Models of Non-Minimal Chaotic Inflation in Supergravity

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Based on:

- C. Pallis and N. Toumbas, *J. Cosmol. Astropart. Phys.* **02**, *019 (2011)* [arXiv:1101.0325];
- C. Pallis and N. Toumbas, *J. Cosmol. Astropart. Phys.* **12**, *002 (2011)* [arXiv:1108.1771];
- C. Pallis and Q. Shafi, *Phys. Rev. D* **86**, *023523 (2012)* [arXiv:1204.0252].

Outline

Introduction
- Non-Minimal Inflation (nMI)
- Realization of nMI Within SUGRA

Quartic Potential for a Gauge Singlet Inflaton
- The Inflationary Potential
- The Inflationary Observables - Results

Quartic Potential for a Gauge non-Singlet Inflaton
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Quadratic Potential
- The Inflationary Potential
- The Inflationary Observables - Results

Conclusions

Coupling non-minimally the Inflaton to Gravity (The non-SUSY Case)

• The Action Of A Scalar Field ϕ with Potential $V(\phi)$ non-Minimally Coupled to the Ricci Scalar Curvature, $\mathcal{R}$, Through A Frame Function $f_{\mathcal{R}}(\phi)$ in the Jordan Frame (JF) is:

$$\mathcal{S} = \int d^4x \sqrt{-g}\left(-\frac{1}{2}m_{\rm P}^2 f_{\mathcal{R}}(\phi)\mathcal{R} + \frac{f_K(\phi)}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),$$

where g is the Determinant Of The Background Friedmann-Robertson-Walker Metric and $f_{\mathcal{R}}(\langle\phi\rangle) \simeq 1$ to Recover Einstein Gravity At Low Energy. We Allow Also for a Kinetic Mixing Through the Function $f_K(\phi)$.

• We can write $\mathcal{S}$ in the Einstein Frame (EF) as follows

$$\mathcal{S} = \int d^4x \sqrt{-\widehat{g}}\left(-\frac{1}{2}m_{\rm P}^2\widehat{\mathcal{R}} + \frac{1}{2} f_K \widehat{g}^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi} - \widehat{V}\left(\widehat{\phi}\right)\right)$$

Performing a Conformal Transformation[1] During Which We define the EF metric:

$$\widehat{g}_{\mu\nu} = f_{\mathcal{R}}\, g_{\mu\nu} \;\Rightarrow\; \begin{cases} \sqrt{-\widehat{g}} = f_{\mathcal{R}}^2\sqrt{-g} \;\; \text{and} \;\; \widehat{g}^{\mu\nu} = g^{\mu\nu}/f_{\mathcal{R}}, \\ \widehat{\mathcal{R}} = \left(\mathcal{R} + 3\Box \ln f_{\mathcal{R}} + 3g^{\mu\nu}\partial_\mu f_{\mathcal{R}}\partial_\nu f_{\mathcal{R}}/2f_{\mathcal{R}}^2\right)/f_{\mathcal{R}} \end{cases}$$

and Introduce the EF Canonically Normalized Field, $\widehat{\phi}$, and Potential, $\widehat{V}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = J^2 = \frac{f_K}{f_{\mathcal{R}}} + \frac{3}{2}m_{\rm P}^2\left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}}\right)^2 \;\;\text{and}\;\; \widehat{V}(\widehat{\phi}) = \frac{V\left(\widehat{\phi}(\phi)\right)}{f_{\mathcal{R}}\left(\widehat{\phi}(\phi)\right)^2}.$$

• The Analysis of nMI in the EF Using The Standard Slow-Roll Approximation is Equivalent[2] With The Analysis in JF.

[1] K. Maeda (1989)

[2] D.S. Salopek, J.R. Bond and J.M. Bardeen (1989); D.I. Kaiser (1995); T. Chiba and M. Yamaguchi (2008).

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Inflationary Observables - Requirements

- The Number of e-foldings, $\widehat{N}_*$, that the Scale $k_* = 0.002/\mathrm{Mpc}$ Suffers During nMI has to be Sufficient to Resolve the Horizon and Flatness Problems of *Standard Big Bang*

$$\widehat{N}_* = \frac{1}{m_{\rm P}^2}\int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_*} d\widehat{\phi}\,\frac{\widehat{V}}{\widehat{V}_{,\widehat{\phi}}} = \frac{1}{m_{\rm P}^2}\int_{\phi_{\rm f}}^{\phi_*} d\phi\, J^2\,\frac{\widehat{V}}{\widehat{V}_{,\phi}} \simeq 22.5 + 2\ln\frac{V(\phi_*)^{1/4}}{1~\mathrm{GeV}} - \frac{4}{3}\ln\frac{V(\phi_{\rm f})^{1/4}}{1~\mathrm{GeV}} + \frac{1}{3}\ln\frac{T_{\rm rh}}{1~\mathrm{GeV}} + \frac{1}{2}\ln\frac{f_{\mathcal{R}}(\phi_{\rm f})}{f_{\mathcal{R}}(\phi_*)}.$$

Where ϕ_* $[\widehat{\phi}_*]$ is The Value of ϕ $[\widehat{\phi}]$ When k_* Crosses Outside The Inflationary Horizon;
$\phi_{\rm f}$ $[\widehat{\phi}_{\rm f}]$ is the Value of ϕ $[\widehat{\phi}]$ at the end of nMI Which Can Be Found From The Condition

$$\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1, \quad \text{With} \quad \widehat{\epsilon} = \frac{m_{\rm P}^2}{2}\left(\frac{\widehat{V}_{,\widehat{\phi}}}{\widehat{V}}\right)^2 = \frac{m_{\rm P}^2}{2J^2}\left(\frac{\widehat{V}_{,\phi}}{\widehat{V}}\right)^2 \quad \text{and} \quad \widehat{\eta} = m_{\rm P}^2\,\frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\widehat{V}} = \frac{m_{\rm P}^2}{J^2}\left(\frac{\widehat{V}_{,\phi\phi}}{\widehat{V}} - \frac{\widehat{V}_{,\phi}}{\widehat{V}}\,\frac{J_{,\phi}}{J}\right).$$

- The Power Spectrum $\Delta^2_{\mathcal{R}}(\phi = \phi_*)$ of the Curvature Perturbations is To Be Consistent with WMAP7 Normalization:

$$\Delta_{\mathcal{R}} = \frac{1}{2\sqrt{3}\,\pi m_{\rm P}^3}\,\frac{\widehat{V}(\widehat{\phi}_*)^{3/2}}{|\widehat{V}_{,\widehat{\phi}}(\widehat{\phi}_*)|} = \frac{|J(\phi_*)|}{2\sqrt{3}\,\pi m_{\rm P}^3}\,\frac{\widehat{V}(\phi_*)^{3/2}}{|\widehat{V}_{,\phi}(\phi_*)|} = 4.93\cdot 10^{-5}$$

- The (Scalar) Spectral Index, $n_{\rm s}$, Its Running, $a_{\rm s}$, And The Scalar-To-Tensor Ratio r are to be Consistent With the Fitting of the WMAP7 Results by the ΛCDM Model:

$$n_{\rm s} = 1 - 6\widehat{\epsilon}_* + 2\widehat{\eta}_* = 0.968\pm0.024, \quad -0.062 \leq \alpha_{\rm s} = \frac{2}{3}\left(4\widehat{\eta}_*^2 - (n_{\rm s}-1)^2\right) - 2\widehat{\xi}_* \leq 0.018 \quad \text{and} \quad r = 16\widehat{\epsilon} < 0.24, \quad \text{at 95\% c.l.}$$

Where $\widehat{\xi} = m_{\rm P}^4 \widehat{V}_{,\widehat{\phi}}\widehat{V}_{,\widehat{\phi}\widehat{\phi}\widehat{\phi}}/\widehat{V}^2 = m_{\rm P}^2\,\widehat{V}_{,\phi}\,\widehat{\eta}_{,\phi}/\widehat{V}\,J^2 + 2\widehat{\eta}\widehat{\epsilon}$ And The Variables With Subscript $*$ Are Evaluated at $\phi = \phi_*$.

- We Have To Check The Hierarchy Between The Ultraviolet cut-off, Λ, of the Effective Theory And The Inflationary Scale.[3] In Particular, The Validity Of The Effective Theory Implies

$$\text{(a)}\quad \widehat{V}(\phi_*)^{1/4} \leq \Lambda \quad \text{or} \quad \text{(b)}\quad \widehat{H}(\phi_*) = \widehat{V}(\phi_*)^{1/2}/\sqrt{3}m_{\rm P} \leq \Lambda \quad \text{with} \quad \Lambda = m_{\rm P}/c_{\mathcal{R}}$$

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The Quartic Potential, $V = \lambda\phi^4/4$

- If $f_{\mathcal{R}}(\phi) = 1$, i.e., With Minimal Coupling to Gravity. We Find:

$$\epsilon \simeq \frac{8m_{\rm P}^2}{\phi^2} \text{ and } \eta \simeq \frac{12m_{\rm P}^2}{\phi^2}. \text{ Therefore } \max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1 \Rightarrow \phi_{\rm f} = 2\sqrt{3}m_{\rm P} \text{ And } N_* \simeq \frac{\phi_*^2}{8m_{\rm P}^2} \Rightarrow \phi_* = 2\sqrt{2N_*}m_{\rm P}.$$

$$\Delta_{\mathcal{R}} \simeq \frac{\sqrt{\lambda}\phi_*^3}{16\sqrt{3}\pi m_{\rm P}^3} = 4.93\cdot 10^{-5} \Rightarrow \lambda \simeq \frac{3}{2}4.93^2\cdot 10^{-10}\pi^2 N_*^{-3} \Rightarrow \underline{\lambda \simeq 2\cdot 10^{-13}} \text{ (!?) for } \widehat{N}_* \simeq 55$$

$$n_{\rm s} \simeq 1 - 3/N_* \simeq 0.947, \quad \alpha_{\rm s} \simeq -3/N_*^2 = 9.5\cdot 10^{-4} \text{ and } r \simeq 16/N_* \simeq 0.28 \; > 0.24\text{:WMAP Requirement}$$

- If $f_{\mathcal{R}}(\phi) = 1 + c_{\mathcal{R}}(\phi/m_{\rm P})^2$, i.e., With the Standard non-Minimal Coupling to Gravity. For $c_{\mathcal{R}} \gg 1$, we Find

$$\widehat{V} = \frac{\lambda\phi^4}{4f_{\mathcal{R}}^2} \simeq \frac{\lambda m_{\rm P}^4}{c_{\mathcal{R}}^2}, \; \widehat{\epsilon} \simeq \frac{4m_{\rm P}^4}{3c_{\mathcal{R}}^2\phi^4} \text{ and } \widehat{\eta} \simeq -\frac{4m_{\rm P}^2}{3c_{\mathcal{R}}\phi^2} \text{ Therefore } \max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1 \Rightarrow \phi_{\rm f} = \sqrt[4]{\frac{4}{3}}\frac{m_{\rm P}}{\sqrt{c_{\mathcal{R}}}}$$

$$\widehat{N}_* \simeq \frac{3c_{\mathcal{R}}\phi_*^2}{4m_{\rm P}^2} \Rightarrow \phi_* = 2m_{\rm P}\sqrt{\frac{\widehat{N}_*}{3c_{\mathcal{R}}}}. \text{ Also } \Delta_{\mathcal{R}} \simeq \frac{\sqrt{\lambda}\widehat{N}_*}{6\sqrt{2}\pi c_{\mathcal{R}}} = 4.93\cdot 10^{-5} \Rightarrow \underline{c_{\mathcal{R}} \simeq 41850\sqrt{\lambda}} \text{ for } \widehat{N}_* \simeq 55$$

$$n_{\rm s} \simeq 1 - 2/\widehat{N}_* \simeq 0.965, \quad \alpha_{\rm s} \simeq -2/\widehat{N}_*^2 \simeq -6.4\cdot 10^{-4} \text{ and } r \simeq 12/\widehat{N}_*^2 \simeq 4\cdot 10^{-3} \text{ In Agreement With Observations.}$$

Status of the Standard Model Higgs Inflation (See Also Talk of Prof. Ross)

In the Presence of a non-Minimal Coupling The Higgs Boson of Standard Model (SM) Can Act as Inflaton. BUT The Required Mass of Higgs Boson Seems To Be Heavier Than The ONE Recently Announced by CMS and ATLAS and the Reheating Process[4] Is Rather Complicate And Uncertain.

[4] J. Garcia-Bellido, D.G. Figueroa and J. Rubio (2008); F. Bezrukov, D. Gorbunov and M. Shaposhnikov (2009)

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$$\Delta_{\mathcal{R}} \simeq \frac{\sqrt{\lambda}\phi_*^3}{16\sqrt{3}\pi m_{\rm P}^3} = 4.93\cdot 10^{-5} \Rightarrow \lambda \simeq \frac{3}{2}4.93^2\cdot 10^{-10}\pi^2 N_*^{-3} \Rightarrow \underline{\lambda \simeq 2\cdot 10^{-13}}\ (!?) \text{ for } \widehat{N}_* \simeq 55$$

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[4] J. Garcia-Bellido, D.G. Figueroa and J. Rubio (2008); F. Bezrukov, D. Gorbunov and M. Shaposhnikov (2009)

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Here $F_{\mathcal{R}}$ is a Dimensionless, Holomorphic Function Whereas F_K is a Dimensionless, Real Function of the Form:

$$F_K\left(|\Phi^\alpha|^2\right) = |\Phi^\alpha|^2/m_{\rm P}^2 + k_{\Phi^\alpha\Phi^\beta}\,|\Phi^\alpha|^2|\Phi^\beta|^2/m_{\rm P}^4$$

With Sufficiently Small Coefficients $k_{\Phi^\alpha\Phi^\beta}$. The Inclusion of the 4th Order Term At Least For The Accompanying Non-Inflaton Field, $\Phi^1 := S$, is Obligatory In Order To Evade A Tachyonic Instability Occurring Along This Direction.

- The Realization of nMI in SUGRA Requires Also $\widehat{V}_{\rm D} = 0$ Which Happens When
  - The Inflaton is a Gauge Singlet $\Phi^2 := \Phi$;
  - The Inflaton is the Radial Part of a Conjugate Pair of Higgs Superfields, $\Phi^2 := \Phi$ and $\Phi^3 := \bar\Phi$, Which are Parameterized so as $\widehat{V}_{\rm D} = 0$

Selecting the Convenient Superpotential and Coupling Function

- In Both Cases Above If We Set $S = 0$ with The Resulting $\widehat{V} = \widehat{V}_{\rm I0}$ is Equal to[7]

$$\widehat{V}_{\rm I0} = e^{K/m_{\rm P}^2} K^{SS^*}\left|W_{,S}\right|^2 = \frac{V_{\rm F}}{f_{S\Phi} f_{\mathcal{R}}^2} \text{ Where } V_{\rm F} = \left|W_{,S}\right|^2,\ f_{S\Phi} = m_{\rm P}^2\Omega_{,SS^*},\ f_{\mathcal{R}} = -\frac{\Omega}{3}, \text{ Since } e^{K/m_{\rm P}^2} = \frac{1}{f_{\mathcal{R}}^3} \text{ and } K^{SS^*} = \frac{f_{\mathcal{R}}}{f_{S\Phi}}.$$

Given That $f_{S\Phi} \ll f_{\mathcal{R}}$, An Inflationary Plateau is Generated When $\widehat{V}_{\rm I0} \simeq V_{\rm F}/f_{\mathcal{R}}^2 \sim \text{const.}$

- Possible Successful Combinations Are
  - $W = \lambda S\Phi^2$ and $F_{\mathcal{R}} = 1 + c_{\mathcal{R}}\Phi^2/m_{\rm P}^2$ (With $D^\alpha = 0$). Then $V_{\rm F} \sim \Phi^4$ and $f_{\mathcal{R}}^2 \simeq c_{\mathcal{R}}^2\Phi^4$. Therefore $\widehat{V}_{\rm I0} \sim \text{const.}$
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Definition of the Model

The Inflaton is Uncharged Under a Local Symmetry and so, $\mathrm{D}^\alpha = 0$.
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$$W = \lambda S\Phi^2,\ F_{\mathcal{R}} = \frac{c_\Phi}{4}\Phi^2$$

Charge Assignments

Superfields:	S	Φ
$U(1)$	2	-1

Note that $F_{\mathcal{R}}$ Violates this $U(1)$ During nMI

$$F_K = \frac{|S|^2}{m_{\rm P}^2} + \frac{|\Phi|^2}{m_{\rm P}^2} - k_S\frac{|S|^4}{m_{\rm P}^4} - 2k_\Phi\frac{|\Phi|^4}{m_{\rm P}^4} - 2k_{S\Phi}\frac{|S|^2|\Phi|^2}{m_{\rm P}^4}$$

The F-term SUGRA Potential

- For $S = 0,\quad \theta = \arg\Phi = 0$ and $c_\Phi \gg 1$,

$\widehat{V} = \widehat{V}_{\rm I0}$ and the Corresponding Hubble Parameter $\widehat{H}_{\rm HI0}$ Become Almost Constant And Are Given By

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Stability of the Inflationary Trajectory

The Scalar Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Mass Squared
1 real scalar	$\widehat{\theta}$	$m_{\theta}^2 \simeq \lambda^2 (1+6c_{\mathcal{R}})\, m_{\rm P}^2 x_\phi^4/3 f_{S\Phi} f_{\mathcal{R}}^3 J^2 \simeq 4H_{\rm I}^2$
2 real scalars	$\widehat{s}_1,\ \widehat{s}_2$	$m_{\widehat{s}}^2 \simeq \lambda^2 m_{\rm P}^2 \left(2 + c_{\mathcal{R}}^2(6k_S f_{\mathcal{R}} - 1)x_\phi^4\right)/6c_{\mathcal{R}}^2 f_{S\Phi}^3 f_{\mathcal{R}}^2$
2 Weyl spinors	$\widehat{\psi}_\pm = \frac{\widehat{\psi}_\Phi \pm \widehat{\psi}_S}{\sqrt{2}}$	$m_{\widehat{\psi}\pm}^2 \simeq \lambda^2 m_{\rm P}^2 (2 - k_{S\Phi}x_\phi^2 + k_{S\Phi}c_{\mathcal{R}}x_\phi^4)^2/18 f_{S\Phi}^3 f_{\mathcal{R}}^2$

We Observe the Following:

- All mass$^2 > 0$. Especially $m_{\widehat{s}}^2 > 0 \ \Leftrightarrow\ k_S > 1/6f_{\mathcal{R}}$,
- All mass$^2 > \widehat{H}_{\rm I}^2$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated.
- The One-Loop Radiative Corrections Have No Significant Effect On The Inflationary Dynamics And Predictions, Since The Slope Of The Inflationary Path Is Generated At The Classical Level.

Approximating the Inflationary Dynamics

For Simplicity we assume that $k_{S\Phi} = 0$ and $k_\Phi = 0$.

- Duration: $\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1 \ \Rightarrow\ \phi_{\rm f} = (4/3)^{1/4} m_{\rm P}\sqrt{1/c_{\mathcal{R}}}$, where $\widehat{\epsilon} \simeq \frac{4m_{\rm P}^4}{3c_{\mathcal{R}}^2\phi^4}$ and $\widehat{\eta} \simeq -\frac{4m_{\rm P}^2}{3c_{\mathcal{R}}\phi^2}$
- Number of e-foldings (ϕ_* Decreases as $c_{\mathcal{R}}$ or λ Increases): $\widehat{N}_* \simeq \frac{3c_{\mathcal{R}}}{4}\frac{\phi_*^2-\phi_{\rm f}^2}{m_{\rm P}^2} \ \Rightarrow\ \phi_* \simeq 2m_{\rm P}\sqrt{\widehat{N}_*/3c_{\mathcal{R}}}$.
- The Power Spectrum Normalization (The Required $c_{\mathcal{R}}$ is a little lower than that in the non-SUSY case):
  $\Delta_{\mathcal{R}} \simeq \frac{\lambda\phi_*^2}{16\sqrt{2}\pi m_{\rm P}^2} \simeq \frac{\lambda\widehat{N}_*}{12\sqrt{2}\pi c_{\mathcal{R}}} 4.93\cdot 10^{-5} \ \Rightarrow\ \underline{c_{\mathcal{R}} \simeq 20925\,\lambda}$ for $\widehat{N}_* \simeq 55$

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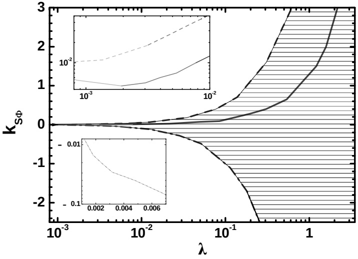
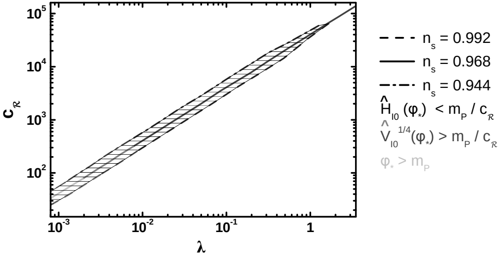
Testing Against Observations

Imposing the Observational Constraints for $k_S = 1$ and $k_\Phi = 0.5$ we Obtain the Following Allowed Regions:

- In the $\lambda - k_{S\Phi}$ Plane



- In the $\lambda - c_{\mathcal{R}}$ Plane



We Observe the Following:

- For $\lambda < 0.05 \Leftrightarrow \phi_* > 0.01 m_P$ a tuning of the order 0.01 is required in the $k_{S\Phi}$-values
- For $\lambda > 0.05 \Leftrightarrow \phi_* < 0.01 m_P$ less tuning as regards the $k_{S\Phi}$-values is required.
- $c_{\mathcal{R}}$ Remains proportional to λ and increases as n_s decreases.

Definition of the Model

The Fields Including the Inflaton Must Be Charged Under a Local Symmetry, e.g. $U(1)_{B-L}$. Then Inflaton can Be Identified With the radial part of the Higgs Fields involved in the Breaking $G_{\rm SM}\times U(1)_{B-L}\to G_{\rm SM}$.

W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry.

$$W=\lambda S\left(\bar\Phi\Phi-M_{\rm BL}^2\right),\ F_{\mathcal R}=\frac{c_{\bar\Phi\Phi}}{2}\bar\Phi\Phi$$

Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
R	1	0	0
$U(1)_{B-L}$	0	1	-1

Note that $F_{\mathcal R}$ Is Invariant Under $U(1)_{B-L}$ and R.

$$F_K=\frac{|S|^2}{m_{\rm P}^2}+\frac{|\Phi|^2}{m_{\rm P}^2}-k_S\frac{|S|^4}{m_{\rm P}^4}-2k_\Phi\frac{|\Phi|^4}{m_{\rm P}^4}-2\bar k_\Phi\frac{|\bar\Phi|^4}{m_{\rm P}^4}-2k_{S\Phi}\frac{|S|^2|\Phi|^2}{m_{\rm P}^4}-2\bar k_{S\Phi}\frac{|S|^2|\bar\Phi|^2}{m_{\rm P}^4}-2k_{\Phi\bar\Phi}\frac{|\Phi|^2|\bar\Phi|^2}{m_{\rm P}^4}$$

A D-Flat Direction of the SUGRA Potential

- If We Use The Parameterization: $\Phi=\phi^{i\theta}\cos\theta_\Phi/\sqrt2$ and $\bar\Phi=\phi^{i\bar\theta}\sin\theta_\Phi/\sqrt2$, Where ϕ is the Inflaton, We Can Easily Deduce That a D-flat Direction Occurs At

$$\theta=\bar\theta=0,\ \theta_\Phi=\pi/4\ \text{ and }\ S=0,\ \ \text{Since}\ \ V_{\rm D}=\frac{g^2}{8}\left(|\Phi|^2-|\bar\Phi|^2\right)^2=0$$

- The Preservation Of this D-Flat Direction Requires the Imposition of a “Conjugation” Symmetry on Kähler Potential

$$\Phi\to\bar\Phi\ \text{ and }\ \bar\Phi\to\Phi\ \Rightarrow\ k_\Phi=\bar k_\Phi\ \text{ and }\ k_{S\Phi}=\bar k_{S\Phi}\ \ \text{Since}\ \ \widehat V_{,\theta_\Phi}\simeq\frac{\lambda^2m_{\rm P}^4x_\phi^6}{4c_{\mathcal R}^3}\left(k_{S\Phi}-\bar k_{S\Phi}\right)=0$$

- Along this Direction, For $c_{\mathcal R}\gg1$, $\widehat V=\widehat V_{\rm I0}$ and the Corresponding Hubble Parameter $\widehat H_{\rm I0}$ Take the Form

$$\widehat V_{\rm I0}=m_{\rm P}^4\frac{\lambda^2(x_\phi^2-4m_{\rm BL}^2)^2}{16f_{S\Phi}f_{\mathcal R}^2}\simeq\frac{\lambda^2m_{\rm P}^4}{16f_{S\Phi}c_{\mathcal R}^2}\ \text{ and }\ \widehat H_{\rm I0}=\frac{\widehat V_{\rm I0}^{1/2}}{\sqrt3m_{\rm P}}\simeq\frac{\lambda m_{\rm P}}{4\sqrt3f_{S\Phi}c_{\mathcal R}}\ \ \text{With}\ \ c_{\mathcal R}=-\frac16+\frac{c_{\Phi\bar\Phi}}{4},$$

$$f_{\mathcal R}=1+c_{\mathcal R}x_\phi^2+(k_\Phi+k_{\Phi\bar\Phi})x_\phi^4/24,\ \ f_{S\Phi}=1-k_{S\Phi}x_\phi^2,\ \ f_\Phi=1-k_\Phi x_\phi^2\ \text{ and }\ m_{\rm BL}=\frac{M_{\rm BL}}{m_{\rm P}},\ x_\phi=\frac{\phi}{m_{\rm P}}.$$

Definition of the Model

The Fields Including the Inflaton Must Be Charged Under a Local Symmetry, e.g. $U(1)_{B-L}$. Then Inflaton can Be Identified With the radial part of the Higgs Fields involved in the Breaking $G_{\rm SM}\times U(1)_{B-L}\to G_{\rm SM}$.

W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry.

$$W=\lambda S\left(\bar\Phi\Phi-M_{\rm BL}^2\right),\quad F_{\mathcal R}=\frac{c_{\bar\Phi\Phi}}{2}\bar\Phi\Phi$$

Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
R	1	0	0
$U(1)_{B-L}$	0	1	-1

Note that $F_{\mathcal R}$ Is Invariant Under $U(1)_{B-L}$ and R.

$$F_K=\frac{|S|^2}{m_{\rm P}^2}+\frac{|\Phi|^2}{m_{\rm P}^2}-k_S\frac{|S|^4}{m_{\rm P}^4}-2k_\Phi\frac{|\Phi|^4}{m_{\rm P}^4}-2\bar k_\Phi\frac{|\bar\Phi|^4}{m_{\rm P}^4}-2k_{S\Phi}\frac{|S|^2|\Phi|^2}{m_{\rm P}^4}-2\bar k_{S\Phi}\frac{|S|^2|\bar\Phi|^2}{m_{\rm P}^4}-2k_{\Phi\bar\Phi}\frac{|\Phi|^2|\bar\Phi|^2}{m_{\rm P}^4}$$

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Stability of the Inflationary Trajectory

- We Introduce the EF Canonically Normalized Fields, $\widehat{\phi}, \widehat{\theta}_+, \widehat{\theta}_-, \widehat{\theta}_\Phi$ and $\widehat{s}$, as follows:

$$\frac{d\widehat{\phi}}{d\phi} = J \simeq \frac{\sqrt{6}}{x_\phi},\ \widehat{\theta}_+ = \frac{J\phi\theta_+}{\sqrt{2}},\ \widehat{\theta}_- = \sqrt{\frac{f_\Phi}{2f_{\mathcal{R}}}}\phi\theta_-,\ \widehat{\theta}_\Phi = \sqrt{\frac{f_\Phi}{f_{\mathcal{R}}}}\phi\left(\theta_\Phi - \frac{\pi}{4}\right) \text{ and } \widehat{s} = \sqrt{\frac{f_{S\Phi}}{f_{\mathcal{R}}}}s, \text{ with } \theta_\pm = \frac{\bar{\theta} \pm \theta}{\sqrt{2}} \text{ and } s = \frac{S}{\sqrt{2}}.$$

The Scalar Mass Spectrum Along The Inflationary Trajectory

Fields	Eigenstates	Masses Squared
2 real scalars	$\widehat{\theta}_\Phi$	$m^2_{\widehat{\theta}_\Phi} = m^2_{\rm P} x^2_\phi \left(g^2 f_{\mathcal{R}} f_{S\Phi} - 2\lambda^2\right)/4 f_{S\Phi} f_\Phi f^2_{\mathcal{R}} \simeq g m^2_{\rm P} x^2_\phi / 4 f_\Phi f_{S\Phi}$
	$\widehat{\theta}_+$	$m^2_{\widehat{\theta}+} = \lambda^2 m^2_{\rm P} x^6_\phi c_{\mathcal{R}} / 12 f^3_{\mathcal{R}} f_{S\Phi} \simeq 4\widehat{H}^2_{\rm I}$
1 complex scalar	$\widehat{S}$	$m^2_{\widehat{S}} = \lambda^2 m^2_{\rm P}\left(12 + x^2_\phi \left(1 + 6 c^2_{\mathcal{R}} x^2_\phi\right)(6k_S - 1) + 36 c^3_{\mathcal{R}} k_S x^6_\phi\right)/144 c^2_{\mathcal{R}} f^3_{S\Phi} f^2_{\mathcal{R}}$
2 gauge bosons	A_{BL}	$m^2_{BL} = f_\Phi g^2 m^2_{\rm P} x^2_\phi / 4 f_{\mathcal{R}}$
4 Weyl spinors	$\widehat{\psi}_\pm = \frac{\widehat{\psi}_{\Phi\pm} + \widehat{\psi}_S}{\sqrt{2}}$	$m^2_{\widehat{\psi}\pm} \simeq \lambda^2 m^2_{\rm P}\left(2 + k_{S\Phi} x^2_\phi (c_{\mathcal{R}} x^2_\phi - 1)\right)/36 f^3_{S\Phi} f^2_{\mathcal{R}} c^2_{\mathcal{R}}$
	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	$m^2_{BL} = f_\Phi g^2 m^2_{\rm P} x^2_\phi / 4 f_{\mathcal{R}}$

We Observe the Following:

- $m^2_{\widehat{S}} > 0 \ \Leftrightarrow\ k_S > 1/6$ and $m^2_{\widehat{\theta}_\Phi} > 0$ Since They Include Terms Proportional to $g \simeq 0.7 > \lambda$,
- All mass$^2 > H^2_{\rm I0}$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated.
- A_{BL} becomes massive Absorbing the d.o.f. of θ_- and so, $U(1)_{B-L}$ is Broken During nMI and Therefore, Topological Defects (Cosmic Strings) are not Produced at its end.
- The One-Loop Radiative Corrections may Have a Significant Effect On The Inflationary Dynamics And Predictions for $f_\Phi \neq 0$, Since an Accidental Cancellation Between m_{BL} and $m_{\widehat{\theta}_\Phi}$ is Removed.

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Upper Bound on $c_{\mathcal{R}}$

We Can Identify The Lowest Mass Scale of the Model in the SUSY Vacuum With The Unification Scale $M_{\rm GUT} \simeq 2\cdot 10^{16}$ GeV, Within the MSSM. i.e.

$$\sqrt{\frac{f_{\Phi 0}}{f_{\mathcal{R}0}}}gM_{\rm BL} = M_{\rm GUT} \Rightarrow m_{\rm BL} \simeq \frac{1}{2\sqrt{2c_{\mathcal{R}}^{\max}-c_{\mathcal{R}}}} \text{ with } \begin{cases} f_{\mathcal{R}0} = f_{\mathcal{R}}(x_\phi = 2m_{\rm BL}) \\ f_{\Phi 0} = f_{\Phi}(x_\phi = 2m_{\rm BL}) \end{cases},\ c_{\mathcal{R}}^{\max} = \frac{g^2m_{\rm P}^2}{8M_{\rm GUT}^2} \text{ and } c_{\mathcal{R}} < 2c_{\mathcal{R}}^{\max} \simeq 1.8\cdot 10^3$$

However, This Bound Can Be Even Lowered Since $\widehat{N}_*$ Becomes Monotonically Decreasing Function of $c_{\mathcal{R}}$ for $c_{\mathcal{R}} > c_{\mathcal{R}}^{\max}$ where $c_{\mathcal{R}}^{\max}$ Can Be Found By The Condition

$$\frac{d\widehat{N}_*}{dc_{\mathcal{R}}} \simeq \frac{3\phi_*^2}{4m_{\rm P}^2}\frac{\left(c_{\mathcal{R}}^{\max}-c_{\mathcal{R}}\right)}{c_{\mathcal{R}}^{\max}} = 0 \Rightarrow c_{\mathcal{R}} \simeq c_{\mathcal{R}}^{\max} \sim 1000.$$

Approximating the Inflationary Dynamics

For Simplicity we assume that $k_{S\Phi} = 0$, $k_\Phi = 0$ and $k_{\Phi\bar\Phi} = 0$.

- Duration: $\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1, \Rightarrow \phi = \phi_{\rm f} = (4/3)^{1/4}\, m_{\rm P}\sqrt{f_{\mathcal{R}0}/c_{\mathcal{R}}}$ where $\widehat{\epsilon} \simeq \frac{4m_{\rm P}^4 f_{\mathcal{R}0}^2}{3c_{\mathcal{R}}^2\phi^4}$ and $\widehat{\eta} \simeq -\frac{4m_{\rm P}^2 f_{\mathcal{R}0}}{3c_{\mathcal{R}}\phi^2}$
- Number of e-foldings (ϕ_* Decreases as $c_{\mathcal{R}}$ or λ Increases): $\widehat{N}_* \simeq \frac{3c_{\mathcal{R}}}{4f_{\mathcal{R}0}}\frac{\phi_*^2-\phi_{\rm f}^2}{m_{\rm P}^2} \Rightarrow \phi_* \simeq 2m_{\rm P}\sqrt{\widehat{N}_* f_{\mathcal{R}0}/3c_{\mathcal{R}}}$.
- The Power Spectrum Normalization:
$\Delta_{\mathcal{R}} \simeq \frac{\lambda\phi_*^2}{16\sqrt{2}\pi m_{\rm P}^2 f_{\mathcal{R}0}} \simeq \frac{\lambda\widehat{N}_*}{12\sqrt{2}\pi c_{\mathcal{R}}} = 4.93\cdot 10^{-5} \Rightarrow \lambda \simeq 8.4\cdot 10^{-4}\pi c_{\mathcal{R}}/\widehat{N}_* \Rightarrow \underline{c_{\mathcal{R}} \simeq 20925\,\lambda}$ for $\widehat{N}_* \simeq 55$.

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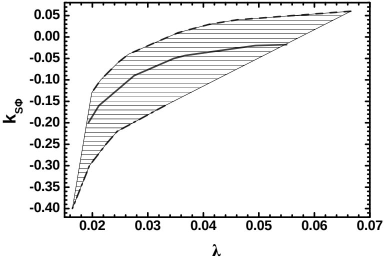
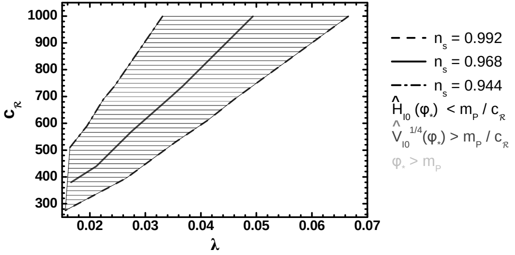
Testing Against Observations (Preliminary Results)

Imposing the Observational Constraints for $k_S = 1,\ k_\Phi = 0.01$ and $k_{\Phi\bar\Phi} = 0.5$, we Obtain the Following Allowed Regions:

- In the $\lambda - k_{S\Phi}$ Plane
- In the $\lambda - c_{\mathcal{R}}$ Plane





We Observe the Following:

- The Allowed Region is Considerably Shrunk w.r.t that Obtained with a Gauge Singlet Inflaton;
- A Tuning of the Order 0.01 is Required as Regards $k_{S\Phi}$ and k_Φ (not $k_{\Phi\bar\Phi}$);
- The Allowed Region is Limited in the $\widehat{H}_{\rm I} < m_{\rm P}/c_{\mathcal{R}}$ Regime;
- $c_{\mathcal{R}}$ Remains Proportional to λ and Increases as $n_{\rm s}$ Decreases.

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We Impose Just a Global $U(1)$ To Uniquely Determine W.

$$W = mS\Phi,\ F_{\mathcal{R}} = \frac{c_{\mathcal{R}}}{\sqrt{2}m_{\rm P}}\Phi$$

Charge Assignments

Superfields:	S	Φ
$U(1)$	1	-1

Note that $F_{\mathcal{R}}$ Violates this $U(1)$ During nMI

$$F_K = \frac{|S|^2}{m_{\rm P}^2} + \frac{|\Phi|^2}{m_{\rm P}^2} - k_S\frac{|S|^4}{m_{\rm P}^4} - 2k_\Phi\frac{|\Phi|^4}{m_{\rm P}^4} - 2k_{S\Phi}\frac{|S|^2|\Phi|^2}{m_{\rm P}^4}$$

The F-term SUGRA Potential

- For $S=0,\quad \theta=\arg\Phi=0$ and $c_{\mathcal{R}}\gg1$,

$\widehat{V}=\widehat{V}_{\rm I0}$ and the Corresponding Hubble Parameter $\widehat{H}_{\rm I0}$ Become Almost Constant And Are Given By

$$\widehat{V}_{\rm I0} = \frac{m^2m_{\rm P}^2\phi^2}{2f_{S\Phi}f_{\mathcal{R}}^2} \simeq \frac{m^2m_{\rm P}^2}{2f_{S\Phi}c_{\mathcal{R}}^2}\quad\text{and}\quad \widehat{H}_{\rm I} = \frac{\widehat{V}_{\rm I0}^{1/2}}{\sqrt{3}m_{\rm P}} \simeq \frac{mm_{\rm P}}{2\sqrt{3}f_{S\Phi}c_{\mathcal{R}}},\quad\text{Where}$$

$$f_{\mathcal{R}} = 1+c_{\mathcal{R}}x_\phi - x_\phi^2/6 - k_\Phi x_\phi^4/6,\quad f_K = 1-4k_\Phi x_\phi^2,\quad\text{and}\quad f_{S\Phi}=1-k_{S\Phi}x_\phi^2\quad\text{with}\quad x_\phi=\phi/m_{\rm P}$$

- Expanding Φ and S as Follows:

$$\Phi = \frac{\phi e^{i\theta}}{\sqrt{2}}\quad\text{and}\quad S=\frac{s_1+is_2}{\sqrt{2}},$$

We Can Introduce The EF Canonically Normalized Fields,

$$\frac{d\widehat{\phi}}{d\phi} = J \simeq \sqrt{\frac{3}{2}}\frac{1}{x_\phi},\quad \widehat{\theta}\simeq J\phi\theta\quad\text{and}\quad \widehat{s_i}\simeq\sqrt{\frac{f_{S\Phi}}{f_{\mathcal{R}}}}s_i\quad\text{with}\quad i=1,2$$

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$$f_{\mathcal{R}} = 1 + c_{\mathcal{R}}x_\phi - x_\phi^2/6 - k_\Phi x_\phi^4/6,\quad f_K = 1 - 4k_\Phi x_\phi^2,\ \text{ and }\ f_{S\Phi} = 1 - k_{S\Phi}x_\phi^2\ \text{ with }\ x_\phi = \phi/m_{\rm P}$$

- Expanding Φ and S as Follows:

$$\Phi = \frac{\phi e^{i\theta}}{\sqrt{2}}\ \text{ and }\ S = \frac{s_1 + is_2}{\sqrt{2}},$$

We Can Introduce The EF Canonically Normalized Fields,

$$\frac{d\widehat{\phi}}{d\phi} = J \simeq \sqrt{\frac{3}{2}}\frac{1}{x_\phi},\quad \widehat{\theta} \simeq J\phi\theta\ \text{ and }\ \widehat{s}_i \simeq \sqrt{\frac{f_{S\Phi}}{f_{\mathcal{R}}}}s_i\ \text{ with }\ i = 1, 2$$

Definition of the Model

The Inflaton Must Be Uncharged Under a Local Symmetry ($D^\alpha = 0$) Since $S = 0$ During nMI.
We Impose Just a Global $U(1)$ To Uniquely Determine W.

Charge Assignments

Superfields:	S	Φ
$U(1)$	1	-1

$$W = mS\Phi, \; F_{\mathcal{R}} = \frac{c_{\mathcal{R}}}{\sqrt{2}m_{\rm P}}\Phi$$

Note that $F_{\mathcal{R}}$ Violates this $U(1)$ During nMI

$$F_K = \frac{|S|^2}{m_{\rm P}^2} + \frac{|\Phi|^2}{m_{\rm P}^2} - k_S\frac{|S|^4}{m_{\rm P}^4} - 2k_\Phi\frac{|\Phi|^4}{m_{\rm P}^4} - 2k_{S\Phi}\frac{|S|^2|\Phi|^2}{m_{\rm P}^4}$$

The F-term SUGRA Potential

- For $S = 0, \quad \theta = \arg\Phi = 0$ and $c_{\mathcal{R}} \gg 1$,

$\widehat{V} = \widehat{V}_{\rm I0}$ and the Corresponding Hubble Parameter $\widehat{H}_{\rm I0}$ Become Almost Constant And Are Given By

$$\widehat{V}_{\rm I0} = \frac{m^2m_{\rm P}^2\phi^2}{2f_{S\Phi}f_{\mathcal{R}}^2} \simeq \frac{m^2m_{\rm P}^2}{2f_{S\Phi}c_{\mathcal{R}}^2} \quad \text{and} \quad \widehat{H}_{\rm I} = \frac{\widehat{V}_{\rm I0}^{1/2}}{\sqrt{3}m_{\rm P}} \simeq \frac{mm_{\rm P}}{2\sqrt{3}f_{S\Phi}c_{\mathcal{R}}}, \quad \text{Where}$$

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Stability of the Inflationary Trajectory

The Scalar Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Mass Squared
1 real scalar	$\widehat{\theta}$	$m^2_{\widehat{\theta}} \simeq c_{\mathcal{R}} m^2 x_\phi / f_{\mathcal{R}}^3 J^2 \simeq 4 H_{\rm I}^2$
2 real scalars	$\widehat{s}_1, \widehat{s}_2$	$m^2_{\widehat{s}} \simeq m^2 \left(2 - c_{\mathcal{R}}^2 x_\phi^2 + 12 c_{\mathcal{R}}^3 k_S x_\phi^2\right) / f_{S\Phi}^3 f_{\mathcal{R}}^2 (2 + 3c_{\mathcal{R}}^2)$
2 Weyl spinors	$\widehat{\psi}_\pm = \frac{\widehat{\psi}_\Phi \pm \widehat{\psi}_S}{\sqrt{2}}$	$m^2_{\widehat{\psi}\pm} \simeq m^2 (6 + x_\phi^2 + 6 c_{\mathcal{R}} k_{S\Phi} x_\phi^3)^2 / 12 f_{S\Phi}^3 f_{\mathcal{R}}^2 (2 + 3c_{\mathcal{R}}^2)$

We Observe the Following:

- $m^2_{\widehat{S}} > 0 \;\Leftrightarrow\; k_S > \left(c_{\mathcal{R}}^2 - 2/x_\phi^2\right)/12 c_{\mathcal{R}}^3$;
- All mass$^2 > \widehat{H}_{\rm I}^2$ and So Any Inflationary Perturbations Of The Fields Other Than The Inflaton Are Safely Eliminated;
- The One-Loop Radiative Corrections Have No Significant Effect On The Inflationary Dynamics And Predictions, Since The Slope Of The Inflationary Path Is Generated At The Classical Level.

Approximating the Inflationary Dynamics

For Simplicity we assume that $k_{S\Phi} = 0$ and $k_\Phi = 0$.

- Duration: $\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1 \;\Rightarrow\; \phi_{\rm f} = 2 m_{\rm P}/\sqrt{3} c_{\mathcal{R}}$, where $\widehat{\epsilon} \simeq \frac{4 m_{\rm P}^2}{3 c_{\mathcal{R}}^2 \phi^2}$ and $\widehat{\eta} \simeq -\frac{4 m_{\rm P}^2}{3 c_{\mathcal{R}} \phi}$
- Number of e-foldings (ϕ_* Decreases as $c_{\mathcal{R}}$ or m Increases): $\widehat{N}_* \simeq \frac{3 c_{\mathcal{R}}}{4} \frac{\phi_* - \phi_{\rm f}}{m_{\rm P}} \;\Rightarrow\; \phi_* \simeq 4 m_{\rm P} \widehat{N}_* / 3 c_{\mathcal{R}}$.
- The Power Spectrum Normalization: $\Delta_{\mathcal{R}} \simeq \frac{m \widehat{N}_*}{6\pi m_{\rm P} c_{\mathcal{R}}} = 4.93 \cdot 10^{-5} \;\Rightarrow\; \underline{m = 4.1 \cdot 10^{13} c_{\mathcal{R}}\,{\rm GeV}}$ for $\widehat{N}_* \simeq 55$

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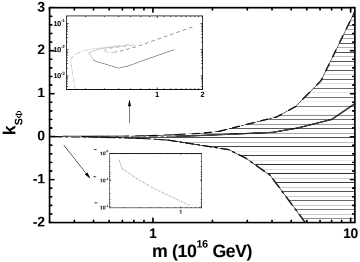
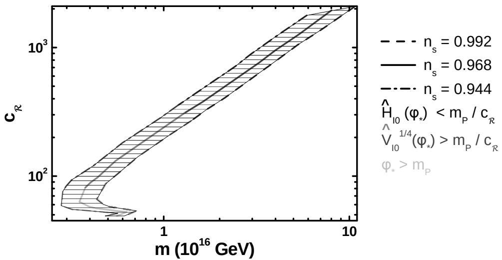
Testing Against Observations

Imposing the Observational Constraints for $k_S = 1$ and $k_\Phi = 0.5$ we Obtain the Following Allowed Regions:

- In the $m - k_{S\Phi}$ Plane
- In the $m - c_{\mathcal{R}}$ Plane





We Observe the Following:

- For $m < 2 \cdot 10^{16}$ GeV $\Leftrightarrow \phi_* > 0.01 m_P$ a tuning of the order 0.01 is required in the $k_{S\Phi}$-values
- For $m > 2 \cdot 10^{16}$ GeV $\Leftrightarrow \phi_* < 0.01 m_P$ Less Tuning As Regards the $k_{S\Phi}$-Values Is Required.
- $c_{\mathcal{R}}$ Remains Proportional To m And Increases As n_s Decreases.

Conclusions

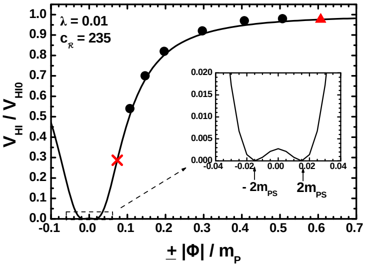
- nMI can Be Realized in SUGRA Adopting a Logarithmic Kähler Potential Including an Holomorphic Function;
- A Tachyonic Instability Occurs Along the Direction of the Accompanying non-inflaton field. This can be cured by a 4rth order Real Term in the Frame Function;
- Less Tuning As Regards the $k_{S\Phi}$-Values Is Required for $\phi_* < 0.01 m_{\rm P}$.

As Regards nMI Driven by a Higgs Field (nMHI) We Conclude The Following:

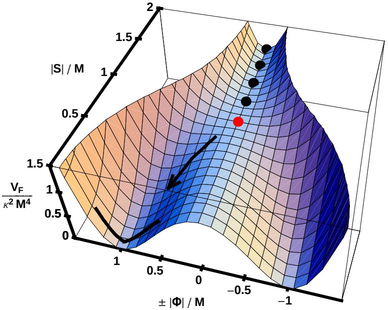
- No Topological Defects Are Produced Since The GUT Symmetry is Broken During nMHI;
- The Spontaneous Breaking Of The GUT Gauge Group to the SM one Can Occur At The SUSY GUT Scale;
- A Conjugation Symmetry Has To Be Imposed on Kähler Potential between the Two Higgs Fields in order the D-Flatness Condition to Remain Valid.

Comparing nMHI and F-term Hybrid Inflation (FHI)

Non-Minimal Higgs Inflation



Standard F-term Hybrid Inflation



In Both Cases:

- We Need The Same Superpotential Terms and, Consequently We Have to Impose the Same R-Symmetry;
- The Flat Inflationary Trajectory Is Generated By Freezing Some Fields To Zero;
- Inflaton Can Decay Into Light Degrees Of Freedom Due To Non-Renormalizable Interaction Terms Arising[8] In The SUGRA Langrangian And Due To The Non-Vanishing Vacuum Expectation Value (VEV) of Infaton.

[8] *M. Endo, M. Kawasaki, F. Takahashi and T.T. Yanagida (2006); M. Endo, F. Takahashi and T.T. Yanagida (2007).*

Differences of GUT Scale nMHI and FHI

Non-Minimal Higgs Inflation

- The Radial Part of the Higgs Field Drives Inflation;
- The GUT Symmetry is Broken during nMI ;
- No Cosmological Defects Are Produced;
- The GUT Scale Can Assume its SUSY value;
- The Flatness Of The Potential Arises Within SUGRA;
- nMI is Largely Independent From Radiative Corrections;
- The Inflationary Observables Lie Within The Range Of The Current Data;
- Possible Naturalness Problem With The Effective Theory.
- Possible Complications in the Reheating Process Occur Due to Instant Preheating[9].

Standard F-term Hybrid Inflation

- A Singlet Field Drives Inflation;
- The GUT Symmetry is Broken In The End of FHI;
- Cosmological Defects May Be Produced;
- The GUT Scale Turns Out To Be Mostly Lower Than Its SUSY value;
- The Flatness Of The Potential Arises Within SUSY;
- FHI Dependents Crucially On Radiative Corrections;
- The Spectral Index Lies Mostly Above The Range Of The Current Data;
- No Naturalness Problem With The Effective Theory;
- Possible Complications In The Reheating Process Occur Due To Tachyonic Preheating[10].

[9] *G.N. Felder, L. Kofman and A.D. Linde (1999).*
[10] *Juan García-Bellido and Ester Ruiz Morales (2002).*