# Introduction to AdS/CFT

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# Overview

- **Lecture 1:** Motivation and Background (confinement, large $N$ , holographic bound, basic CFT, anti de-Sitter space)

- **Lecture 2:** Main statement of AdS/CFT, derivation, how to do computations, generalizations

# Introduction

# The AdS/CFT correspondence

The AdS/CFT correspondence is a duality (an exact equivalence) between two seemingly different theories

1. A four dimensional **quantum field theory** (a gauge theory-like QCD)

2. **Gravity** (string theory) in a higher dimensional spacetime

- Discovered in 1997 by J. Maldacena and elaborated by Gubser, Klebanov, Polyakov, Witten,...

## Applications

Use Gravity to learn about QFT

- QCD $\Rightarrow$ strong coupling phenomena (confinement, chiral symmetry breaking etc.)
- Fluid dynamics, condensed matter systems

Use QFT to learn about Gravity

- Black Holes (singularities, entropy, Hawking radiation....)
- Cosmology (Big Bang, inflation, c.c. problem, ...)

## Fundamental Physics

- (so far the only) Non-perturbative definition of string theory/quantum gravity

- **SPACE AND TIME ARE EMERGENT CONCEPTS !!!**

- Is our world a hologram?

# Origins

1. **Confinement in gauge theories, large $N$ expansion**

2. **Black Holes and Holography**

# Confinement in QCD

- QCD: $SU(3)$ gauge theory + fermions

- In the UV: quarks + gluons

- Coupling constant runs with energy scale (asymptotic freedom)

- Theory becomes strongly coupled at low energies

- Strong coupling $\Rightarrow$ "**color confinement**" : asymptotic states are $SU(3)$ singlets

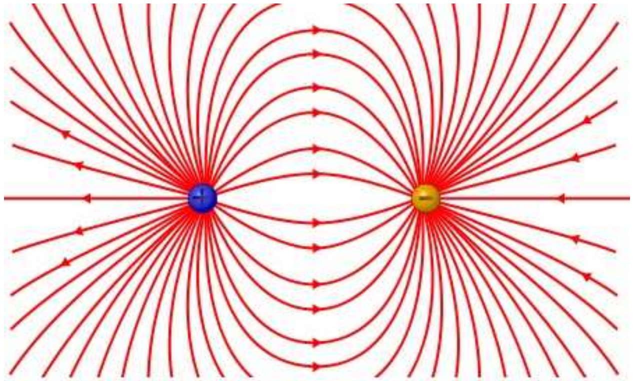
- In the IR we see mesons, baryons, glueballs etc.

- Confinement difficult to understand analytically:

**NO EXPANSION PARAMETER**

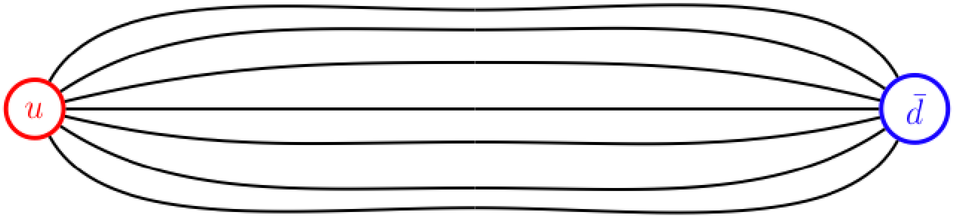
## Flux-tubes and string theory

- Consider two charges in $U(1)$ gauge theory



field lines spread out $\Rightarrow$ force $\sim 1/r^2$

- In non-abelian gauge theories




chromoelectric field independent of $r$ $\Rightarrow$ energy in field linear with r $\Rightarrow$ confinement

# Flux-tubes and string theory

- Fluxtube behaves like a string of constant tension

- Mesons can be understood as excitations of the fluxtube $\sim$ open strings (Regge trajectories)

- Glueballs $\sim$ closed strings

## The large $N$ expansion

- QCD has no obvious expansion parameter
- What if we replace $SU(3) \to SU(N)$?
- 't Hooft: theory simplifies in the large $N$ limit
- In order to have good behavior we need to scale

$$N \to \infty$$

$$g_{YM} \to 0$$

keeping

$$\lambda \equiv g_{YM}^2 N$$

fixed. The parameter $\lambda$ is called the "'t Hooft coupling"

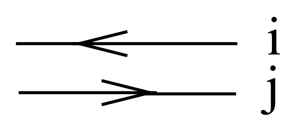
- Consider $U(N)$ gauge theory. The gauge field has the form

$$A_\mu = A_\mu^I T^I$$

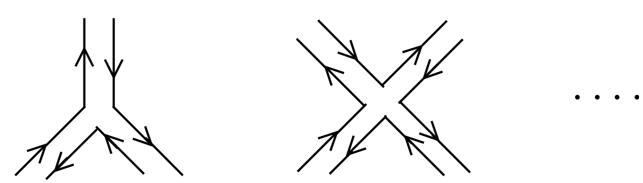
where $T^I$ are the generators of the Lie algebra in the adjoint representation

- The adjoint representation can be understood as $\overline{\mathbf{N}} \otimes \mathbf{N}$. Hence we can trade the index $I \to (i, \bar{j})$

- The gluon propagator can then be represented as

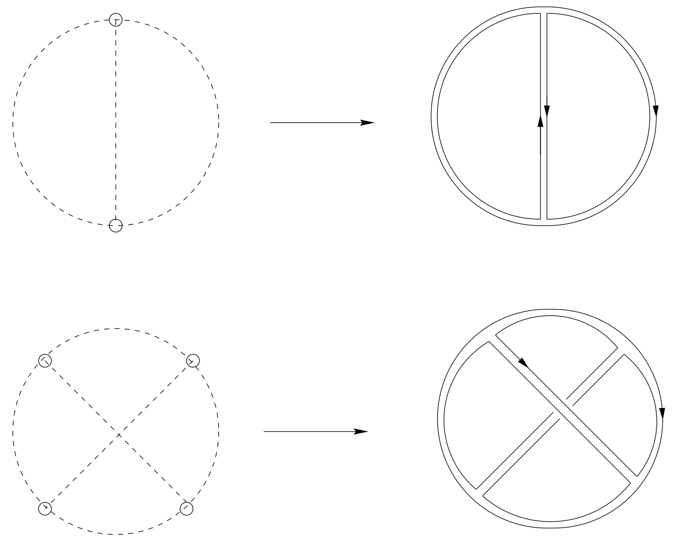



and vertices of the gauge theory are



# Double-line diagrams at large $N$

• Feynman diagrams can be rewritten as double-line diagrams where the arrows have to be connected consistently



• Different diagrams contribute with different power of $N$ in the large $N$ limit.
• The double-line notation makes the counting of factors of $N$ easier.

## Double-line diagrams as "discretized" surfaces

Gauge index loop $\Leftrightarrow$ Face of surface

Propagator $\Leftrightarrow$ Edge of surface

Interaction Vertex $\Leftrightarrow$ Vertex of surface

## Counting powers of $N$

The lagrangian has the form

$$\mathcal{L} = -\frac{1}{4g_{YM}^2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

where $1/g_{YM}^2 = \frac{N}{\lambda}$.

• Every propagator carries a factor of $\frac{\lambda}{N}$. Every vertex carries a factor of $\frac{N}{\lambda}$. The summation over each closed line gives a factor of $N$

• If we have a diagram with $V$ vertices, $E$ propagators and $F$ loops we find that it scales like

$$N^{V-E+F}\lambda^{E-V}$$

the quantity $V - E + F = \chi$ is the Euler character of a surface corresponding to the diagram

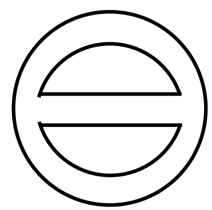
• For closed, oriented surfaces $\chi = 2 - 2g$ where $g$ is the genus

- Power of $N$ depends only on **topology** of the diagram. If $g$ is the genus then the $N$-dependence is

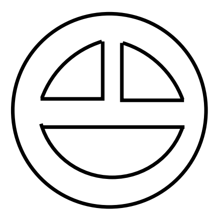
$$N^{2-2g}$$

**EXAMPLES**

**genus 0:**

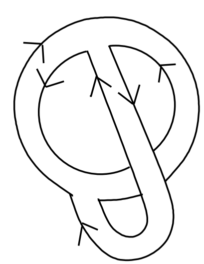


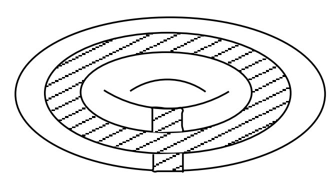
$$N^3(g^2) \sim N^2(g^2N)$$



$$N^4(g^2)^2 \sim N^2(g^2N)^2$$

**genus 1:**





Torus

$$N^0(g^2N)$$

## Simplifications at large $N$

- Only planar (genus zero) diagrams contribute

- There is a systematic $1/N$ expansion

- Gauge singlets (mesons, glueballs etc.) become stable and free

- Large $N$ limit is a "classical limit"

- While theory simplifies, still non-trivial dynamics $\Rightarrow$ we still have confinement

## Large $N$ expansion and string theory

• The genus expansion of large $N$ gauge theories $\sim$ genus expansion of string theory, if we identify

$$\frac{1}{N} \sim g_s$$

• This suggests that a large $N$ gauge theory is dual to a string theory

• At large $\lambda$ the "holes" in double line diagrams close $\Rightarrow$ they become smooth surfaces (string worldsheet)

• String theory is inconsistent in four-dimensions, hence the dual string theory lives in higher dimensions

- Schwarzchild black hole:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

- Event Horizon at $r = 2GM$, singularity at $r = 0$.
- Black Hole dynamics + Hawking radiation $\Rightarrow$

**BLACK HOLES HAVE ENTROPY**

$$S = \frac{A}{4G}$$

This has far-reaching implications for the nature of space-time.

## The holographic bound

• Consider region of spacetime of size $R$. How many degrees of freedom do we need to describe it?

• $\#$ of degrees of freedom $\sim$ (maximal) entropy contained in region.

• In conventional local systems we entropy scales like **volume**

$$S \sim R^3$$

# A Gedanken experiment

- Imagine adding matter to region. Entropy cannot decrease.

- If sufficient amount of matter $\Rightarrow$ collapse and black hole formation

- Entropy of final black hole goes like the **area** of the region!

$$S \sim R^2$$

in contrast our expectations for systems with local degrees of freedom

## Quantum gravity is holographic

• Black hole entropy + 2nd law of thermodynamics $\Rightarrow$ in theories of gravity $\#$ of degrees of freedom scales like the area, not volume

How is this possible?

• A natural mechanism to guarantee this would be to assume that somehow the degrees of freedom necessary to describe physics in a region $M$, live on the boundary of the region $\partial M$.

• These degrees of freedom on the boundary completely encode what happens in the interior.

• **Gravity is holographic.**

## AdS/CFT: gravity and gauge theories

The two aforementioned ideas

1. That large $N$ gauge theories can be described by string theories.
2. That quantum gravity is holographic.

have found a precise realization with the discovery of the AdS/CFT correspondence

**Large $N$ gauge theory in $d$ dimensions**

$$\Leftrightarrow$$

**Quantum gravity (string theory) in $\geq d+1$dimensions**

- Applications and fundamental physics
- Simplest case: gauge theory is conformal and gravity is in AdS
- $\mathcal{N} = 4\ \mathrm{Yang-Mills} \quad \Leftrightarrow \quad \mathrm{IIB\ string\ theory\ on\ AdS_5 \times S^5}$

# Conformal Field Theory

# Scale invariance

• Most QFTs have scales (masses, couplings, etc.) $\Rightarrow$ non-trivial RG-flow

• Cutoff $\Rightarrow$ quantum violation of scale invariance

• Dynamically generated scales (like $\Lambda_{QCD}$)

**HOWEVER**

• There are QFTs which have no scale and where $\beta = 0 \Rightarrow$ exact scale invariance.

• New symmetry generator: dilatation operator $D$

$$[D, P_\mu] = -iP_\mu, \qquad [D, M_{\mu\nu}] = 0$$

- In most cases scale invariant QFTs are invariant under a larger symmetry group, the conformal group

- In addition to $P_\mu, M_{\mu\nu}$ and $D$ it contains new symmetry generators

$$\text{special conformal transformations} \qquad K_\mu$$

$$\begin{aligned} \text{Poincare} \qquad & x^\mu \to \Lambda^\mu_\nu \, x^\nu + a^\mu \\ \text{scale} \qquad & x^\mu \to \lambda \, x^\mu \\ \text{special conformal} \qquad & x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2b\cdot x + b^2 x^2} \end{aligned} \tag{1}$$

- In $d$ spacetime dimensions the conformal group is isomorphic to $SO(d,2)$.

# No S-matrix in CFTs, correlation functions

• In theories with no mass gap $\Rightarrow$ no well defined asymptotic states

• $\Rightarrow$ No S-matrix. What are the physical observables?

• Correlation functions of "local operators" (gauge invariant ones)

$$\phi^k, \quad \phi\partial_\mu\phi, \quad F_{\mu\nu}F^{\mu\nu}, \qquad \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right), \quad \ldots$$

• In CFTs we would like to compute

$$\langle \mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)\rangle$$

where $\mathcal{O}_i$ are local operators like those mentioned above

• "Solving the CFT" $\Leftrightarrow$ computing such correlation functions

## Classification of local operators

- Usual QFT $\Rightarrow$ classify states under Poincare group
- in CFT $\Rightarrow$ classify local operators under conformal group

$$[D, \mathcal{O}(0)] = -i\Delta\mathcal{O}(0)$$

$\Delta$ is the "conformal dimension of the operator".

From the algebra we have

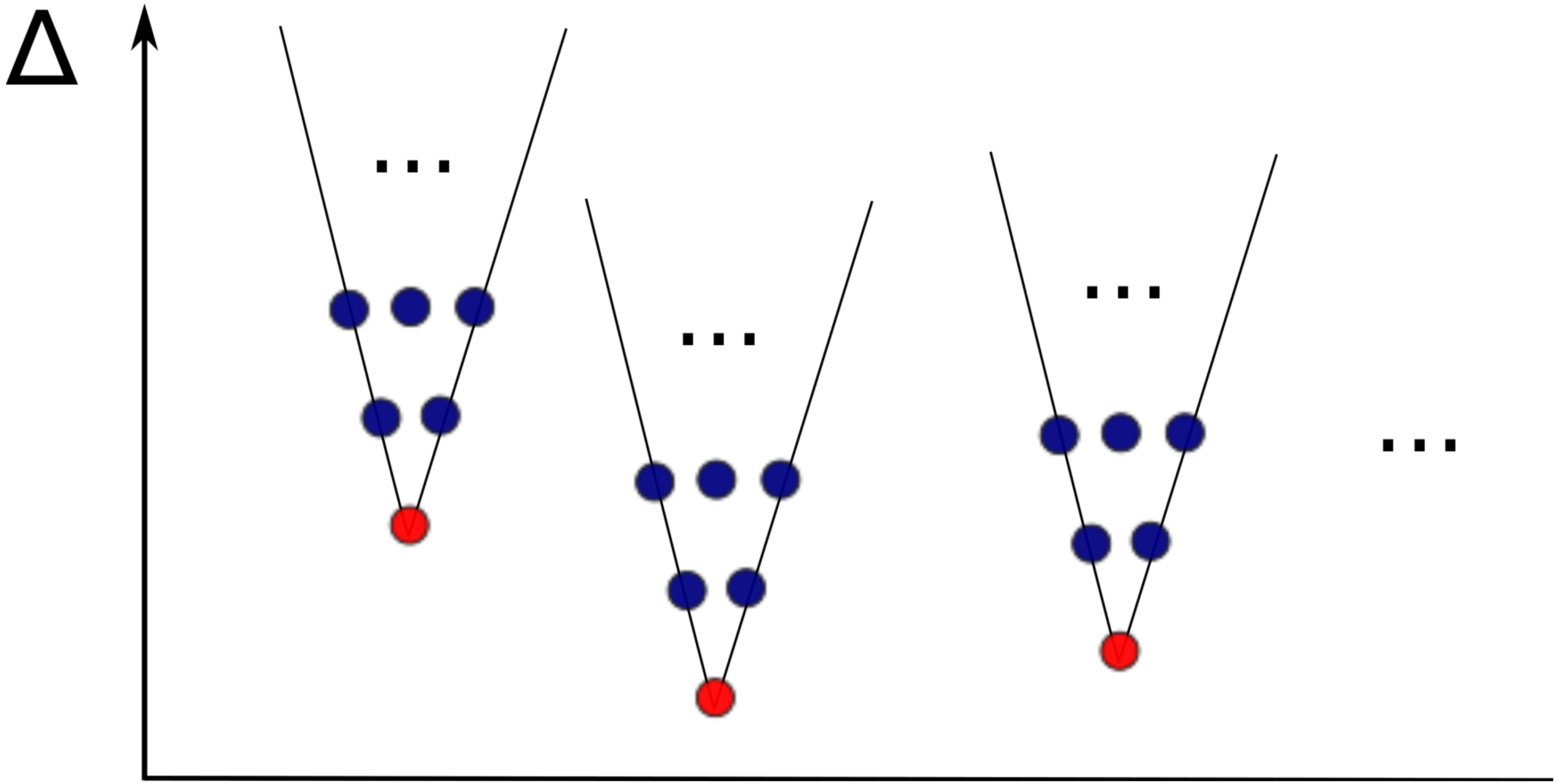
$$[D, P_\mu] = -iP_\mu, \qquad [D, K_\mu] = iK_\mu$$

so $P_\mu$ raises the dimension of an operator while $K_\mu$ lowers it.

- Local operators annihilated by the $K_\mu$'s are called **conformal primaries**. The are characterized by $\Delta$ and their spin.

- All other local operators can be derived from primaries by acting with $P_\mu \sim -i\partial_\mu$. They are called **descendants**.

# Spectrum of operators in CFT



# Correlation functions in CFTs

• Conformal invariance fixes form of 2-point function of conformal primaries

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

• and also the 3-point function

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\rangle = \frac{C}{|x-y|^{\Delta}|y-z|^{\Delta}|x-z|^{\Delta}}$$

• 4- and higher-point correlation functions are constrained but NOT fixed by conformal invariance

## The $\mathcal{N} = 4$ SYM theory

- 4d QFT with maximum amount of supersymmetry (16 supercharges). The field content is

$$\begin{array}{lll} \text{gauge field} & A_\mu & \\ \text{fermions} & \lambda^i, & i = 1, ..., 4 \\ \text{scalars} & \Phi^I, & I = 1, ..., 6 \end{array} \tag{2}$$

all in the adjoint of the gauge group $G$.

- The Lagrangian of the theory has the schematic form

$$\mathcal{L} = -\frac{1}{4g^2} \text{Tr} \left( F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi^I)^2 + \overline{\lambda} \not{D} \lambda + [\Phi^I, \Phi^J]^2 + \ldots \right)$$

- For given gauge group $G \Rightarrow$ Unique 4d QFT with $\mathcal{N} = 4$ SUSY

## Basic properties of the $\mathcal{N} = 4$ SYM

- The theory is conformal ($\beta$ function is exactly zero).

- Theory is invariant under the superconformal group. Its bosonic subgroup is

$$SO(4,2) \times SO(6)$$

The $SO(6) = SU(4)$ is the R-symmetry of the theory.

- Exact $SL(2,Z)$ duality. Define complexified coupling $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$. Theory invariant under

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad (a,b,c,d) \in SL(2,Z)$$

# Anti de-Sitter space

## Geometry of AdS

- AdS$_{d+1}$ is the maximally symmetric spacetime in $d+1$ dimensions

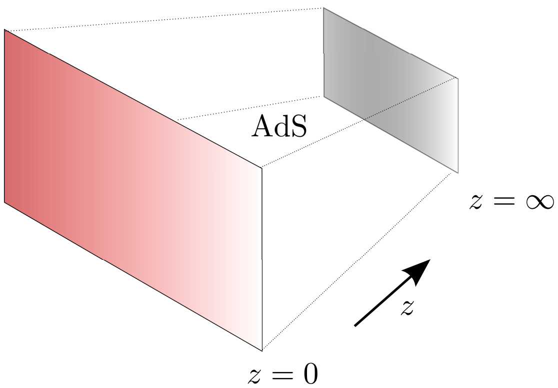
- It has constant negative curvature and is a solution of Einstein equations with negative cosmological constant

- The isometry group of AdS$_{d+1}$ is $SO(d,2)$ (notice that it coincides with the conformal group in $d$ dimensions!)

- There are various coordinate systems which can be used, each with its own advantages

• One useful coordinate system is the so-called "Poincare patch", where the coordinates are $(z, t, \vec{x})$ and the metric has the form

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

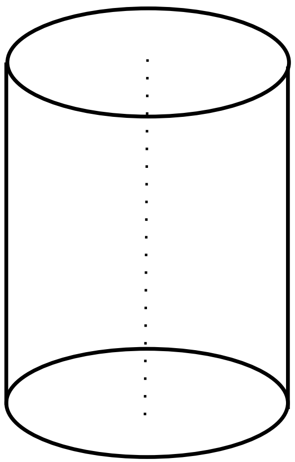



• We have Minkowski-space slices along $t, \vec{x}$ which are **warped** along the direction $z$

• Only the Poincare invariance along $d$ directions and scaling is manifestly visible (not the full isometry group $SO(d, 2)$).

• Another useful coordinate system is the "global patch", with coordinates $t, \rho, \Omega_{d-1}$ and the metric

$$ds^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_{d-1}^2$$



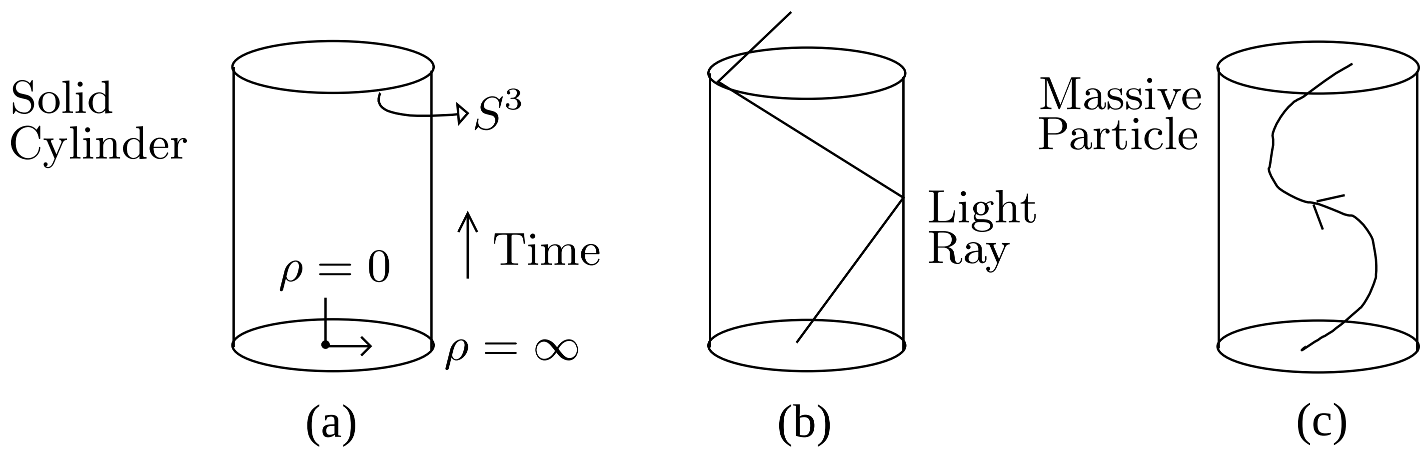
Now the $SO(d) \times \mathbf{R}$ isometry is manifest

# Particles in AdS

- Attractive gravitational potential towards the "center" i.e. $\rho = 0$

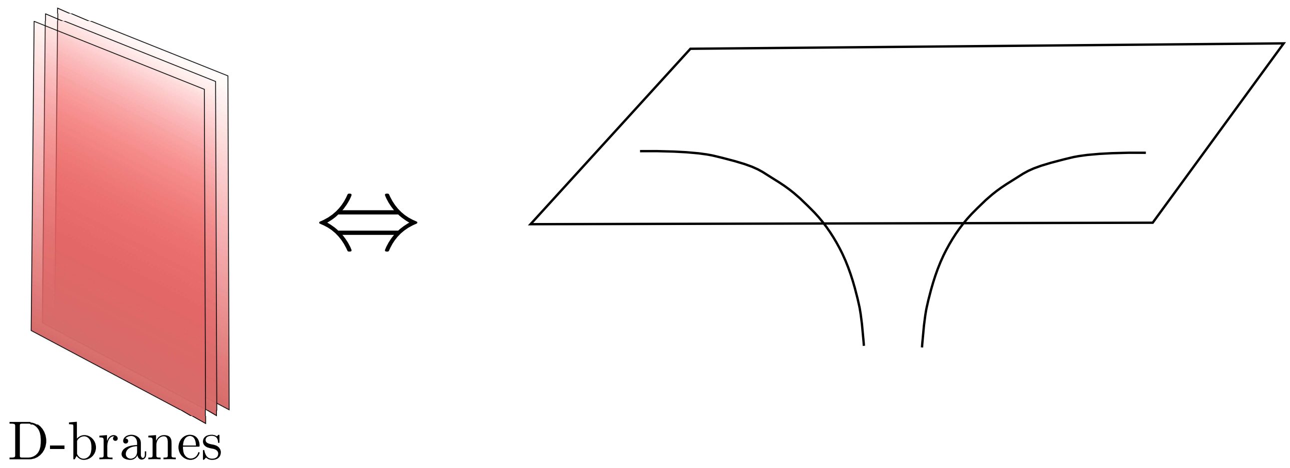
- Penrose diagram of AdS $\Rightarrow$ "conformal boundary is $S^{d-1} \times R$

- Massless particles can reach the boundary $(\rho = \infty)$ in finite time, massive particles never reach the boundary



## String theory on AdS

- (Super)string theory is consistent in 10 dimensions
- Consider the space $\mathrm{AdS}_5 \times \mathrm{S}^5$ (with $F_5$ flux)
- It is a consistent background for IIB string theory
- At low energies $\Rightarrow$ IIB supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$
- Is equivalent to the 4d $\mathcal{N} = 4$ gauge theory



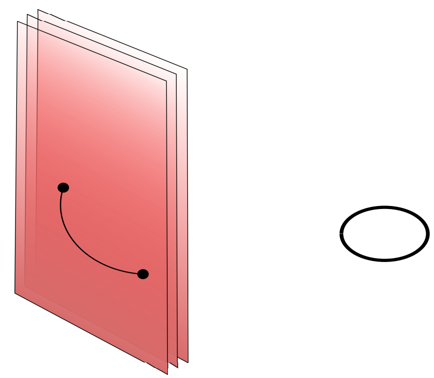

- For example, 3-brane solution

$$ds^2 = \left(1 + \frac{R^4}{r^4}\right)^{-1/2}(-dt^2 + d\vec{x}^2) + \left(1 + \frac{R^4}{r^4}\right)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

- Same R-charge, mass, SUSYs...
- Scattering computations

## The decoupling argument

- Consider IIB string theory in 10d flat space and stack of $N$ D3-branes

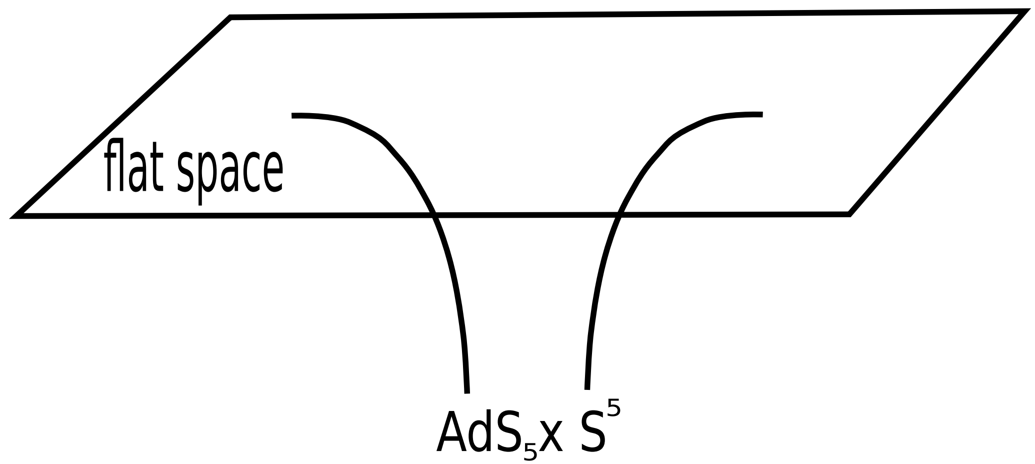


D-branes

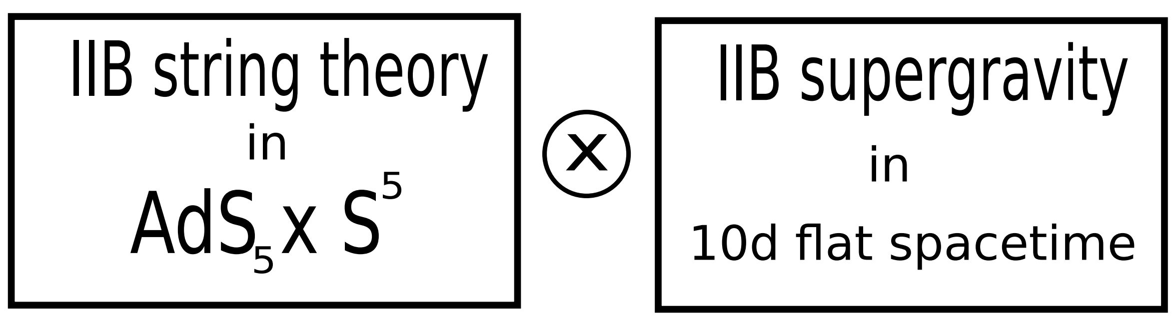
- Low energy limit $\Rightarrow$

N=4 Super Yang-Mills in 4d $\otimes$ IIB supergravity in 10d flat spacetime

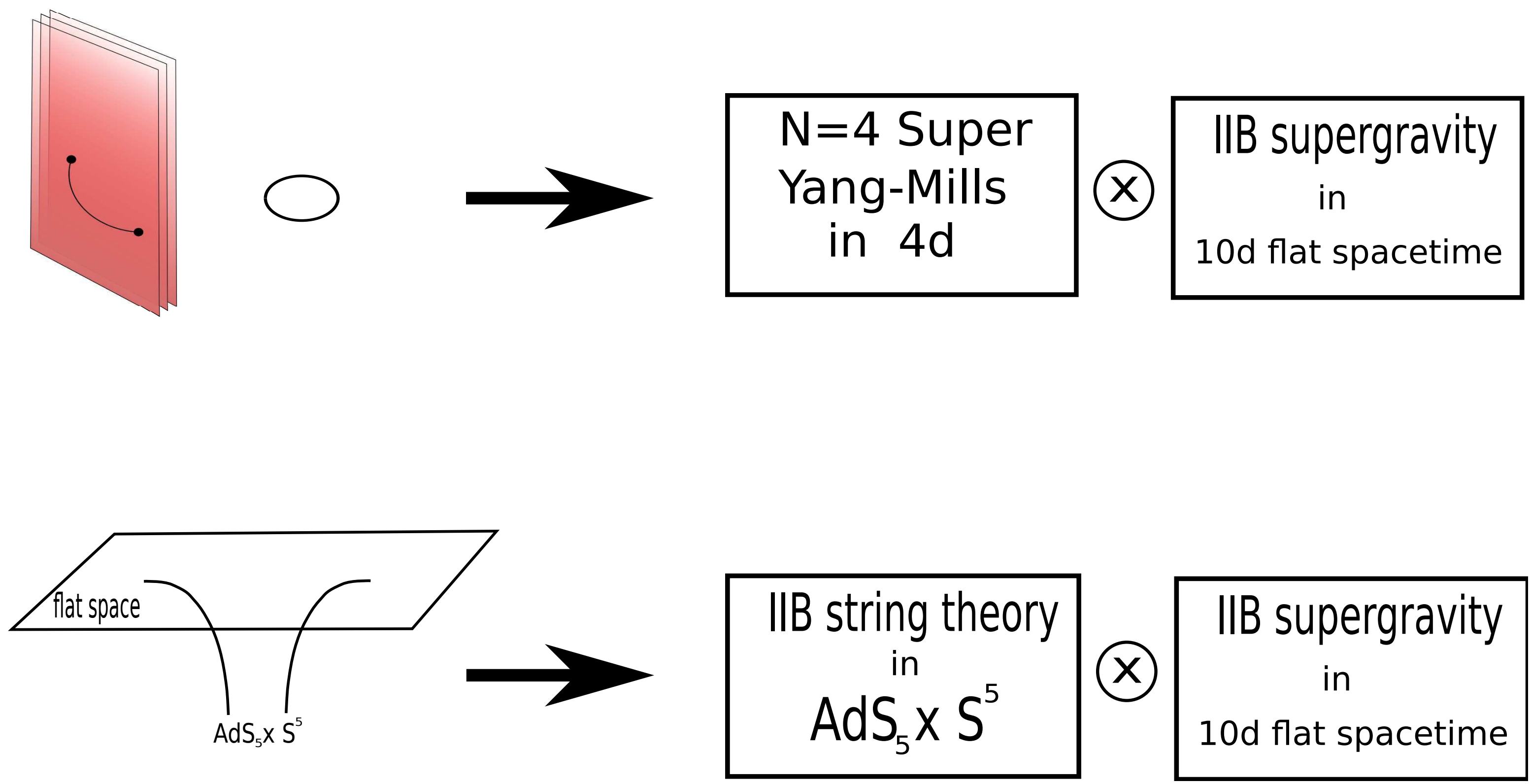
- Supergravity solution for 3-branes




- Low energy limit ⇒



# The decoupling argument



# Main statement

$\mathcal{N} = 4$ SYM gauge theory in 4d

$\Leftrightarrow$

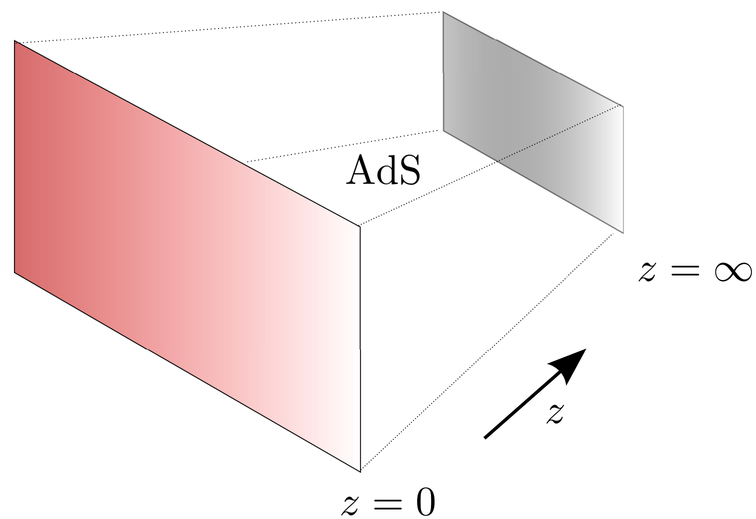
IIB string theory in $AdS_5 \times S^5$

**Parameters:**

| | | |
|---|---|---|
| $N$ units of $F_5$ flux | $\Leftrightarrow$ | $SU(N)$ gauge group |
| $g_s$ | $\Leftrightarrow$ | $g_{YM}^2$ |

- Anti de Sitter space (AdS)

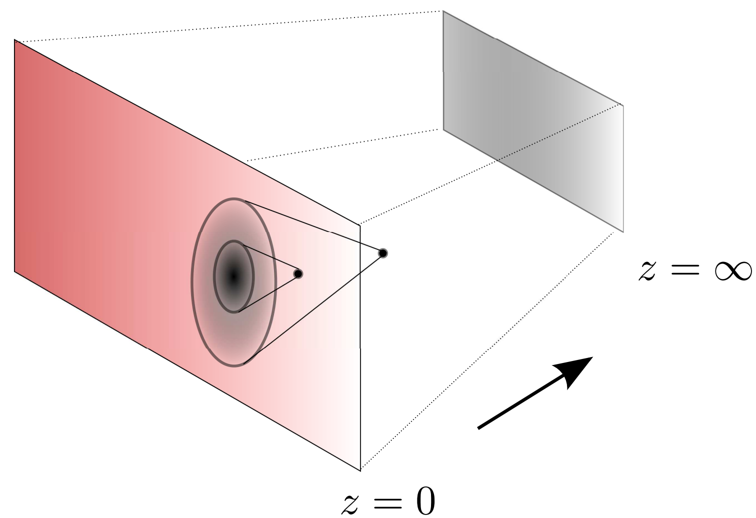
$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$




- Quantum filed theory lives on the the "boundary" of AdS ($z = 0$).

## RG-flow and the holographic dimension

- Extra dimension ⇒ "scale" in quantum field theory



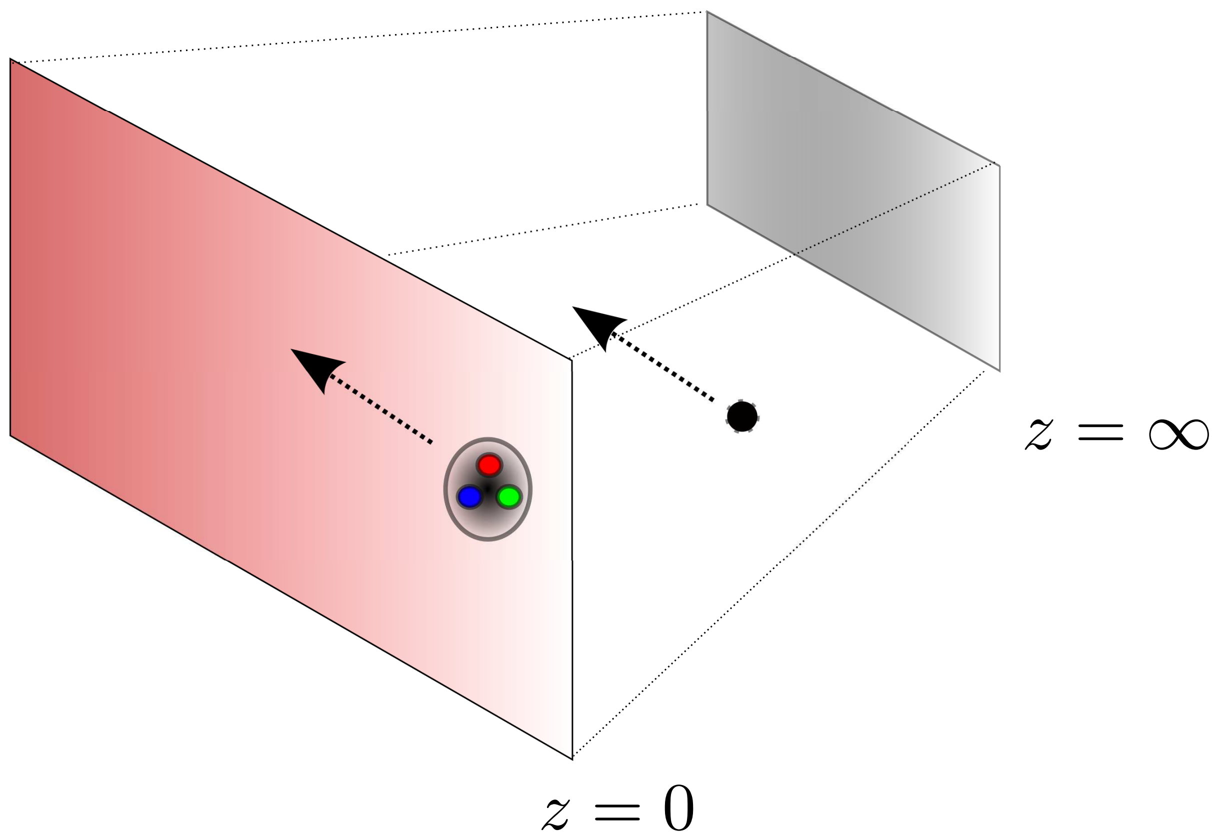

- Radial evolution ⇒ RG-flow

- Scale invariant theories ⇒

AdS geometry $ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$
invariant under $(t, \vec{x}, z) \to (\lambda t, \lambda \vec{x}, \lambda z)$.

# Hilbert spaces

- Particles in AdS $\Leftrightarrow$ "glueballs, hadrons... in gauge theory




- Matching of spectrum, scattering

# Basic Identifications

| STRING THEORY | | GAUGE THEORY |
|---|---|---|
| $SO(2,4)$ Isometry of $\mathrm{AdS}_5$ | $\Leftrightarrow$ | $SO(2,4)$ Conformal group in 4d |
| $SO(6) \approx SU(4)$ Isometry of $\mathrm{S}^5$ | $\Leftrightarrow$ | $SU(4)$ R Symmetry of $\mathcal{N}=4$ |
| 32 supercharges | $\Leftrightarrow$ | $32 = 16 + 16(Q's \text{ and } S's)$ |
| $\tau = C_0 + ie^{-\phi}$ | $\Leftrightarrow$ | $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g_{YM}^2}$ |
| $SL(2,Z)$ duality of IIB | $\Leftrightarrow$ | $SL(2,Z)$ duality of $\mathcal{N}=4$ |

# Regimes

- Working in the 't Hooft limit $N \to \infty$ , $\lambda = g_{YM}^2 N = \text{fixed}$ we find

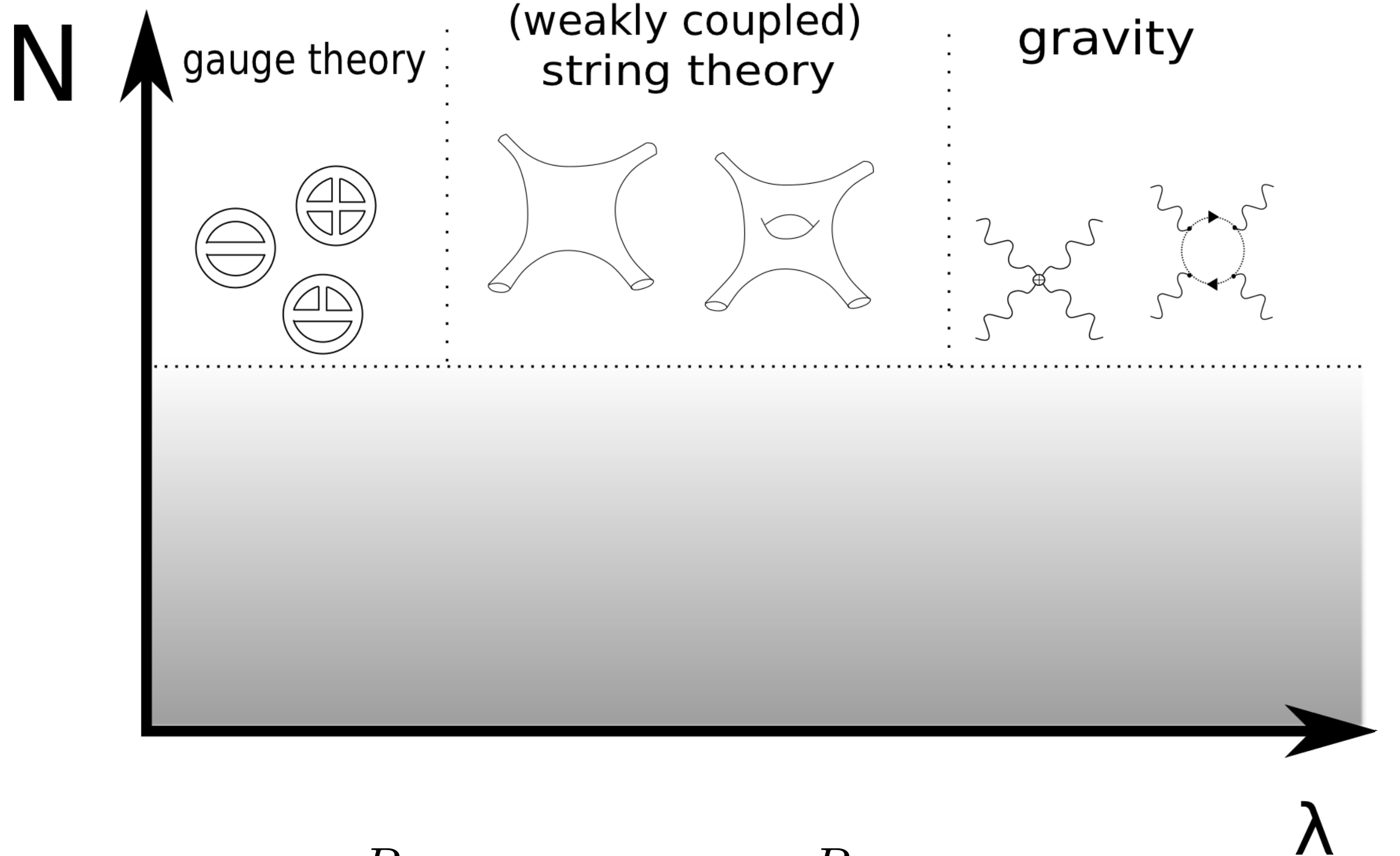
$$g_s \sim \frac{1}{N}$$

- Call $R$ the radius of AdS$_5$. Then we have

$$\frac{R}{l_s} \sim \lambda^{1/4}$$

$$\frac{R}{l_p} \sim N^{1/4}$$

# Regimes




$$\frac{R}{l_s} \sim \lambda^{1/4} \quad , \quad \frac{R}{l_p} \sim N^{1/4}$$

# Field-Operator correspondence

- Fields in gravity $\Leftrightarrow$ local operators in dual QFT

$$\phi(x,z) \quad \Leftrightarrow \quad \mathcal{O}(x)$$

$$A_\mu(x,z) \quad \Leftrightarrow \quad J_\mu(x)$$

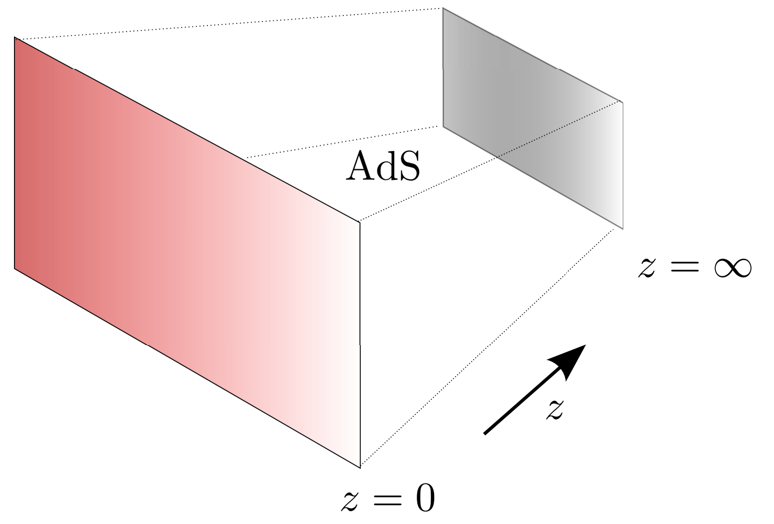
$$g_{\mu\nu}(x,z) \quad \Leftrightarrow \quad T_{\mu\nu}(x)$$

- Precise mapping of quantum numbers. For example, for scalars we have

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + (mR)^2}$$

for $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

## Correlation functions




Boundary value $\tilde{\phi}(x) = lim_{z \to 0} \phi(x, z) \Leftrightarrow$ "source" for dual operator $\mathcal{O}$ in QFT

$$\langle e^{\int dx \tilde{\phi}(x) \mathcal{O}(x)} \rangle_{QFT} = \mathcal{Z}_{string}(\phi \to \tilde{\phi})$$

# Correlation functions

- In the gravity limit $(N, \lambda \gg 1)$ we have

$$\langle e^{\int dx \widetilde{\phi}(x) \mathcal{O}(x)} \rangle_{QFT} = e^{-S_{gravity}(\widetilde{\phi})}$$

Which means

1. Fix boundary values $\tilde{\phi}$
2. Find classical solution of EOMS with this boundary value
3. Evaluate classical action on this solution

- This gives us $S_{gravity}(\widetilde{\phi})$ and by the equation above, the generating functional of CFT correlators

**QUANTUM correlators in strongly coupled QFT from** (semi-) **CLASSICAL gravitational computations !**

## Example, 2-point functions

- Consider scalar field obeying

$$(\Box - m^2)\phi = 0$$

Look for solutions of the form $\phi(x,z) = f(z)e^{ikx}$.

- Two linearly independent solutions, for $z \to 0$

$$f(z) = A(k)\left(z^{d-\Delta} + \dots\right) + B(k)\left(z^{\Delta} + \dots\right)$$

where $\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + (mR)^2}$.

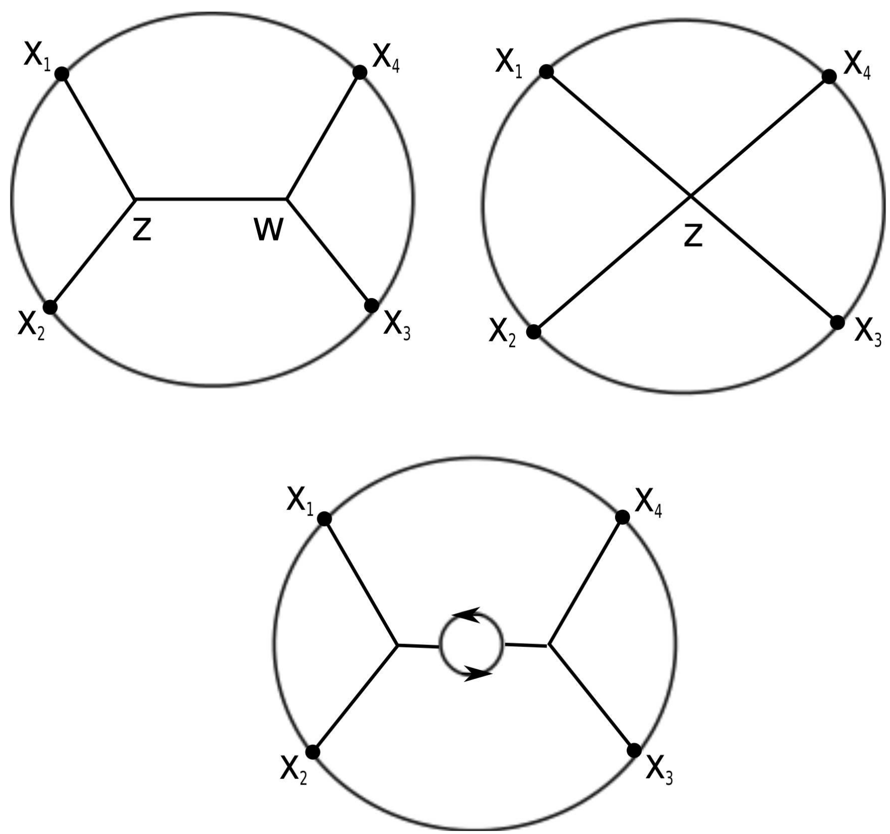
- CFT 2-point function (in momentum space)

$$G(k) = \frac{B(k)}{A(k)}$$

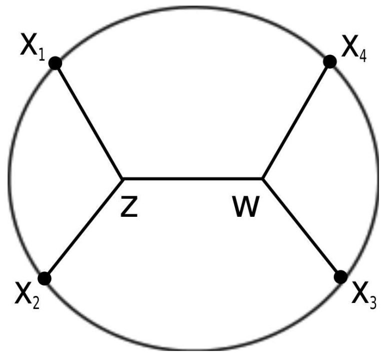
- Imposing regularity in the interior fixed this ratio.

# Witten Diagram Expansion

• More general correlators can be computed by semi-classical gravity Feynman diagrams in AdS = “Witten diagrams”




• Loop expansion $\sim 1/N$ expansion




- Bulk-to-boundary propagator

$$K(x,z) = \left(\frac{z_0}{z_0^2 + (\vec{z}-\vec{x})^2}\right)^{\Delta}$$

- Bulk-to-bulk propagator

$$G(z,w) = s^{\Delta}\,{}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}, \Delta-1; s^2\right)$$

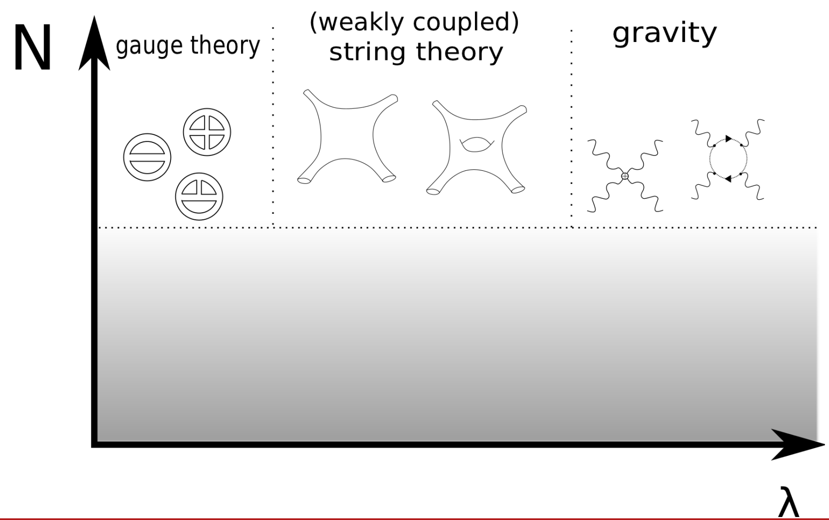
- Full diagram is

$$\int \frac{d^{d+1}z}{z_0^{d+1}} \int \frac{d^{d+1}w}{w_0^{d+1}} K(x_1,z)K(x_2,z)G(z,w)K(x_3,w)K(x_4,w)$$

# Matching of the spectrum

- Field-operator correspondence $\phi \Leftrightarrow \mathcal{O}$.
- Mass-dimension relation $mR \sim \Delta$
- Spectrum of IIB string theory on AdS$_5 \times$S$^5$ should match with spectrum of operators in $\mathcal{N} = 4$ SYM

At what value of the coupling????



## Spectrum at strong coupling

- For $\lambda \gg 1$ we have IIB string theory on AdS$_5\times$S$^5$ with $R/l_s \sim \lambda^{1/4}$

- We expect massive string states with $m_s R \sim \lambda^{1/4}$, so the corresponding operators have

$$\Delta_{stringy} \sim \lambda^{1/4}, \qquad \lambda \gg 1$$

- Massless modes of IIB string theory $\Rightarrow$ IIB SUGRA $\Rightarrow$ Kaluza-Klein tower of supergravitons on $S^5$ $\Rightarrow$, of mass $m_{KK} \sim \frac{k}{R}$ $\Rightarrow$ tower of supermultiplets with

$$\Delta = k, \qquad k = 2, 3, \ldots$$

## Spectrum at weak coupling

- Fundamental fields of $\mathcal{N}=4$ SYM are $A_\mu, \lambda^i, \Phi^I$ in adjoing of $SU(N)$

- We want **local, gauge invariant** operators

- Traces of products of fields (single-trace operators)

$$\mathrm{Tr}(\Phi^I \Phi^J \lambda_k \ldots), \quad \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu} \Phi^I \Phi^J \Phi^K ...) \ldots$$

or

- Products of traces (multi-trace operators)

$$\mathrm{Tr}(\Phi^I \Phi^J)\mathrm{Tr}(\Phi^I \Phi^J \Phi^K)\mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}), \ldots$$

- Single- and multi-particle states in gravity

- Counting single-trace operators at weak coupling

- Toy model: consider 2 adjoint valued fields $A, B$ of dimension $\Delta = 1$.

- We have single trace operators of the form

$$\mathrm{Tr}(AAA), \mathrm{Tr}(ABBA), \ldots$$

- At weak coupling conformal dimensions are additive. Ask: how many single trace operators are there of dimension $\Delta$ ?

$$\mathrm{Tr}(AAABBBA.....BBABA)$$

with $\Delta$ "letters".

## Strings from gauge fields

• Counting single-trace operators at weak coupling

• Toy model: consider 2 adjoint valued fields $A, B$ of dimension $\Delta = 1$.

• We have single trace operators of the form

$$\mathrm{Tr}(AAA), \mathrm{Tr}(ABBA), \ldots$$

• At weak coupling conformal dimensions are additive. Ask: how many single trace operators are there of dimension $\Delta$ ?

$$\mathrm{Tr}(AAABBBA.....BBABA)$$

with $\Delta$ "letters". Number of such operators grows like $2^{\Delta}$

• Exponential growth in  of states (Hagedorn growth) $\Rightarrow$ Stringy spectrum!!

• When we increase $\lambda$ the dimensions of single-trace operators become $\Delta(\lambda)$

**Short vs Long multiplets**

• $\mathcal{N} = 4$ algebra and unitarity imply "BPS bound"

$$\Delta \geq R$$

operators with $\Delta = R$ are "short" or "BPS" operators and receive no quantum corrections. For them $\Delta$ independent of $\lambda$

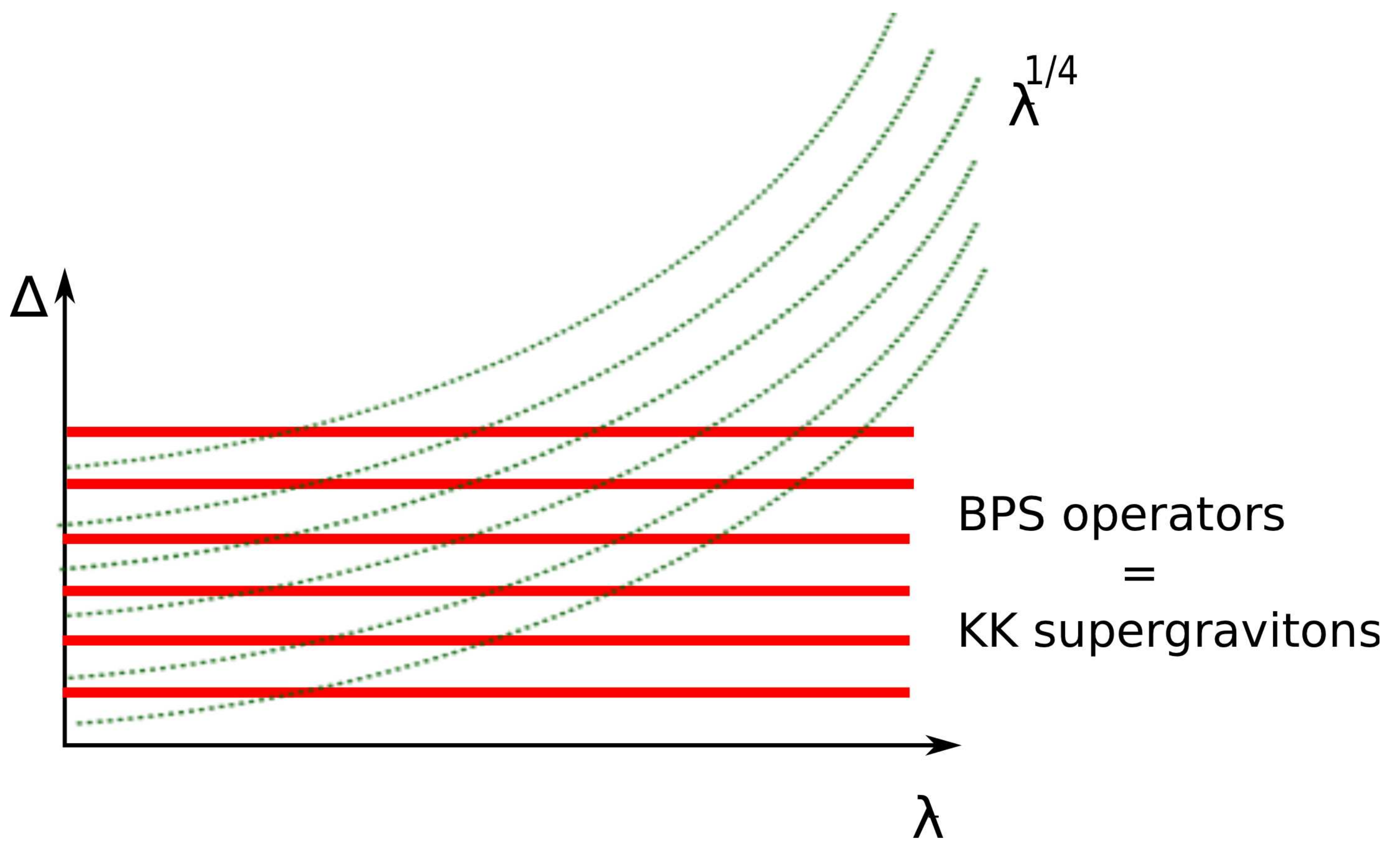
operators with $\Delta > R$ are "long" and $\Delta$ varies with $\lambda$

• In the $\mathcal{N} = 4$ SYM the BPS operators are the chiral primaries

$$\mathrm{Tr}(Z^k), \qquad k = 2, 3, 4...$$

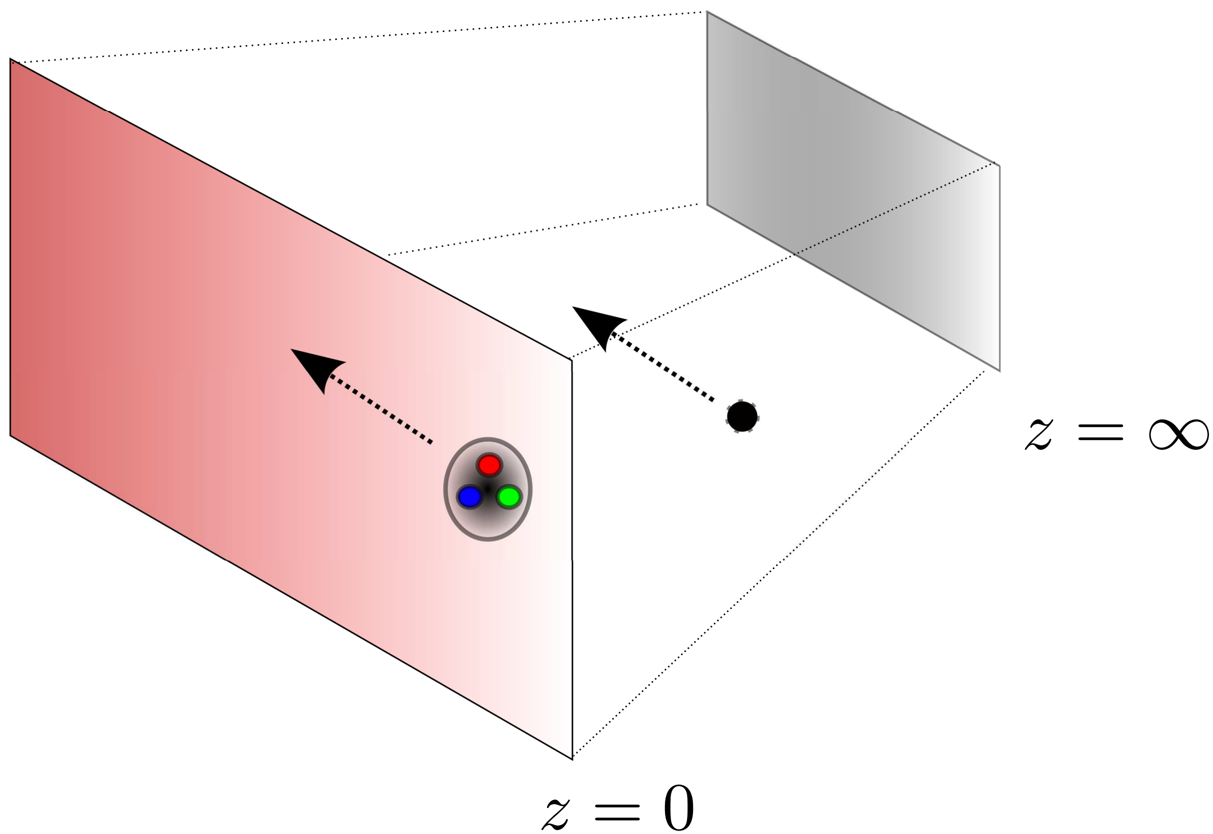
with $\Delta = k$. Here $Z = \Phi^1 + i\Phi^2$.

# Picture of the spectrum



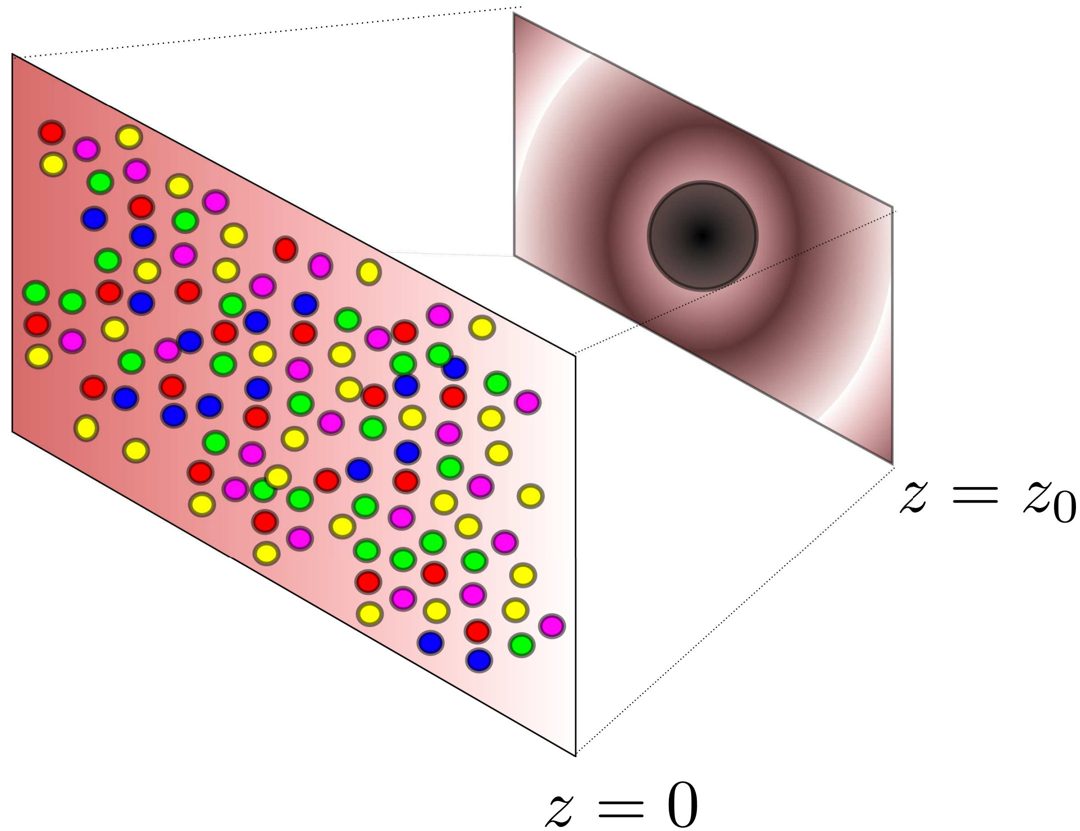
# Hilbert spaces

- Particles in AdS ⇔ "glueballs, hadrons... in gauge theory

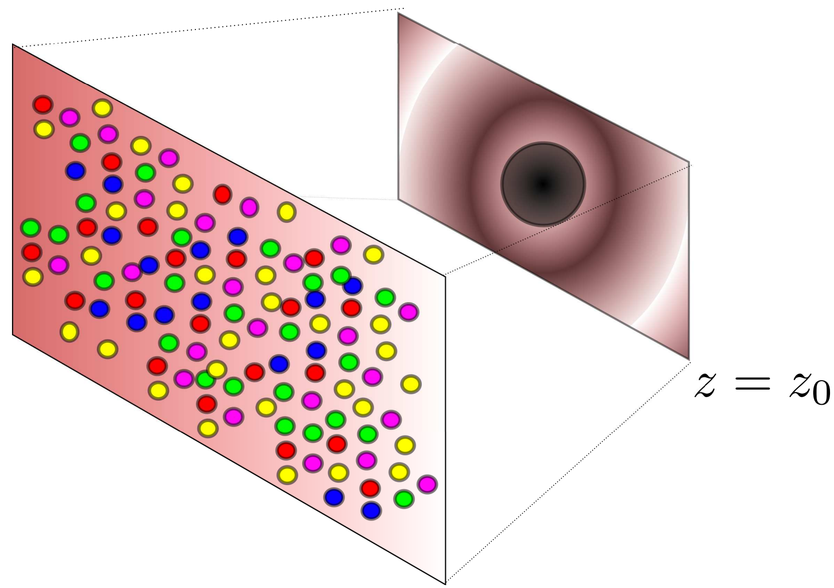


# Black Holes and Quark Gluon Plasma

- Black Hole in AdS $\Leftrightarrow$ Quark Gluon Plasma in the gauge theory

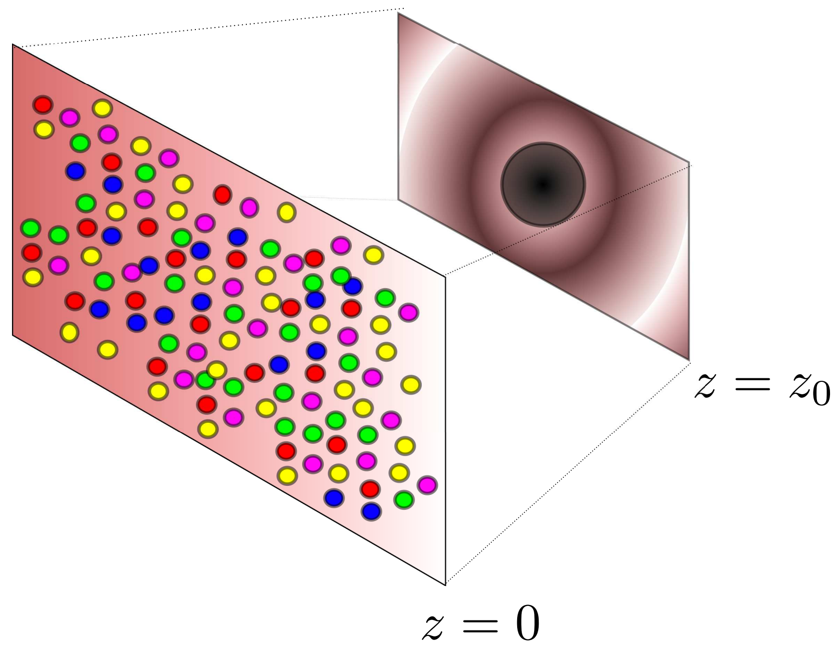


# Black Holes and Quark Gluon Plasma




$$ds^2 = -\frac{f(z)}{z^2}dt^2 + \frac{1}{f(z)z^2}dz^2 + \frac{1}{z^2}d\vec{x}^2 \quad , \quad f(z) = 1 - \frac{z^4}{z_0^4}$$

- Temperature of plasma ⇔ Hawking temperature of BH
- Energy of plasma ⇔ Mass of BH
- Entropy of plasma ⇔ Horizon Area

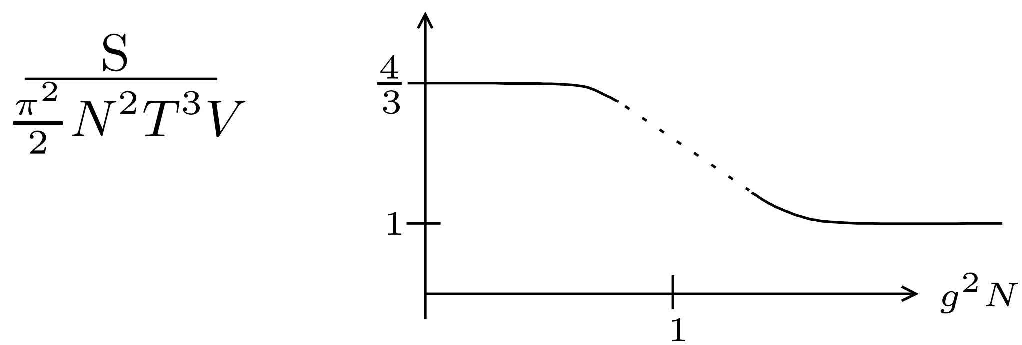



⇒ Equation of State for quark-gluon plasma at strong coupling!

$$S = \frac{\pi^2}{2} N^2 T^3 V$$

⇒ Understanding microscopic structure of black hole quantum states

• Estimate black hole entropy from weakly coupled gauge theory




• Correct entropy up to a factor of 3/4 !

• Numerical simulation of (lower dimensional examples of) gauge theory $\Rightarrow$ exact agreement

# More on Black Holes

- Thermal correlation functions
- Real-time processes, transport coefficients
- QGP formation

...

- Hawking radiation/Information paradox
- Infalling observer

...

## Generalizations

- Higher/lower dimensions
- Non-conformal
- Less/Non supersymmetric
- Finite $N$?

...

- flat, de Sitter ??

Thank you!