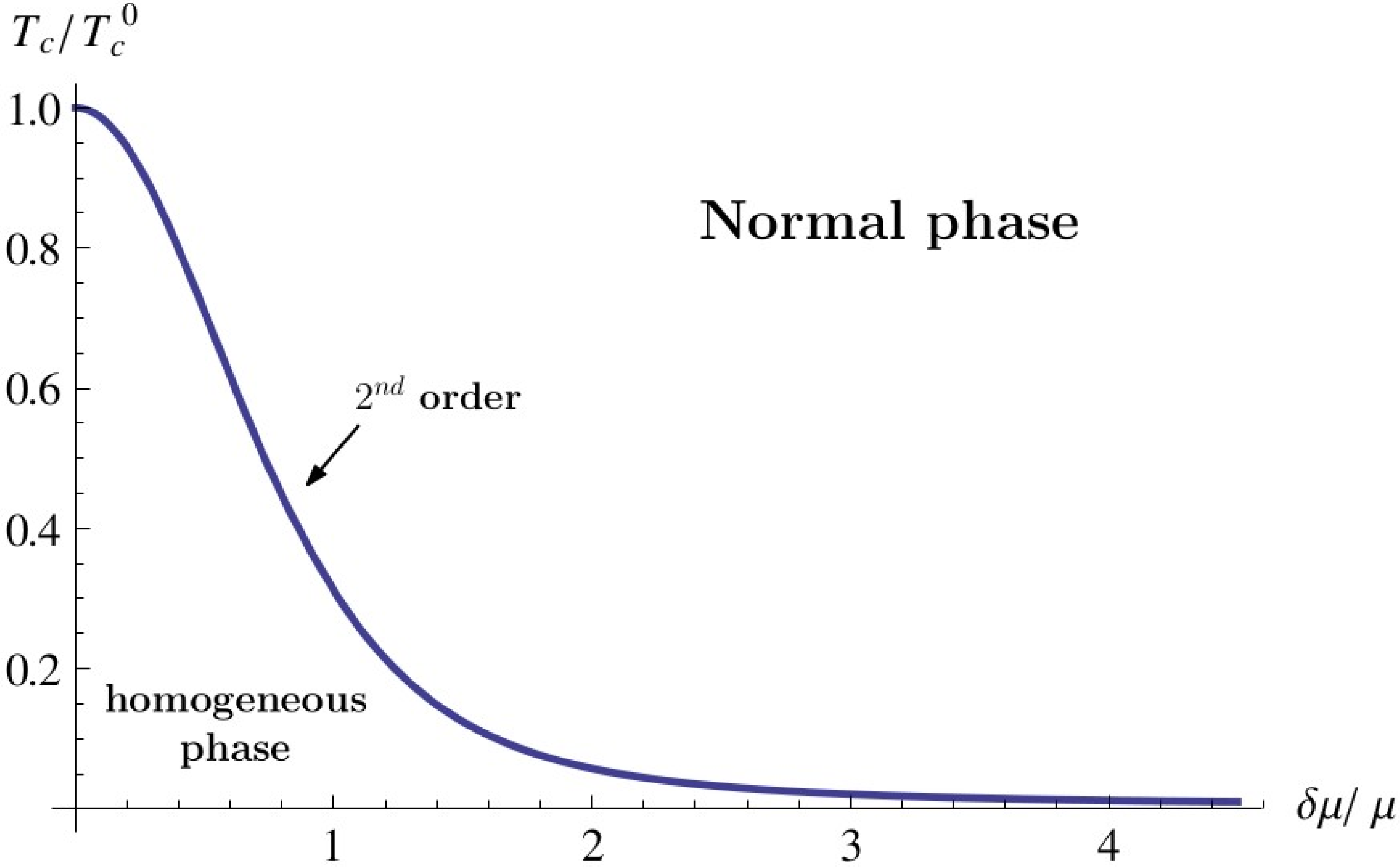
- $T > T_c$: no hair, no condensate.
- $T < T_c$: $\langle\mathcal{O}\rangle \neq 0$ breaks spontaneously $\mathrm{U}(1)_{\mathrm{E.M.}}$
- Around T_c we have a 2-nd order phase transition

Phase Diagram at weak coupling



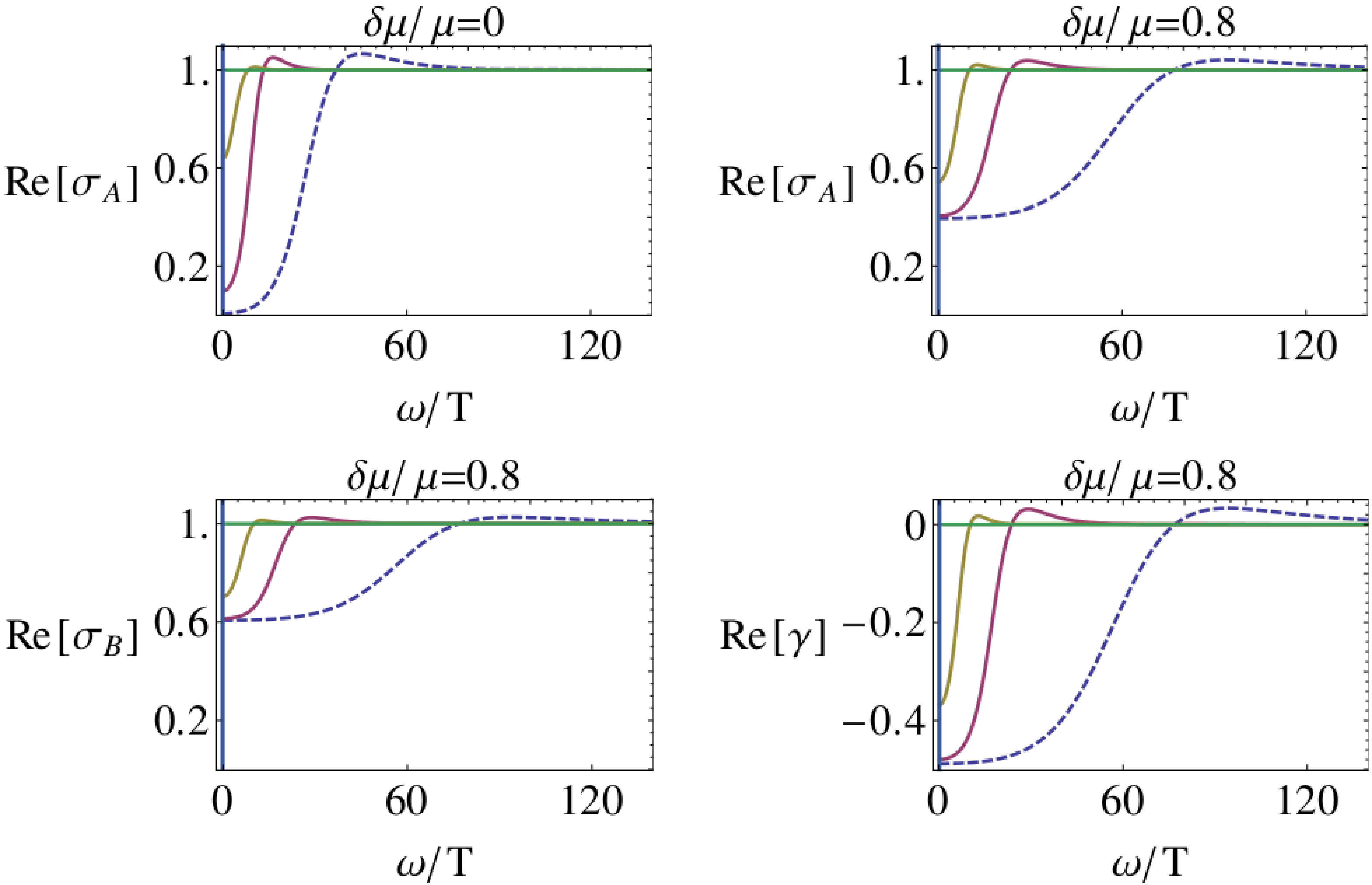
Above a maximum value for $\delta\mu$ the system is too unbalanced to develop a superconducting phase (*Chandrasekar-Clogston bound*).

Phase diagram at strong coupling



The holographic minimal model presents NO Chandrasekar-Clogston bound and NO inhomogeneous (LOFF) phase. BUT the features of the phase diagram depend (strongly) on the parameters (*e.g. the potential for the scalar field, backreaction strength, ...*).

Conductivities (Normal phase)



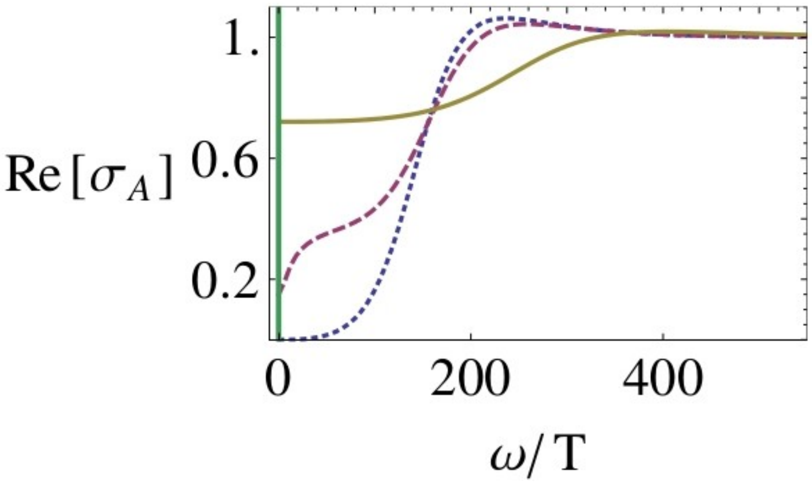
Carriers?

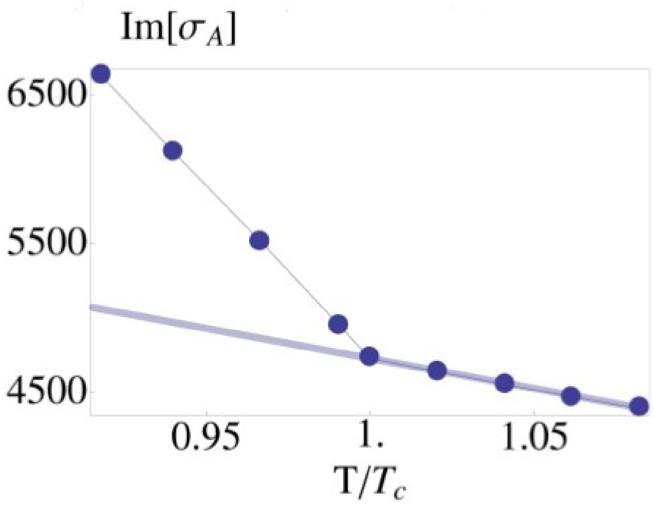
$$\begin{pmatrix} \sigma_A & \alpha T & \gamma \\ \alpha T & \kappa T & \beta T \\ \gamma & \beta T & \sigma_B \end{pmatrix}$$

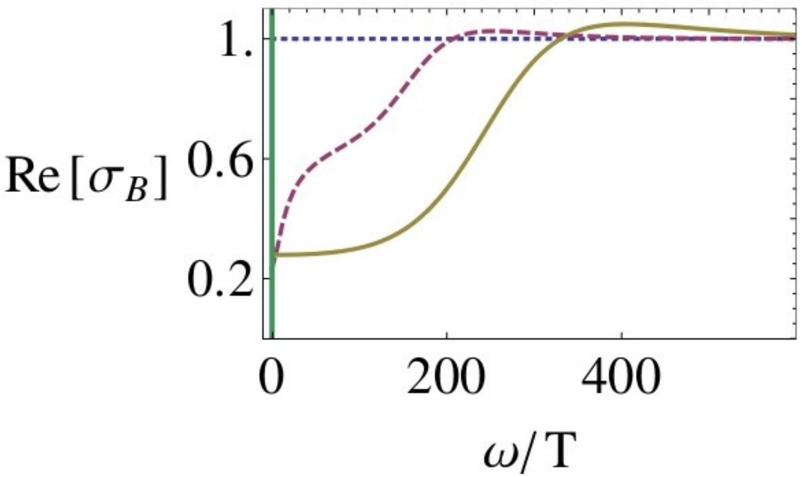
$$\sigma_A = f(\omega)\rho^2 + 1$$
$$\sigma_B = f(\omega)\delta\rho^2 + 1$$
$$\gamma = f(\omega)\rho\,\delta\rho$$

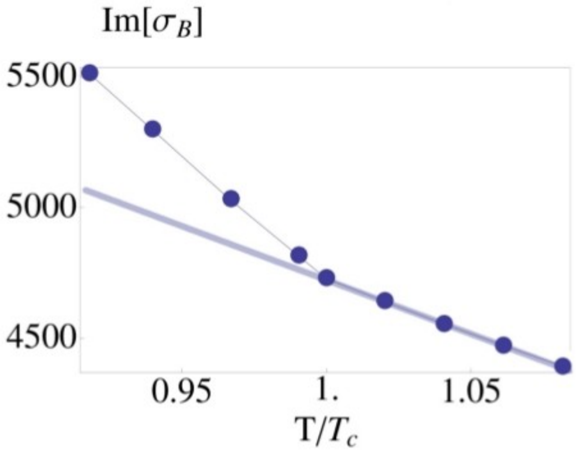
The spin-electric part of the conductivity matrix can be parametrized in terms of a single, suggestive function, the **optical mobility function** → $f(\omega)$

Superconducting Phase









Authentic superconductive contribution to the DC conductivity

Summary

- Backreaction and gravity lead in general to spin-electric mixed effects
- Strongly coupled spintronics (Mott's model) and carrier-like mobility function

- No C.C. bound for the minimal model
- No LOFF phase for the minimal model

Future perspective

- Generalizations
  - Momentum dependent fluctuations
    (Neg. refraction, Additional Light Waves, ...)
  - Spontaneous Ferromagnetic/Superconductor
  - ...
- Space dependent studies
  - Direct look at LOFF
  - Momentum relaxation and holographic lattice
  - ...

THANKS!

Comparison with hep-th/1208.4582

Different realization of the effective magnetic field

$$\mathcal{A}_x = \mathcal{B}x \ , \quad \mathcal{A}_t = \mathcal{A}_x = \mathcal{A}_z = 0$$

$$B_t = \delta\mu - \delta\rho z + \ldots \ , \quad B_x = B_y = B_z = 0$$

NEW INGREDIENT: Interaction between the complex scalar and the magnetic field

$$S_{\text{int}} = \xi \int d^4x \ \sqrt{-g} \ |\mathcal{F}^{AB}\partial_B \phi|^2$$

Sufficient condition for instability

Near horizon → free scalar on AdS2 with eff. mass

$$m^2_{(\text{eff})} = m^2 - \frac{2q^2}{1 + \frac{\delta\mu^2}{\mu^2}}$$

$$L^2_{(2)} m^2_{(\text{eff})} = \frac{L^2}{6} m^2_{(\text{eff})} < -\frac{1}{4} \quad \Rightarrow \quad m^2_{(\text{eff})} < -\frac{3}{2}$$

$$\left(1 + \frac{\delta\mu^2}{\mu^2}\right)\left(m^2 - \frac{3}{2}\right) < 2q^2$$

Smaller radius → smaller "AdS box" → higher "confinement energy" → more stable

What do we know about high Tc superconductors?

(Leggett 2006)

- Superconductivity in the copper oxides is a result of the formation of Cooper pairs. (unit flux: h/2e)
- The principal locus of superconductivity is the copper oxide planes.
- To a 0th order approximation, pairs form independently in the different copper oxide multilayers.
- The dominant mechanism of Cooper-pair formation in the copper oxide materials does not involve a net saving of ionic kinetic energy as in BCS superconductor. (zero isotope effect)
- The spin state of the pairs is a singlet.

Spontaneous U(1) breaking and superconductivity

$$A_\mu \to A_m u + \partial_\mu \alpha$$ Goldstone $$G \to G + \alpha$$

$$A - dG$$ → *Gauge invariant*

$$L = \int d^d x \, [F \cdot F + \mathcal{L}_g(A - dG)]$$

$$J^0 = \rho = \frac{\delta \mathcal{L}_g}{\delta A^0} = -\frac{\delta \mathcal{L}_g}{\delta(\partial_t G)}$$ *Canonical conjugation*

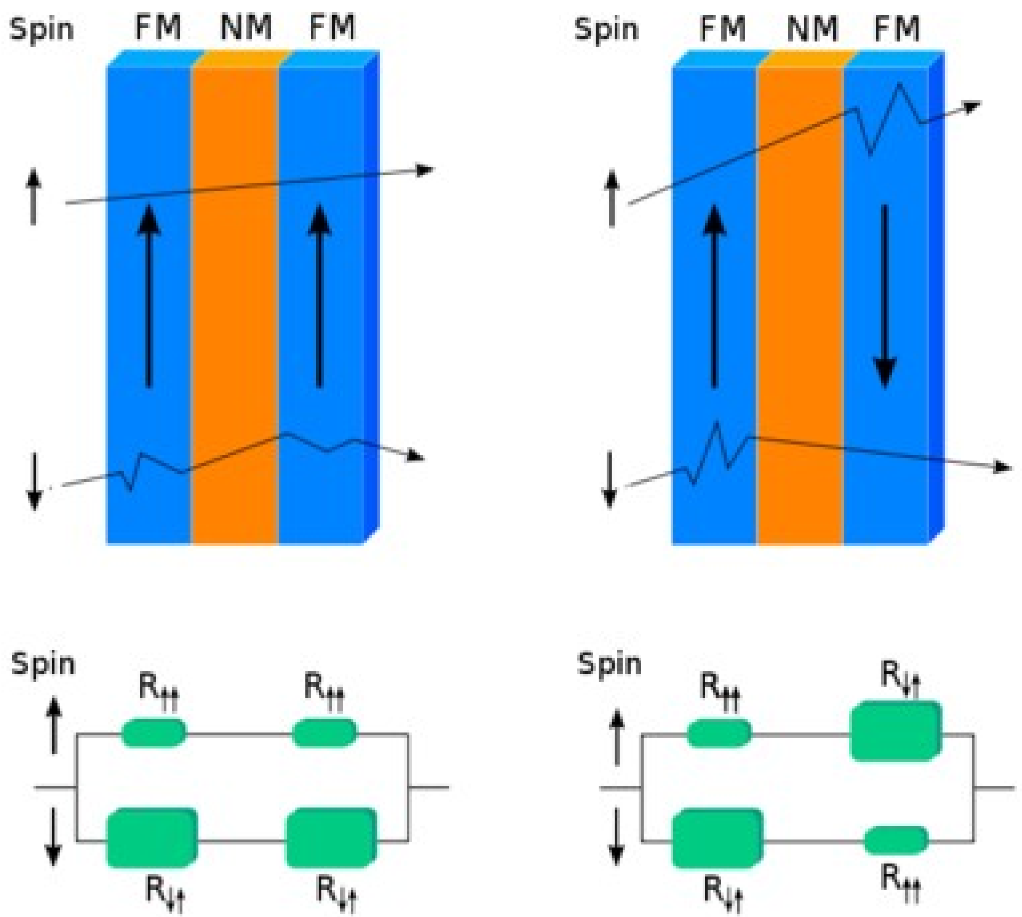
$$\partial_t G = -\frac{\mathcal{H}}{\delta \rho}$$ → $$\boxed{\partial_t G = -V}$$

Stationarity implies zero voltage!

Gian Magneto-Resistance

Preliminary studies on alloys containing impurities with strongly spin-dependent cross section (Fert 1966)

Layered material with FM layers:
Significant change in the electric resistance depending whether adjacent FM layers are parallelly or anti-parallelly polarized
(Fert, Grünberg 1988)



First experimental evidence

$CeCoIn_5$

- Heavy fermion superconductor
- Layered (*quasi 2D system*)

The first order transition at upper magnetic, critical field indicates that the Pauli paramagnetic effect dominates over the orbital effect (Maki parameter $\alpha \approx 5$).

Novel high field superconducting state at the low-T /high-H corner of the H–T to be (*possibly*) identified as the FFLO state

---

Cold atoms? →

Cuprates

- Bismuth, Strontium, Calcium Copper Oxide (BSCCO)
  - first cuprate superconductor (1988)

$$\mathrm{Bi_2Sr_2Ca}_{n-1}\mathrm{Cu}_n\mathrm{O}_{2n+4+x}$$

- Yttrium, Barium, Copper Oxide (YBCO)
  - first material to have showed a Tc value above the boiling point of liquid nitrogen (77K)

Layered materials with *perovskite* structure

String embeddings and UV completion (fund. fermions)

System with N_c D2 branes and a pair of space-filling "flavor" brane/antibrane

- Two gauge fields
- Complex scalar stretching between the branes
- Complex scalar with the "right" charges
- Scalar naturally related to a fundamental fermion bilinear

Analogy with holographic QCD models → Chiral condensate

String embeddings and UV completion (adj. fermions)

Gluino bilinears breaking some $\mathrm{U}(1)_R$

- KK truncation of 11d SUGRA on a 7-manifold *(as for the balanced holographic superconductor)*
- 7-manifold isometries $\rightarrow$ R-symmetry
- **Need for a second scalar field!**

Motivations

- QCD at finite (*but not asymptotic*) density, e.g. *neutron stars*
  - *Perturbative approach fails*
  - *Lattice approach affected by the "sign problem"*

- Unbalanced *cold atoms* systems
  - *BCS – BEC crossover*

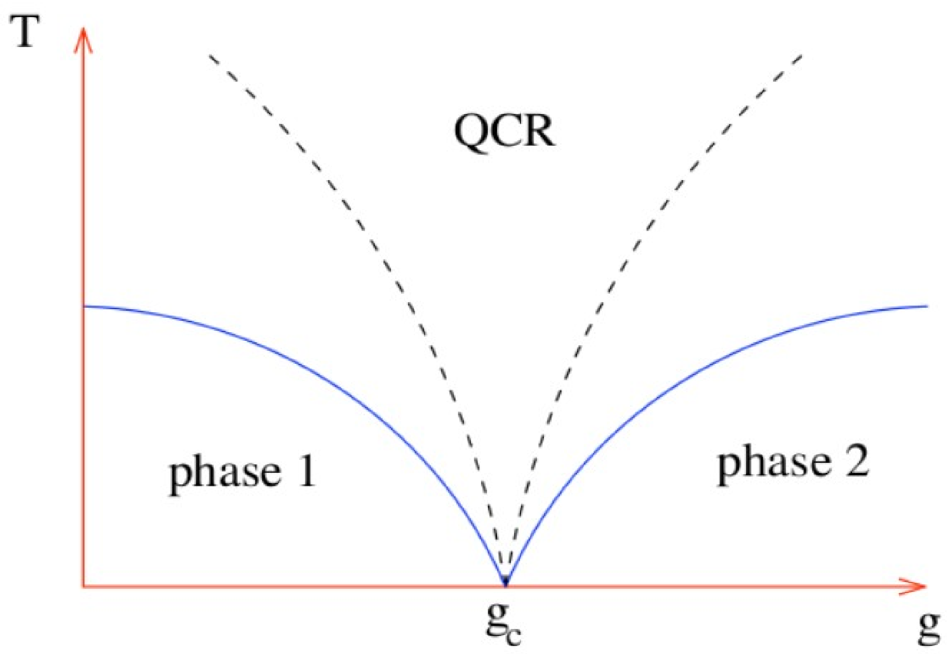
P-wave holographic superconductor

- *model 1:* back-reacted Einstein-Yang-Mills in 4+1 d
- *model 2*: 9+1 dim D3/D7 brane setup with 2 coincident D7 brane probes

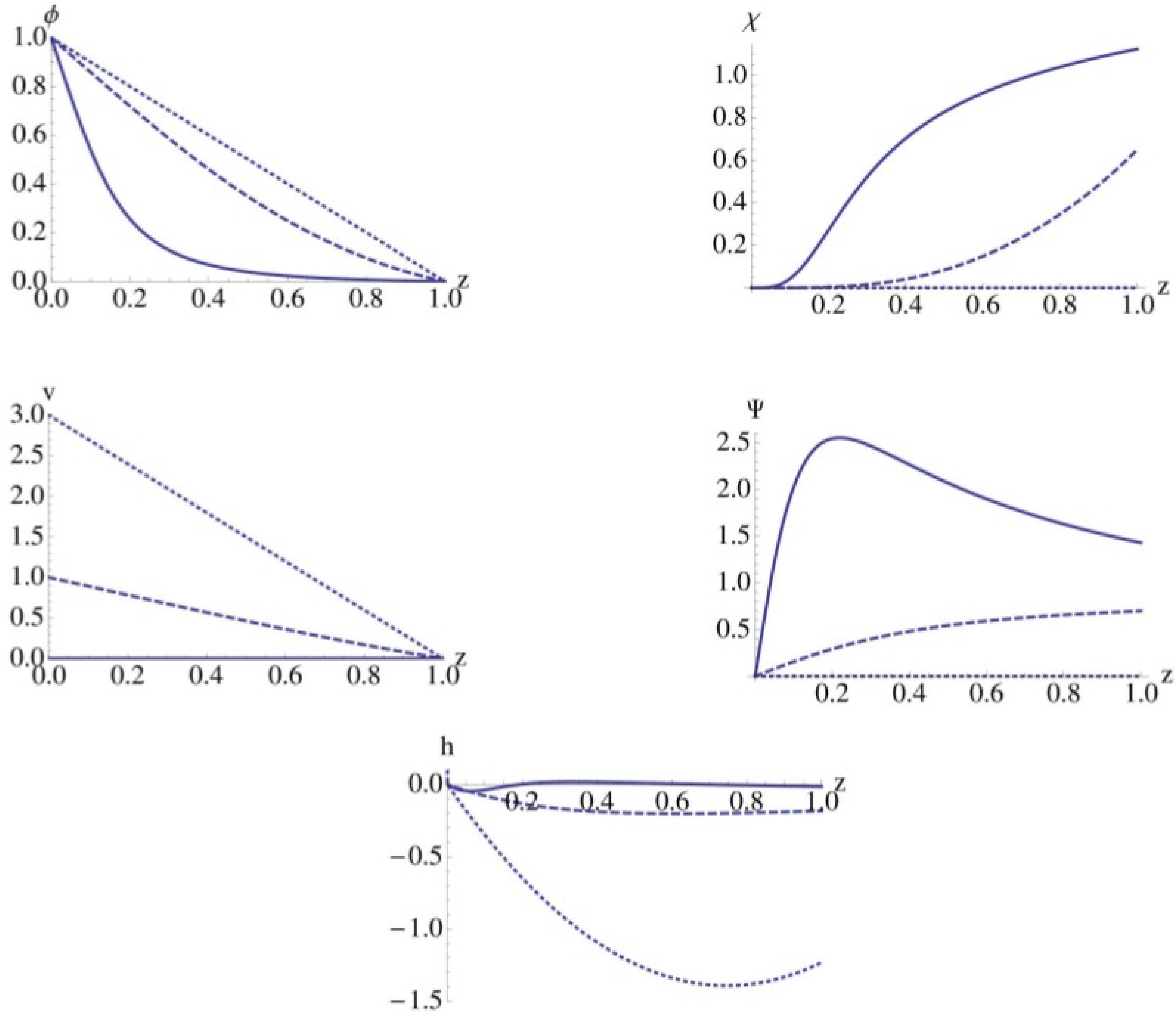
Differences in the phase diagrams:

How does the order of the phase transition depend on the interactions?

Quantum Critical Point



- Occurs for unconventional superconductors (*e.g. planar cuprates, heavy fermions,...*)
- The physics at the QCP is scale invariant → no quasi-particle description
- Strongly coupled CFT → dual AAdS gravitational model
- At T>0 (within the quantum critical region) AAdS Black Hole



ϕ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
z
χ
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
z
v
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
z
Ψ
2.5
2.0
1.5
1.0
0.5
0.2
0.4
0.6
0.8
1.0
z
h
0.0
0.2
0.4
0.6
0.8
1.0
z
−0.5
−1.0
−1.5

Equations of motion

$$\psi'' + \psi'\left(\frac{g'}{g} + \frac{2}{r} - \frac{\chi'}{2}\right) - \frac{V'(\psi)}{2g} + \frac{e^{\chi}q^2\phi^2\psi}{g^2} = 0$$

$$\phi'' + \phi'\left(\frac{2}{r} + \frac{\chi'}{2}\right) - \frac{2q^2\psi^2}{g}\phi = 0$$

$$\frac{1}{2}\psi'^2 + \frac{e^{\chi}(\phi'^2 + v'^2)}{4g} + \frac{g'}{gr} + \frac{1}{r^2} - \frac{3}{gL^2} + \frac{V(\psi)}{2g} + \frac{e^{\chi}q^2\psi^2\phi^2}{2g^2} = 0$$

$$\chi' + r\psi'^2 + r\frac{e^{\chi}q^2\phi^2\psi^2}{g^2} = 0$$

$$v'' + v'\left(\frac{2}{r} + \frac{\chi'}{2}\right) = 0$$