Corfou, 9-10 September 2012

QCD in 2012

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Research supported by LHCPhenonet

Plan

1 - Introduction
  - Non perturbative QCD
    - Lattice QCD
    - Confinement
    - Heavy Ion physics

2 - Asymptotic Freedom
  - Perturbative QCD
    - Basic concepts and results

3 - QCD in the LHC time

QCD stands as a very solid building block of the SM

The unbroken gauge symmetry of the SM is $SU(3)xU(1)_Q$

QCDxQED

For many years the field theory of reference was QED, now QCD is a more complex and intriguing framework

Due to asymptotic freedom, actually QCD is a better defined theory than QED (which has a bad UV limit)

Comparison with experiment is excellent

Steady progress in techniques to extract precise predictions (higher order perturbative, non perturbative, lattice, event generators and simulations)

QCD is an unbroken SU(3) gauge theory with triplet quarks

$$L = -\frac{1}{4}\sum_{A=1}^{8} F^{A\mu\nu}F^{A}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j(i\widehat{D} - m_j)q_j$$

Defs: $[t^A, t^B] = iC^{ABC}t^C$ $\qquad Tr[t^A t^B] = \frac{1}{2}\delta^{AB}$

(C_{ABC}: SU(3) struture constants, t^A: generator representation)

$$g_\mu = \sum_{A=1}^{8} g^A_\mu t^A$$

(g^A_μ is a gluon field)

$\widehat{D} = D_\mu \gamma^\mu$; $D_\mu = \partial_\mu + ie_s g_\mu$ (D: covariant derivative)

$\alpha_s = \frac{e_s^{\ 2}}{4\pi}$ (e_s: SU(3) gauge coupling)

$$F^A_{\mu\nu} = \partial_\mu g^A_\nu - \partial_\nu g^A_\mu - e_s C_{ABC} g^B_\mu g^C_\nu$$

QCD is a "simple" theory

$$L = -\frac{1}{4}\sum_{A=1}^{8} F^{A\mu\nu}F^{A}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j(i\widehat{D} - m_j)q_j$$

but with an extremely rich dynamical content:

- Confinement
- Complex hadron spectrum (light and heavy quarks)
- Spontaneous breaking of (approx.) chiral symm.
- Phase transitions

  [Deconfinement (q-g plasma), chiral symmetry restauration,……...]
- Highly non trivial vacuum topology

  [Instantons, $U(1)_A$ symm. breaking, strong CP violation (?)]
- Asymptotic freedom

• • •

How do we get predictions from QCD?

- Non perturbative methods

•Lattice simulations (great continuous progress)

•Effective lagrangians

  * Chiral lagrangians
  * Heavy quark effective theories
  * Soft Collinear Effective Theory (SCET)

****

•QCD sum rules

•Potential models (quarkonium)

- Perturbative approach

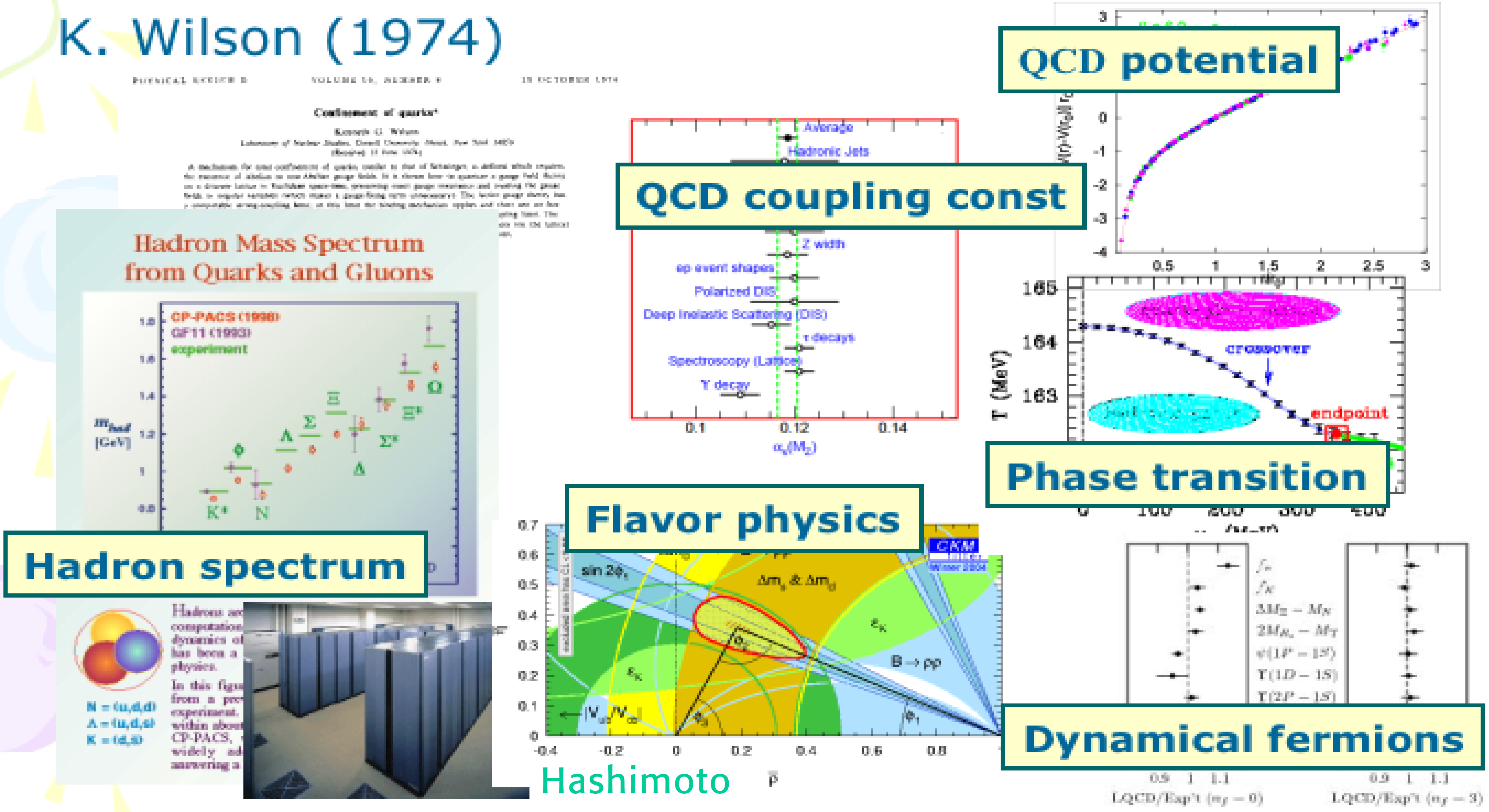
Based on asymptotic freedom.

It still remains the main quantitative connection to experiment.

The main tool for non perturbative QCD
in continuous progress

38 years of lattice QCD

K. Wilson (1974)



QCD potential

QCD coupling const

Phase transition

Flavor physics

Hadron spectrum

Dynamical fermions

Hashimoto

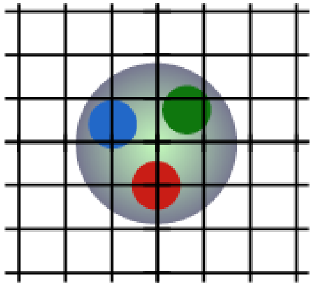
Major progress in recent years

A review:
Kronfeld, 1203.1204

Much more powerful computers now allow for:

Finer lattice spacing a-> 0 (continuum limit)

Improved lagrangians [$o(a^2)$]



Larger volume L = Na, larger N

in most cases, corrections exp. down: e^{-kV}

Smaller quark masses (realistic π mass, m_π^2 -> 0)

large q masses numerically simpler:
smaller wavelenghts need smaller V

extrapolation guided by resummation of chiral logs

Unquenching (taking quark loops into account)

Fermions on the lattice

imaginary time

a generic average:

$$\langle \bullet \rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\psi\mathcal{D}\bar{\psi}\;[\bullet]\exp\left(-S\right)$$

$$\bar{\psi}\mathfrak{M}\psi$$

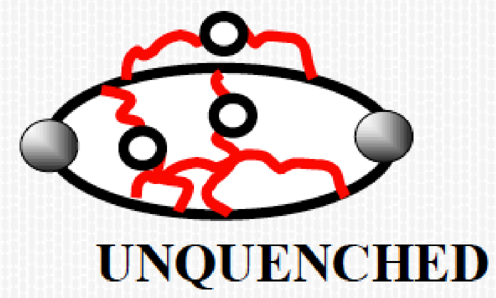
By integrating fermions away:

unquenching:
effect of sea

$$\langle \bullet \rangle = \frac{1}{Z}\int \mathcal{D}A\,[\bullet']\det\mathfrak{M}\exp\left(-S_{\mathrm{gauge}}\right)$$

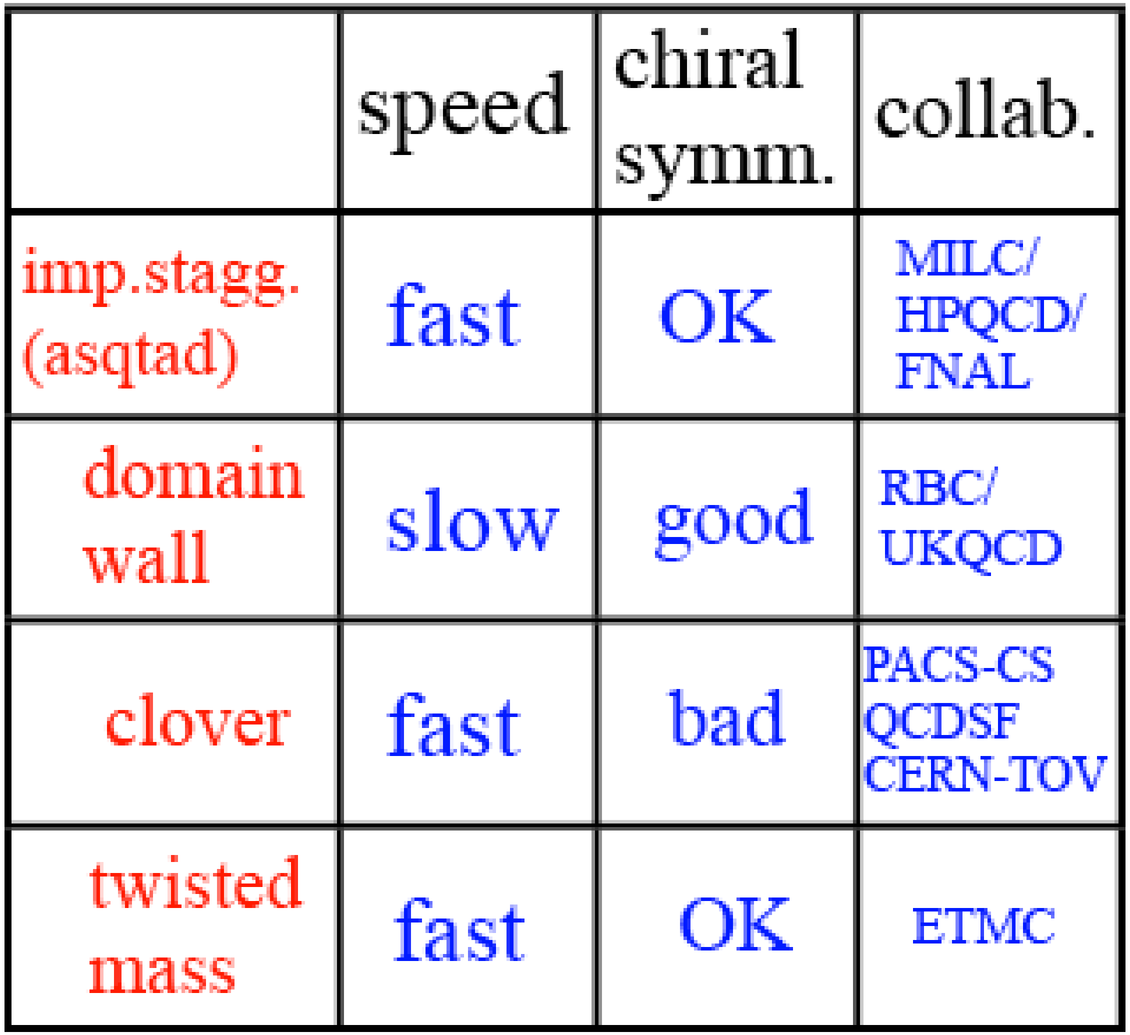
$$\psi_i\bar{\psi}_j \longrightarrow [\mathfrak{M}^{-1}]_{ij}$$

propagator of valence
quark in coloured medium



most realistic
results for
n_f=2+1
(u,d)+s

Different methods for fermions on the lattice



	speed	chiral symm.	collab.
imp.stagg. (asqtad)	fast	OK	MILC/ HPQCD/ FNAL
domain wall	slow	good	RBC/ UKQCD
clover	fast	bad	PACS-CS QCDSF CERN-TOV
twisted mass	fast	OK	ETMC

Roma

a compromise between efficiency and theoretical purity is needed

Kronfeld

Chiral extrapolation

- Lattice simulation is limited in a heavier quark mass region $m_q \sim (0.5\text{-}1) m_s$.

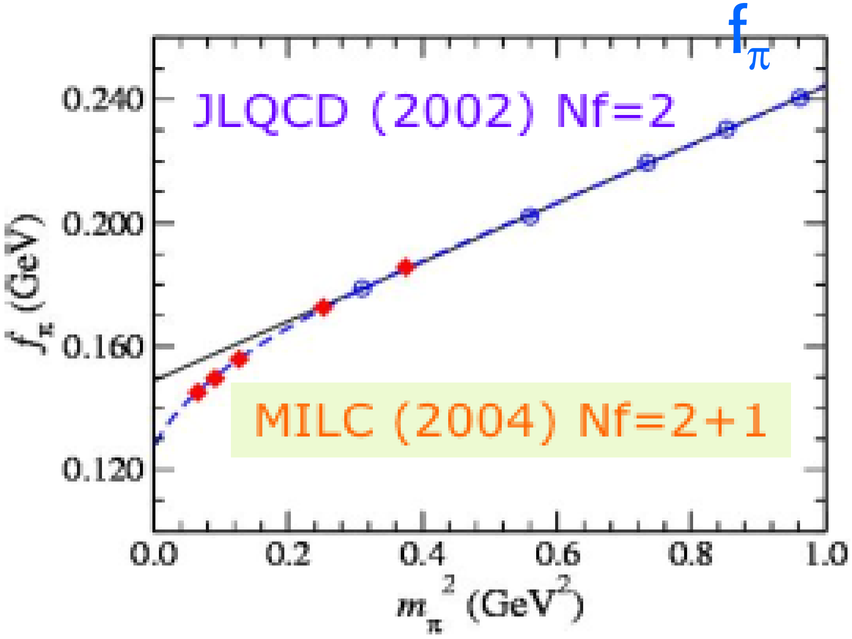
ChPT predicts the chiral log near the chiral limit.

$$c \log(m_q/1\text{GeV})$$

with a fixed coefficient.

Staggered simulation can push the quark mass much lower.

$$\langle 0|\partial^\mu A_\mu|\pi\rangle = f_\pi m_\pi^2$$

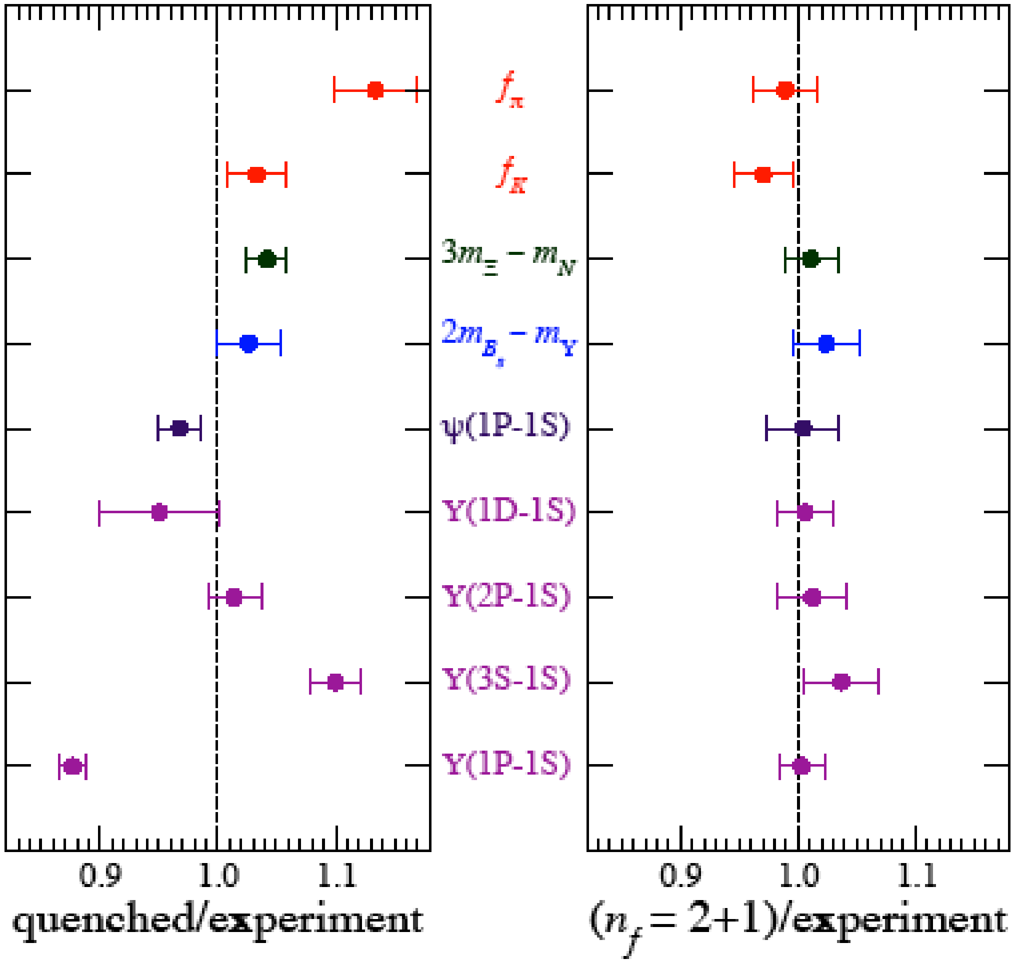


The quenched approximation (QA) is superseded: what was rough agreement in QA is now precise with unquenching

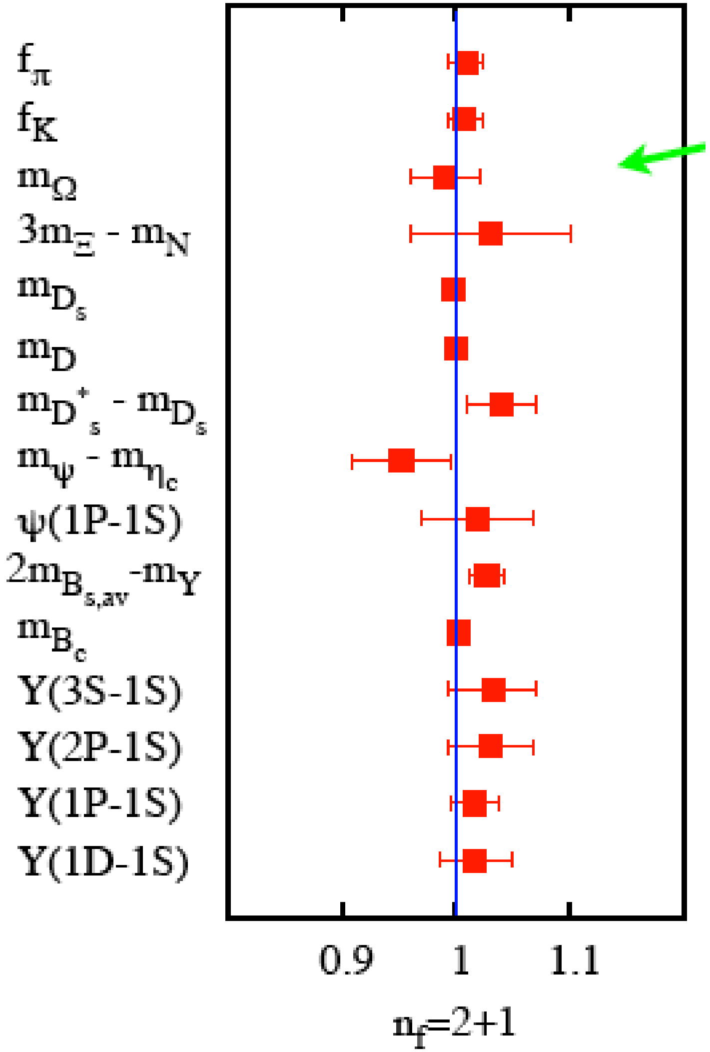
old

quenched

unquenched

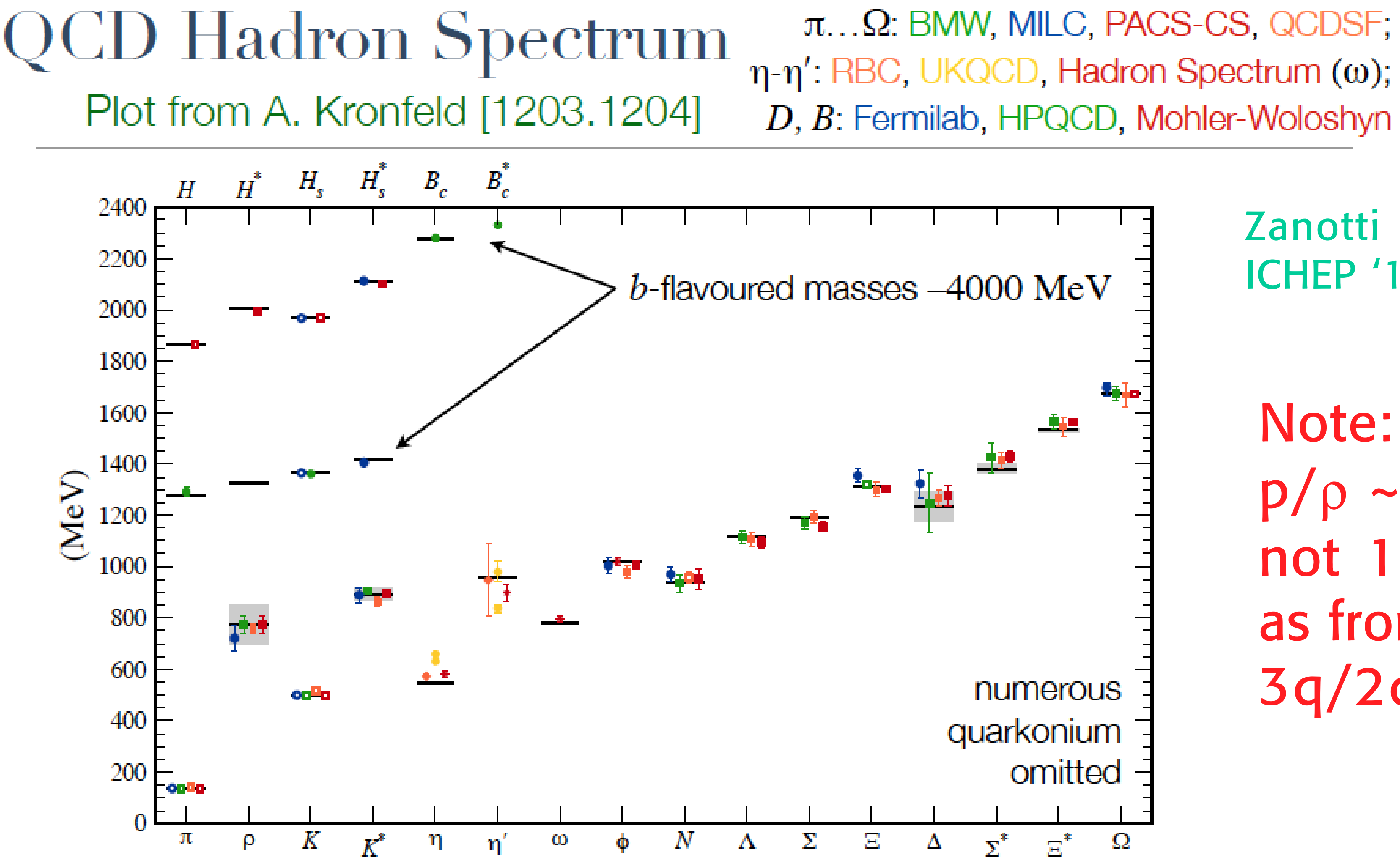


new



‘07

Unquenched lattice simulations reproduce spectrum well



Zanotti
ICHEP '12

Note:
p/ρ ~ 1.2
not 1.5
as from
3q/2q

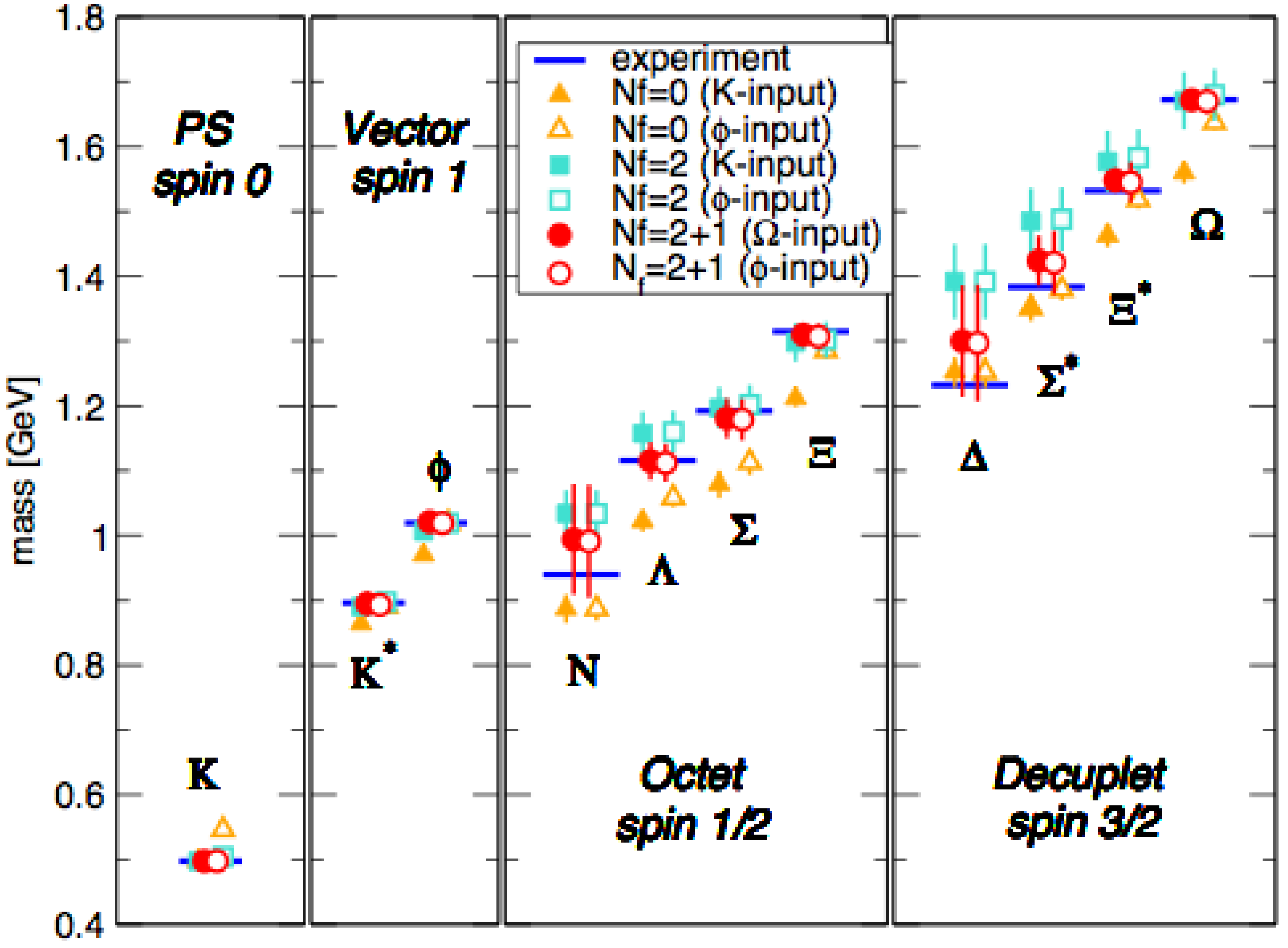
Excellent agreement between different collaborations/lattice formulations

Unquenched lattice simulations reproduce spectrum well

Kuromashi

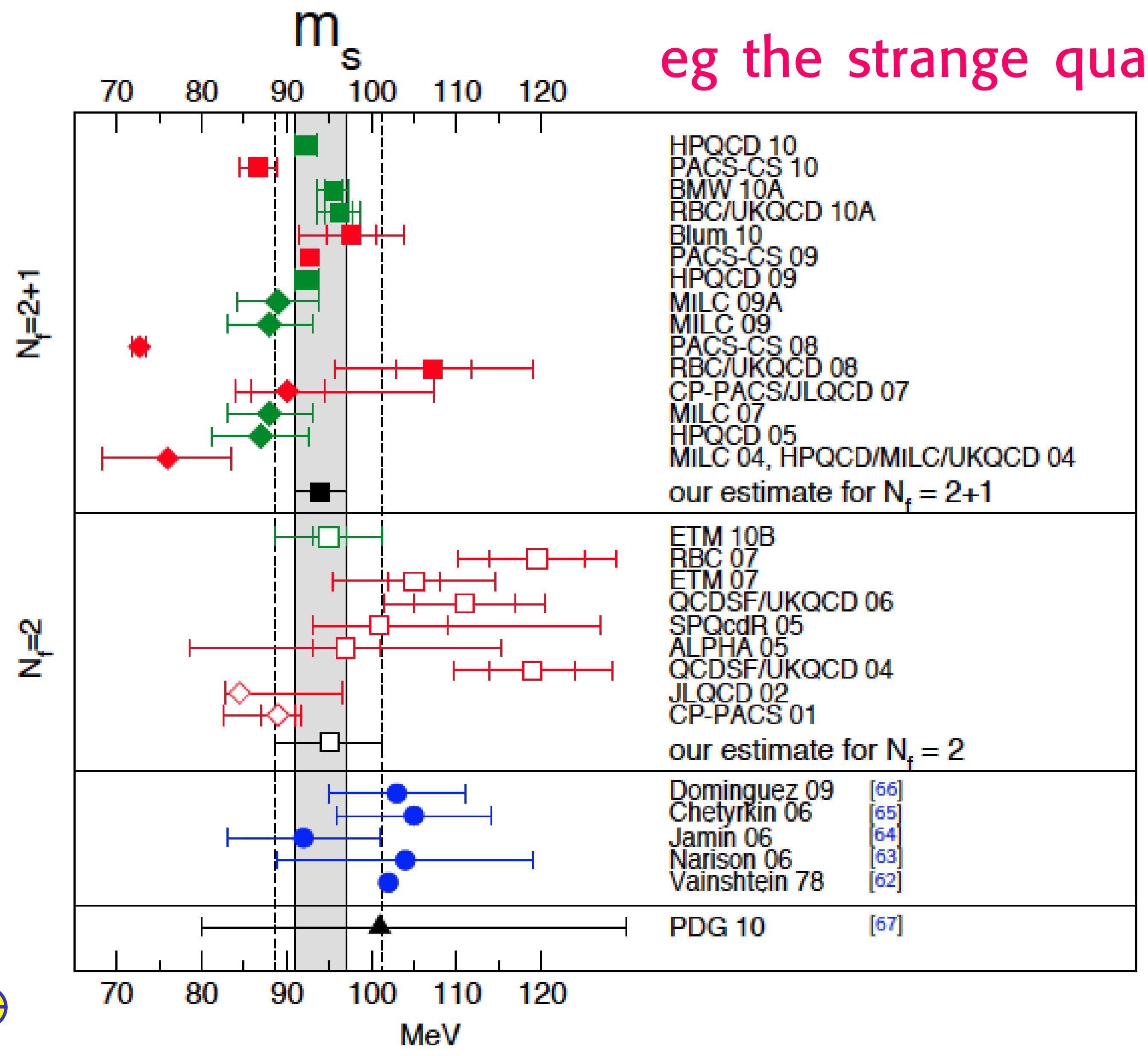
Wilson N_f=2+1

Here the focus is on strange particles



Quark Masses

eg the strange quark



$$m_u = 2.19(15) \text{ MeV}$$

$$m_d = 4.67(20) \text{ MeV}$$

$$m_s = 94(3) \text{ MeV}$$

$$m_s/m_{ud} = 27.4(4)$$

Lattice is playing an increasingly important role in flavour physics

Davies LP'07

Lattice inputs
(2+1 sea quarks):

B_K

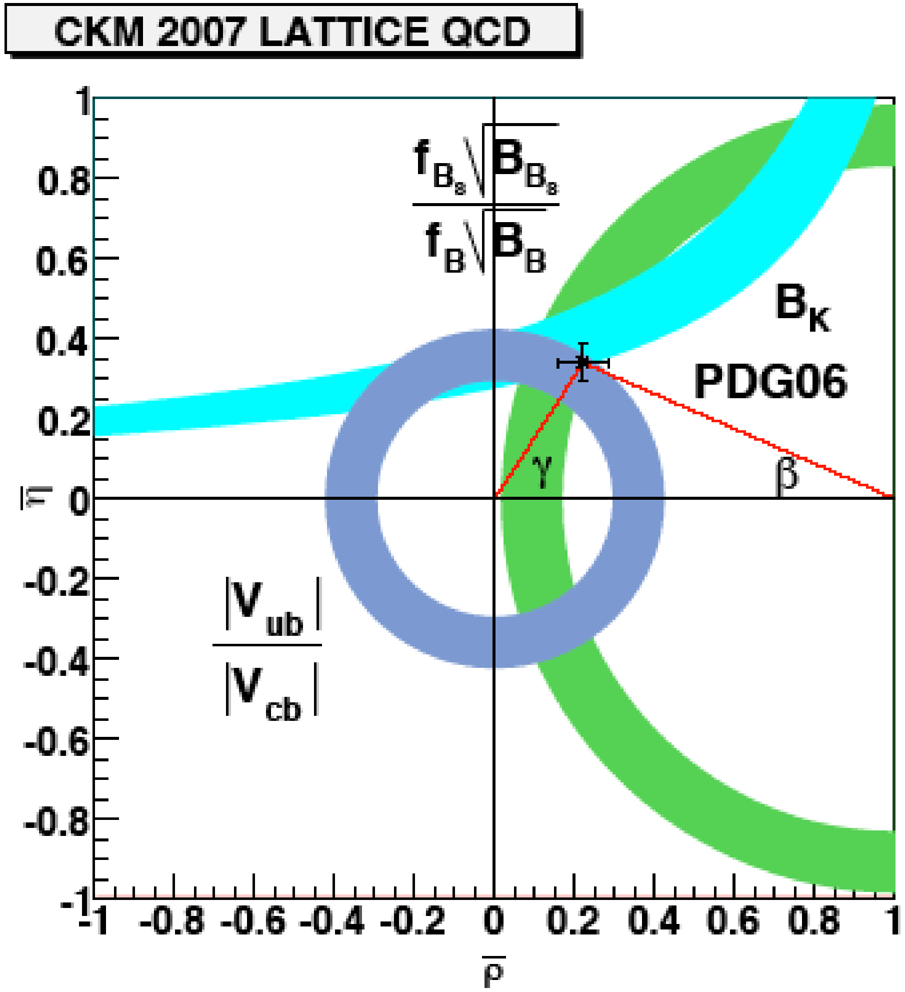
f_K/f_π , $f_+(K \to \pi l \nu)$

$F(B \to D^* l \nu)$

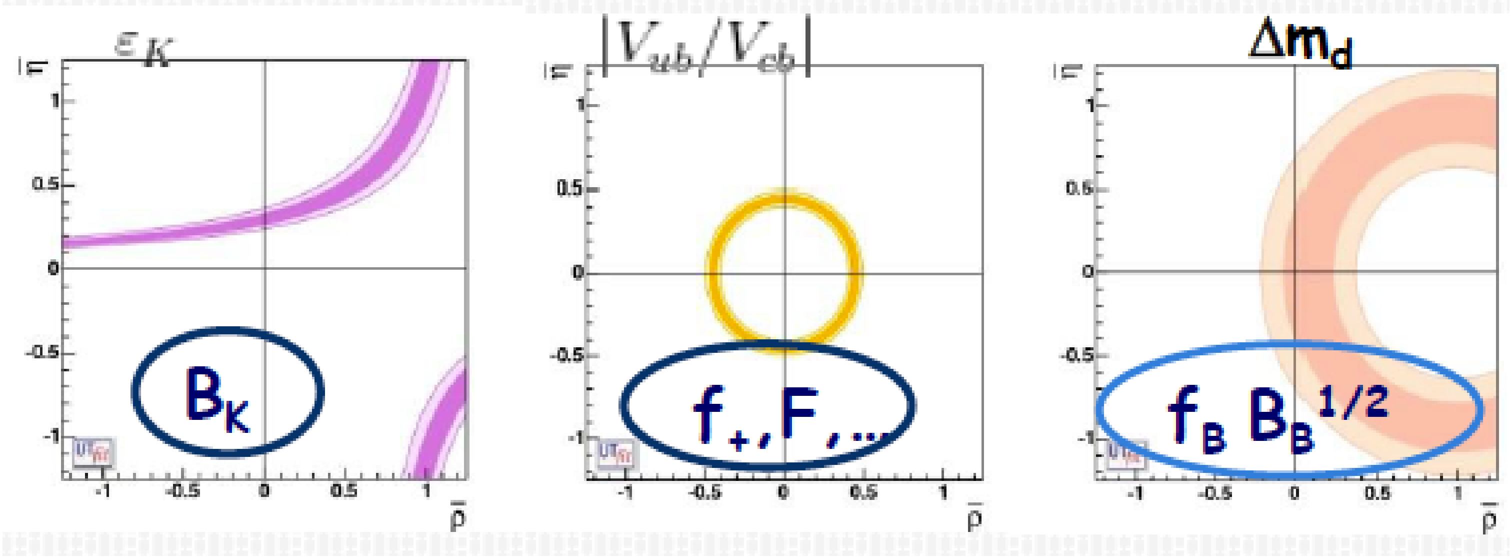
$f_+(B \to \pi l \nu)$

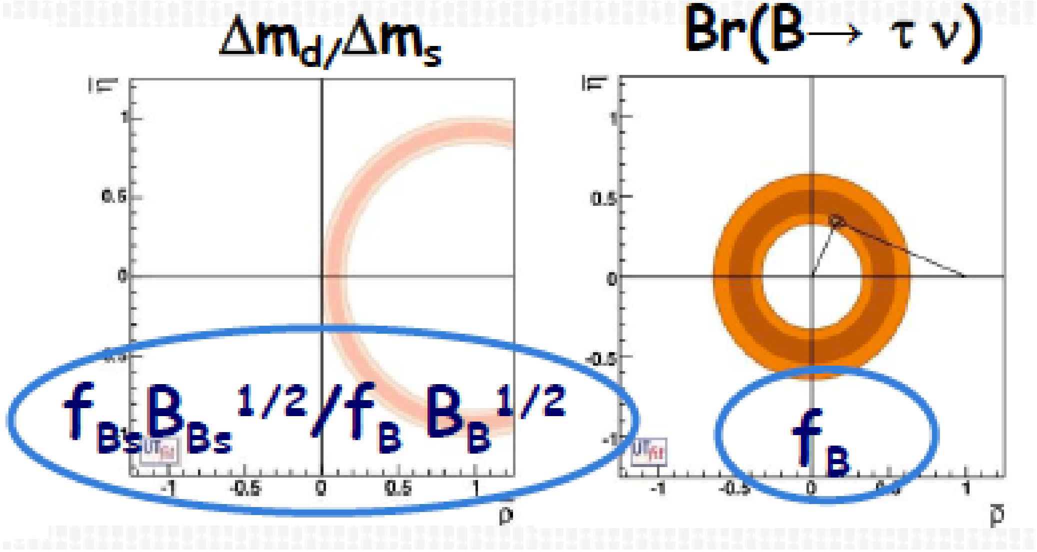
$$\frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}}$$

]



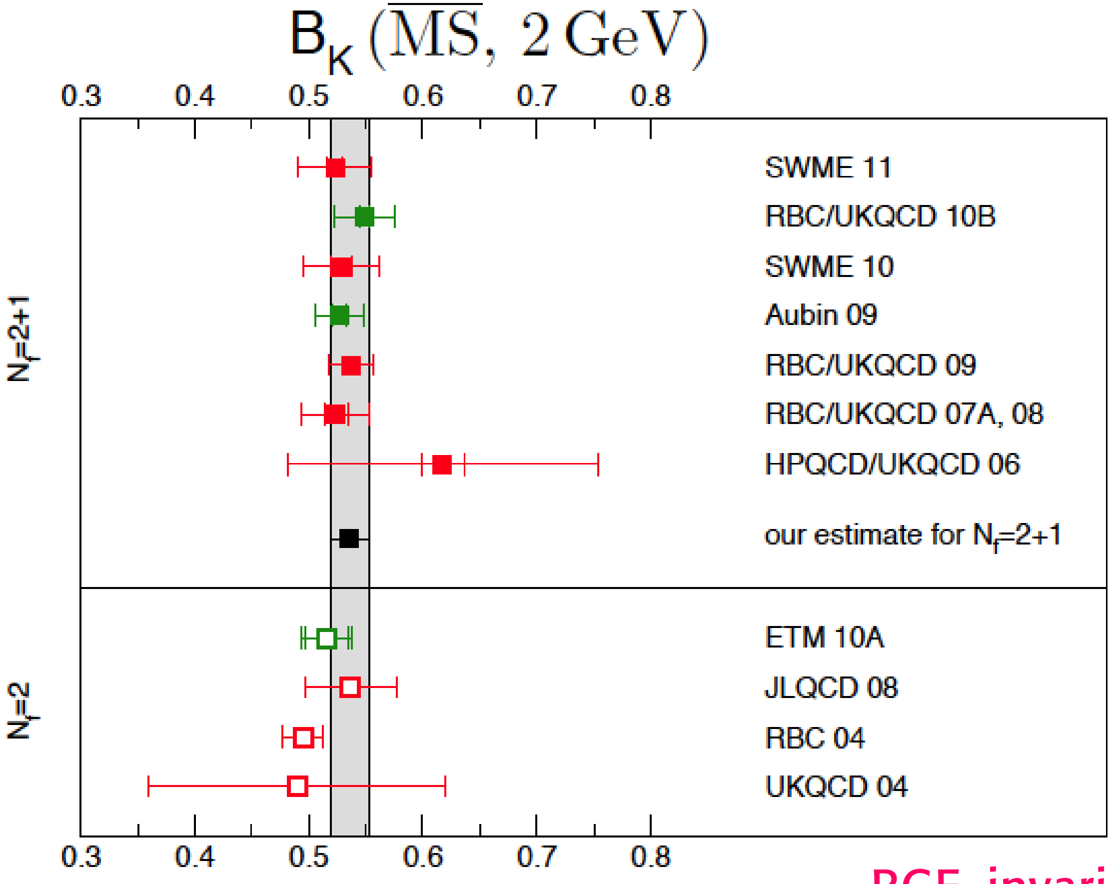
Examples


ε_K
$|V_{ub}/V_{cb}|$
Δm_d
$\bar{\eta}$
$\bar{\rho}$
-1
-0.5
0
0.5
1
B_K
$f_+, F, \ldots$
$f_B\ B_B^{1/2}$


$\Delta m_d/\Delta m_s$
$Br(B\rightarrow\ \tau\ \nu)$
$\bar{\eta}$
$\bar{\rho}$
-1
-0.5
0
0.5
1
$f_{Bs}B_{Bs}^{1/2}/f_B\ B_B^{1/2}$
f_B

Kaon Mixing

- Summary of Lattice results for B_K from FLAG [1011.4408]



Zanotti
ICHEP '12

RGE invariant

$$B_K = 0.536 \pm 0.017\,, \quad \hat{B}_K = 0.738 \pm 0.020 \qquad (N_f = 2+1)$$

Confinement: no free coloured particles

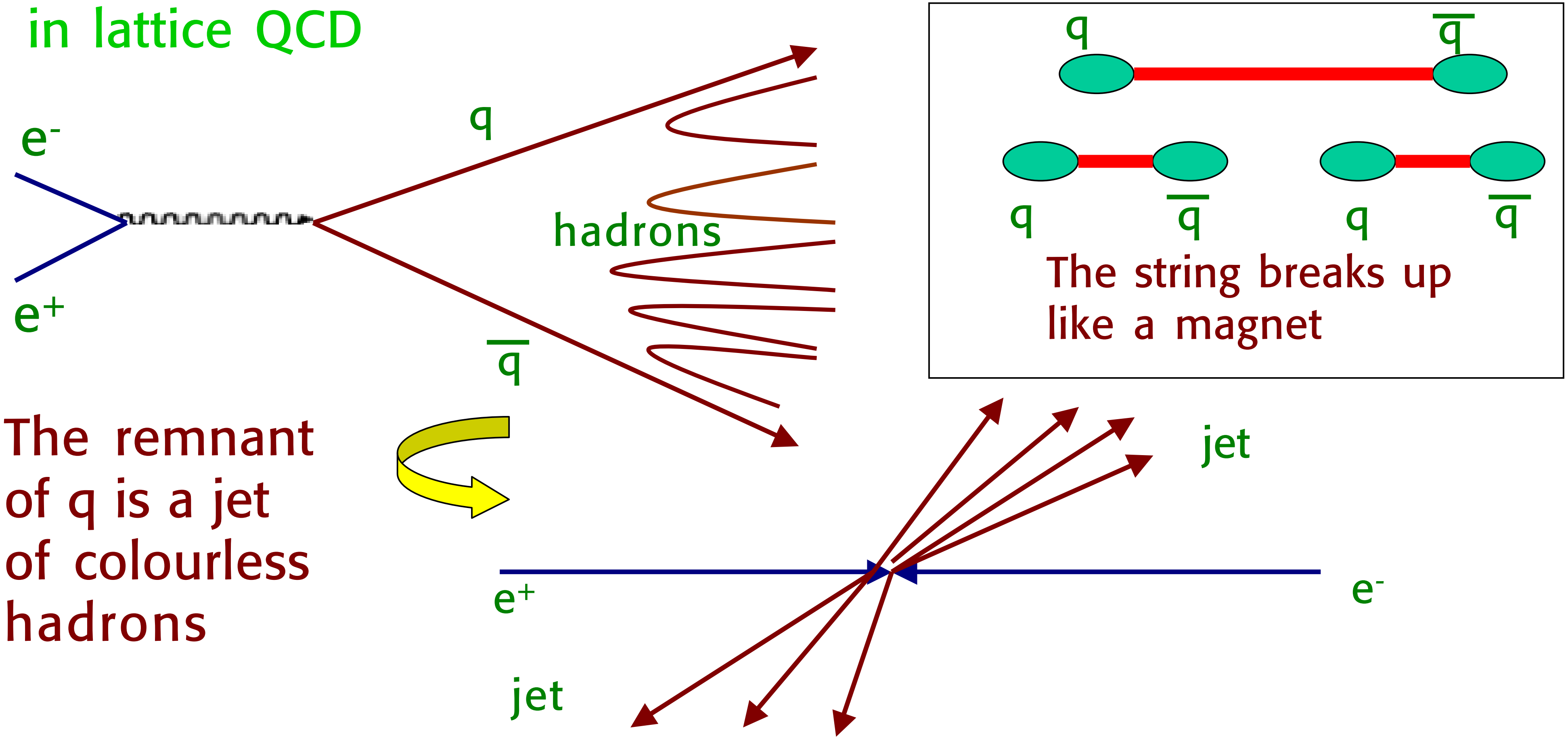
q-$\bar{q}$ potential:

$$V(r) \approx C_F\left[\frac{\alpha_s(r)}{r}+\ldots+\sigma r\right]$$

short

long dist.

Has been studied in lattice QCD

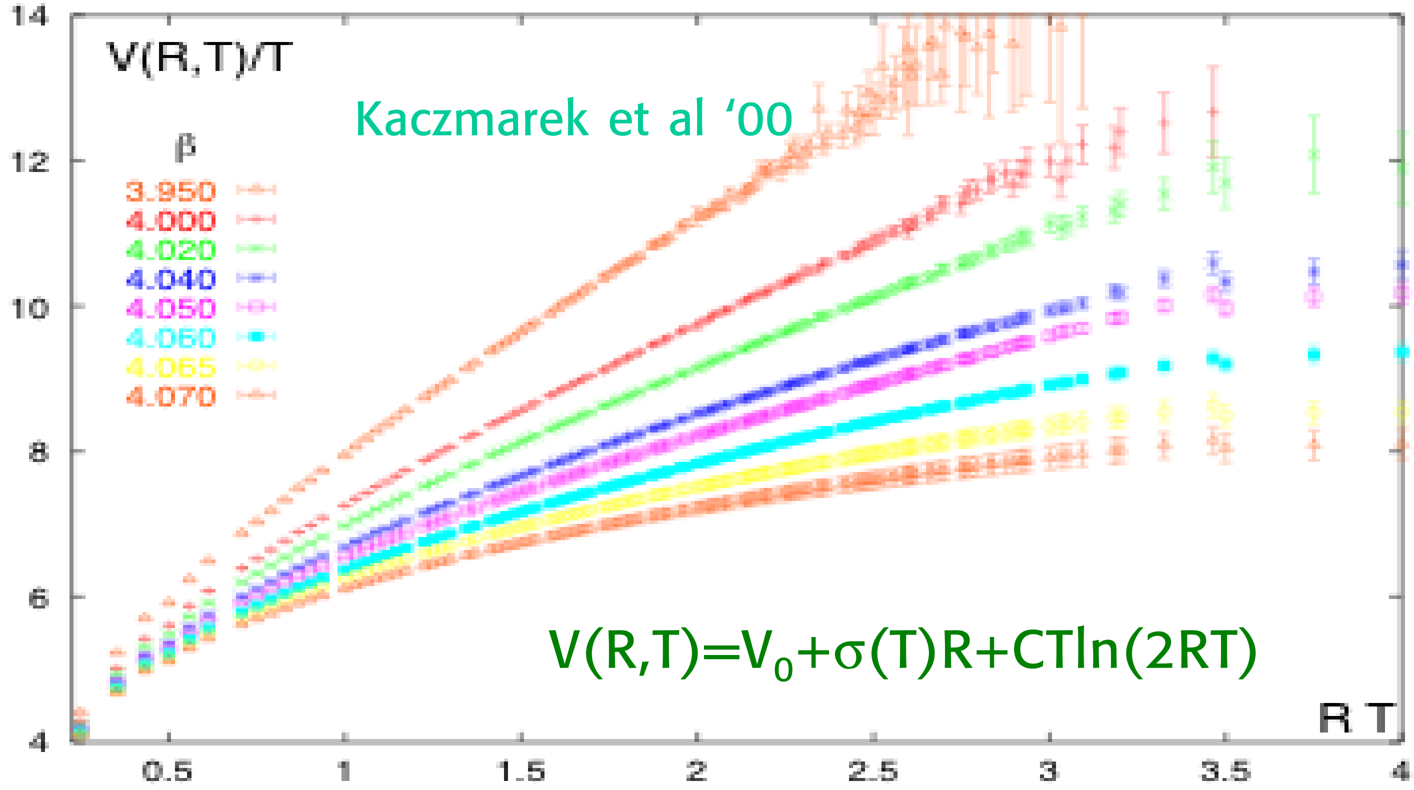


The string breaks up like a magnet

The remnant of q is a jet of colourless hadrons

Lattice QCD offers the most convincing evidence of confinement

quenched approx.



Kaczmarek et al '00

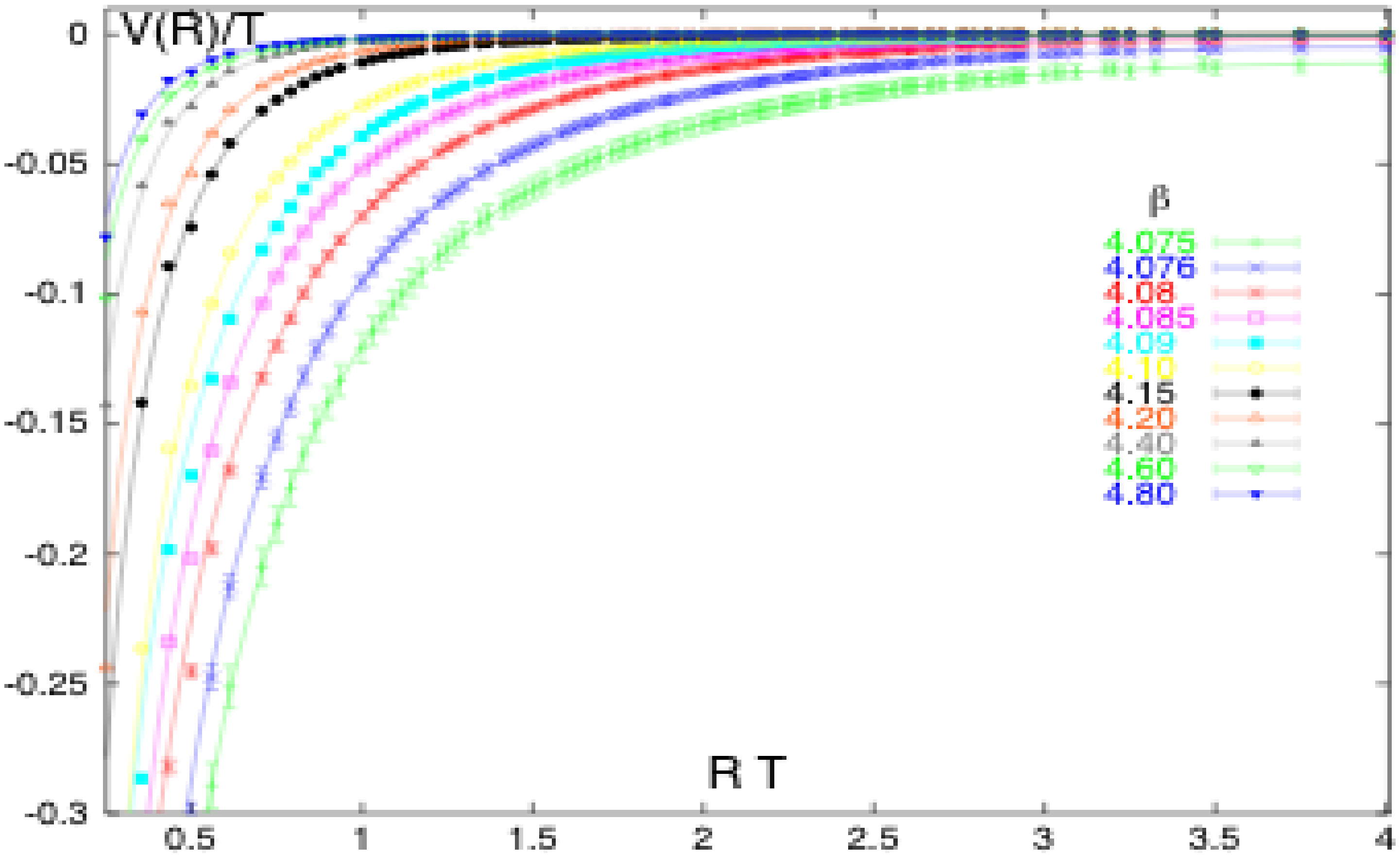
$V(R,T)=V_0+\sigma(T)R+CT\ln(2RT)$

Potential between static quarks on the lattice

Potential in units of kT (k=1) as function of R in units 1/T, for different β=1/T

The linearly rising term slope vanishes at T_C

At T>T$_C$ the slope at large R remains zero

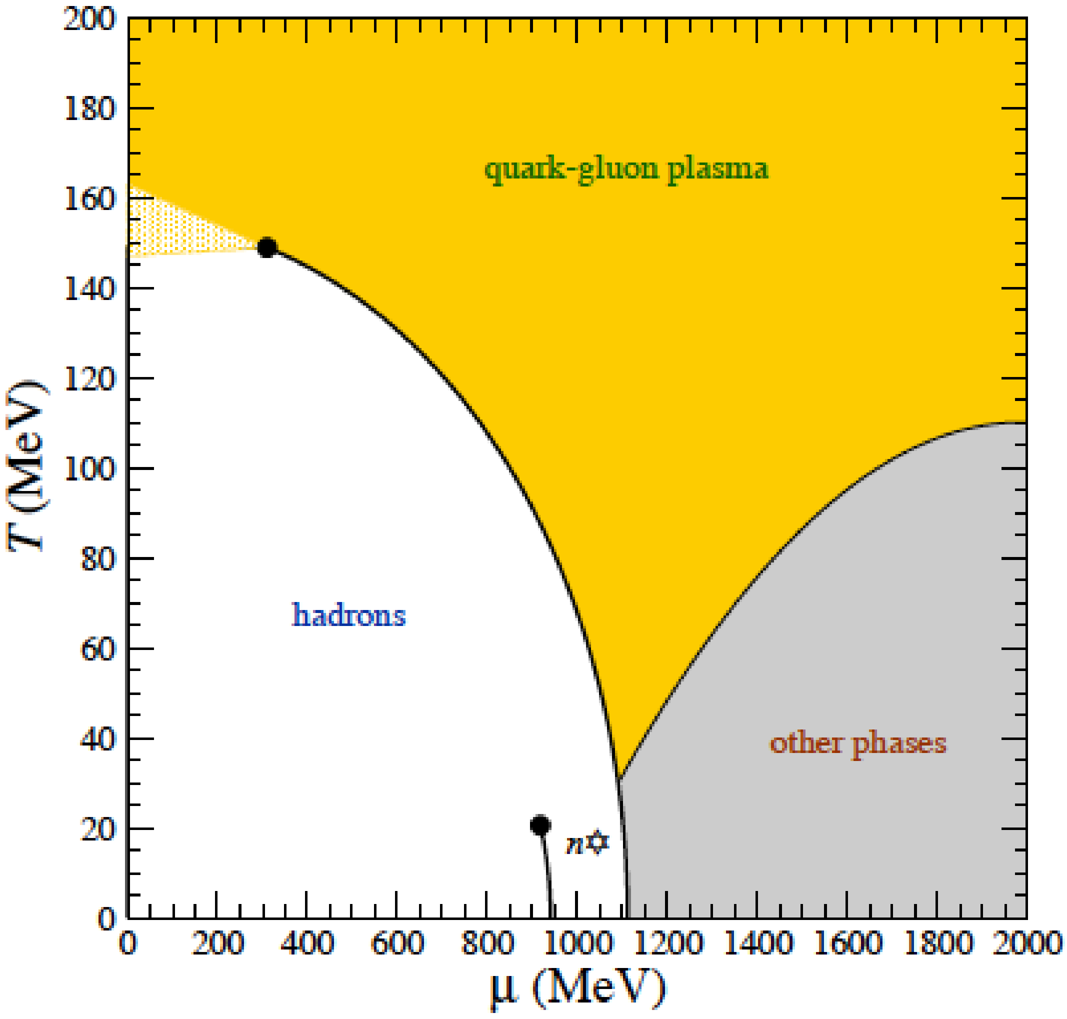


T$_C$ depends on the number of quark flavours

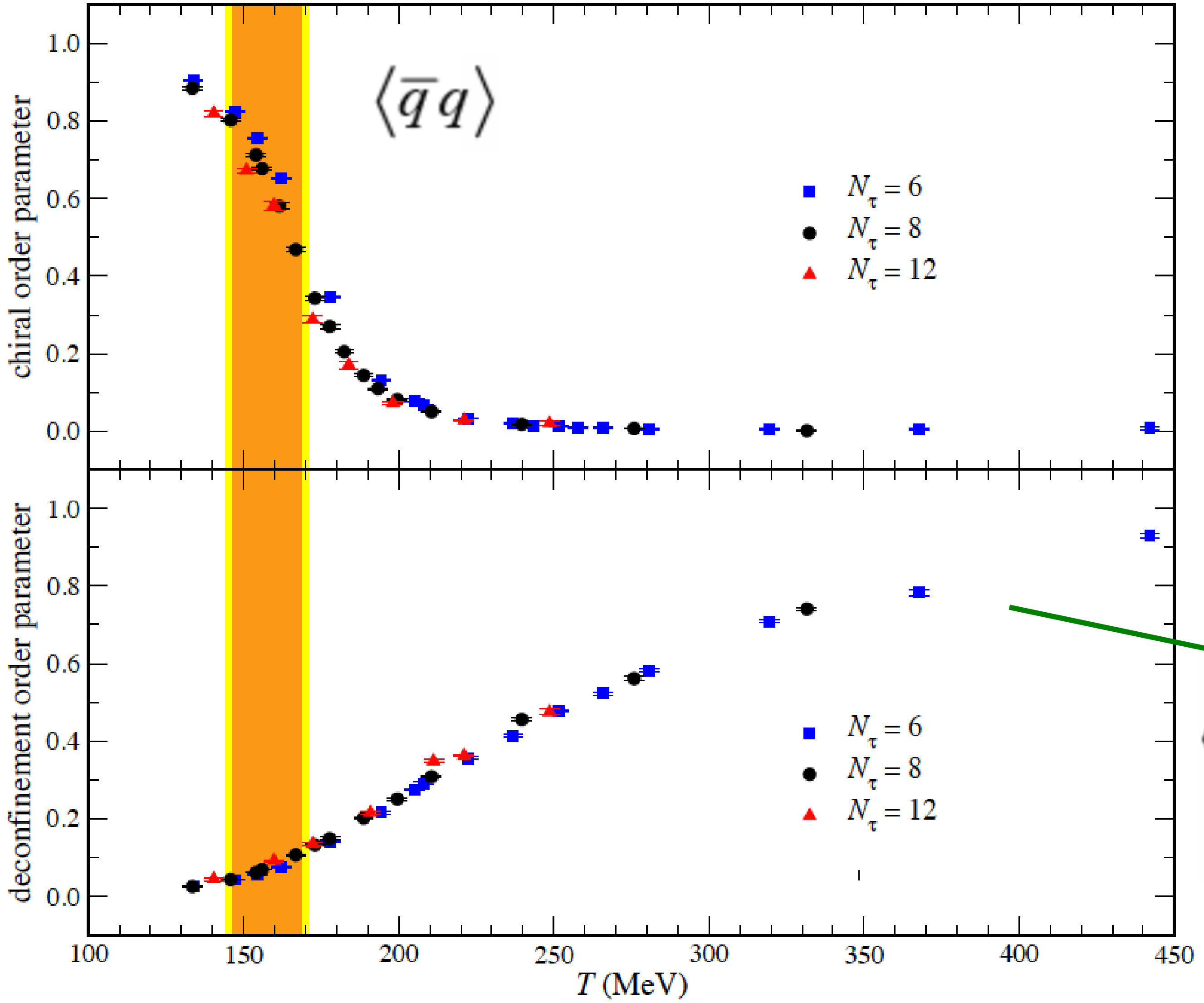
T$_C$ ~ 175 MeV

The QCD phase diagram

Studied on the lattice and probed by colliding heavy ions at AGS, SPS, RHIC, LHC (ALICE, ATLAS, CMS)



Lattice QCD predicts a rapid transition, with correlated deconfinement and chiral restauration

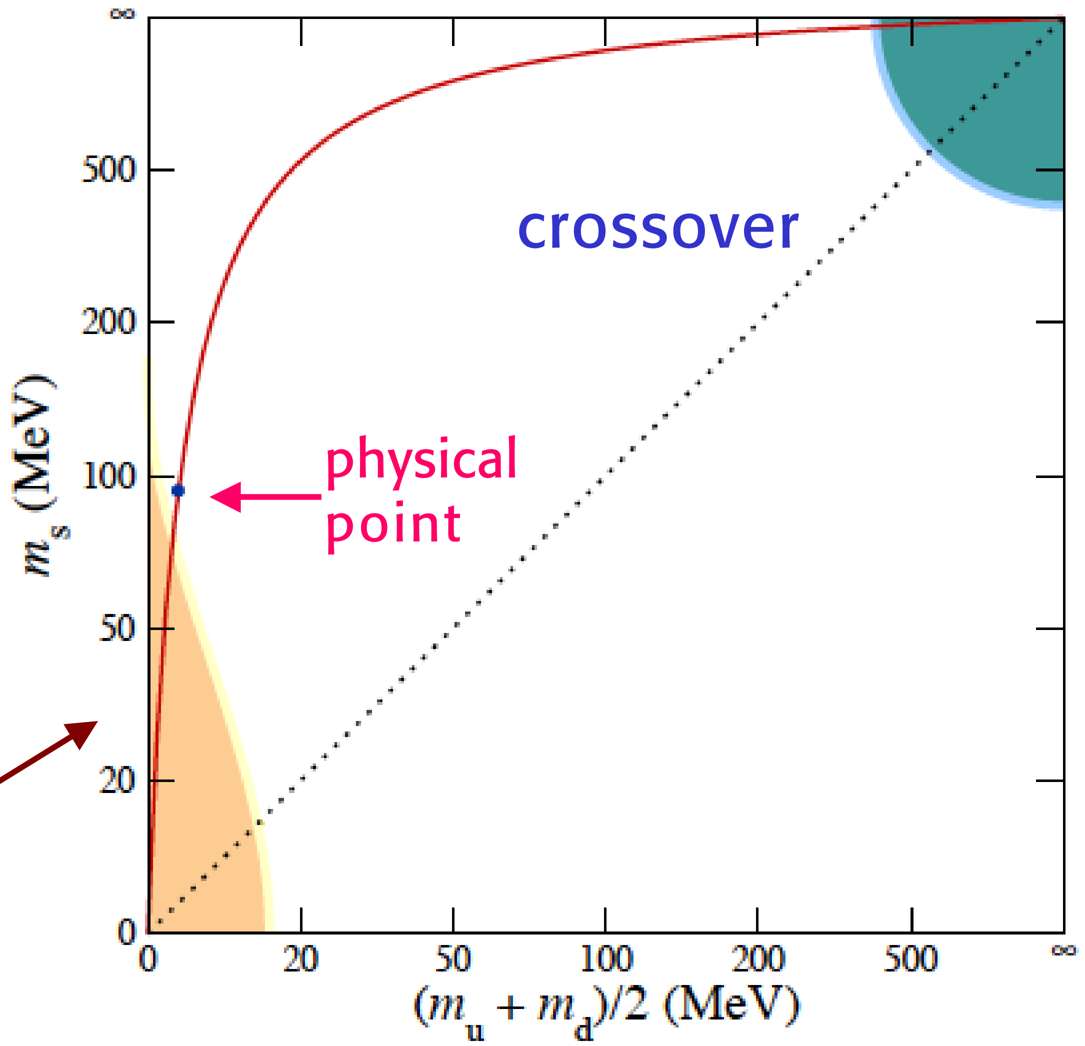


Bazavov et al,
1111.1710

But not a phase transition, rather a smooth cross over

$$\langle L \rangle = \left\langle \mathrm{tr} \exp(i \int_0^{1/T} A_4 \, d\tau) \right\rangle$$
$$\sim \exp(-F_Q / T)$$

The order of the phase transition is a function of m_q



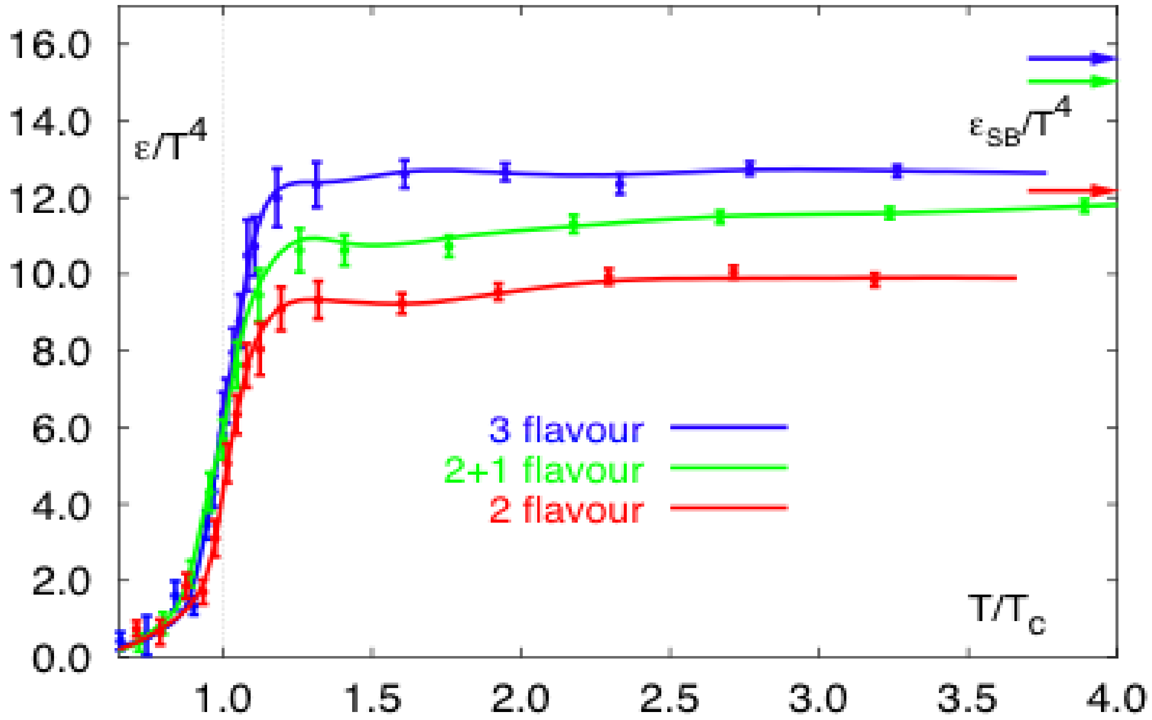
1st order

crossover

physical point

1st order

- **energy density increases sharply by the latent heat of deconfinement**



For $N_f = 2, 2+1$:

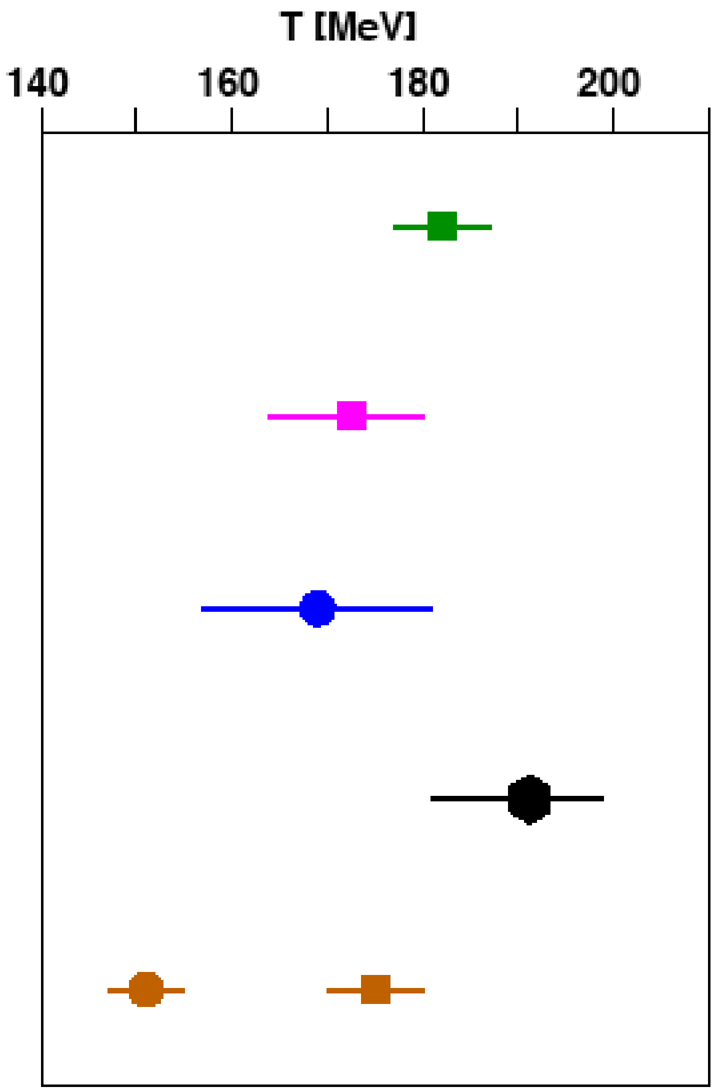
$T_c \simeq\ 175\ \mathrm{MeV}$

$\epsilon(T_c) \simeq 0.5 - 1.0\ \mathrm{GeV/fm^3}$

Summary of recent results on T_c

use T=0 scale: r0=0.469fm

Karsch LAT'07



Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)
(improved Wilson, Nt=8, 10; input: r0=0.5 fm)
(added Nt=12, Lattice'07) (rescaled to r0)

Y. Maezawa et al., hep–lat/0702005 (QM'2006)
(improved Wilson, Nt=4, 6; input: m–rho)
(no cont. exp. yet)

Nf=2=1:

C. Bernard et al., Phys.Rev. D71, 034504 (2005)
(improved staggered (asqtad), Nt=4,6,8, input r1)
(rescaled to r0)

M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006)
(staggered (stout), Nt=4,6,8,10; input fK)
(converted to r0)

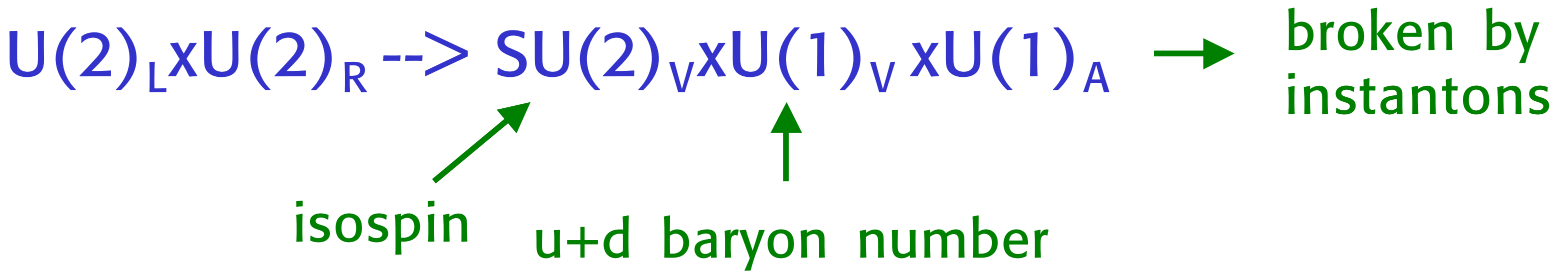
● chiral ■ deconfinement ⬢ chiral+deconfinement

Chiral symmetry and its breaking

In the limit m_u, m_d -> 0 the QCD Lagrangian is $U(2)_L x U(2)_R$ symmetric

But no parity doublets in the hadron spectrum:
the symmetry is spontaneously broken by $\bar{q}q$ condensates

The 3 pions are the would be Goldstone bosons from the breaking of the axial SU(2):



$$U(2)_L x U(2)_R \dashrightarrow SU(2)_V x U(1)_V x U(1)_A \longrightarrow$$

broken by instantons

isospin

u+d baryon number

The quark condensate has been computed on the lattice:

$$\bar{u}u+\bar{d}d = [234 \pm 4 \pm 17\ \mathrm{MeV}]^3 \quad (\overline{\mathrm{MS}}\ \text{scheme at 2 GeV})$$

Fukaya et al 1012.4052

Strong CP violation: possible new physics ?

The axial anomaly breaks the singlet axial current

$$\partial_\mu j_5{}^\mu = \frac{\alpha_s}{4\pi} Tr(F_{\alpha\beta}\tilde{F}^{\alpha\beta}) \qquad \tilde{F}^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$$

As an effect a term is added to the lagrangian

$$\Delta L = \theta \frac{\alpha_s}{4\pi} Tr(F_{\alpha\beta}\tilde{F}^{\alpha\beta})$$

where θ arises from the topology of the vacuum in non abelian gauge theories which is far from trivial:

$\theta = \theta_{\text{instantons}} + \text{Arg Det m}$ m quark mass matrix θ is expected to be o(1). But it would contribute to the neutron electric dipole moment:

$$d_n(e \cdot cm) \simeq 3 \cdot 10^{-16}\theta$$

From experiment: → $|\theta| \leq 10^{-10}$

The "strong CP problem" consists in finding an explanation:

- Non rinormalisation theorem in SUSY
- An ad hoc symmetry (Peccei-Quinn) spont. broken --> axion
- Something not understood on vacuum topology?

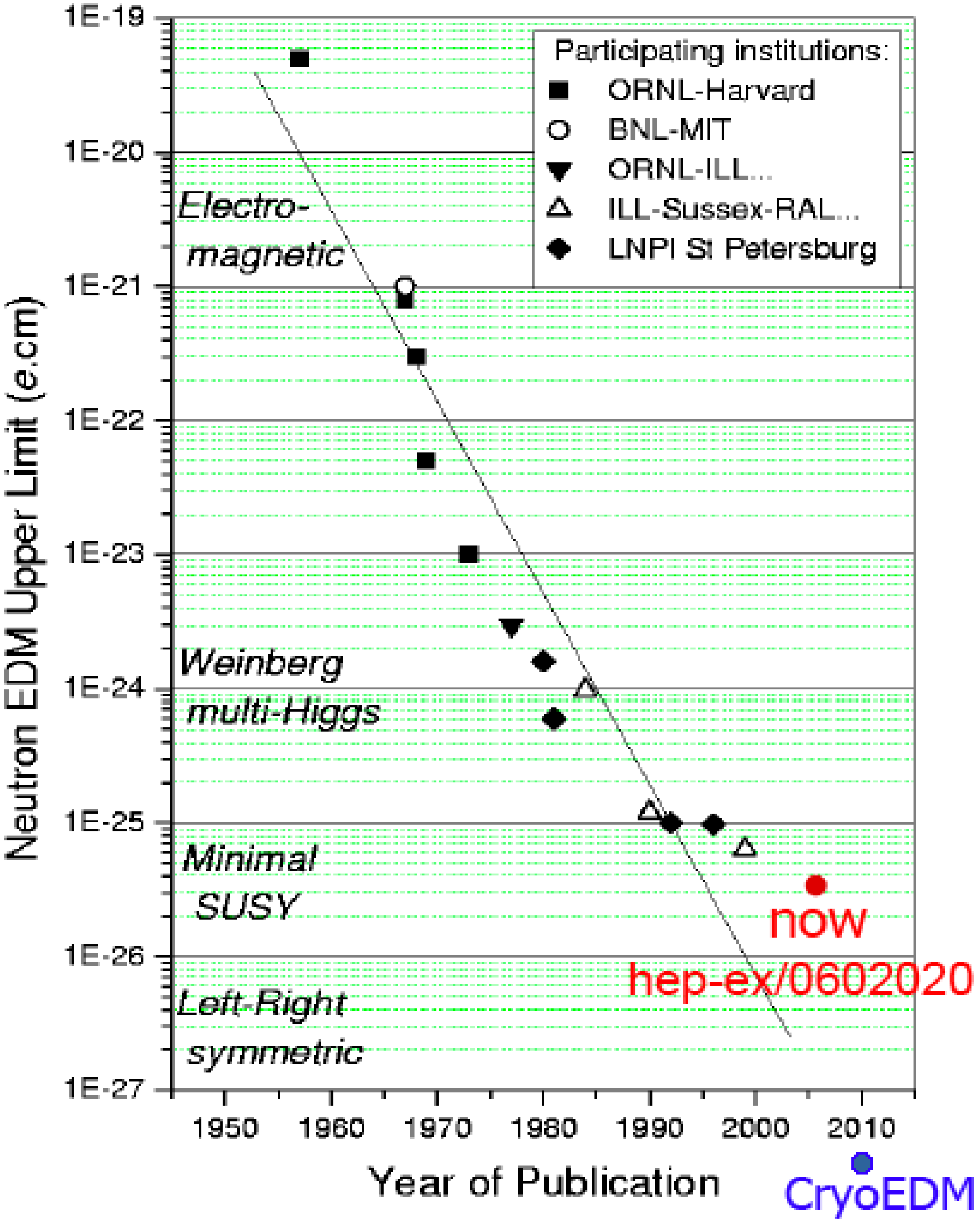
••••••

CPV in FC channels is dominated by CKM

What in flavour conserv. channels?

present limit on nEDM from Grenoble

$|d_n| < 3\ 10^{-26}$ e cm (90%cl)



LHC Heavy Ion Experiments (ALICE, ATLAS, CMS)

7 TeV p-p com energy corresponds to 7x82 = 574 TeV Pb-Pb

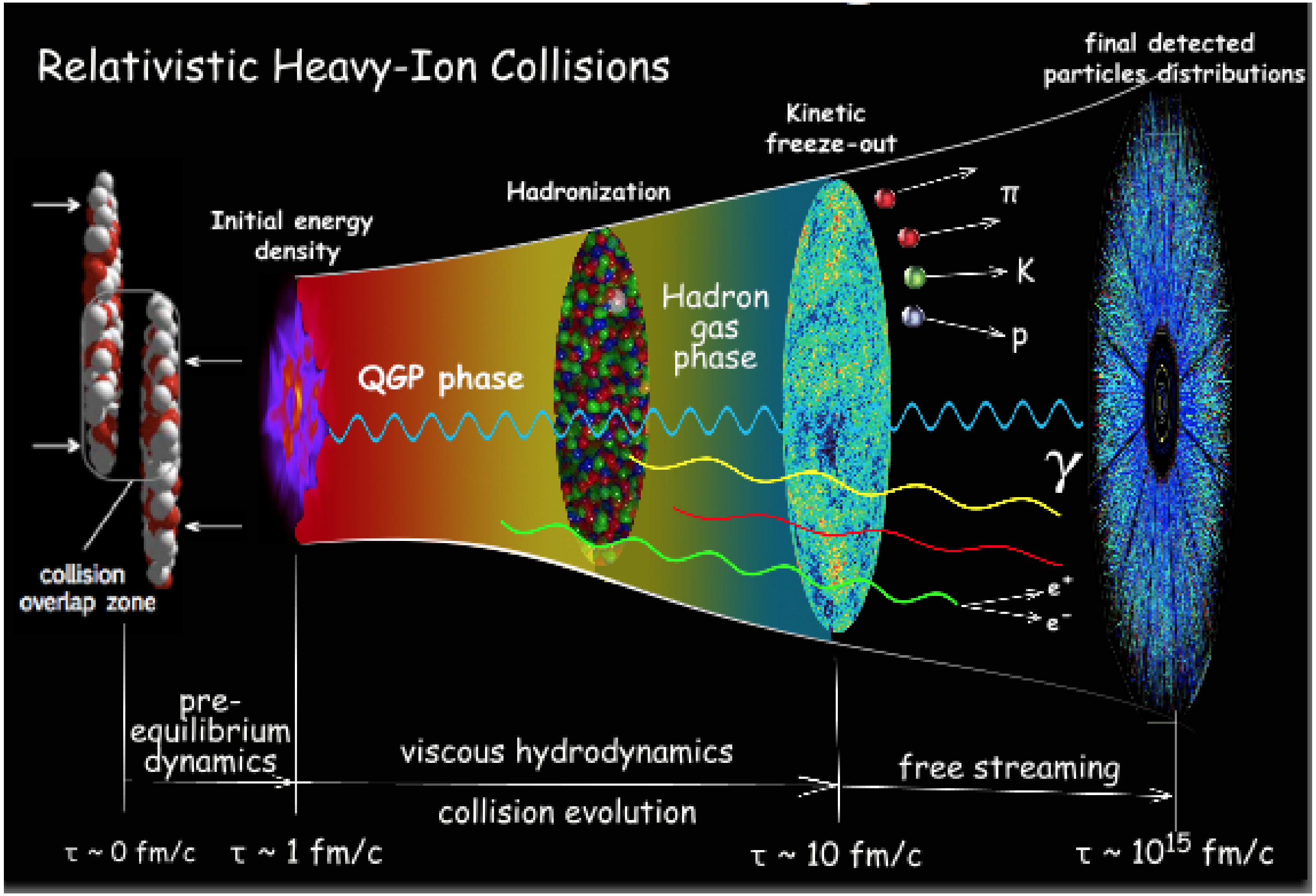
Pb has 82 protons and 208 nucleons: 574/208 = 2.76 TeV

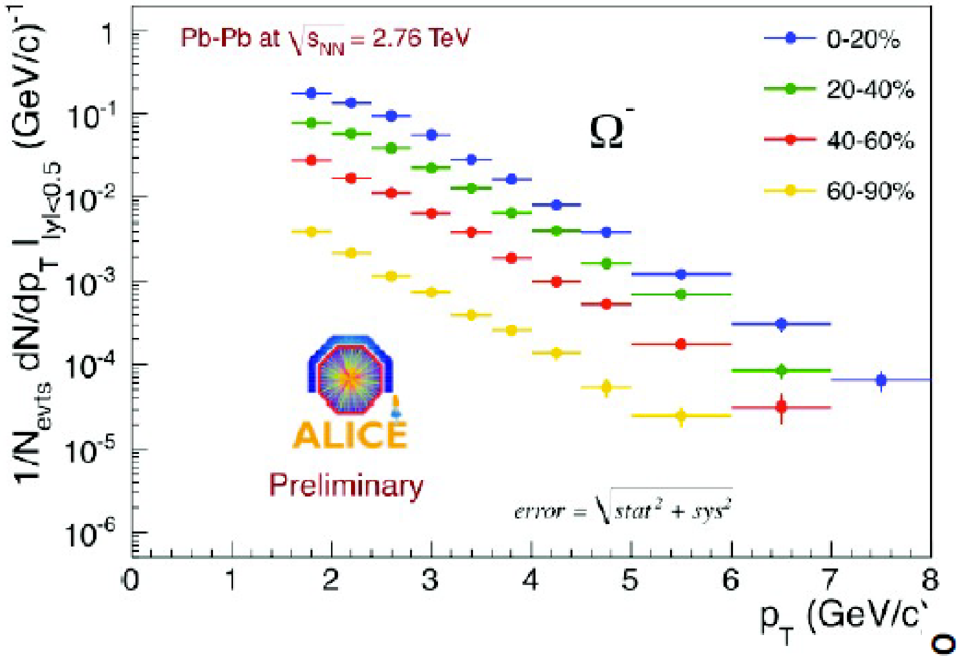
Or 2.76 TeV for each NN pair

From the measured ch. particle multiplicity/unit rapidity $dN_{ch}/d\eta \sim 1600$ (in most central collisions) one estimates:

$$\varepsilon_0 \sim 146 \text{ GeV/fm}^3, T \sim 640 \text{ MeV} \sim 4\, T_c$$

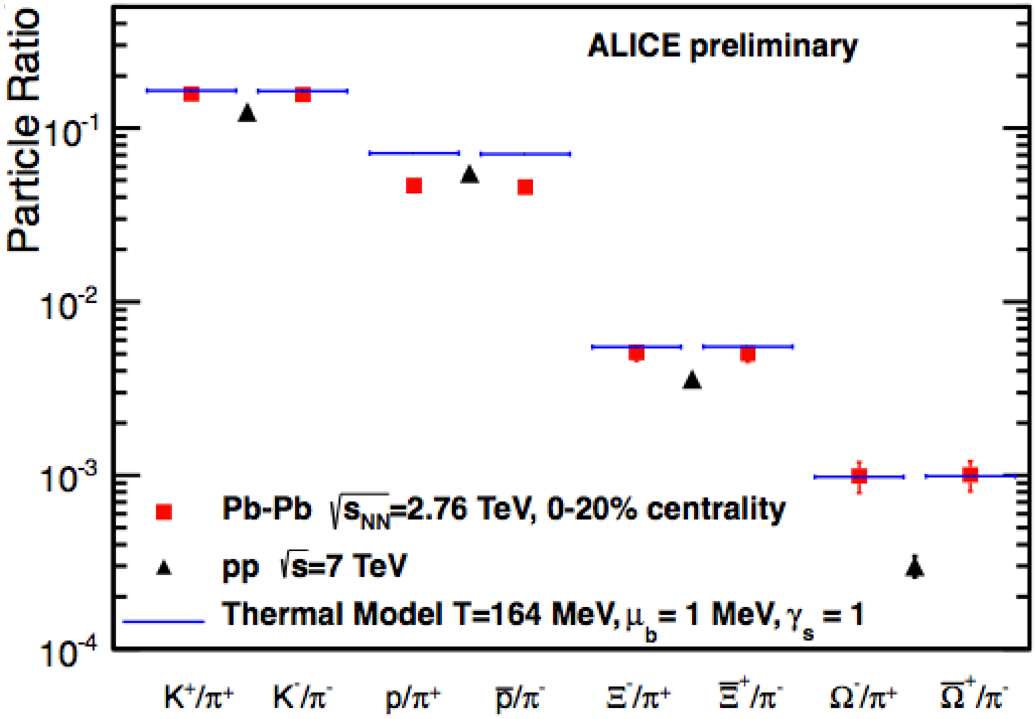
A review: B. Muller, Schukraft, Wyslouch ArXiv:1202.3233


Relativistic Heavy-Ion Collisions
final detected particles distributions
Kinetic freeze-out
Hadronization
Initial energy density
QGP phase
Hadron gas phase
π
K
p
γ
e+
e−
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c



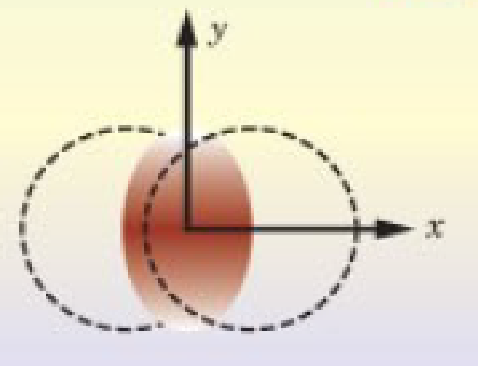
Strangeness enhancement for central events [0-20% means most central]

$$n_h/n_l \sim \exp -[m_h-m_l]/kT$$



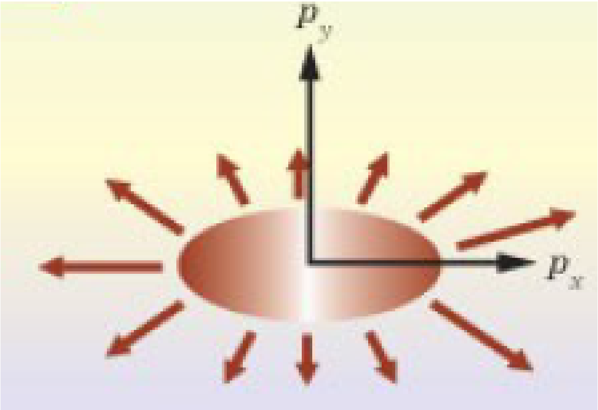
statistical model reproduces yields well (some p deficit?)

Elliptic flow: a tool to study the primeval final state



coord. space

→



mom. space

$$\frac{dN}{dp_t\,dy\,d\phi} = N_0\cdot\left[1+\sum_{i=1} 2\,v_i\left(y,\,p_t\right)\cos\left(i\,\phi\right)\right]$$

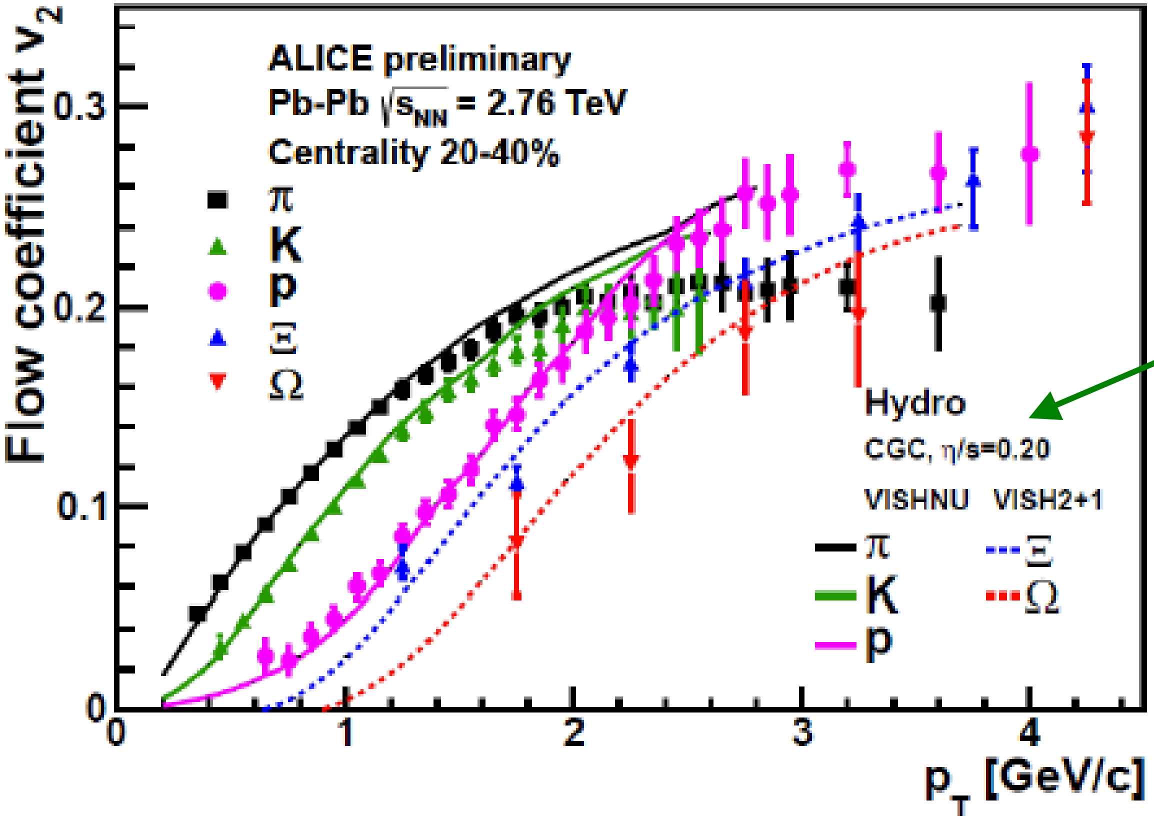
$v_{1\text{-}6}$ now measured

$$\frac{dN}{d\phi} \sim \left(1 + 2v_2 \cos\left[2\left(\phi - \phi_0\right)\right] + ...\right)$$

dominant anisotropy parameter

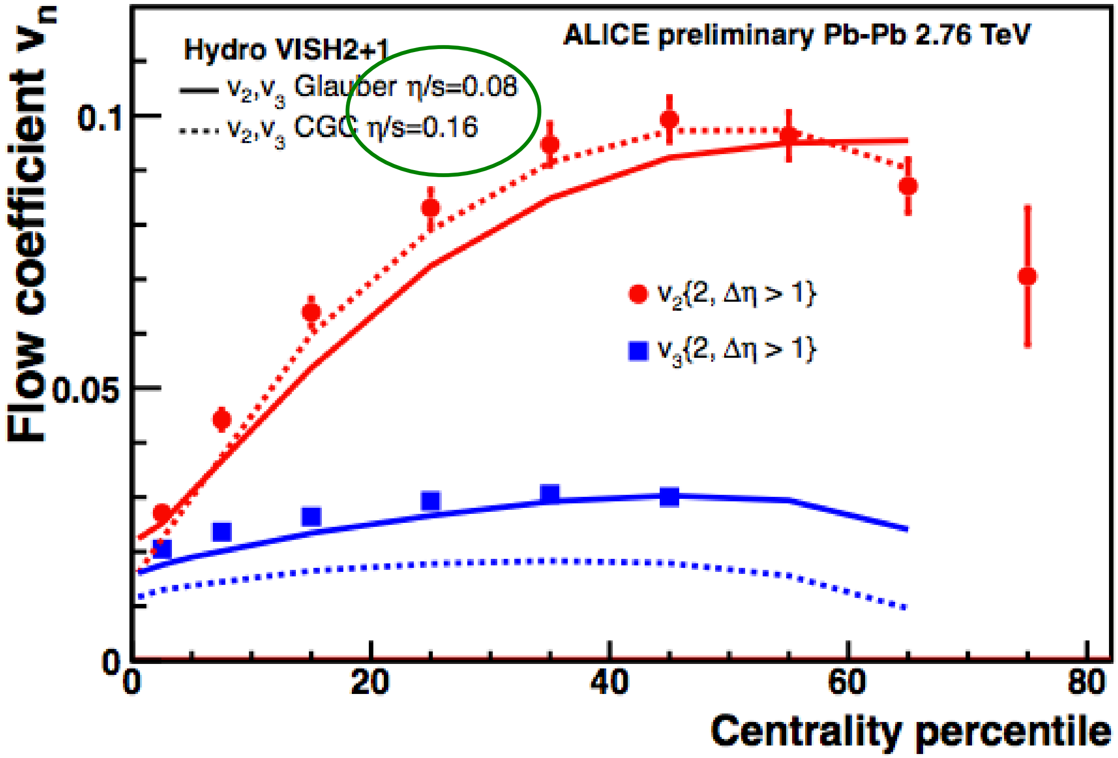
Hydrodynamic calc'ns depend on η/s (shear viscosity/entropy density).

Luzon (QM12)



η/s~0.2

η/s can be determined from the p_T or the centrality distributions with compatible results



Small values of η/s are obtained: 0.07 - 0.43. More precision possible in near future.

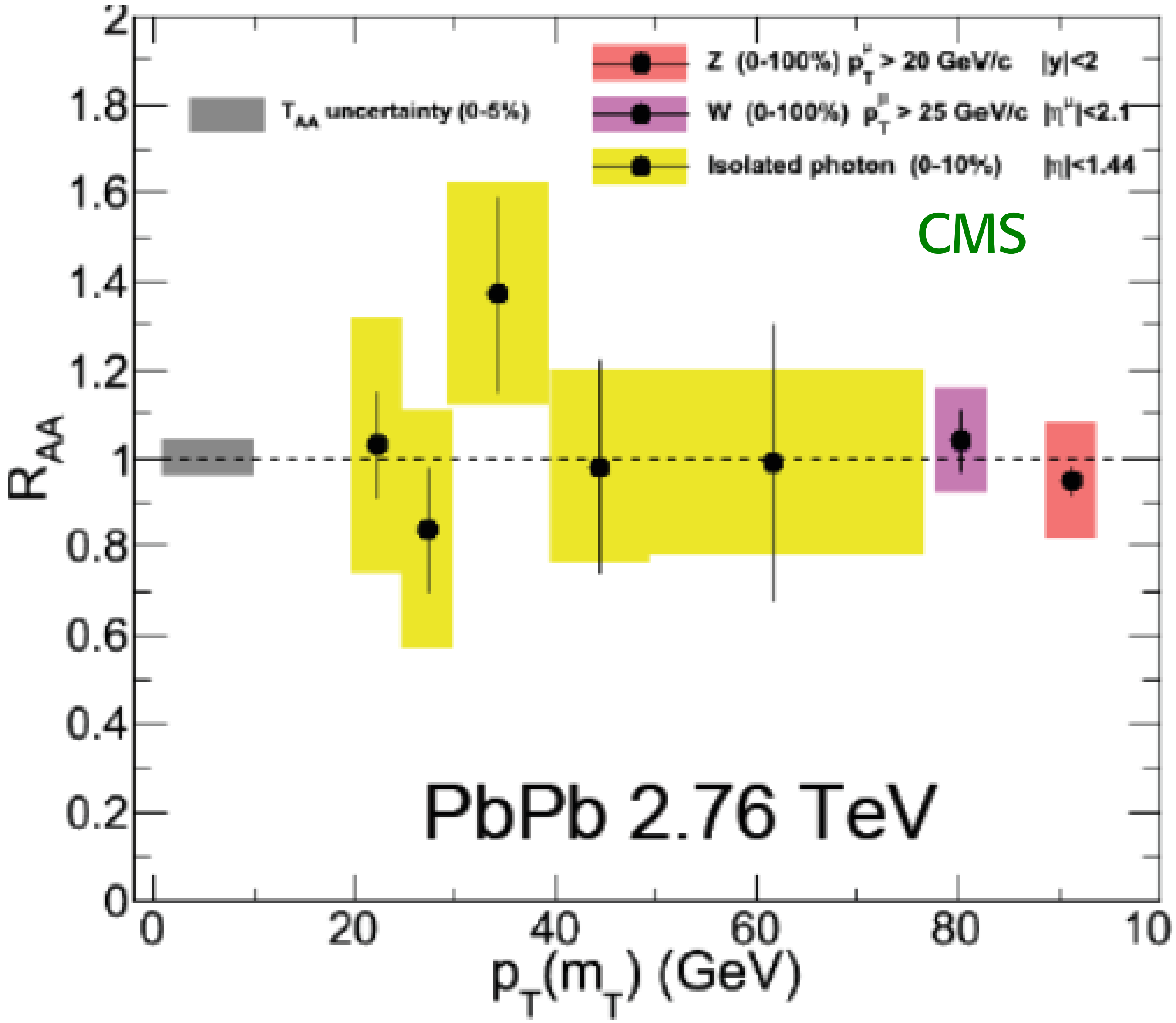
For a perfect quantum fluid $\eta/s \sim 1/4\pi \sim 0.08$

On the basis of the AdS/CFT correspondence it is conjectured that this is a lower limit in real QCD

In summary:

- the hot dense matter formed is close to a perfect fluid

EW probes (W, Z, γ) are not suppressed in the medium

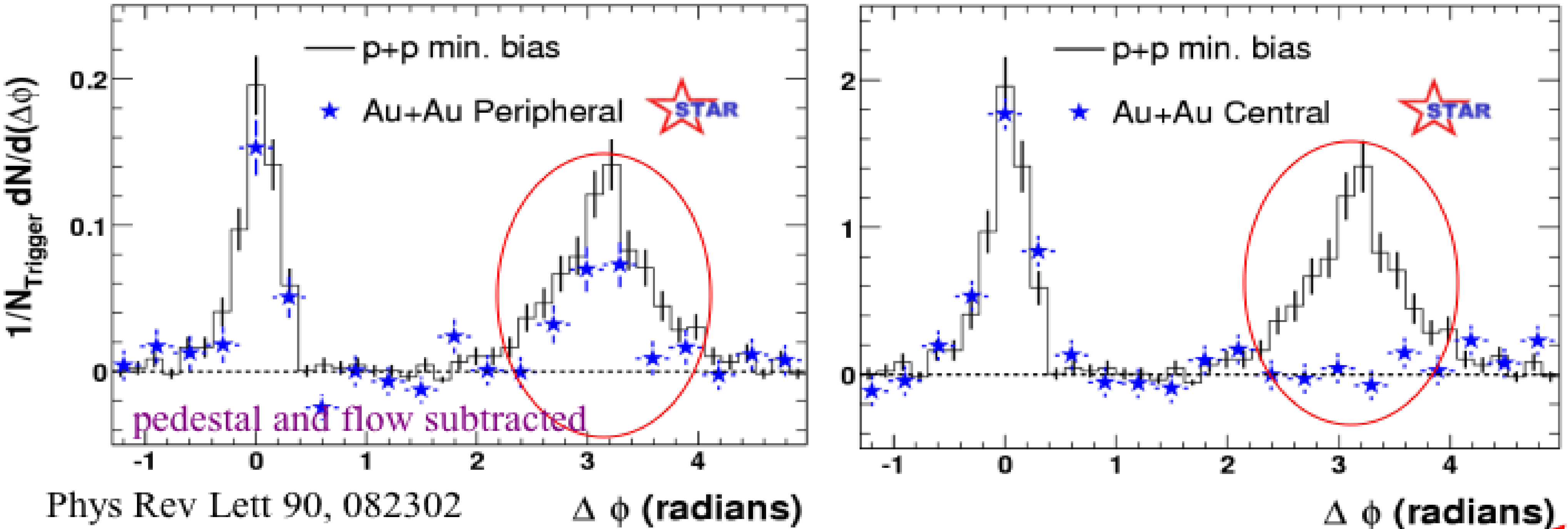


RAA:
ratio PbPb/pp

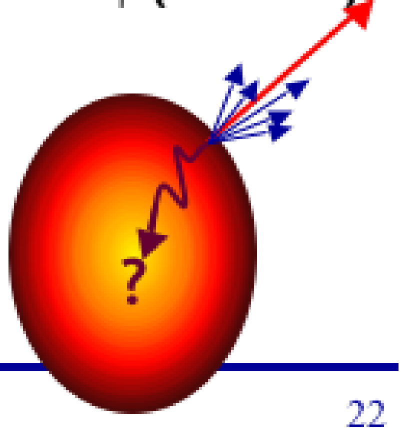
Azimuthal distributions in Au+Au

Au+Au peripheral

Au+Au central

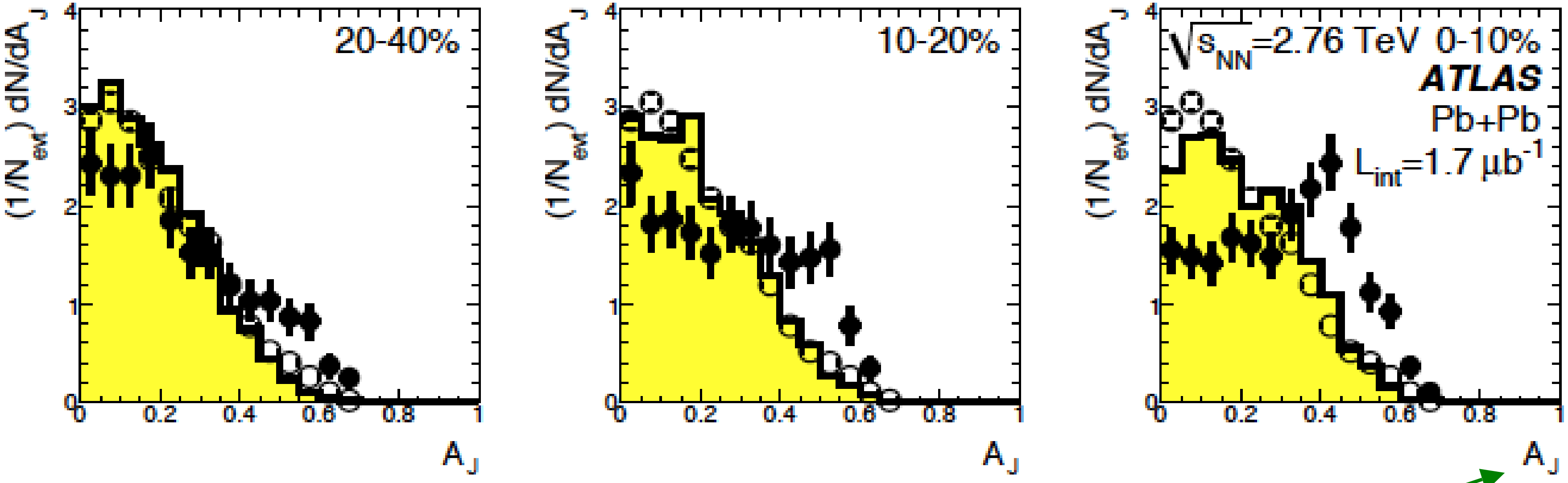


Phys Rev Lett 90, 082302



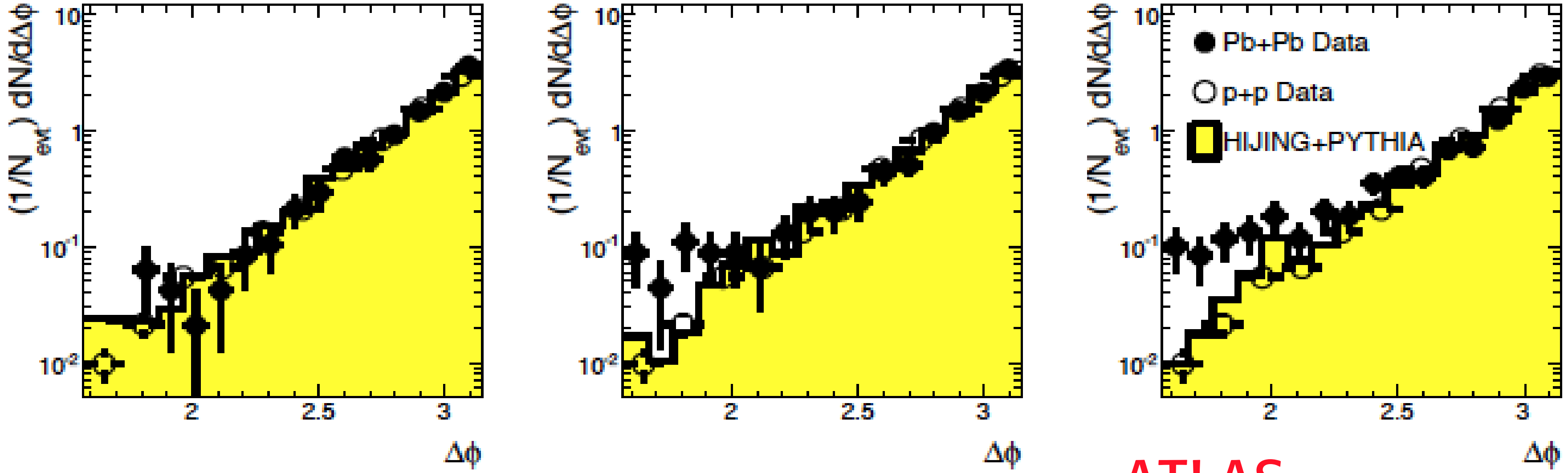
Near-side: peripheral and central Au+Au similar to p+p

Strong suppression of back-to-back correlations in central Au+Au



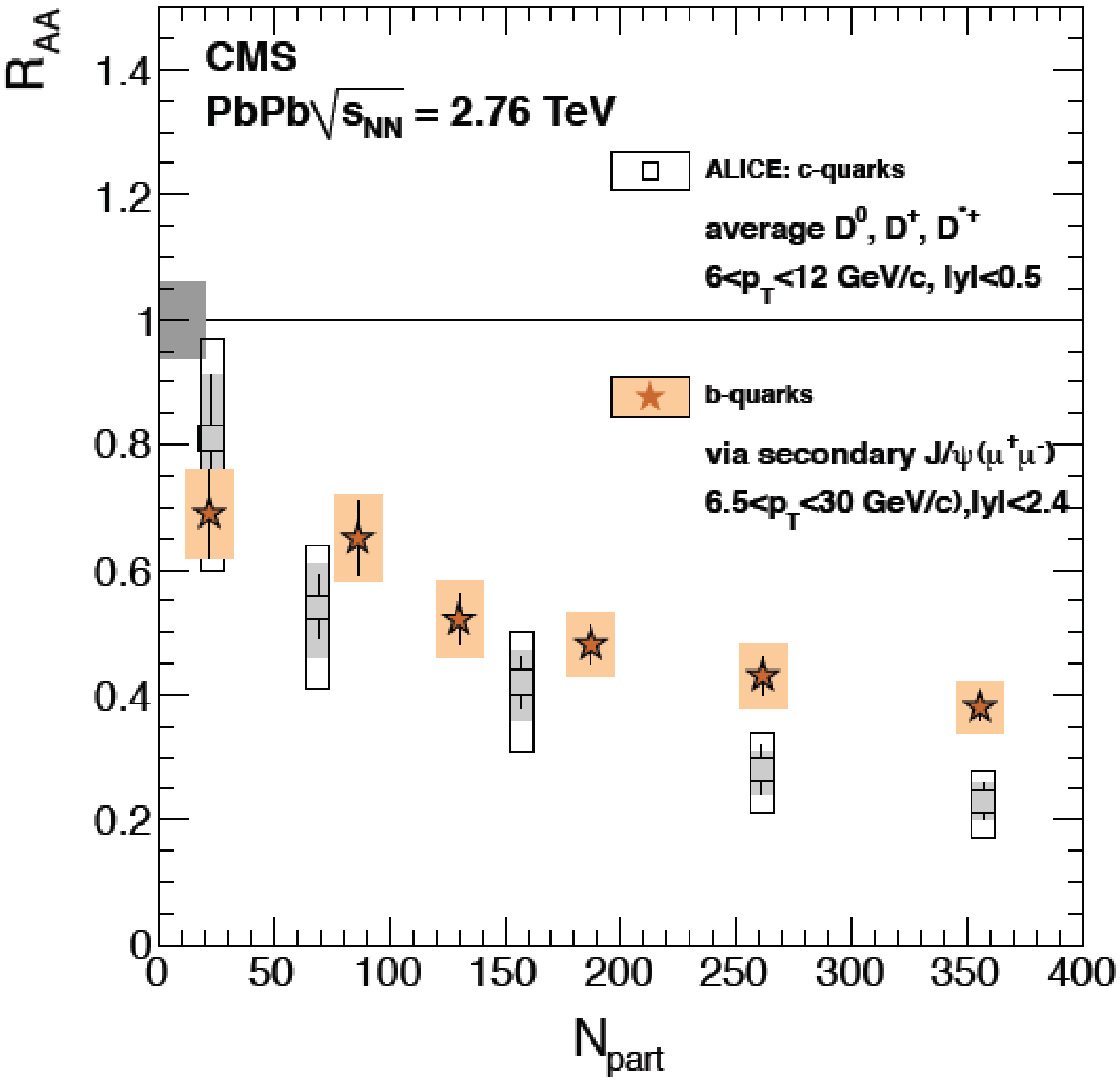
In most central events the energy unbalance of the 2 jets is increased

$$A_J = \frac{E_T^1 - E_T^2}{E_T^1 + E_T^2}$$



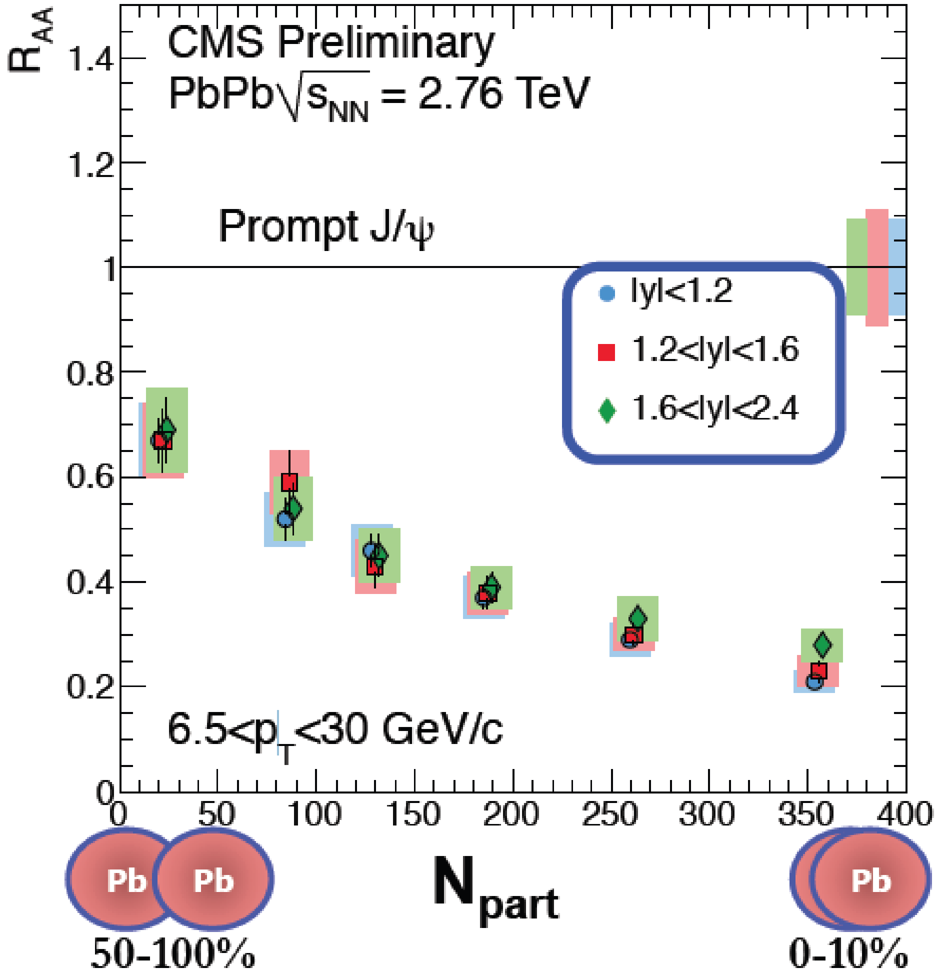
ATLAS

Open charm and beauty (D and B mesons) are suppressed

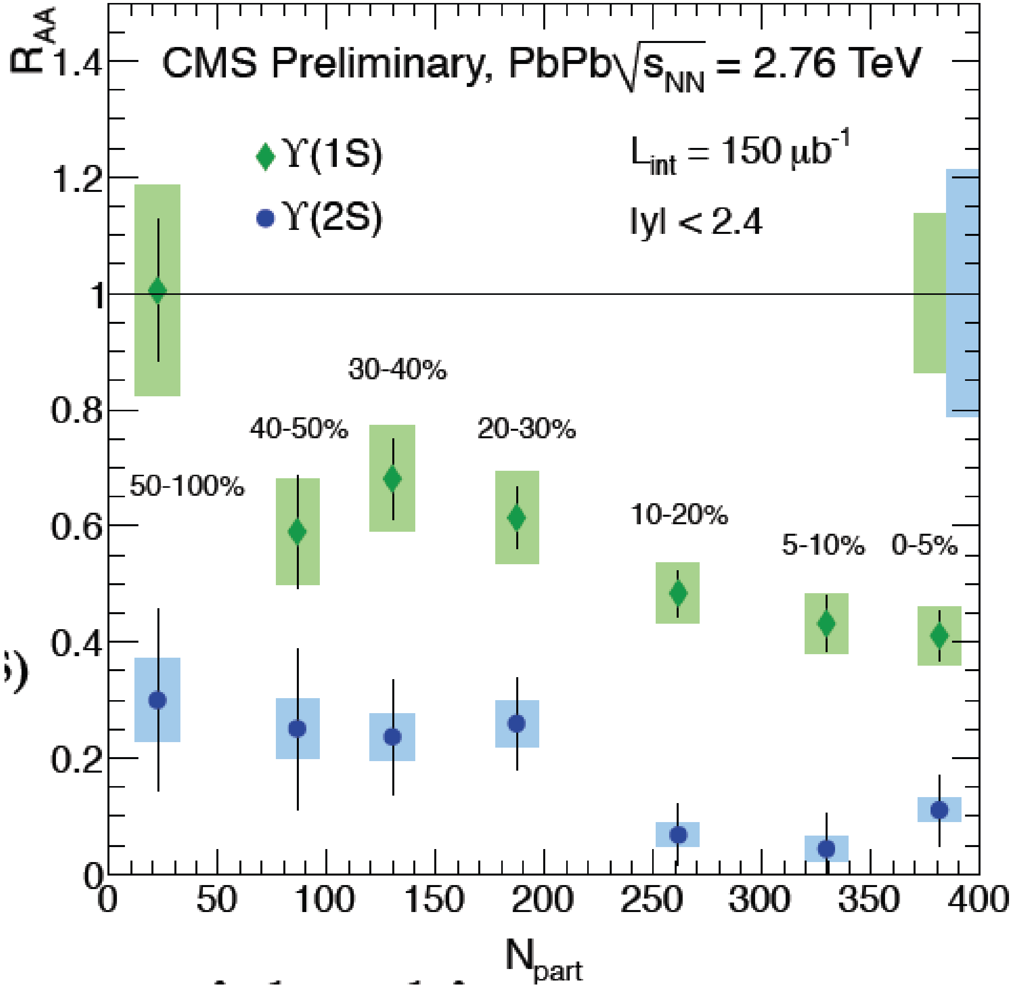


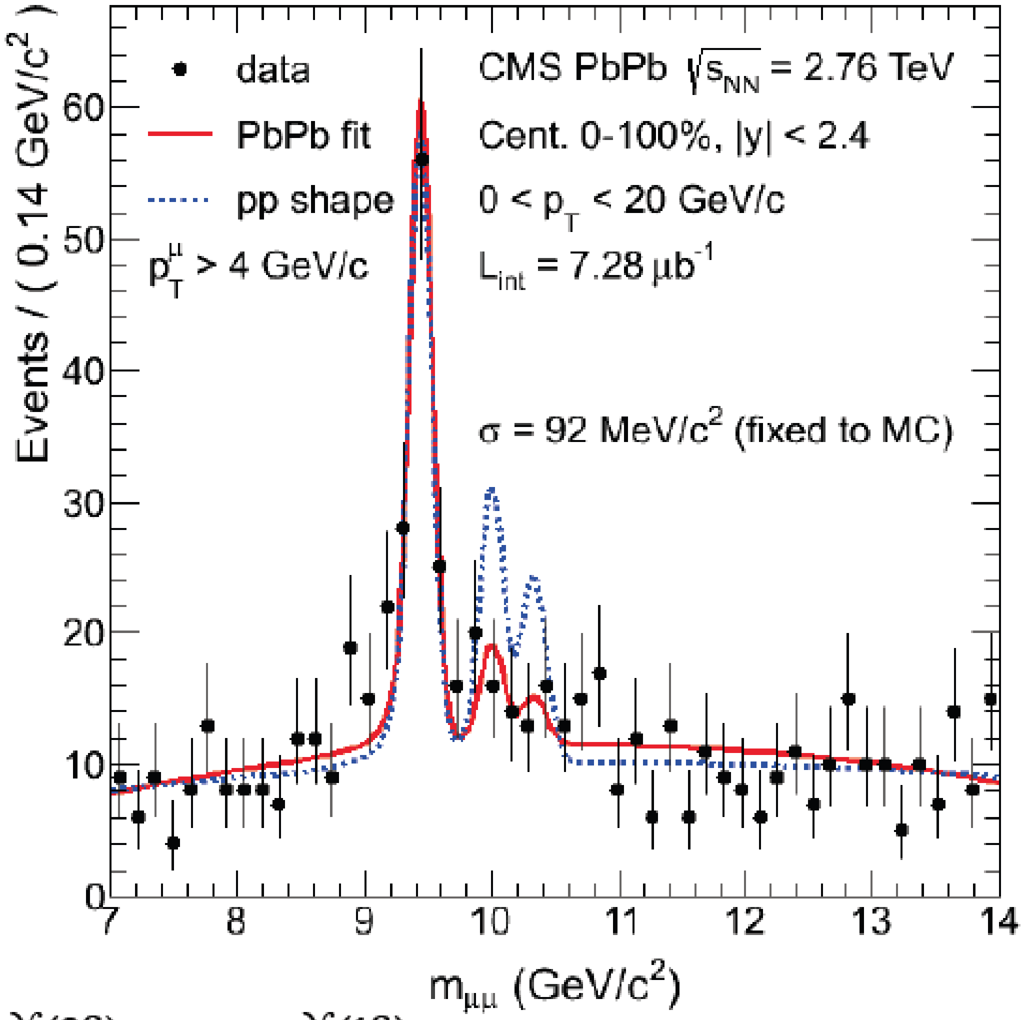
J/ψ suppression at the start (SPS) was thought to be a clear indicator of colour screening.

Interpretation of data at RHIC and the LHC demands both screening and recombination
(late formation of J/ψ from charm quarks in the medium)



b-onium suppression (less affected by recombination)





$$R_{AA}^{\Upsilon(3S)} < R_{AA}^{\Upsilon(2S)} < R_{AA}^{\Upsilon(1S)}$$

Conclusion

Heavy Ion collisions have demonstrated the formation of a strongly interacting, hot, near perfect liquid.

The reconstructed temperature and energy density are compatible with what expected for quark-gluon plasma.

Additional properties of this liquid like shear viscosity, equation of state and sound velocity are under continuing study.

QCD is an unbroken SU(3) gauge theory with triplet quarks

$$L = -\frac{1}{4}\sum_{A=1}^{8} F^{A\mu\nu}F_{\mu\nu}^{A} + \sum_{j=1}^{n_f}\bar{q}_j(i\widehat{D} - m_j)q_j$$

Defs: $[t^A, t^B] = iC^{ABC}t^C$ $\qquad Tr[t^At^B] = \frac{1}{2}\delta^{AB}$

(C_{ABC}: SU(3) struture constants, t^A: generator representation)

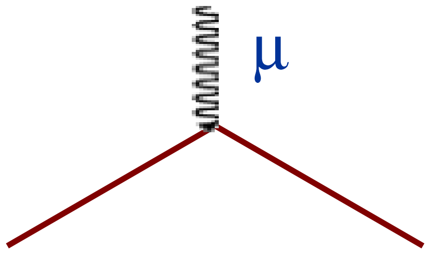
$$g_\mu = \sum_{A=1}^{8} g_\mu^A t^A$$ (g^A_μ is a gluon field)

$\widehat{D} = D_\mu\gamma^\mu$; $D_\mu = \partial_\mu + ie_s g_\mu$ (D: covariant derivative)

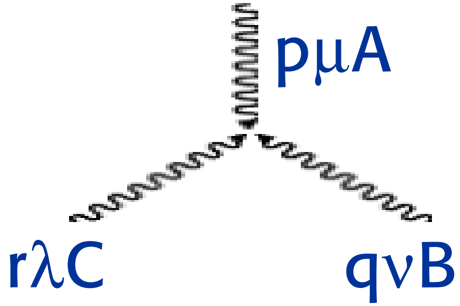
$\alpha_s = \frac{e_s^2}{4\pi}$ (e_s: SU(3) gauge coupling)

$$F^A_{\mu\nu} = \partial_\mu g^A_\nu - \partial_\nu g^A_\mu - e_s C_{ABC} g^B_\mu g^C_\nu$$

Physical QCD vertices

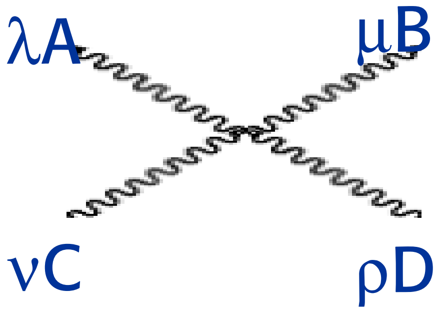


$-ie_s\gamma^{\mu}t^A$



$p+q+r=0$

$e_s C_{ABC}\left[g_{\mu\nu}(p-q)_{\lambda}+\text{perm}\right]$



Note: e_s^2

$-ie_s^2\left[\, C_{ABF}C_{CDF}\,(g_{\lambda\nu}\,g_{\mu\rho}-g_{\lambda\rho}\,g_{\mu\nu})+\text{perm}\right]$

Classical gauge th. lagrangian

Quantisation
↓
Gauge fixing terms
Ghosts

Feynman rules

Perturbation Theory
↓

Infinities

Regularisation
Renormalisation
↓
Cutoff K
Redefinition of m, α_s,
Z_i (wave funct.n norm'ns)

Perturbative quantum gauge th.

Perturbative QCD and scale invariance

In the QCD lagrangian

$$L = -\frac{1}{4}\sum_{A=1}^{8} F^{A\mu\nu}F^{A}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j(i\widehat{D} - m_j)q_j$$

quark masses are the only parameters with dimensions.

Naively we would expect massless QCD to be scale invariant (dimensionless observables should not depend on the absolute energy scale, but only on ratios of energy variables)

The massless limit should be relevant for the asymptotic large energy limit of processes which are non singular for m -> 0.

This naïve expectation is false!

For massless QCD the scale symmetry of the classical theory is destroyed by regularisation and renormalisation which introduce a dimensional parameter in the quantum version of the theory (Λ_{QCD}).

[When a symmetry of the classical theory is necessarily destroyed by quantisation, regularisat.n and renorm.n one talks of an "anomaly"]

While massless QCD is finally not scale invariant, the departures from scaling are asymptotically small, logarithmic and computable (in massive QCD there are additional mass corrections suppressed by powers of m^2/E^2).

Hard processes

At the "parton" level (q and g) we can apply the asymptotics from massless QCD to processes with the following properties:

- all relevant energy variables are large

$E_i = x_i Q$ $Q \gg m$ x_i : scaling variables

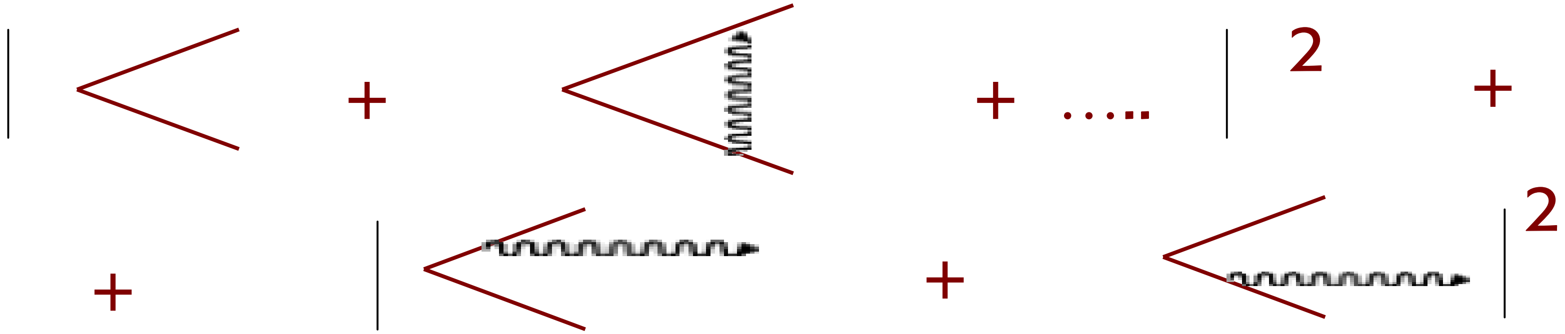
- no infrared and collinear singularities ("infrared safe")
- finite for m -> 0 (no mass singularities.)

To satisfy these criteria processes must be sufficiently "inclusive":

- add all final states with massless gluon emission
- add all mass degenerate final states (e.g. q-qbar pairs)

Bloch-Nordsieck Theorem:

Infrared singularities cancel between real and virtual diagrams when all resolution indistinguishable final states are added up.



Kinoshita-Lee-Nauenberg Theorem:

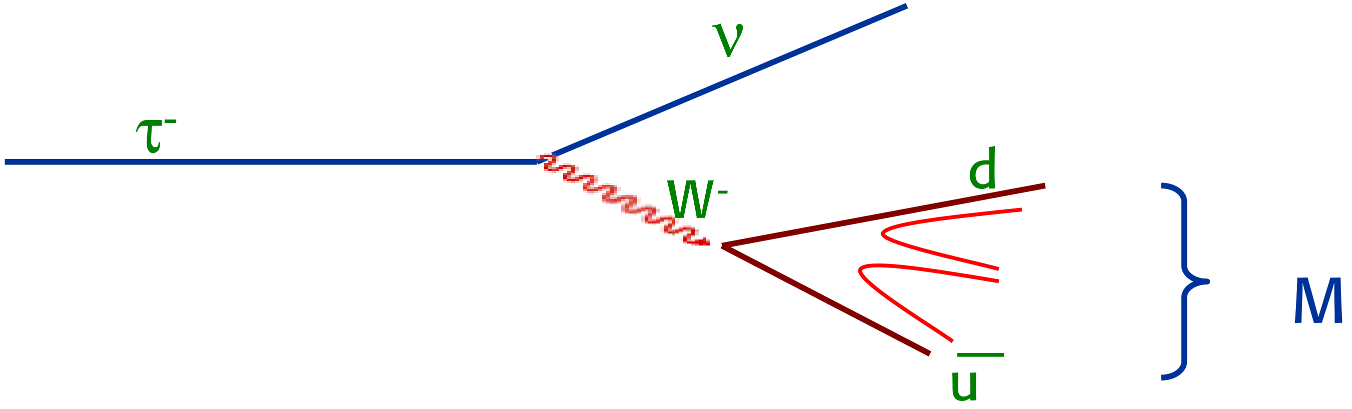
Mass singularities are absent if all degenerate states are added up (including collinear qqbar pairs for massless q).
If an inclusive final state is taken, only the mass singularities from the initial lines remain.

(Will be absorbed inside the initial parton densities)

Note: We compute inclusive rates for partons and take them as equal to rates for hadrons.

Partons and hadrons are considered as two equivalent sets of complete states.

This is called "global duality" and is rather safe in the totally inclusive case.
It is less so for distributions, like $d\sigma/dM$ in the invariant mass M ("local duality") where it is reliable only if smeared over a sufficiently large bin of M.



Regularisation and Renormalisation

In general:

- A dimensional "cut off" K is introduced (must be gauge invariant)

- The dependence on the cut-off is eliminated by a redefinition of m, e_s and Z using suitable renormalisation conditions.

$$\mathit{Propagator} = \frac{Z}{p^2 - m^2} + \mathit{no} - \mathit{pole}$$

Renormalized mass: position of the propagator pole.
Wave funct'n renormalization Z: residue at the pole.

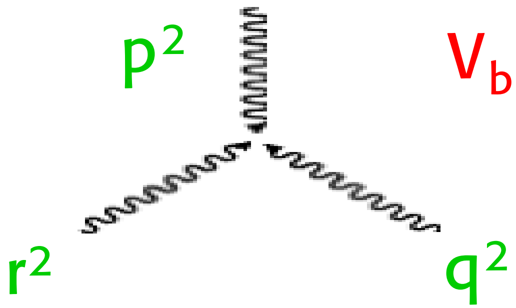
The renormalized coupling e_s is, for example, defined in terms of a renormalized 3-point vertex at some momenta.

In particular in massless QCD:

If we start with $m_0=0$ the mass is not renormalized because it is protected by a symmetry (chiral symm.) -> m=0

The coupling e_s can be defined in terms of the 3-gluon coupling at a scale $-\mu^2$:

Ward id. guarantee the same result starting from any other vertex

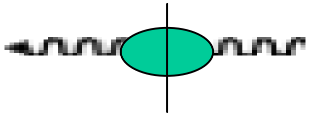


$$V_{bare}(p^2,q^2,r^2)=Z\,V_{ren}(p^2,q^2,r^2)$$

$(Z=Z_g^{-3/2}$ for V 1PI)

$$V_{ren}(-\mu^2,-\mu^2,-\mu^2)=e_s$$

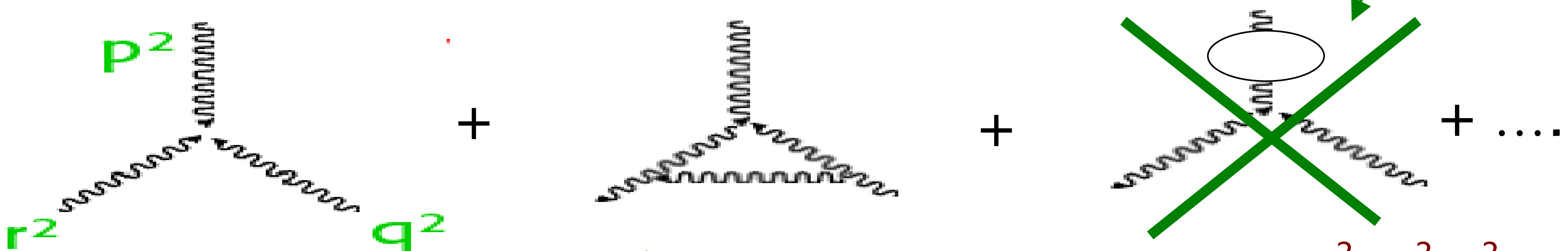
- The scale μ cannot be zero (infrared singularity)!
- $-\mu^2<0$: no absorptive parts



Similarly Z_g can be defined by the inverse propagator at $p^2=-\mu^2$

$$P^{-1}{}_{bare}=Z_g{}^{-1}P^{-1}{}_{ren}$$

Computing all 1PI diagrams (with cutoff K)



at 1 loop

Not 1PI

$p^2=q^2=r^2$

$$V_{bare} = e_0\left[1 + c\alpha_s \log\frac{K^2}{p^2} + \dots\right] =$$

$$= \left[1 + c\alpha_s \log\frac{K^2}{-\mu^2} + \dots\right] e_0 \left[1 + c\alpha_s \log\frac{-\mu^2}{p^2}\right] = Z_V{}^{-1} e_0[..] =$$

$$= \left[1 + d\alpha_s \log\frac{K^2}{-\mu^2} + \dots\right] e \left[1 + c\alpha_s \log\frac{-\mu^2}{p^2}\right] = Z_g{}^{-\frac{3}{2}} V_{ren}$$

V_0 starts with e_0
ie tensor structure
factorized

$e_0 = Z_g^{-3/2} Z_V e$

Note: V_{Bare} depends on K but not on μ

$$\frac{dV_{Bare}}{d\log\mu^2} = 0$$

Both Z and V_{ren} depend on μ

Renormalisation group equation

(We write α for α or α_s in QED or QCD)

In general:

$$G_{Bare}(K^2, \alpha_0, p_i^2) = Z\, G_{ren}(\mu^2, \alpha, p_i^2)$$

so that:

$$\frac{dG_{Bare}}{d\log\mu^2} = \frac{d}{d\log\mu^2}[ZG_{ren}] = 0$$

or

$$Z\left[\frac{\partial}{\partial\log\mu^2} + \frac{\partial\alpha}{\partial\log\mu^2}\frac{\partial}{\partial\alpha} + \frac{1}{Z}\cdot\frac{\partial Z}{\partial\log\mu^2}\right]G_{ren} = 0$$

Finally the RGE can be written as:

$$\left[\frac{\partial}{\partial\log\mu^2} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \gamma(\alpha)\right]\cdot G_{ren} = 0$$

⊕ This is a relation among physical quantities (no cutoff K)

Consider the RGE: $\left[\frac{\partial}{\partial \log \mu^2} + \beta(\alpha)\frac{\partial}{\partial \alpha} + \gamma(\alpha)\right] \cdot G_{ren} = 0$

applied to some hard process at a large scale Q : G_{ren} -> $F(t,\alpha,x_i)$ where x_i are scaling variables (omitted in the following), and

$$t = \log \frac{Q^2}{\mu^2}$$

Assume F is adimensional, then in the naïve scaling limit F would be independent of t.

We want to solve the RGE equation:

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha)\frac{\partial}{\partial \alpha} + \gamma(\alpha)\right] \cdot F(t,\alpha) = 0$$

- with a given boundary cond.: $F(0,\alpha)$ specified.

Given the general RGE:

$$\left[-\frac{\partial}{\partial t}+\beta(\alpha)\frac{\partial}{\partial \alpha}+\gamma(\alpha)\right]\cdot F(t,\alpha) = 0 \qquad t = \log\frac{Q^2}{\mu^2}$$

The solution, with boundary cond. F(0,α), is:

$$F(t,\alpha)=F[0,\alpha(t)]\exp\int_{\alpha}^{\alpha(t)}\frac{\gamma(\alpha')}{\beta(\alpha')}d\alpha'$$

where the "running coupling" α(t) is defined by:

$$t = \int_{\alpha}^{\alpha(t)}\frac{1}{\beta(\alpha')}d\alpha'$$

Note: at t=0, $\alpha(0) = \alpha$. One has: $\frac{\partial}{\partial t}\alpha(t) = \beta(\alpha(t))$

The important point is the appearance of the running coupling that determines the asympt. behaviour.

The running coupling

QCD or QED

$$\alpha = \frac{e^2}{4\pi} \quad \Bigg| \quad t = \log \frac{Q^2}{\mu^2}$$

The running coupling $\alpha(t)$ is fixed by the beta function:

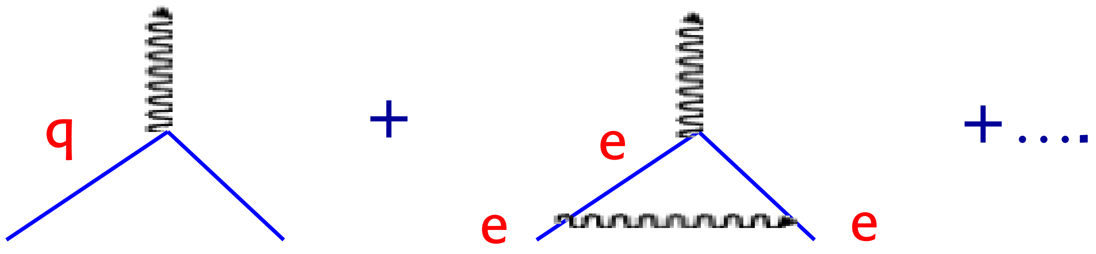
$$\frac{\partial \alpha}{\partial \log \mu^2} = \beta(\alpha)$$

or

$$\frac{\partial}{\partial t}\alpha(t) = \beta(\alpha(t))$$

The μ dependence starts at 1-loop:

Recall: in QCD Ward id. guarantee the same result starting from vertex



$$\frac{\partial e}{\partial \log \mu^2} \propto e^3 \quad \Rightarrow \quad 2e\frac{\partial e}{\partial \log \mu^2} \propto e^4 \propto \alpha^2$$

$$\beta(\alpha) = \pm b\alpha^2[1 + b'\alpha + \ldots] \quad (b>0)$$

By explicit calculation at 1-loop one finds:

N_C=1 for leptons,
3 for quarks

QED: $\beta(\alpha) \sim + b\alpha^2 + \ldots$

$$b = \sum_i \frac{(N_C Q^2)_i}{3\pi}$$

The sum is over all fermions of charge Qe

QCD: $\beta(\alpha) \sim - b\alpha^2 + \ldots$

$$b = \frac{11 N_C - 2 n_f}{12\pi}$$

Here N_C =3

n_f is the number of quark flavours

Recall: $\frac{\partial}{\partial t}\alpha(t) = \beta(\alpha(t))$

$$t = \log \frac{Q^2}{\mu^2}$$

If $\alpha(t)$ is small, we can compute b in pert. th. The sign in front of b decides whether: $\alpha(t)$ increases with t or Q^2 (QED) or $\alpha(t)$ decreases with t or Q^2 (QCD).

QCD is "asymptotically free". In 4-dim all and only non-abelian gauge theories are asympt. free.

Going back to the equation:

$$t = \int_{\alpha}^{\alpha(t)} \frac{1}{\beta(\alpha')} d\alpha' \qquad \alpha(0)=\alpha \qquad t=0 \to Q=\mu$$

We replace $\beta(\alpha) \sim \pm b\alpha^2$, integrate and do a small algebra. We find:

$$\alpha(t) = \frac{\alpha}{1 + b\alpha t} \sim \alpha(1 - b\alpha t + \ldots) \qquad \text{QCD}$$

$$\alpha(t) = \frac{\alpha}{1 - b\alpha t} \sim \alpha(1 + b\alpha t + \ldots) \qquad \text{QED}$$

In QCD we have:

$$\alpha(t) = \frac{1}{\frac{1}{\alpha} + bt} = \frac{1}{b \log \frac{\mu^2}{\Lambda^2} + b \log \frac{Q^2}{\mu^2}} = \frac{1}{b \log \frac{Q^2}{\Lambda^2}}$$

Λ_{QCD}

Note

- α decreases logaritmically in Q^2
- a dimensional parameter $\Lambda = \Lambda_{QCD}$ replaces μ.

$$\beta(\alpha) \sim \pm b\alpha^2(1+b'\alpha+\ldots.)$$

In general the pert. coeff.s of $\beta(\alpha)$ depend on the def. of α, the renorm. scheme etc. But both b and b' are indep.

Here is a sketch of the proof: $\alpha' \cong \alpha(1+k\alpha+\ldots)$

$$\frac{d}{d\log\mu^2}\alpha' \cong \frac{d\alpha}{d\log\mu^2}(1+2k\alpha+\ldots) = \pm b\alpha^2(1+b'\alpha+\ldots)(1+2k\alpha+\ldots) =$$

$$= \pm b\alpha'^2(1+b'\alpha'+\ldots) = \beta(\alpha')$$

QCD: $b' = \dfrac{153-19n_f}{2\pi(33-2n_f)}$ for N_C=3

Taking b' into account: $\alpha_0^{-1}(Q^2) = b\log\dfrac{Q^2}{\Lambda^2}$

$$\alpha(Q^2) = \alpha_0(Q^2)\left[1-b'\alpha_0(Q^2)\log\log\frac{Q^2}{\Lambda^2}+\ldots..\right]$$

Summarising: the running coupl. $\alpha(Q^2)$ is the crucial quantity:

$$\frac{d\alpha(Q^2)}{d\log Q^2} = \beta[\alpha(Q^2)]$$

$$\boxed{\alpha(Q^2) = \frac{1}{b\log\frac{Q^2}{\Lambda^2_{QCD}}}(1+.....)}$$

$$\beta(\alpha) = -b\alpha^2[1+b'\alpha+....] \quad (b>0)$$

$$b = \frac{11N_C - 2n_f}{12\pi}$$

MS$_{(bar)}$, n_f=5: $$\beta(\alpha) \cong -0.610\alpha^2\left[1+1.261\frac{\alpha}{\pi}+1.475\left(\frac{\alpha}{\pi}\right)^2+9.836\left(\frac{\alpha}{\pi}\right)^3+...\right]$$

4th: van Ritbergen, Vermaseren, Larin (1997)

~ 50.000 4-loop diagrams!!

Λ_{QCD} is the scale that breaks scale inv. in massless QCD

Λ_{QCD} = 218±24 MeV (N_f=5)

The ρ mass etc are due to Λ_{QCD} not to m_q

No hierarchy problem in QCD!

$$\Lambda_{QCD} = M_{Pl}\exp\left(\frac{-1}{2b\alpha(M^2_{Pl})}\right)$$

Dependence of Λ from n_f

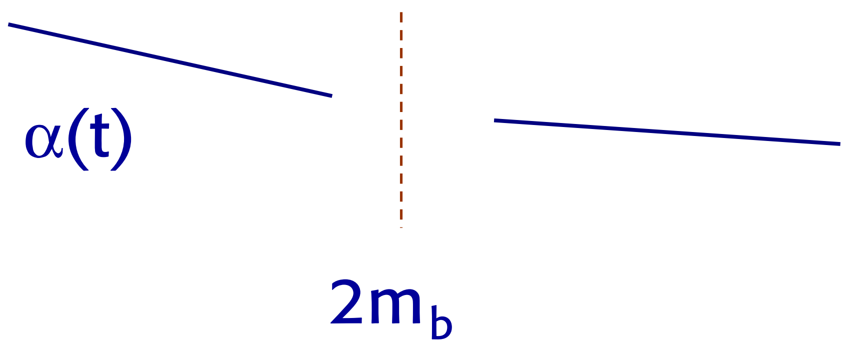
QED and QCD are theories with decoupling: quarks with mass m>Q do not contribute to the running of α up to the scale Q.

So for $2m_c<Q<2m_b$ the relevant asymptotics is for $n_f=4$, while for $2m_b<Q<2m_t$ $n_f=5$.

Going across the $2m_b$ threshold, the $\beta(\alpha)$ coeff.s change, so the $\alpha(t)$ slope changes. But $\alpha(t)$ is continuous so that Λ_4 and Λ_5 are different:

$$\alpha(Q^2) \cong \frac{1}{b\log\frac{Q^2}{\Lambda^2}}(1+....)$$

$$b = \frac{11N_C - 2n_f}{12\pi}$$



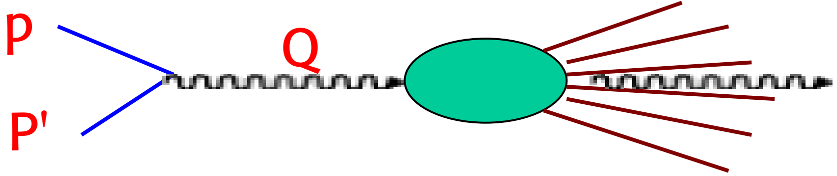
From matching $\alpha(Q^2)$

——> Λ_5~0.65 Λ_4

Examples of important hard processes

- e^+e^- -> hadrons

$(p+p')^2=s=Q^2$



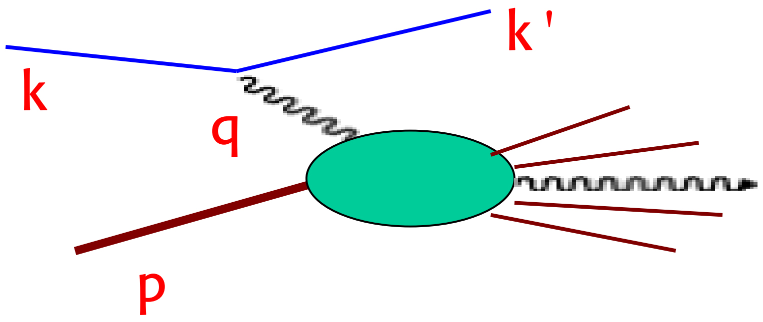
At parton level the final state is

$q\bar{q}$ + n gluons + n' $q\bar{q}$ pairs

(i.e. totally inclusive). The conversion of partons into hadrons does not affect the rate (some smearing over a Q bin can be needed for probability 1)

- l + N -> l' + hadrons

(Deep Inelastic Scattering: DIS)



$$(k-k')^2 = q^2 = -Q^2$$

$$x = \frac{Q^2}{2(pq)}$$

The simplest application is to the process:

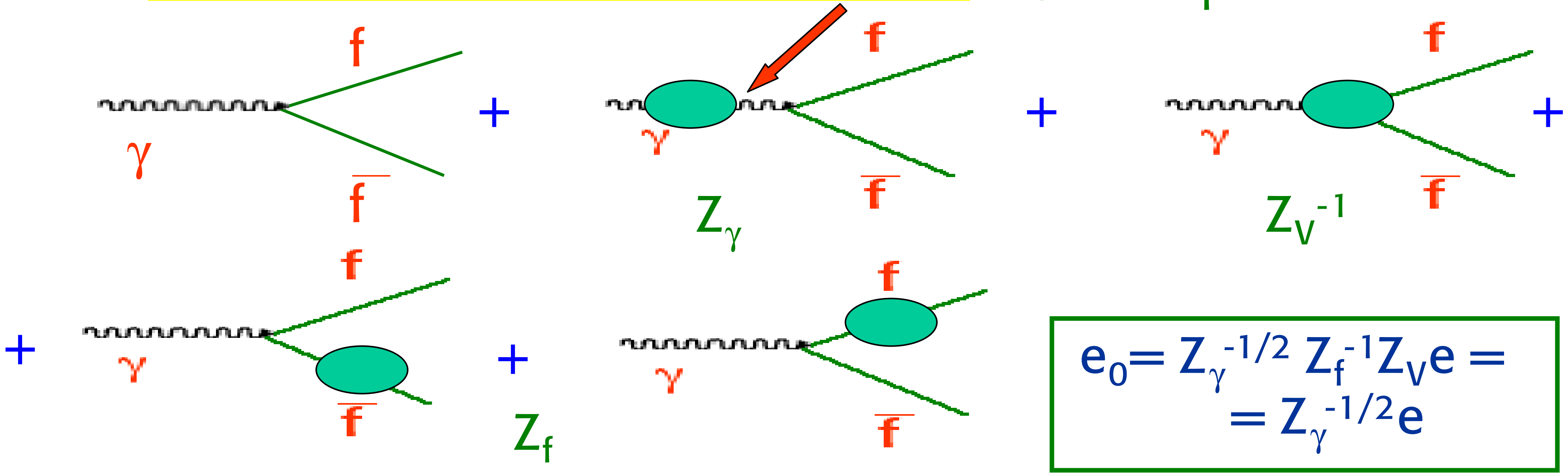
$R=\sigma(e^+ e^- \to \text{hadrons})/\sigma(e^+ e^- \to \mu^+ \mu^-)$ $\rightarrow$ $F(t,\alpha_s)$

$t=\log Q^2/\mu^2$

For this process $\gamma(\alpha)=0$: renorm. of charge is the same for quarks and leptons!

Charge renorm in QED at 1 loop:

Here is the connected Green 3-p function

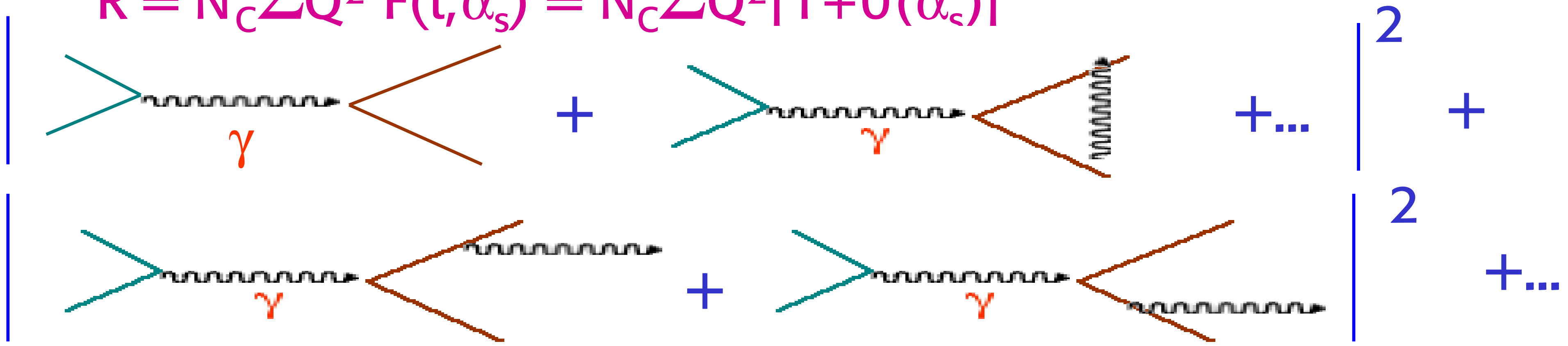


$$e_0 = Z_\gamma^{-1/2} Z_f^{-1} Z_V e = \\ = Z_\gamma^{-1/2} e$$

Only Z_γ (marked with arrow) survives.

Z_V^{-1} and Z_f cancel by Ward identity. No α_s terms (gluon exchange) at 1 loop in the γ-blob Z_γ.

$R = N_C \Sigma Q^2 \; F(t,\alpha_s) = N_C \Sigma Q^2 [1+0(\alpha_s)]$



The RGE prediction is: $t = \log \frac{Q^2}{\mu^2}$

$$F(t, \alpha_s) = F[0, \alpha_s(t)]$$

with

$$\alpha_s(t) = \frac{\alpha_s}{1 + b\alpha_s t} \cong \alpha_s \cdot (1 - b\alpha_s t + \ldots)$$

that is at 2-loops (no $\alpha_s t$, $\alpha_s^2 t^2$ terms, coeff $\alpha_s^2 t$ fixed…):

$$F[0, \alpha_s(t)] = 1 + c_1 \alpha_s (1 - b\alpha_s t + \ldots) + c_2 \alpha_s^2 + \ldots..$$

$c_1 = \frac{1}{\pi}$; c_2, c_3, c_4 also known (dep. on def. α_s)

In $\overline{\mathrm{MS}}$ with n_f=5 for e^+e^- ($a_s = \alpha_s(Q^2)/\pi$)

$$R(Q^2)=3\,\Sigma_f Q_f^2\ [1+a_s+1.4097a_s^2-12.76709a_s^3-80.0075a_s^4+\ldots]$$

Note: the sub-leading coeff.s depend on scale choice:
if instead of Q was Q/2 they would change.

Similar perturbative results at 3-loops exist for Γ(Z->hadrons)/ Γ(Z->leptons), $\Gamma(\tau\text{->}\nu_\tau$+hadrons)/ $\Gamma(\tau\text{->}\nu_\tau$+leptons), etc

The pattern of power corrections is controlled by the light-cone operator expansion:

$$F = \text{pert.} \quad + r_{-2}\frac{m^2}{Q^2} + r_{-4}\frac{\langle 0|Tr[\sum F^A_{\mu\nu}F^{\mu\nu}_A]|0\rangle}{Q^4} + \ldots + r_{-6}\frac{\langle 0|O_6|0\rangle}{Q^6} + \ldots\ldots$$

Light Cone Operator Product Expansion

$R_{e+e-} \sim \Pi(Q^2)$ $\sigma_{e+e-} \sim L_{\mu\nu}T^{\mu\nu}$

$$T_{\mu\nu} = \sum_n \langle 0|J^\dagger{}_\mu(0)|n\rangle\langle n|J_\nu(0)|0\rangle(2\pi)^4\delta^4(q-p_n) =$$

$$= \int e^{iqx}\langle 0|J^\dagger{}_\mu(x)J_\nu(0)|0\rangle dx = (-g_{\mu\nu}Q^2 + q_\mu q_\nu)\Pi(Q^2)$$

For Q^2 -> infinity the x^2 -> 0 region is dominant. To all orders in pert. th. the OPE can be proven. Schematically, dropping Lorentz indices, near $x^2 \sim 0$:

Wilson: Brandt, Preparata

$$J^\dagger(x)J(0) \cong I(x^2) + E(x^2)\sum_{n=0}^{\infty} c_n(x^2)x^{\mu_1}\ldots x^{\mu_n}\cdot O^n_{\mu_1\ldots\mu_n}(0) +$$

$$+ \text{less sing.terms}$$

$I(x^2)$, $E(x^2)$,..,$c_n(x^2)$,.., c-number sing.
O^n: string of local operators.

$E(x^2)$ is the sing. of free field th., $I(x^2)$, $c_n(x^2)$ contain powers of $\log(\mu x)$ in interaction. $I(x^2)$ is the most sing. in x^2 .
Some O^n are already present in free field th., more appear in interaction.

$\Pi(Q^2)$ is related to the Fourier transform. Less sing. terms in x^2 ("higher twist") lead to power suppressed terms in $1/Q^2$.

$$F = \text{pert.} \quad + r_{-2}\frac{m^2}{Q^2} + r_{-4}\frac{\langle 0|Tr[\sum F^A_{\mu\nu}F^{\mu\nu}_A]|0\rangle}{Q^4} + ... + r_{-6}\frac{\langle 0|O_6|0\rangle}{Q^6} + ...$$

Note: $g_\mu g^\mu$ not gauge invariant

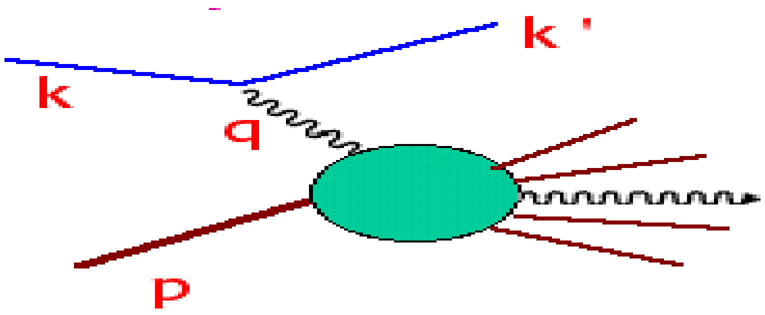
The pert. terms come from $I(x^2)$. Down by $1/Q^2$ are mass terms (e.g. m_b^2/Q^2). Dimension 4, 6... operators are suppressed by $1/Q^4$, $1/Q^6$ …

Deep Inelastic Scattering has played a capital role in the development of QCD

$l + N \to l' + X, \quad l = e, \mu, \nu$

- Many structure functions
- $F_i(x,Q^2)$: two variables
- Neutral currents, charged currents
- Different beams and targets
- Different polarization

From the beginning: Establishing quarks and gluons as partons
Constructing a field theory of strong int.ns
and along the years: Quantitative testing of QCD
Totally inclusive
QCD theory of scaling violations crystal clear
(based on ren. group and operator exp.)
Q^2 dependence tested at each x value)
Measuring q and g densities in the nucleon
Instrumental to compute all hard processes
Measuring α_s
Always presenting new challenges, e g:
Structure functions at small x; heavy flavour structure functions;
polarized parton densities, g_1, g_2, h_1...; non forward pdf's
Diffraction



$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

(θ: l-l' lab. angle)

$$m\nu = (p\cdot q) \qquad x = \frac{Q^2}{2m\nu}$$

Structure functions $\sigma = l_{\mu\nu}\cdot W^{\mu\nu}$

$l_{\mu\nu}$: leptonic

$W^{\mu\nu}$: hadronic

$$W_{\mu\nu} \cong \int e^{iqx}\langle p|J^{\dagger}_{\mu}(x)J_{\nu}(0)|p\rangle dx = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(Q^2,\nu) +$$

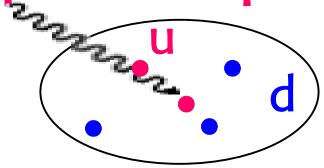
$$+\left(p_\mu - \frac{m\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{m\nu}{q^2}q_\nu\right)W_2(Q^2,\nu)/m^2 +$$

$$-\frac{i}{2m^2}\varepsilon_{\mu\nu\lambda\rho}p^\lambda q^\rho W_3(Q^2,\nu)$$

Early crucial breakthroughs

- Approximate Scaling Bjorken
- Success of Naive Parton Model Feynman

From constituent quarks (real? fictitious?) to parton quarks (real!)



- $R = \sigma_L/\sigma_T$ ---> 0 Spin 1/2 quarks
- ~50% of momentum carried by neutrals Gluons
- Quark charges:

$\sigma_L \sim 0$

$F = 2F_1 \sim F_2/x$

...... = small sea

$F\gamma p = 4/9\ u(x) + 1/9\ d(x) + ...$

$F\gamma n = 4/9\ d(x) + 1/9\ u(x) + ...$

$F\nu p \sim F\bar{\nu} n = 2\ d(x) + ...$

$F\nu n \sim F\bar{\nu} p = 2\ u(x) + ...$

$$\int (u - \bar{u})dx = 2$$

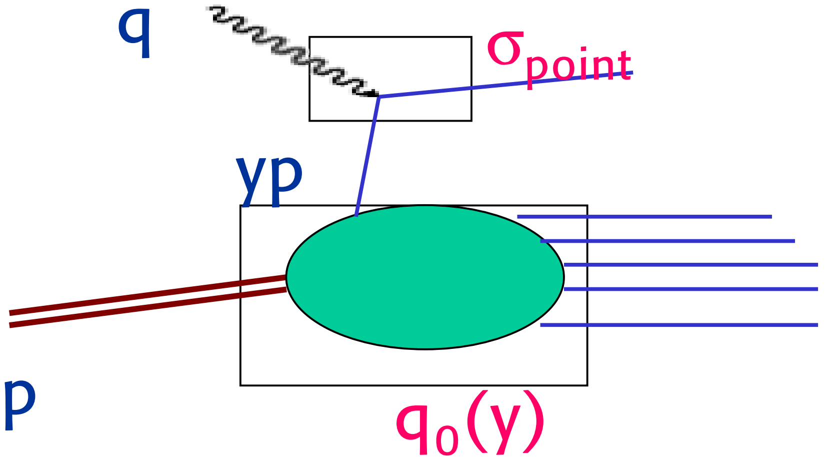
$$\int (d - \bar{d})dx = 1$$

$$\int (s - \bar{s})dx = 0$$

F= F(x), u=u(x), d= d(x):
naive parton model (scaling)

In QCD there are log scaling violations induced by $\alpha_s(Q^2)$.

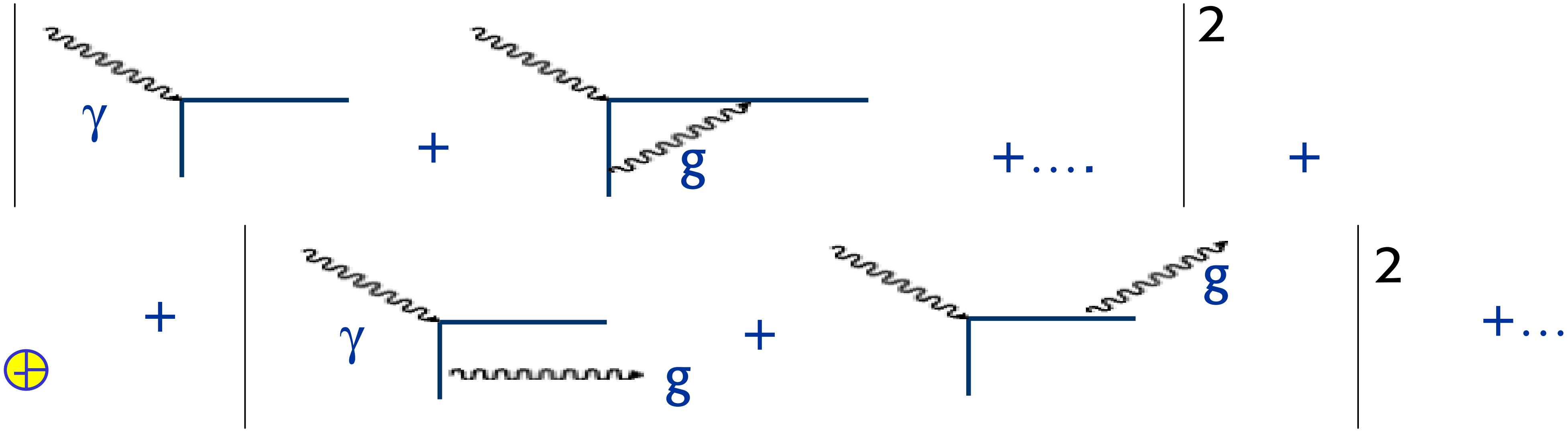
$$2F_1(x) = \int_x^1 dy \frac{q_0(y)}{y} \sigma_{point}\left(\frac{x}{y}\right) + o\left(\frac{1}{Q^2}\right)$$



Born: σ_{point}-> $e^2\delta(x/y-1)$

$2F_1 = e^2 q_0(x)$

QCD modifies σ_{point} at $o(\alpha_s)$



The result is of the form (y>x)

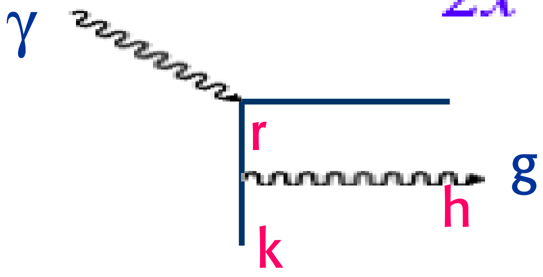
$$t = \log \frac{Q^2}{\mu^2}$$

$$\sigma_{point} = e^2\left[\delta\left(\frac{x}{y}-1\right) + \frac{\alpha_s}{2\pi}\left(tP\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right)\right]$$

The log is from the collinear sing. of the incoming quark leg. In a special gauge, (axial or physical gauge) the dominant real diagram is:

$$E_k = \frac{yQ}{2x}$$

p_T: r transverse mom. (propag)^2 ~ $(1/p_T)^4$
Num~$(p_T)^2$ (helicity non cons. at θ=0)



$$Propag = \frac{1}{(k-h)^2} = \frac{1}{-2\cdot E_h\cdot E_k\cdot(1-\cos\theta)} = \frac{1}{-4E_hE_k\cdot\left(\sin\frac{\theta}{2}\right)^2} \approx \frac{1}{p_T^2}$$

$$\sigma \approx \int^{Q^2} \frac{1}{p_T^2}\, dp_T^2 \approx \log Q^2$$

We factorise the mass sing. into the quark parton density (non perturbative):

(All integrals from x to 1)

$$2F_1 = \int dy \frac{q_0(y)}{y} e^2 \left[\delta\left(\frac{x}{y}-1\right) + \frac{\alpha_s}{2\pi}\left(tP\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right)\right]$$

$$= \int dy \frac{q_0(y)+\Delta q(y,t)}{y} e^2 \left[\delta\left(\frac{x}{y}-1\right) + \frac{\alpha_s}{2\pi} f\left(\frac{x}{y}\right)\right]$$

We replace: $q_0(x)$ -> $q(x,t)= q_0(x)+\Delta q(x,t)$: effective, Q^2-dep. parton density.

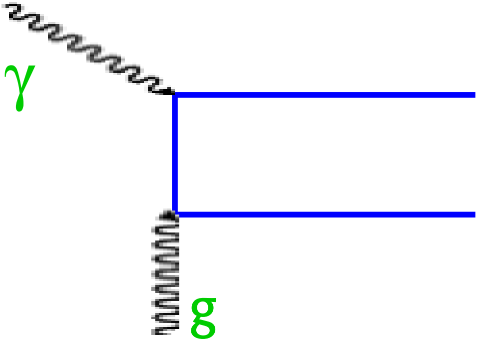
$$\Delta q(x,t) = \frac{\alpha_s}{2\pi} t \int_x^1 \frac{dy}{y} q_0(y) P\left(\frac{x}{y}\right)$$

According to the RGE, now α_s -> $\alpha_s(t)$

$$2F_1 = \int dy \frac{q(y,t)}{y} e^2 \left[\delta\left(\frac{x}{y}-1\right) + \frac{\alpha_s(t)}{2\pi} f\left(\frac{x}{y}\right)\right] = e^2 q(x,t) + o(\alpha_s(t))$$

$$\frac{d}{dt} q(x,t) = \frac{\alpha_s(t)}{2\pi} \int dy \frac{q(y,t)}{y} P\left(\frac{x}{y}\right) + o(\alpha_s(t)^2)$$

The t-evolution eqs. become non diagonal as soon as gluon partons are also included:



The full set becomes

Recall: $\int_x^1 dy \frac{q(y,t)}{y} P\left(\frac{x}{y}\right) = [q \otimes P](x,t)$

$$\frac{d}{dt} q_i(x,t) = \frac{\alpha_s(t)}{2\pi} [q_i \otimes P_{qq}] + \frac{\alpha_s(t)}{2\pi} [g \otimes P_{qg}]$$

$$\frac{d}{dt} g(x,t) = \frac{\alpha_s(t)}{2\pi} [\sum q_i \otimes P_{gq}] + \frac{\alpha_s(t)}{2\pi} [g \otimes P_{gg}]$$

⇓

The quark density with fraction y times the probab. of a gluon in a quark with fraction x/y of the parent long. mom.

The LO form of the splitting functions can be derived directly from the QCD vertices (process indep.: factorisation)

$$P_{qq}(x) = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] + o(\alpha_s(t))$$

Def.: $$\int_0^1 \frac{f(x)}{(1-x)_+}dx = \int_0^1 \frac{f(x)-f(1)}{1-x}dx$$

Note quark conserv. fixes the δ terms of P_{qq}

$$\int_0^1 P_{qq}(x)dx = 0$$

$$\int (u-\bar{u})dx = 2$$

Similarly for P_{gg} via momentum conservation

$$P_{gq}(x) = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] + o(\alpha_s(t))$$

$$P_{qg}(x) = \frac{1}{2}[x^2 + (1-x)^2] + o(\alpha_s(t))$$

$$P_{gg}(x) = 6\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{33-2n_f}{6}\delta(1-x) + o(\alpha_s(t))$$

Splitting functions

For many years all splitting funct.s P have been known to NLO accuracy: $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \ldots\ldots.$

GLAP, Floratos et al; Gonzales-Arroyo et al; Curci et al; Furmanski et al

Then the complete, analytic NNLO results have been derived for the first few moments (N<13,14).

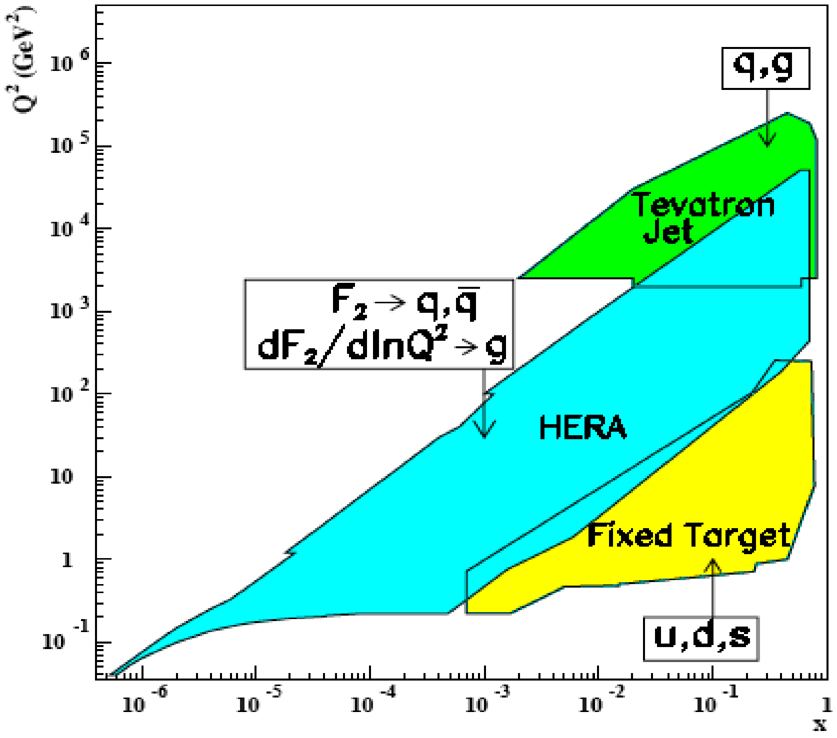
Larin, van Ritbergen, Vermaseren+Nogueira

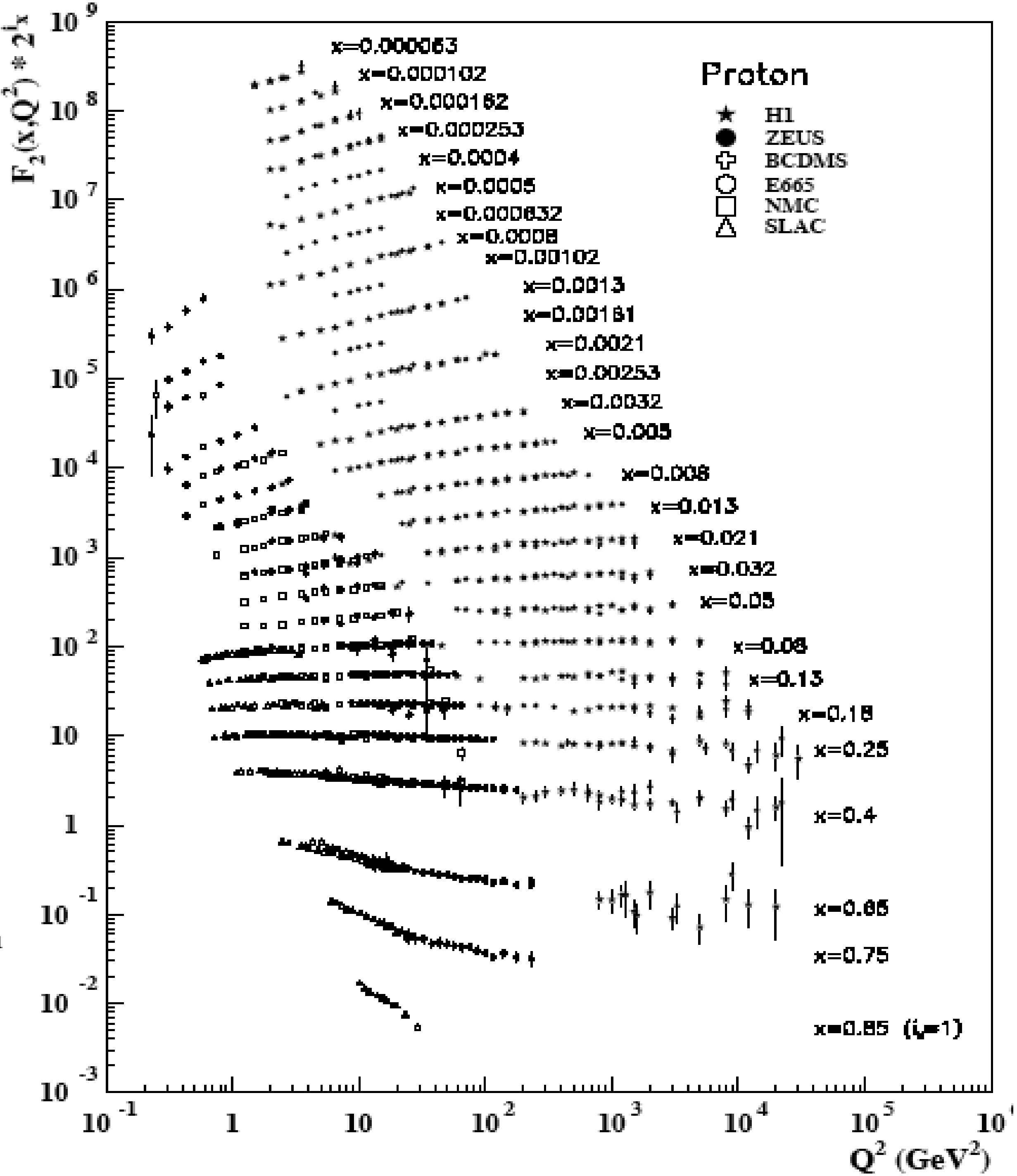
Finally, in 2004, the calculation of the NNLO splitting functions has been totally completed $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \alpha_s^3 P_3 + \ldots\ldots.$

Moch, Vermaseren, Vogt

A really monumental, fully analytic, computation

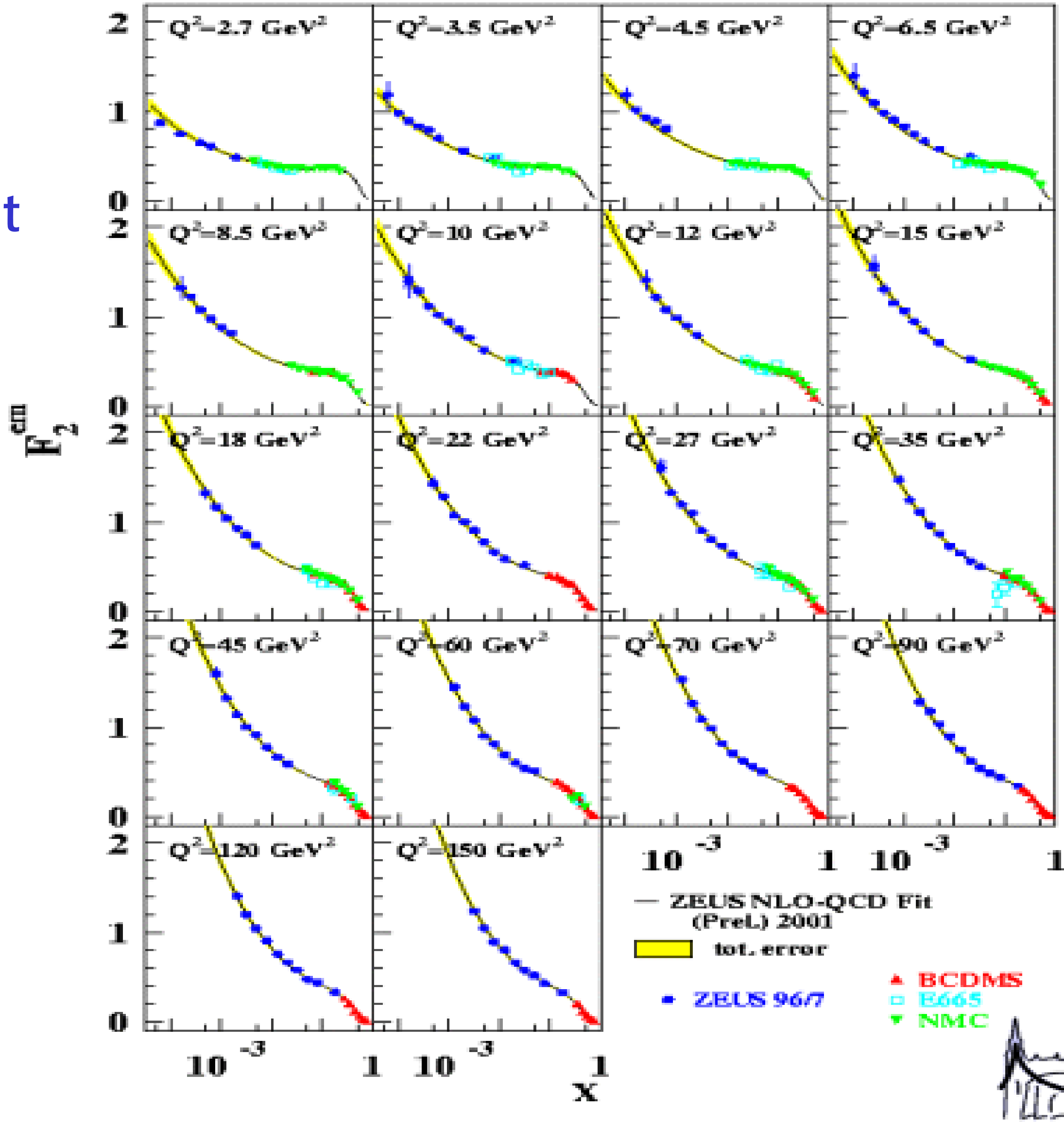
The scaling violations are clearly observed
and the (N)NLO QCD fits are remarkably good.





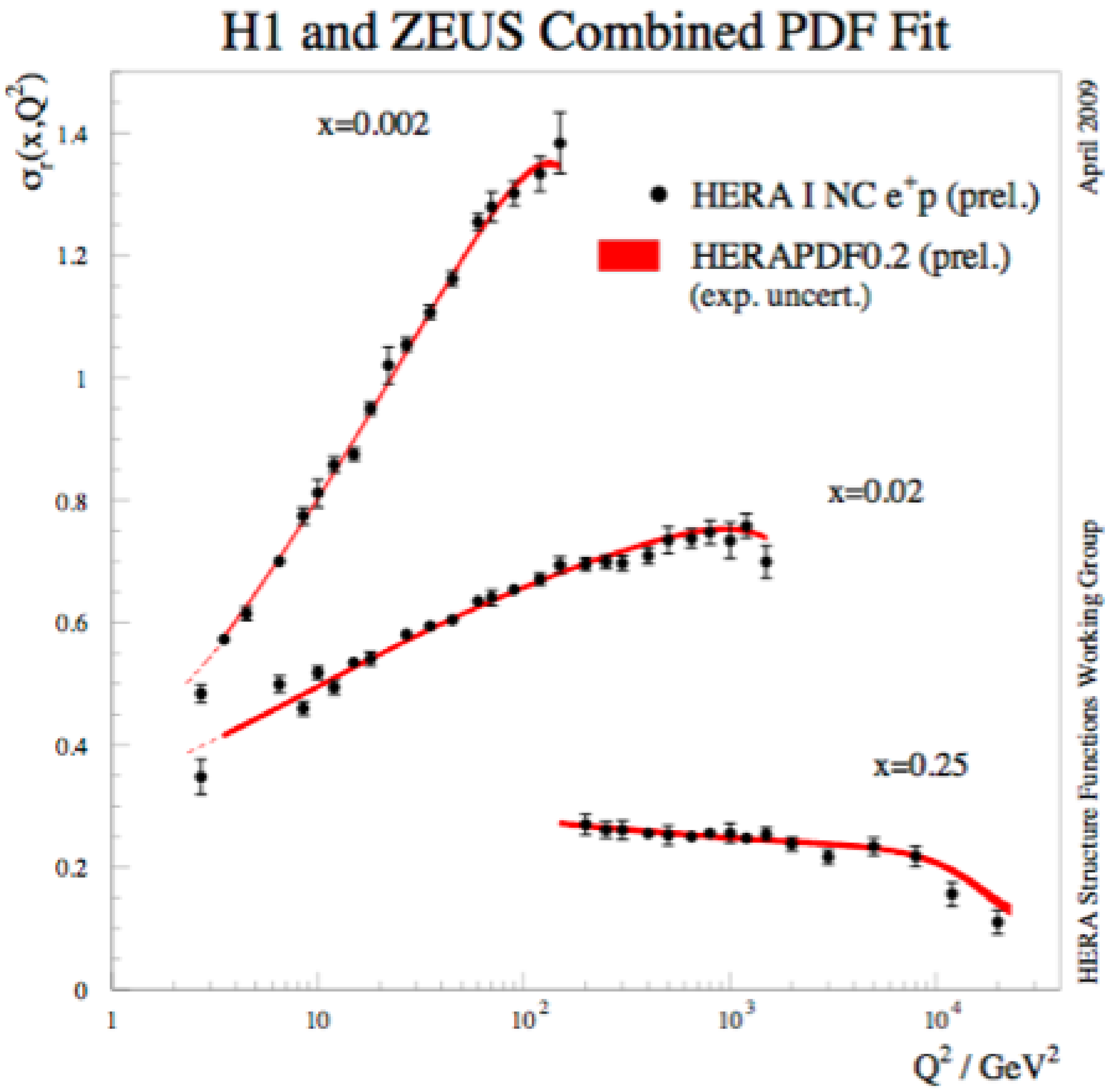
Proton Structure Function $F_2(x,Q^2)$

Example of NLO QCD evolution fit

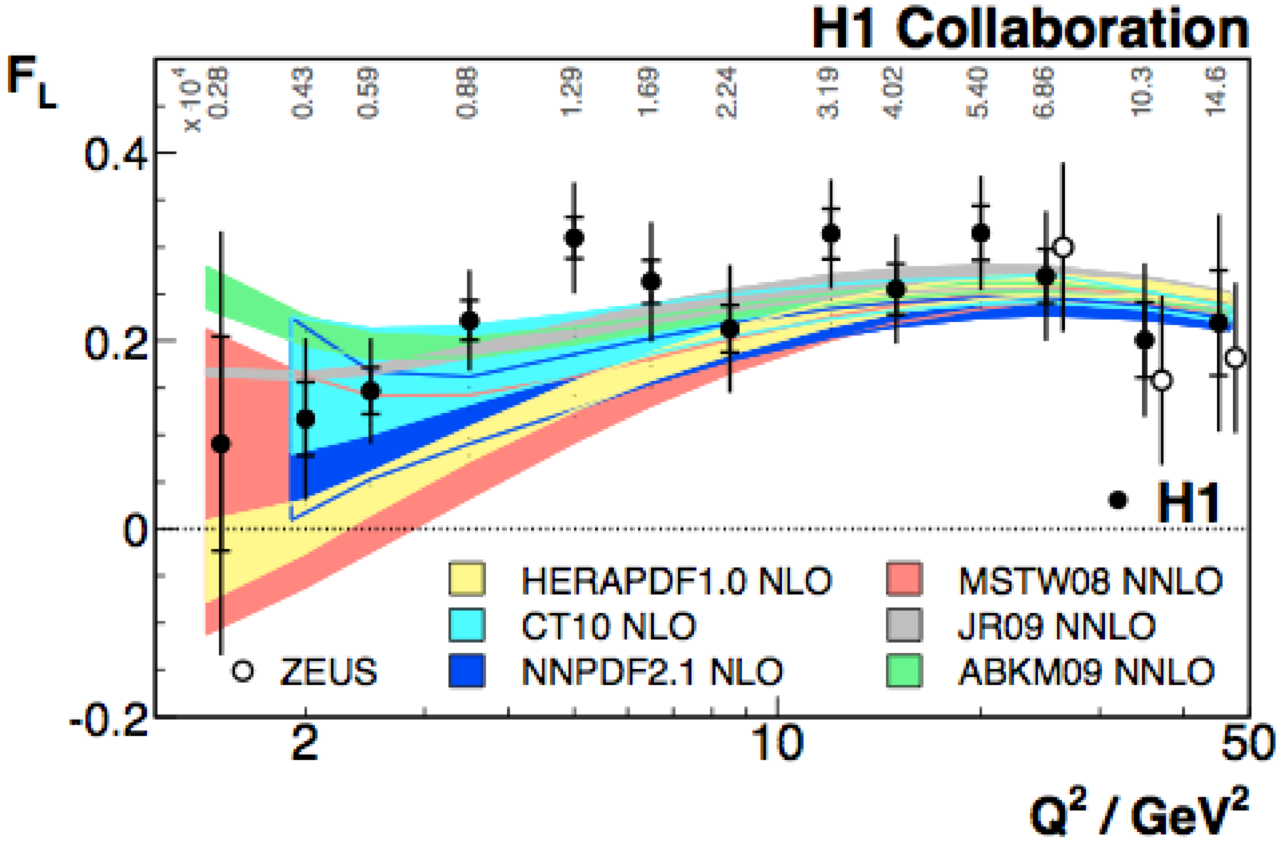


Incr. Q^2

This is how the scaling violations appear now after 40 years of DIS measurements



It took ~40 years to get meaningful data on the longitudinal structure function!!



$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left[\frac{8}{3} F_2(y,Q^2) + \frac{40}{9} yg(y,Q^2)(1-\frac{x}{y}) \right]_{n_f=4}$$

Altarelli, Martinelli '78

Corfou, 9-10 September 2012

QCD in 2012

Guido Altarelli

Universita' di Roma Tre/CERN

Research supported by LHCPhenonet

Plan

1 - Introduction
  - Non perturbative QCD
    - Lattice QCD
    - Confinement
    - Heavy Ion physics

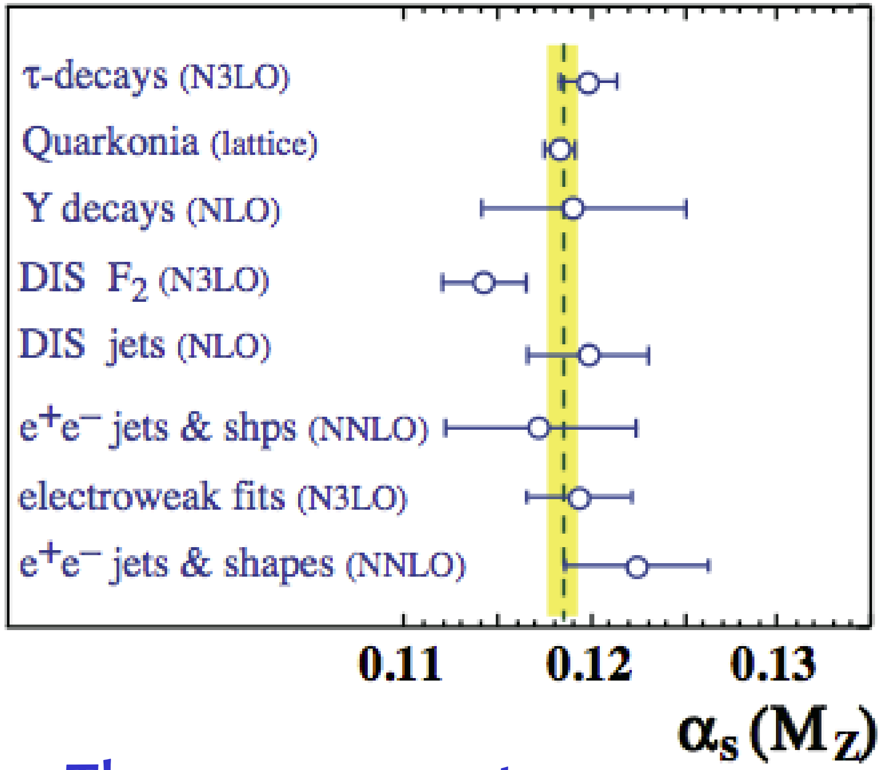
2 - Asymptotic Freedom
  - Perturbative QCD
    - Basic concepts and results

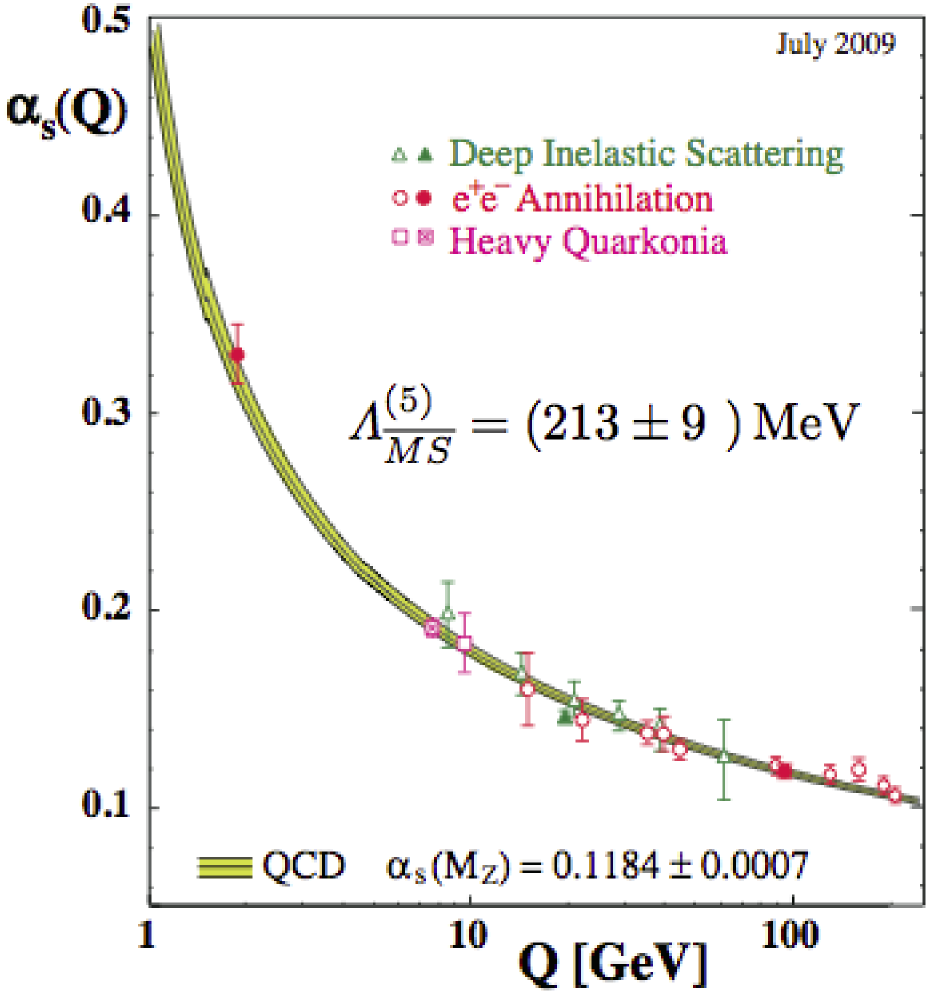
3 - QCD in the LHC time

Measurements of $\alpha_s(m_Z)$

PDG'10 summary on $\alpha_s(m_Z)$ $\overline{MS}$

The official compilation due to Bethke is reproduced here:





The agreement among so many different ways of measuring α_s is a strong quantitative test of QCD

New preliminary Bethke value '11
$\alpha_s(m_Z) = 0.1183 \pm 0.0010$

However for some entries the stated errors are taken directly from the original works and are not transparent enough (e.g. the lattice determination)

In my opinion one should select few theoretically cleanest processes for measuring α_s and consider all other ways as tests of the theory

Note that in QED α is measured from one single very precise, very clean observable (at present the electron g-2)

The cleanest processes are the totally inclusive ones (no hadronic corrections) with light cone dominance, like Z decay, scaling violations in DIS and perhaps τ decay (but for τ the energy scale is low)

The main inclusive methods for α_s at LEP/SLC are:

- inclusive Z decay, R_l, σ_l, σ_h, Γ_Z
- inclusive τ decay

$$R_{l,\,\tau} = \frac{\Gamma(Z,\tau \Rightarrow hadrons)}{\Gamma(Z,\tau \Rightarrow leptons)} \approx R^{EW}(1+\delta_{QCD}+\delta_{NP})$$

δ_{QCD} is known to (N)NNLO accuracy:

recently completed

$$\delta_{QCD} = c_1\left(\frac{\alpha_s(Q)}{\pi}\right) + c_2\left(\frac{\alpha_s(Q)}{\pi}\right)^2 + c_3\left(\frac{\alpha_s(Q)}{\pi}\right)^3 + c_4\left(\frac{\alpha_s(Q)}{\pi}\right)^4 + \ldots$$

δ_{NP} are power suppressed $(1/Q^2)^n$ terms governed by the OPE.

Here $Q=m_Z$ or m_τ

Clearly the Z case is apriori more reliable because $m_Z \gg m_\tau$.

Inclusive Z decays

(assuming the SM, m_{texp}, m_{Hexp}):

R_l only (traditionally used for no good reason): $\alpha_s(m_Z)=0.1226\pm0.0038$

a bit large!

σ_l is more sensitive to α_s:

$\alpha_s(m_Z)=0.1183\pm0.0030$

$$R_l = \frac{\Gamma_h}{\Gamma_l}$$

$$\Gamma_Z = (\Gamma_h + 3\Gamma_l + \Gamma_{inv})$$

$$\sigma_h = \frac{12\pi}{m_Z^2}\frac{\Gamma_l\Gamma_h}{\Gamma_Z^2}$$

$$\sigma_l = \frac{12\pi}{m_Z^2}\cdot\frac{\Gamma_l^2}{\Gamma_Z^2}$$

Better, one can use all info from R_l, Γ_Z, σ_h, σ_l …and in general take $\alpha_s(m_Z)$ as a parameter to be fitted from the EW precision tests One obtains (with only $c_{1\text{-}3}$ included):

LEP1 only: $\alpha_s(m_Z)=0.1187\pm0.0027$

All EW Data (also m_W...): $\alpha_s(m_Z)=0.1186\pm0.0026$

Apriori the main theor. errors are higher QCD orders ($c_{4\ldots}$).
Error from power corrections very small.
In addition, th. error from possible new physics (eg in Zbb vertex).

Inclusive hadronic Z and τ decay at $o(\alpha_s^4)$ (NNNLO!!)

Baikov, Chetyrkin, Kuhn '08
Baikov, Chetyrkin, Kuhn, Rittinger '12

~20.000 diagrams

$o(\alpha_s^4)$ terms complete for τ and Z hadronic decay

For example, Z decay, $R = \Gamma_h/\Gamma_l$

$$R=R_0\ [1+a_s+0.76264\ a_s^2-15.49a_s^3-68.2a_s^4+\ldots]$$

$n_f=5$, $a_s=\alpha_s(m_Z^2)/\pi$

Now no more significant error from higher orders!

Can be used to improve α_s from Z

$\alpha_s(m_Z^2)=$ 0.1186---> 0.1190 ± 0.0025

Note that the error shown is dominated by the exp. errors. For example having now fixed m_H does not decrease the error significantly

α_s from R_τ

$$R_\tau = \frac{\Gamma(\tau \Rightarrow \nu_\tau + hadrons)}{\Gamma(\tau \Rightarrow \nu_\tau + leptons)}$$

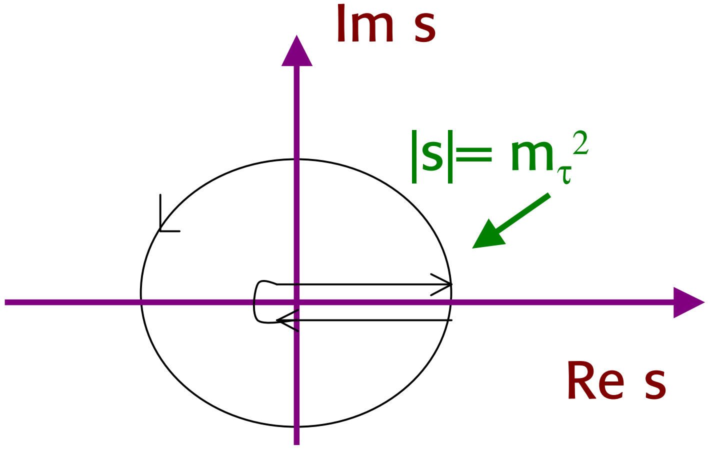
R_τ has a number of advantages that, at least in part, compensate the smallness of m_τ=1.777 GeV:

- R_τ is even more inclusive than $R_{e+e-}(s)$.

$$R_\tau = \frac{1}{\pi}\int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 Im\,\Pi_\tau(s)$$

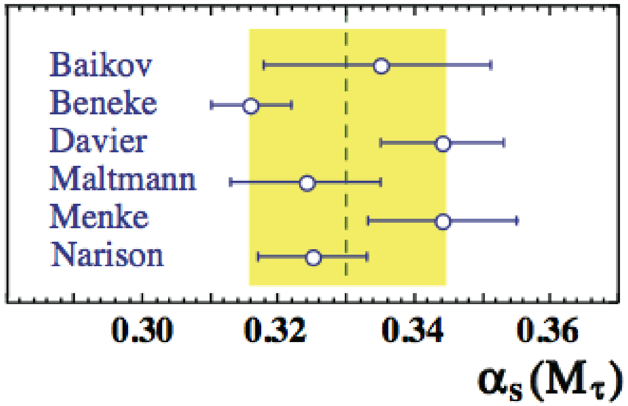
- one can use analiticity to go to $|s|= m_\tau^2$

$$R_\tau = \frac{1}{2\pi i}\oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 \Pi_\tau(s)$$



- factor $(1-s/m_\tau^2)^2$ kills sensitivity to Re s= m_τ^2 (thresholds)

Still the quoted result (by Bethke '09) looks a bit too precise



Bethke'09 $\alpha_s(m_Z)=0.1197\pm0.0016$

This precision is obtained by taking for granted that corrections suppressed by $1/m_\tau^2$ are negligible.

$$R_\tau \sim R_\tau^0[1+\delta_{pert}+\delta_{np}]$$

This is because in the massless theory:

$$\delta_{np} = \frac{ZERO}{m_\tau^2} + c_4 \cdot \frac{<O_4>}{m_\tau^4} + c_6 \cdot \frac{<O_6>}{m_\tau^6} + \ldots$$

In fact there are no dim 2 operators (e.g. $g_\mu g^\mu$ is not gauge invariant) except for light quark m^2 (m~few MeV if parton quarks are relevant, m~few 100 MeV if constituents) .

Most people believe that partons are relevant. I am not sure that the gap is not filled by ambiguities of $o(\Lambda^2/m_\tau^2)$ from δ_{pert}.

eg effect of ultraviolet renormalons

GA, Nason, Ridolfi '95; Chetyrkin, Narison,Zakharov '98

α_s from DIS : more complicated

The scaling violations of non-singlet str. functs. would be ideal: less dependence on input parton densities

$$\frac{d}{dt}\log F(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 dy \frac{F(y,t)}{yF(x,t)} P_{qq}\left(\frac{x}{y}\right)$$

But

- for F_p-F_n exp. errors add up in the difference,
- $F_{3\nu N}$ is not terribly precise

  (ν data only from CCFR, NuTeV)
- neglecting sea and glue in F_2 for $x > x_0$ decreases the sample, introduces a dependence on x_0 and an error from residual singlet terms.

Non singlet electron/muon production

From a recent analysis of eP and eD data, neglecting sea and gluons at $x > 0.3$ (error to be evaluated)

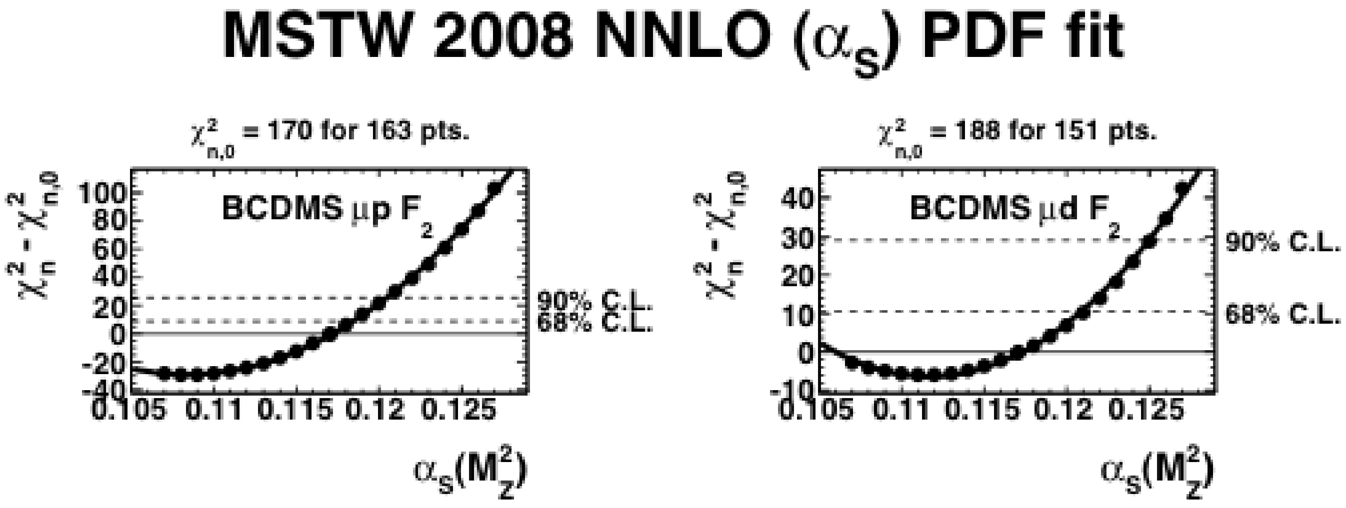
- Non singlet DIS: $\alpha_s(m_Z)=0.1148\pm0.0019$ (exp)+? (NLO)
  $\alpha_s(m_Z)=0.1134\pm0.0020$ (exp)+? (NNLO)

Bluemlein, Bottcher, Guffanti '07

- a rather small central value
- not much difference between NLO and NNLO

According to G. Watt the contribution of singlet to F_2 at $x \sim 0.3$ is still ~ 10%

BCDMS data push towards small α_s



According to Watt 162/280 exp points at $x > 0.3$ are dominated by BCDMS

When one measures α_s from scaling viols. in F_2 from e or μ beams, data are abundant, exp. errors small but:

α_s ⟷ gluon correlation $dF/d\log Q^2 \sim \alpha_s\, g$

There is a strong feedback on α_s of the parametrisation of g.

A too rigid param'n of gluon may strongly bias α_s

The Neural Network approach suppresses g parametrization errors (The NNPDF Coll. '11)

DIS only $\alpha_s(m_Z)=0.1166\pm0.0008(\mathrm{exp}) + 0.0009\ (\mathrm{th})$ (NNLO)

Including Tevatron jets may be important to constrain g at large x (and then, via momentum conservation, also at small x). But jets rates only known at NLO accuracy

⊕ With jets and DY $\alpha_s(m_Z)=0.1173\pm0.0007(\mathrm{exp}) + 0.0009\ (\mathrm{th})$

Recent $\alpha_s(m_Z)$ determinations from DIS at NNLO

$$\alpha_s(m_Z) = 0.1129 \pm 0.0014 \text{ (exp)}+?$$

Alekhin, Blumlein, Klein, Moch '09

$$\alpha_s(m_Z) = 0.1158 \pm 0.0035 \text{ (exp)}+?$$

Jimenez-Delgado, Reya '08

Ambiguities:

- Heavy quarks
- F_L
- Higher orders

From combined H1+ZEUS data

$$\alpha_s(m_Z) = 0.1147 \pm 0.0012 \text{ (exp)}+?$$

Alekhin, Blumlein, Moch '10

For HERA data the NLO evolution should be improved by a correct treatment of small x effects (negative g at small x and Q^2 is a symptom)

Global fit to α_s and PDF

dominated by DIS but not only DIS

$$\alpha_s(m_Z) = 0.1171 \pm 0.0014(\mathrm{exp}) + ?$$ (NNLO)

Martin, Stirling, Thorne, Watt '09

MRST attribute their larger value of α_s to a more flexible parametrisation of the gluon and claim that the Tevatron jets are needed to fix g at large x

In conclusion, for $\alpha_s(m_Z)$ from DIS

Bethke takes $\alpha_s(m_Z) = 0.1142 \pm 0.0023$ from non-singlet and this is what he puts in his average from DIS

recall: $\alpha_s(m_Z)=0.1134\pm0.0020$ (exp)+? (NNLO)

Bluemlein, Bottcher, Guffanti '07

Problems: neglect singlet at $x>x_0$, small data sample, BCDMS...

From the previous discussion it appears that for singlet there are problems related to the gluon determination and parametrization

$\alpha_s(m_Z)$ tends to slide towards low values if the g problem is not fixed [$\alpha_s(m_Z)$ ~ 0.113-0.116]

The NNPDF approach or fixing the g on the Tevatron jets increases $\alpha_s(m_Z)$ [$\alpha_s(m_Z)$ ~ 0.117]

Still an open problem!

⊕ I would take from DIS: $\alpha_s(m_Z) = 0.116 \pm 0.002$ (NNLO)

Summarising

Z decay $\alpha_s(m_Z) = 0.1190 \pm 0.0025$ (NNNLO)

τ decay $\alpha_s(m_Z) = 0.1197 \pm 0.0016 \pm ?$ (NNLO)

DIS $\alpha_s(m_Z) = 0.116 \pm 0.002$ (NNLO)

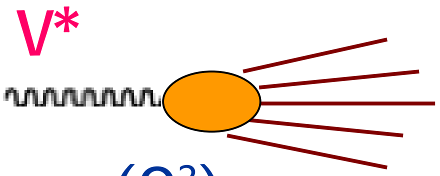
Combining Z decay and DIS

$$\boxed{\alpha_s(m_Z) = 0.1172 \pm 0.0016}$$ my choice

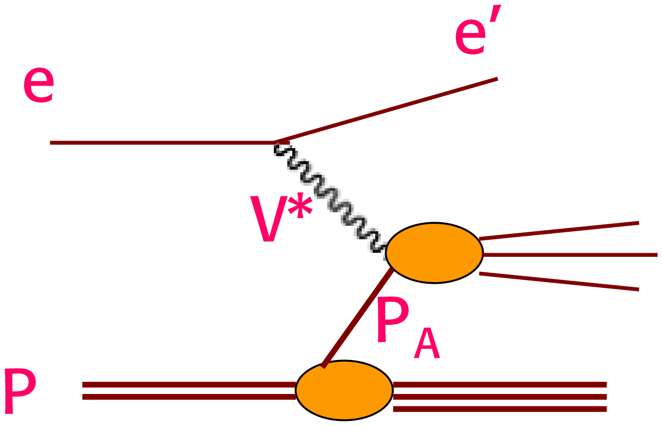
Adding the τ (optimistically forgetting the extra th error)

$$\alpha_s(m_Z) = 0.1184 \pm 0.0011$$

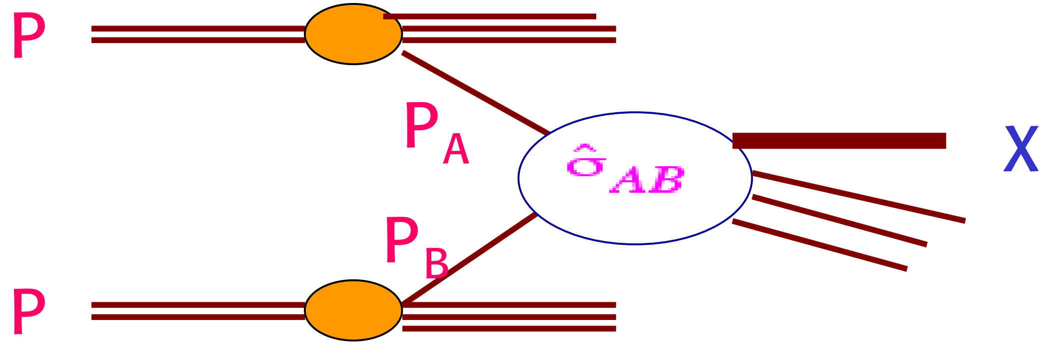
Compare with Bethke $\alpha_s(m_Z) = 0.1183 \pm 0.0010$



$\alpha_s(Q^2)$



$\alpha_s(Q^2)$ & $q(x,Q^2)$, $g(x,Q^2)$



The basic experimental set ups:

- no initial hadron (....LEP, ILC, CLIC)
- 1 hadron (....HERA, ... LHeC)
- 2 hadrons (....SppS, Tevatron, LHC)

Progress in particle physics needs their continuous interplay to take full advantage of their complementarity

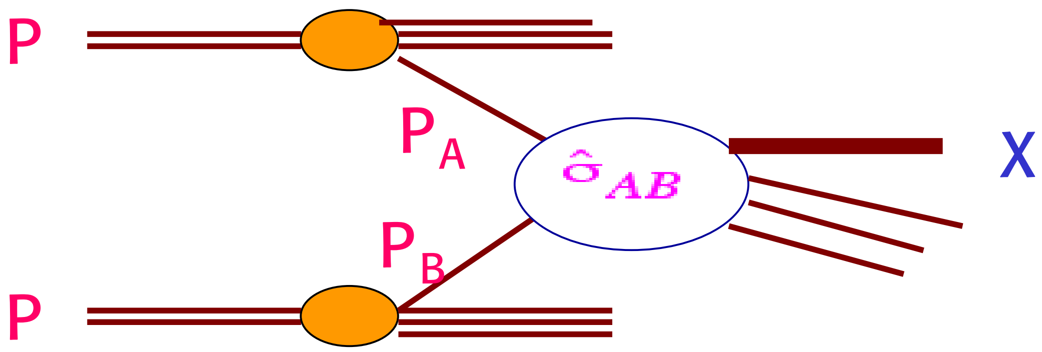
Parton densities extracted from DIS are used to compute hard processes, via the Factorisation Theorem (FT):

density of parton A

$$\sigma(s) = \sum_{A,B} \int dx_1 dx_2\, p_A(x_1, Q^2)\, p_B(x_2, Q^2)\, \hat{\sigma}_{AB}(x_1 x_2 s, Q^2)$$

reduced X-section

For example, at hadron colliders



V=γ*,W,Z

X=V, jets, QQ, H.....

Q=b,c,t

- Very stringent tests of QCD
- Feedback on constraining parton densities

Is the FT proven?

In pert. theory up to NNLO has been explicitly checked to hold.

At all orders detailed studies only for DY

Collins, Soper, Sterman '85,'88

A large amount of theoretical work was devoted to directly prepare the interpretation of LHC experiments

- New and improved generators for event simulation
- Resummations
- New techniques for advanced QCD and EW calculations
- Calculations for signals, backgrounds and interpretation

e.g. the top quark FB asymmetry at the Tevatron has generated much work (axi-gluons, FC Z'...)

QCD event simulation A big boost in view of the LHC

General algorithms for computer NLO calculations

the dipole Catani, Seymour,..... FKS formalisms Frixione, Kunszt, Signer

the antenna pattern Kosower....

Beyond
general purpose
HERWIG
PYTHIA, SHERPA

Matching matrix elements and parton showers

LO ME: ALPGEN, MadGraph, MLM, (L)-CKKW Mangano.....

NLO ME: MC@NLO
POWHEG, MENLOPS

Frixione, Webber.....
Frixione, Nason, Oleari.....
Hamilton, Nason

Perturbative (+ resumm.s)

$$d\sigma = A\alpha_S^N [\, 1 + (c_{1,1}L + c_{1,0})\alpha_S + (c_{2,2}L^2 + c_{2,1}L + +c_{2,0})\alpha_S^2 + \dots \,]$$

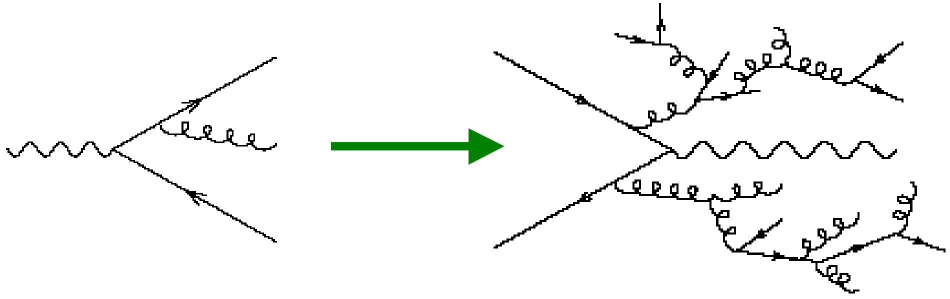
L= large log eg L=log(p_T/m)

Complementary virtues:
the hard skeleton plus
the shower development
and hadronization

Parton showers

collinear emissions factorize

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \times \frac{\alpha_S}{2\pi}\frac{dt}{t}P_{qq}(z)dz\frac{d\varphi}{2\pi}$$
$$t = (p_q + p_g)^2 \longrightarrow 0$$



hadronization added

On going progress in automatisation

Resummation of large logs

Beyond the RGE [$\log Q^2/\mu^2$] there are often other large logs L

Examples of L:

$\log p_T^2/Q^2$ in p_T distrib.'ns for W, H (Sudakov logs)
$\log 1/x$ for small x structure functions in DIS
$\log 1/(1-x)$ Thrust distributions, large x in DIS.......

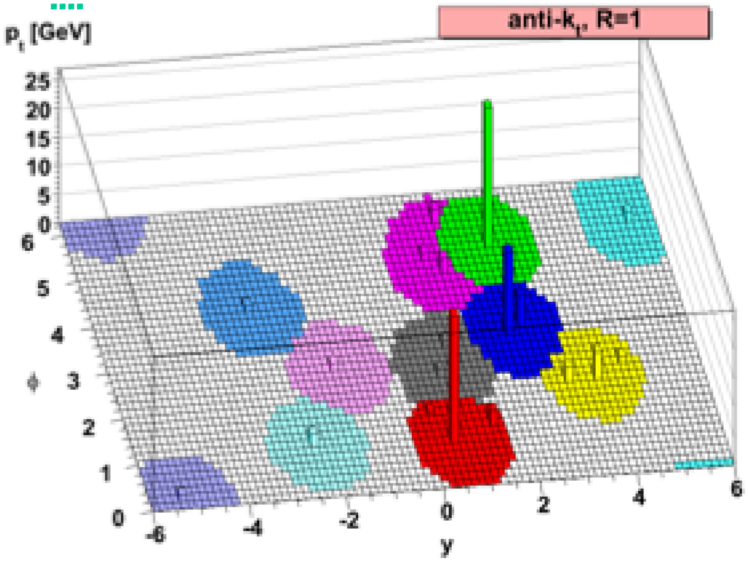
When $\alpha_s(Q^2)L^2$ or $\alpha_s(Q^2)L$ are large, the sequences $(\alpha_s(Q^2)L^{1 \text{ or } 2})^n$ have to be resummed (the LL or NLL coefficients can often be computed to all orders).

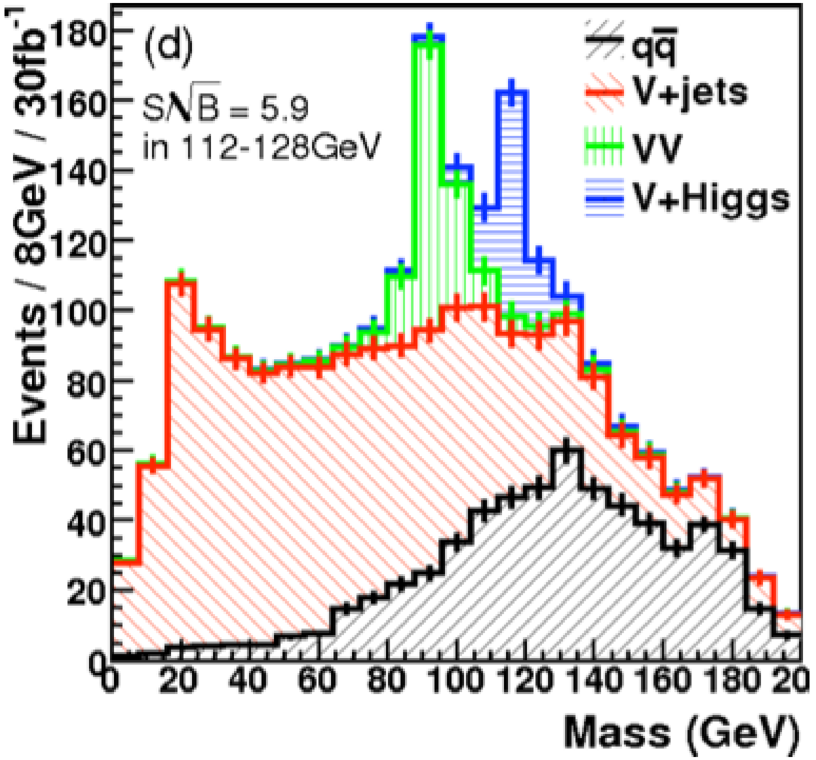
Leading logarithmic

Important recent work on jet recombination algorithms SISCone, anti-k_T

G. Salam et al

Cacciari
Zanderighi
....





It is essential that a correct jet finding is implemented by LHC experiments for an optimal matching of theory and experiment

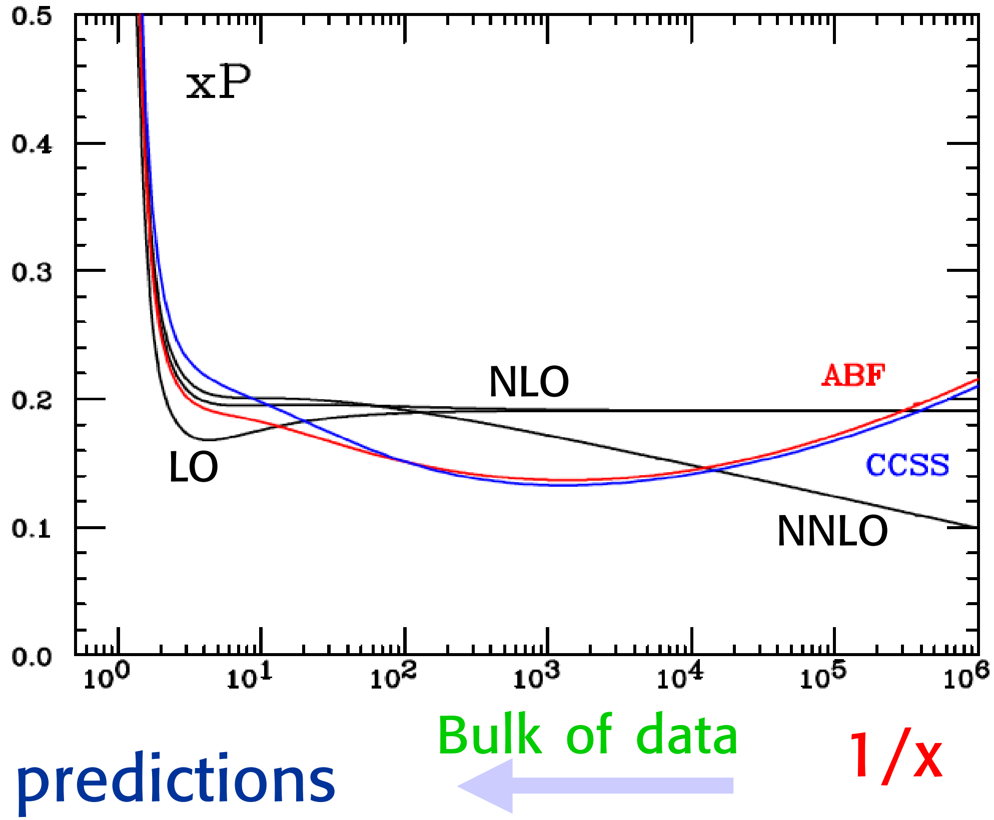
Singlet splitting function at small x

The problem of correctly including BFKL at small x has been solved

Ciafaloni, Colferai, Salam, Stasto (CCSS)
Altarelli, Ball, Forte (ABF)

Momentum cons.+ symmetry + running coupling effect

→ soft simple pole in anom. dim



- BFKL sharp rise tamed
- resummed result close to NLO in HERA region
- new expansion stable

Makes the ground solid for LHC predictions (eg b production)

QCD for LHC: very difficult calculations needed

New powerful techniques for loop calculations

Basic idea: Loops can be fully reconstructed from their unitarity cuts

First proposed by Bern, Dixon, Kosower '93-'97
Revived by Britto, Cachazo, Feng '04
Perfected by Ossola, Papadopoulos, Pittau '06

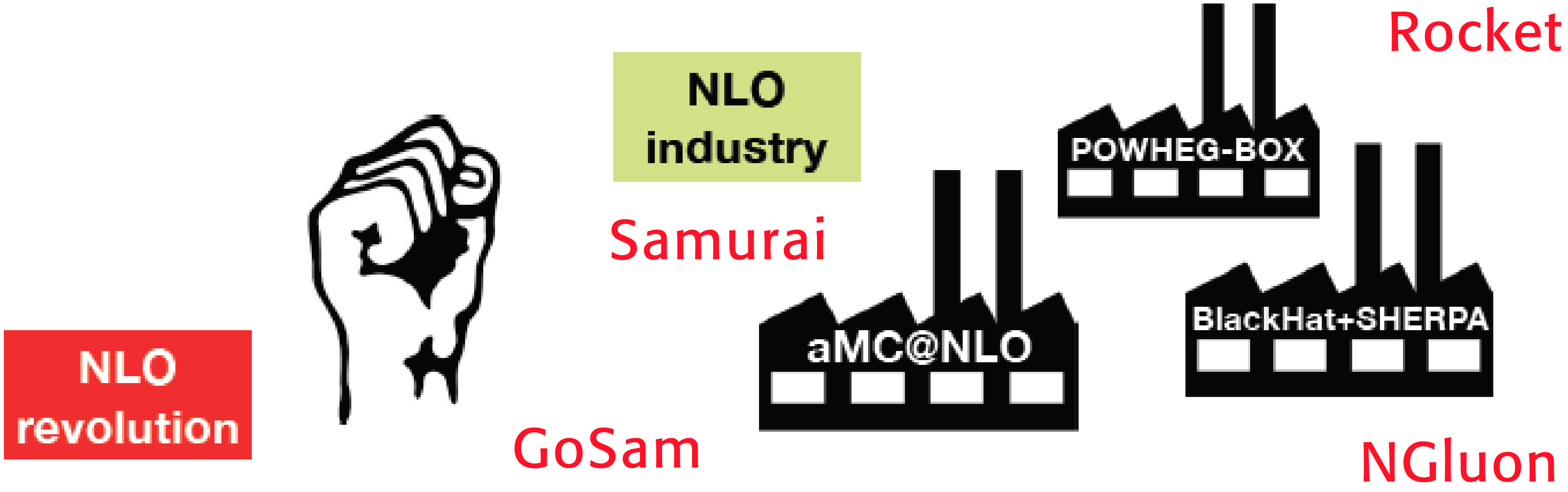
Generalized d-dimension unitarity
K. Ellis, Giele, Kunszt, Melnikov '08-'09

A review:
One-loop calculations in quantum field theory:
from Feynman diagrams to unitarity cuts
K. Ellis, Kunszt, Melnikov, Zanderighi ArXiv: 1105.4319

The Industrial Age of NLO

Campbell '12

- In recent years, much reference to "NLO revolution"
  - development of new wave of tools in anticipation of LHC
  - especially numerical techniques: straightforward generation of new results for complicated final states
- 2011-12: time for putting these revolutionary ideas to work



Examples of recent NLO calculations in pp collisions

ttbb Bredenstein et al ‘09-’10, Bevilacqua et al ‘09
ttW K. Ellis, Campbell ’12
W+3jets Berger et al ‘09, R.K.Ellis , Melnikov, Zanderighi ‘09,
Z,γ^* +3jets Berger et al ‘10
WW+2jets Melia et al ‘10-’11, Jager, Zanderighi ‘12
WWbb Denner et al ‘10
tt+2jets Bevilacqua et al ‘10-’11
bbbb, jjjj Greiner et al ‘11, Bern et al ‘11
W, Z+4jets Berger et al ‘11, Bern et al ‘12; W+5jets Bern et al ‘12

....

And the Higgs cross section and distributions are known to NNLO Harlander, Kilgore ‘02; Anastasiou, Melnikov ‘02; Ravindran et al ‘03; Anastasiou, Melnikov, Petriello ‘04, Bozzi et al ‘07

⊕ A terrific amount of work by QCD theorists for LHC

Parton densities extracted from DIS (with feedback from other hard processes) are available for further use.

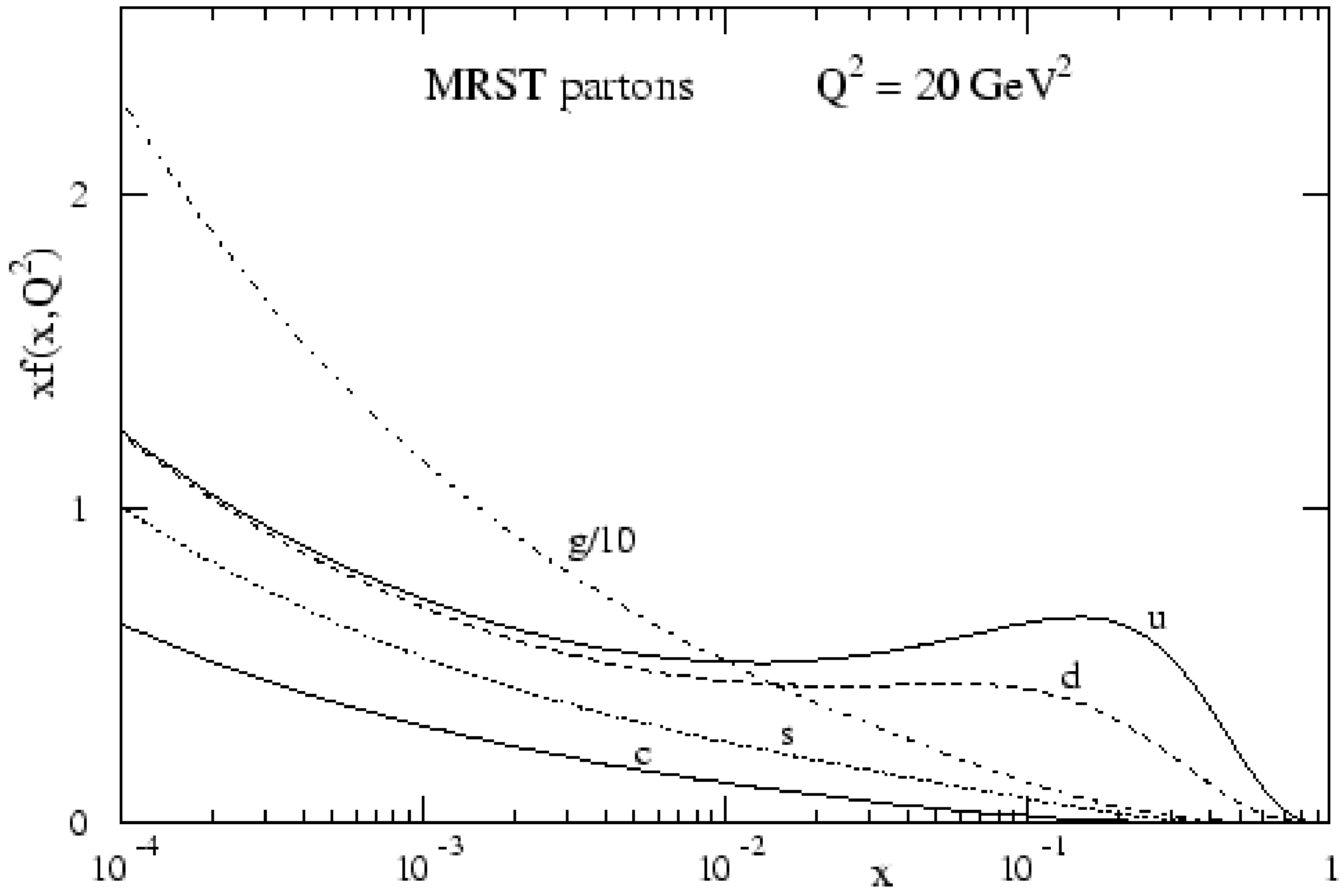
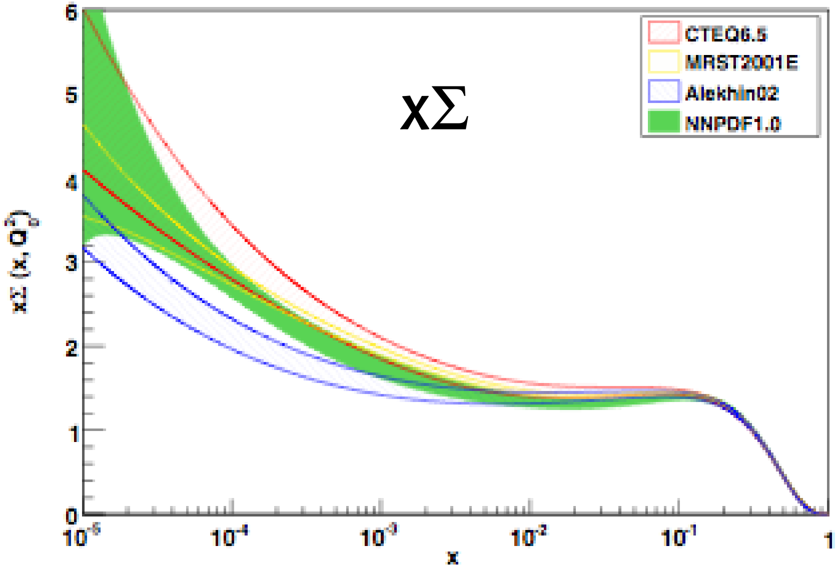
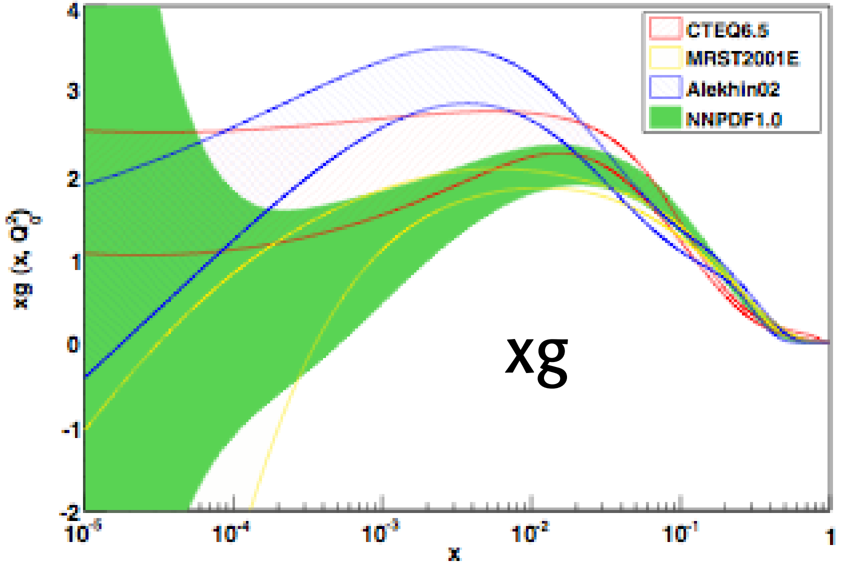


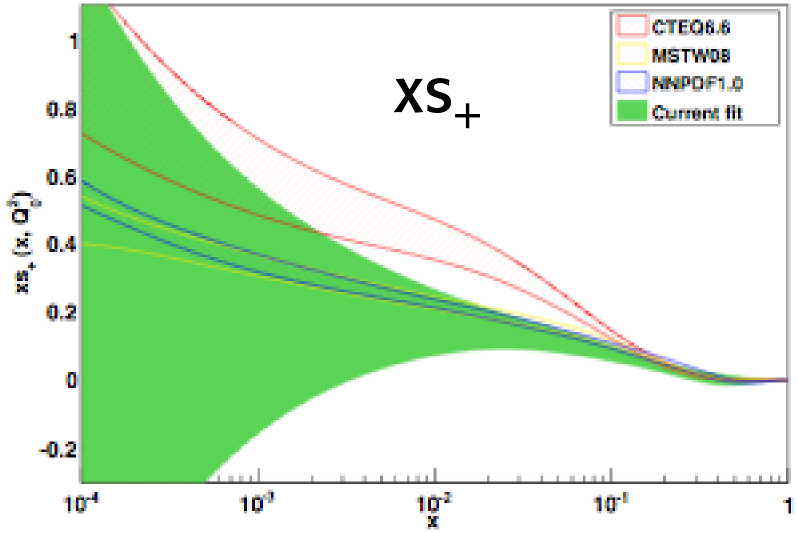
Fig. 19: Parton distributions by the MRST group.

NNPDF: R. Ball et al '08

M. Ubiali







J. Rojo

Neural Network pdf
less dep. on parametrization.

a large ensemble of pdf allowed

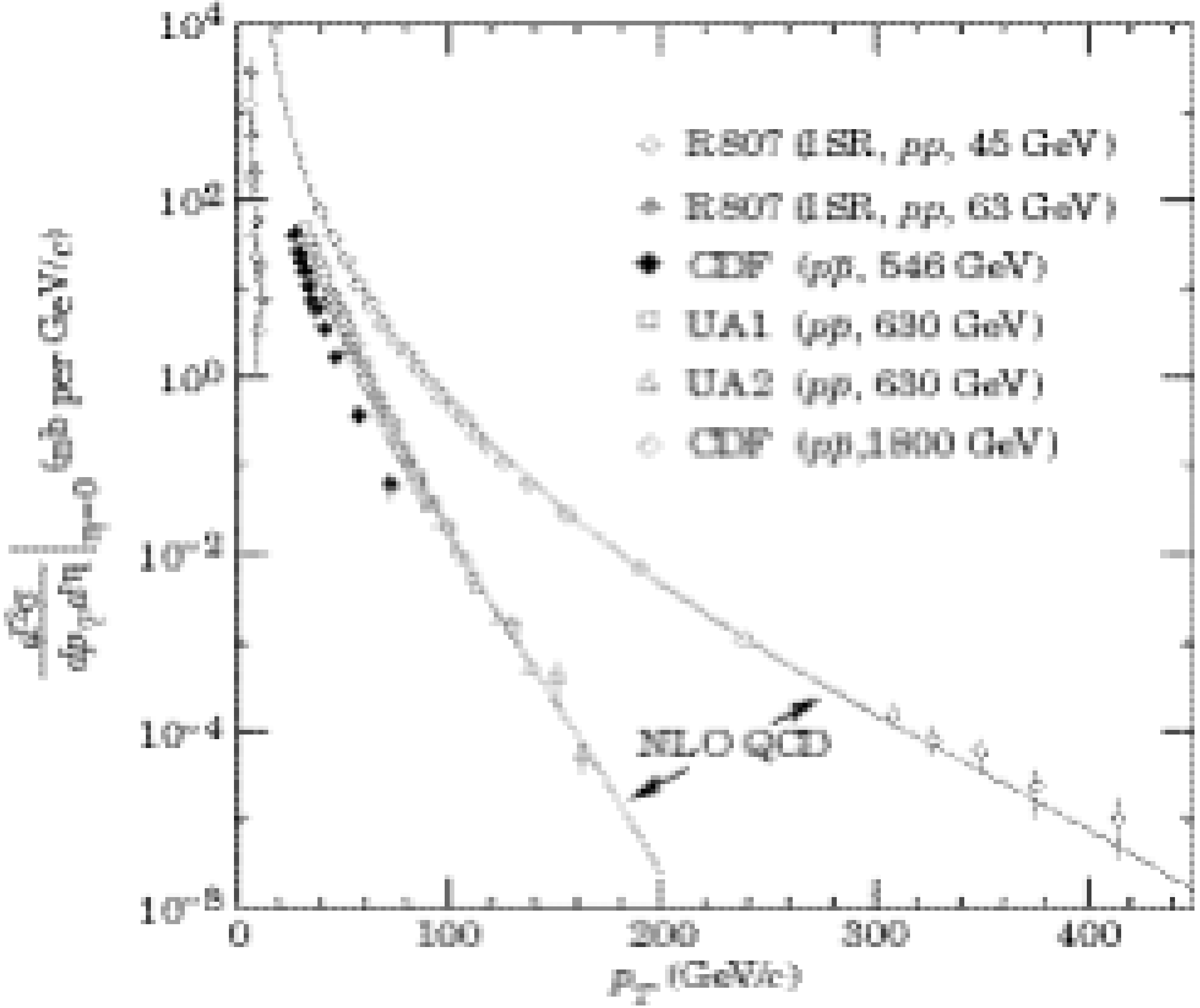
Uncertainties larger than for
CTEQ, MRST, Alekhin
in unmeasured region

Jet Production in pp or pp^{bar} interactions

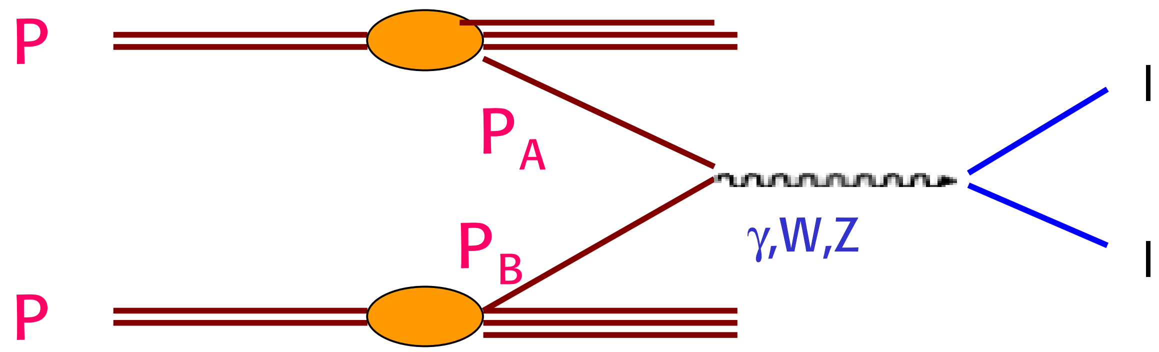
$p_1 p_2$-> jet +X: all scalar products large

$(p_1+p_2)^2=s$; $(p_{1,2}\text{-jet})^2=t,u$ also large -> the jet must be at large p_T

NLO QCD fits
no free parameters
except exp.
norm' n
Note: many orders
of magnitude!



W, Z and Drell-Yan lepton pair production at hadron colliders.

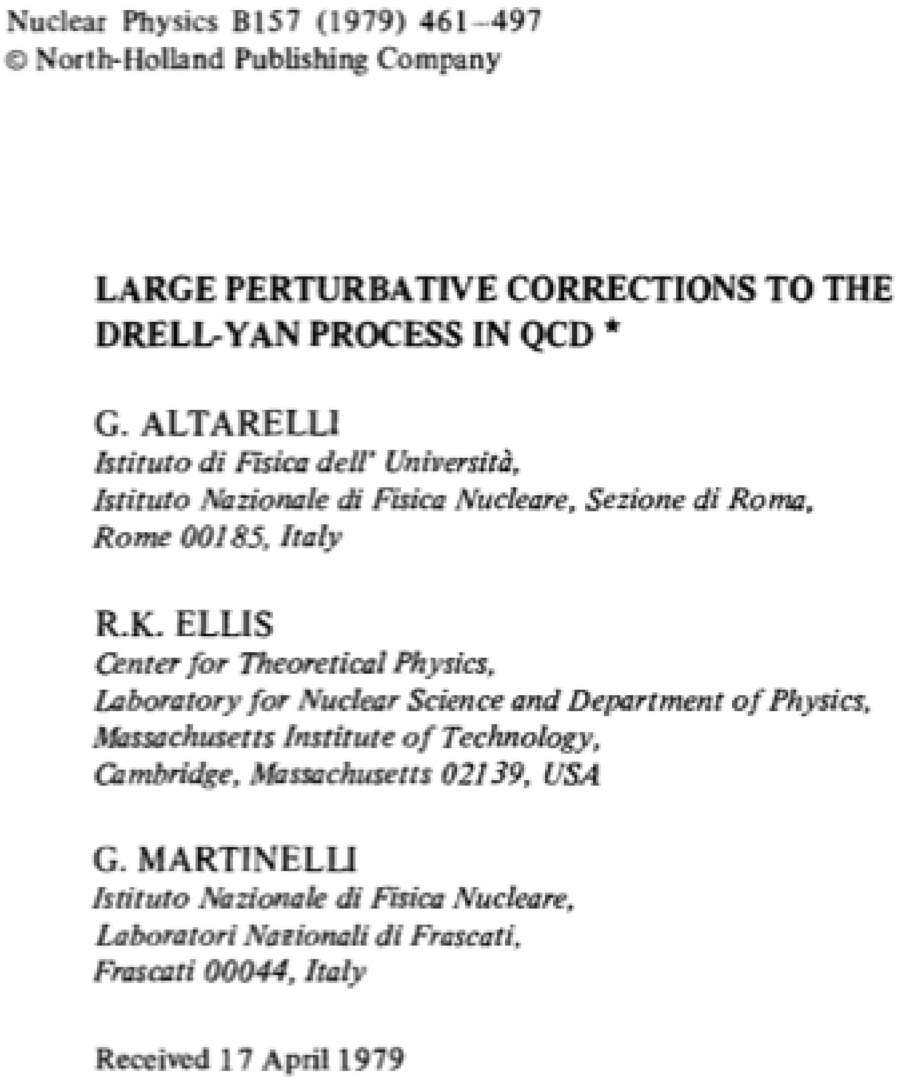


o(1): Drell, Yan; o(α_s): Altarelli, K.Ellis, Martinelli; Kubar-Andre, Paige; o(α_s^2): Hamberg, van Neerven, Matsuura+Zijestra

1979

The K-factor paper

The first NLO calculation in QCD



Nuclear Physics B157 (1979) 461–497
© North-Holland Publishing Company

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

G. ALTARELLI
Istituto di Fisica dell' Università,
Istituto Nazionale di Fisica Nucleare, Sezione di Roma,
Rome 00185, Italy

R.K. ELLIS
Center for Theoretical Physics,
Laboratory for Nuclear Science and Department of Physics,
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Cambridge, Massachusetts 02139, USA

G. MARTINELLI
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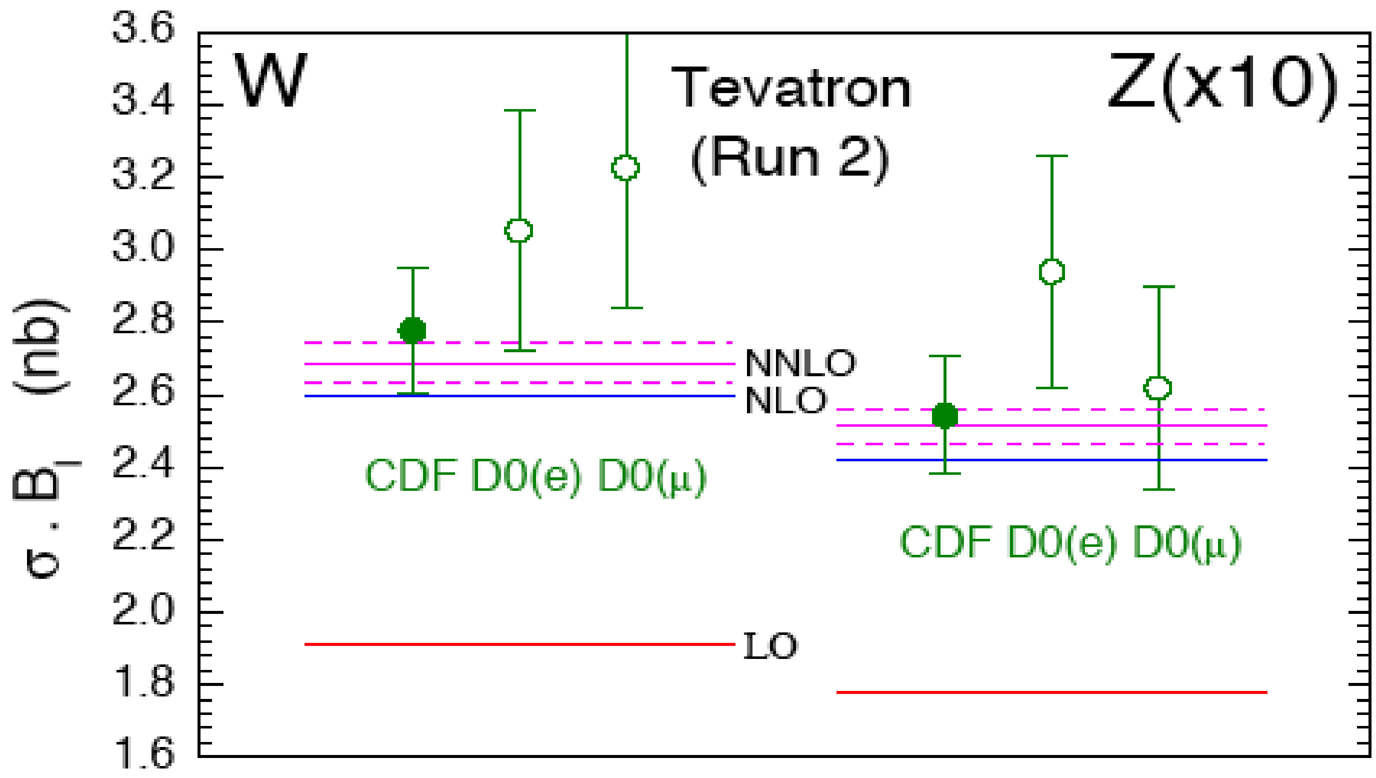
Laboratori Nazionali di Frascati,
Frascati 00044, Italy

Received 17 April 1979

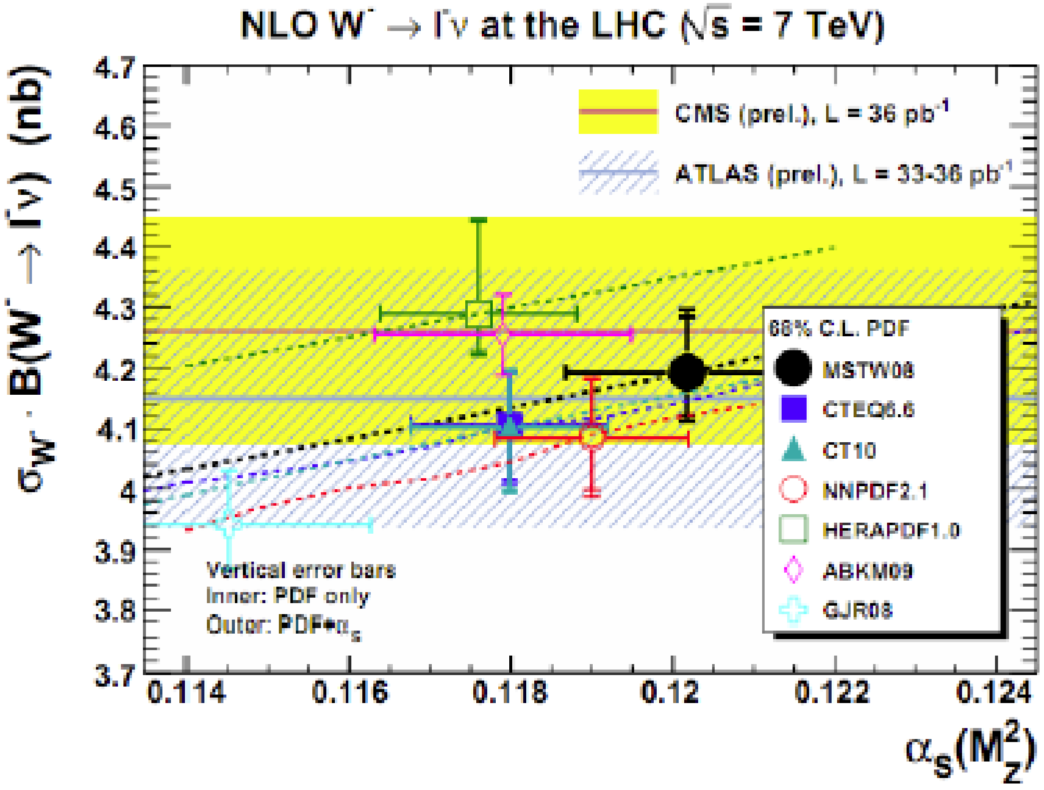
The prediction for $\sigma B_{W,Z}$ is obtained using parton densities from DIS, the measured Λ and Br. ratios from the EW theory

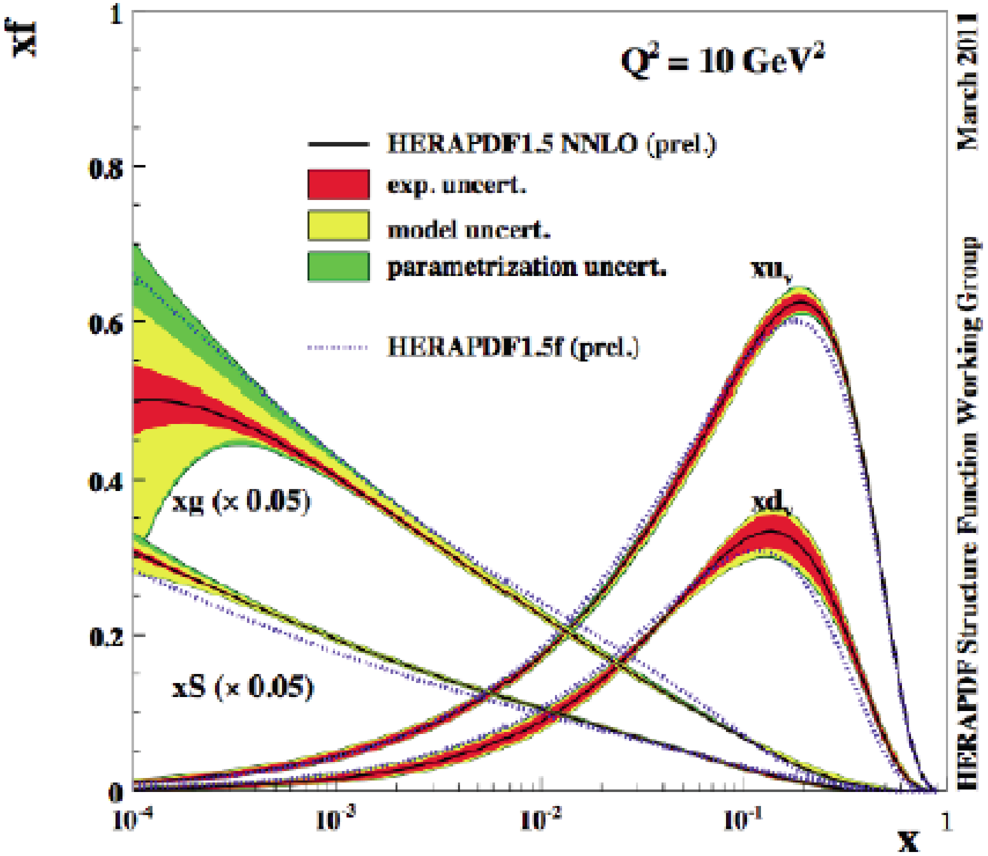


p_T distribution has also been a classic laboratory

see later

An important task: preparing the optimal pdf's for the LHC

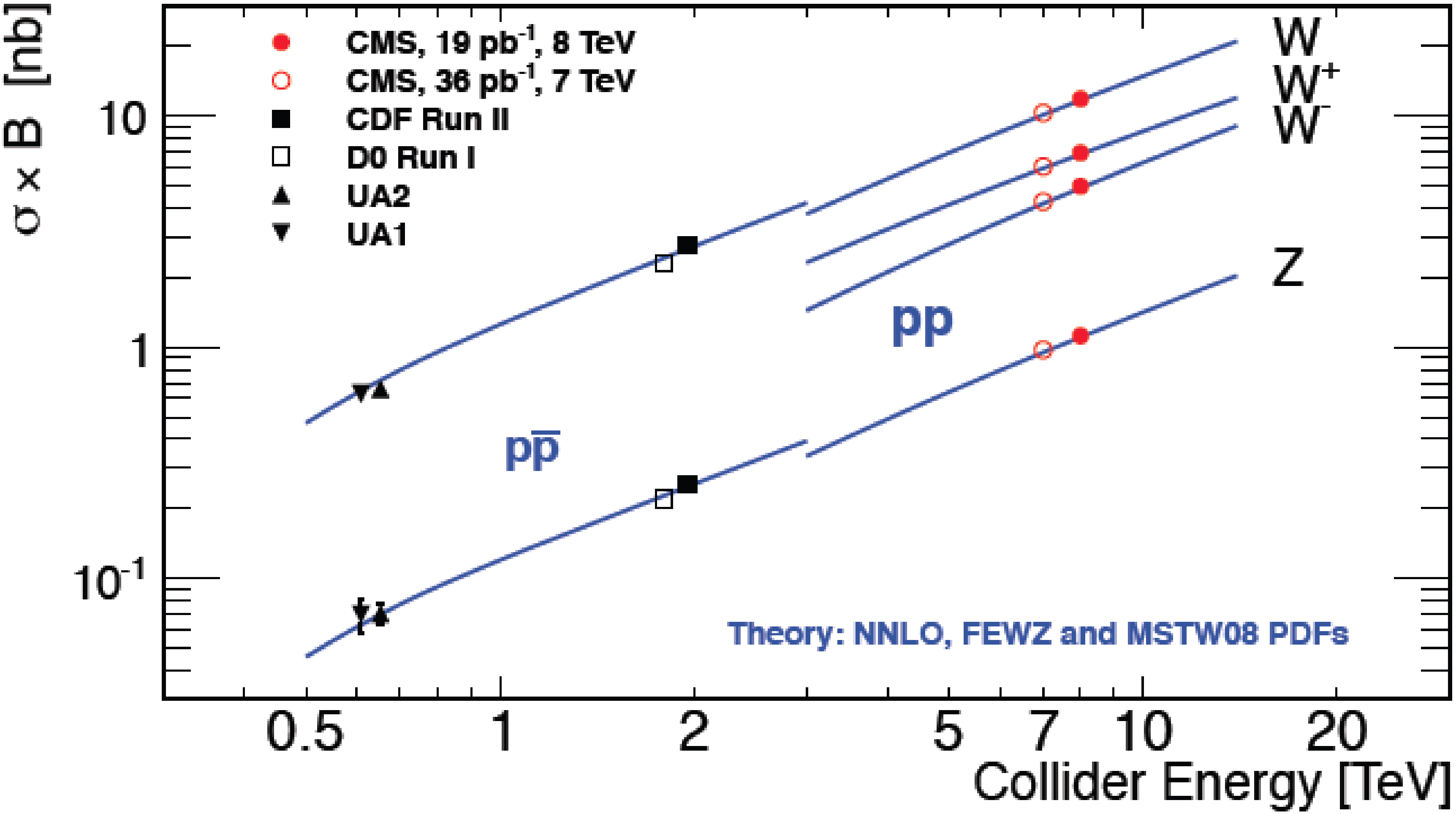




Dedicated groups

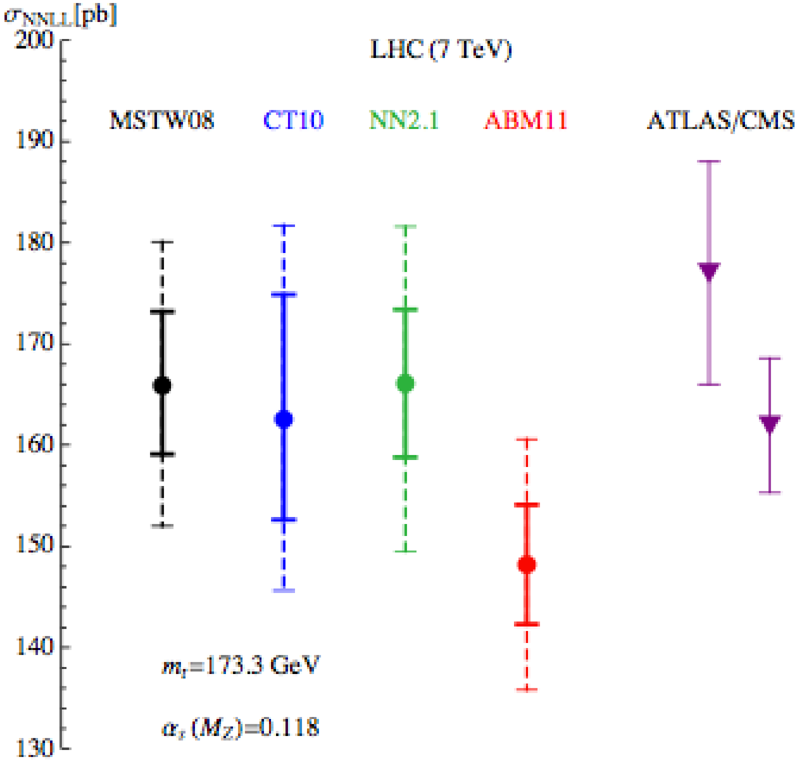
MSTW, CTEQ, NNPDF, HERAPDF,.....

ICHEP ‘12

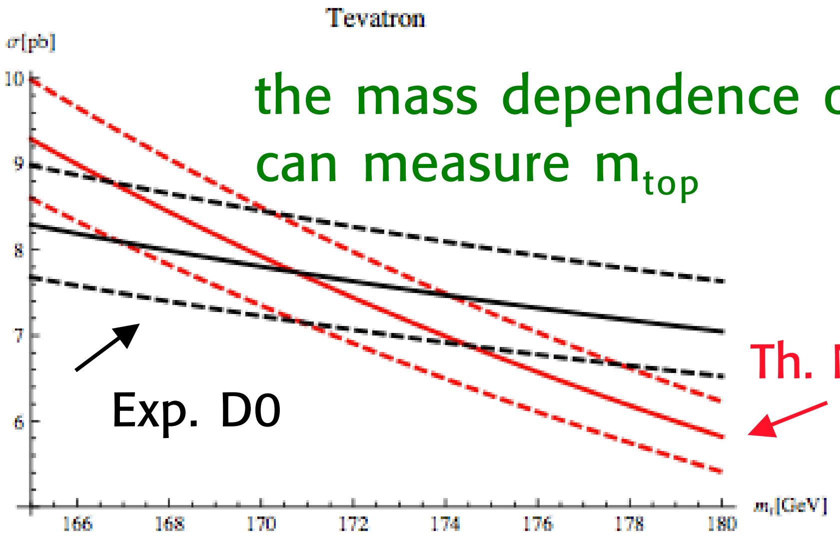


CMS, 19 pb-1, 8 TeV
CMS, 36 pb-1, 7 TeV
CDF Run II
D0 Run I
UA2
UA1
σ × B [nb]
10
1
10-1
0.5
1
2
5
7
10
20
Collider Energy [TeV]
W
W+
W-
Z
pp̄
pp
Theory: NNLO, FEWZ and MSTW08 PDFs

$t\bar{t}$ cross section known to NNLO plus resummation of soft Coulomb effects



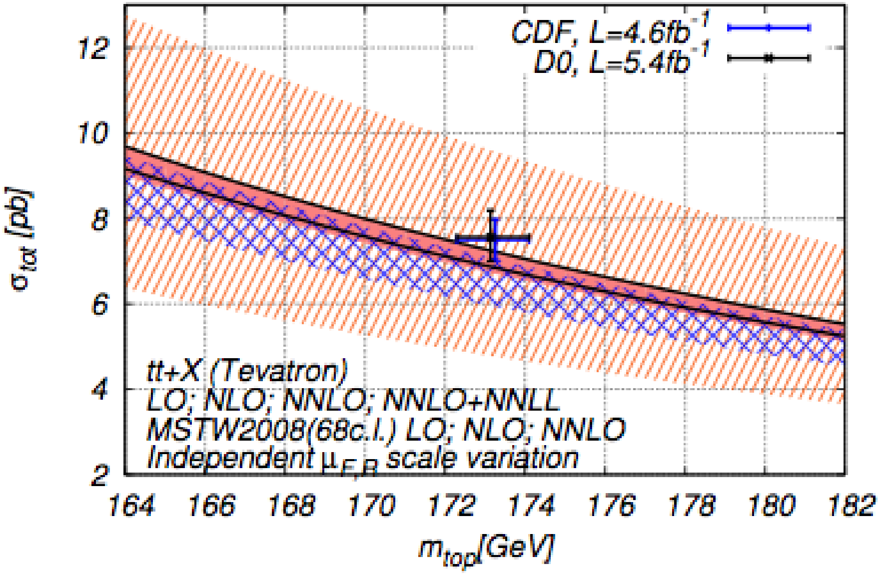
Beneke et al '11, '12
Ahrens et al '11



the mass dependence of σ can measure m_{top}

Th. NNLO

Exp. D0



Barnreuther, Czakon, Mitov '12

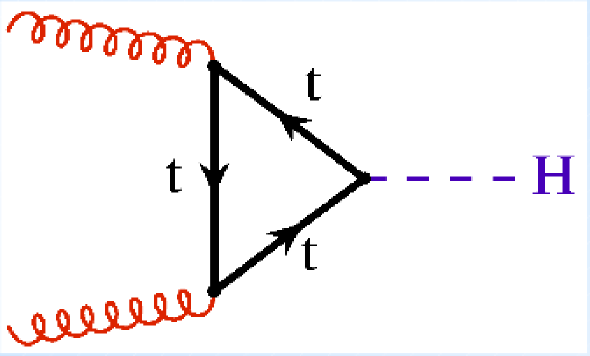
The Higgs cross sections and distributions are at the center of the stage now

see for a review

Handbook of LHC Higgs cross sections
Dittmaier, Mariotti, Passarino and Tanaka editors
ArXiv 1101.0593,1201.3084

Higgs production via g+g -> H

Very important for the LHC



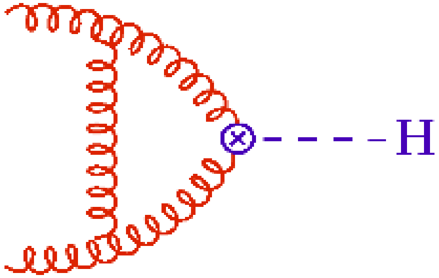
Effective lagrangian (m_t -> infinity)

$$\mathscr{L} = C_1 H G^{\mu\nu} G_{\mu\nu}$$

C_1 known to α_s^4

Chetyrkin, Kniehl, Steinhauser'97

NLO corr.s computed with effective lagrangian

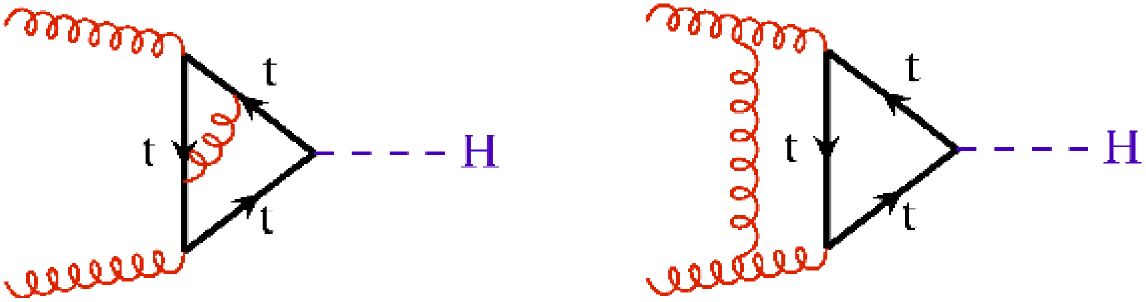


Dawson

Djouadi, Spira, Graudenz, Zerwas

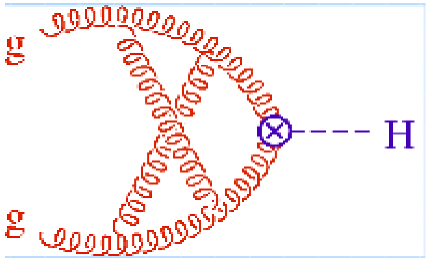
AND the full theory

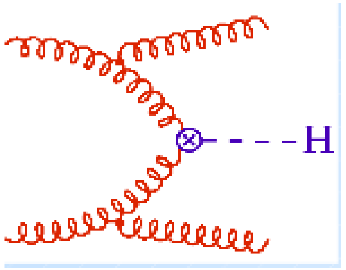
Djouadi, Spira, Graudenz, Zerwas



They agree very well

More recently the NNLO calculation was completed (analytic)

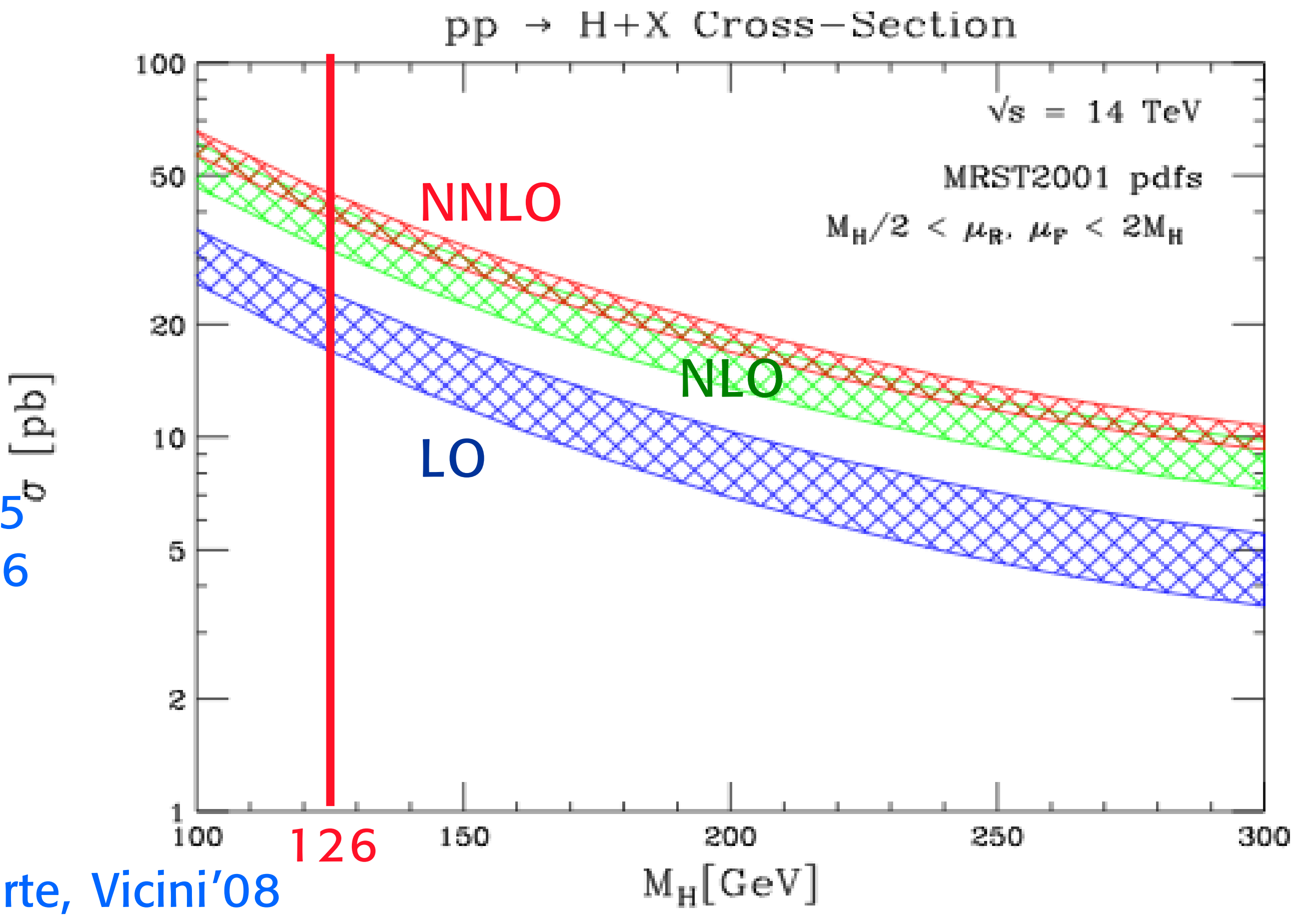




Catani, de Florian, Grazzini ’01.
Harlander, Kilgore ’01, ‘02
Anastasiou, Melnikov’02
Ravindran, Smith, van Neerven ’03

Also NLO y and p_T distributions have been computed

De Florian, Grazzini, Kunszt ‘99
Glosser, Schmidt’02
Anastasiou, Melnikov, Petriello’05
Ravindran, Smith, van Neerven’06



Recent progress:
Resummation of large partonic-energy logs

DeMarzani, Ball, Del Duca, Forte, Vicini’08

Higgs-related advances

Campbell
ICHEP’12

signals

NLO	**H+γ+2 jets** via VBF	Arnold et al
NLO+PS	**gg→H+1,2 jets**	Ellis et al
	ttH	Garzelli et al
	$H^{\pm}$ t	Klasen et al
	MSSM gg→H	Bagnaschi et al
NNLO	**WH**	Ferrera et al
	H→bb differential rate	Anastasiou et al
	bb→H	Harlander et al, Buehler et al
	non-minimal H via VBF	Bolzoni et al
	non-minimal gg→H	Furlan

Higgs p_T distribution: $[\log(p_T/m_H)]^n$ resummed

Bozzi, Catani, De Florian, Grazzini'03-'08

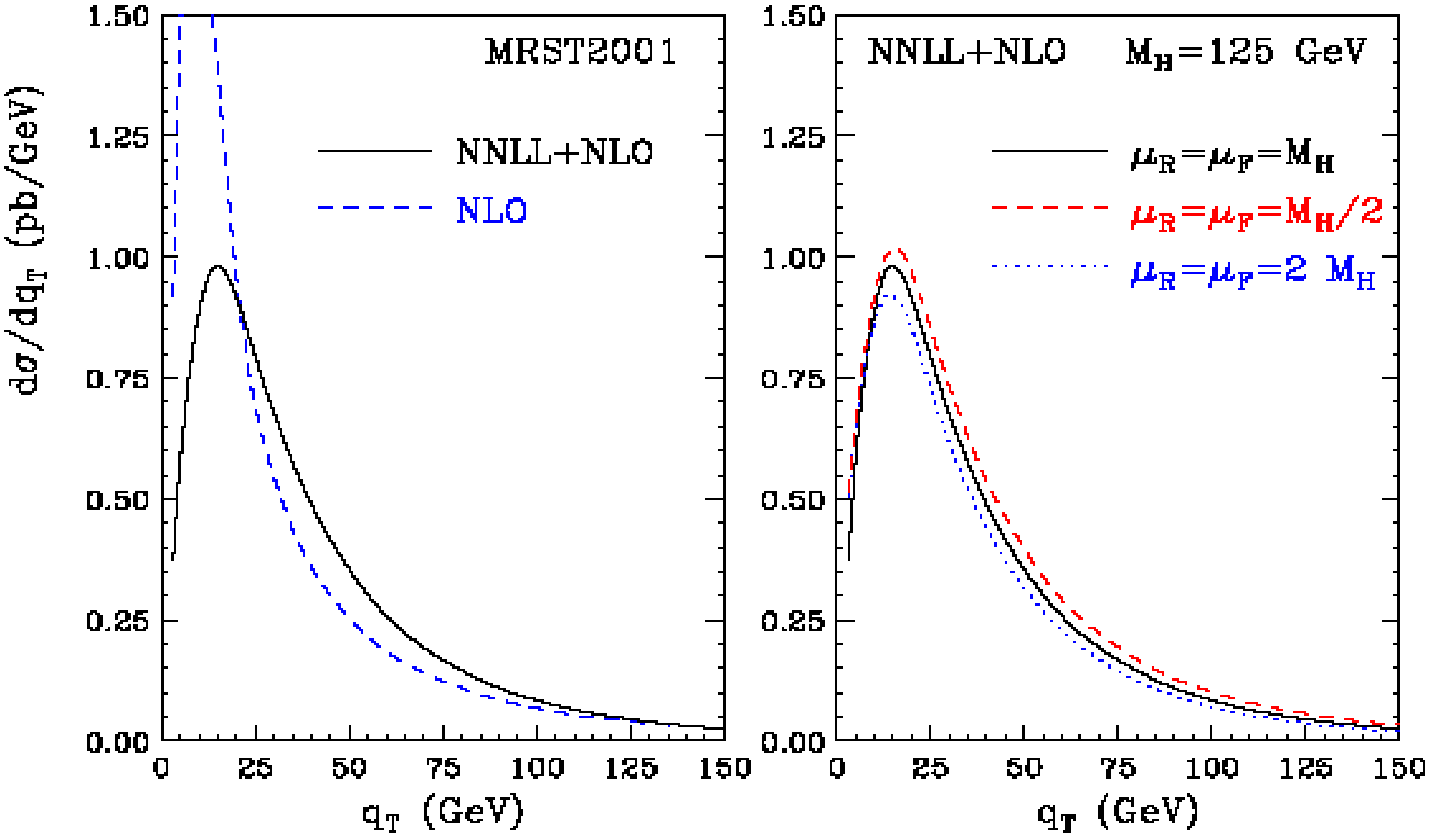
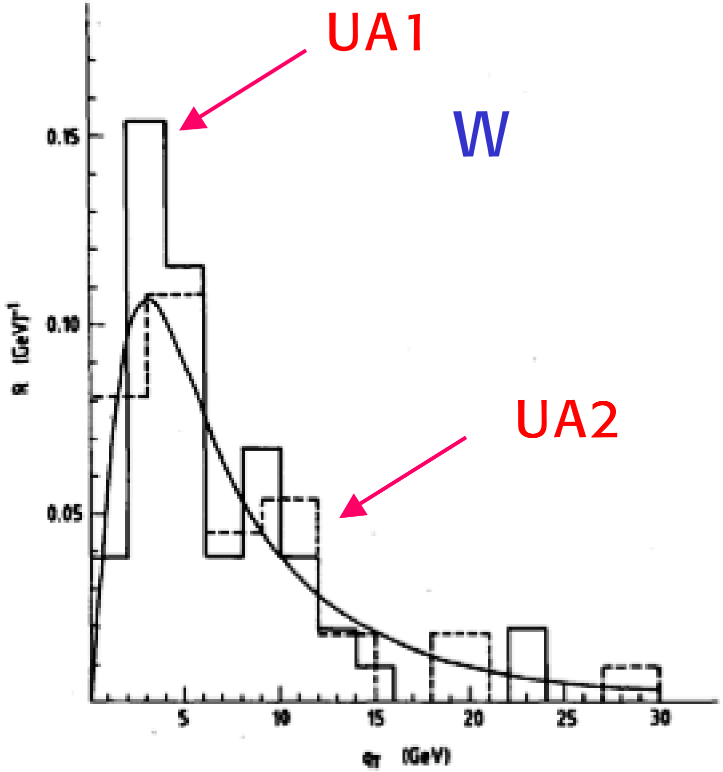


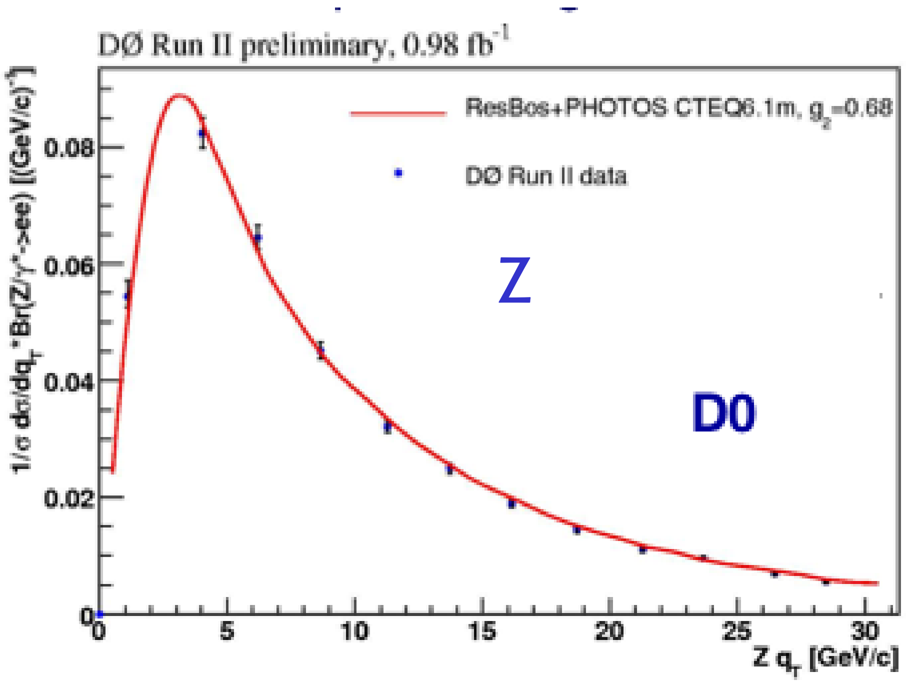
Figure 7. Resummed pQCD prediction for the Higgs transverse momentum distribution at the LHC, from Bozzi *et al.*[25]

~28 years ago at CERN we computed the W and Z p_T distribution in QCD



GA, K.Ellis, M. Greco, G.Martinelli '84

Here all relevant ingredients were first assembled and matched. Later mainly refinements were added



In agreement with perturbative QCD augmented by Collins-Soper-Sterman (CSS) resummation at low q_T

J. Collins, D. Soper, G. Sterman, Nucl. Phys. B250 (1985) 199.

ResBos describes data well up to ~ 30 GeV

F. Landry, R. Bock, P.Nadolsky, C.P. Yuan
Phys. Rev. D 67, 073016 (2003)

NNLO describes better above 30 GeV

K. Melnikov and F. Petriello Phys. Rev. D74 114017 (2006)

The avantgarde of contemporary QCD research

see L. Dixon
talk at
ICHEP '12

N=4 SUSY QCD and AdS/CFT correpondence

N=4 SUSY QCD has $\beta(\alpha) = 0$ and is loop finite

In limit N_C -> infinity with $\lambda = g^2 N_C$ fixed, planar diagrams are dominant

The large λ limit corresponds by AdS/CFT duality to the weakly coupled string (gravity) theory on $AdS_5 x S_5$

There is progress towards a solution of planar N=4 SUSY QCD amplitudes

N = 8 Supergravity, related to N = 4 SUSY Yang-Mills, has been proven finite up to 4 loops. It could possibly lead to a finite field theory of gravity in 4 dimensions

Conclusion

QCD is a non abelian unbroken gauge quantum field theory of fundamental physical relevance

Its physics content is very large and our knowledge esp. in the non perturbative domain is still very limited but progress both from experiment (HERA, Tevatron, RHIC, LHC) and from theory is continuous

Very good agreement with experiment

EXTRA

Why SU(N_C=3)$_{\mathrm{Colour}}$?

Observed: hadrons

Mesons: qq^*

Baryons: qqq

Colour singlets

The group must:

- admit complex reprs.: q different from q^*

(qq must not be a singlet) SU(n>2), SO(4n+2), E(6)

(In SU(3) $q \sim 3$, $q^* \sim 3^*$, $3X3=6+3^*$, $3X3^*=8+1$)

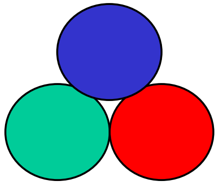
- allow a totally antisymm. qqq singlet

(qqq is totally symm. in space [s-wave], spin and SU(3) [SU(6) ~56])

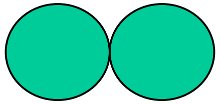
(In SU(3) $\varepsilon_{abc}q^a q^b q^c \sim 1$)

Hadron spectroscopy

All observed hadrons are colourless composites of quarks



Baryons: qqq



Mesons: $q\bar{q}$

For example:

Proton p: uud

Pion π^+: $u\bar{d}$

Colour is essential for Fermi statistics

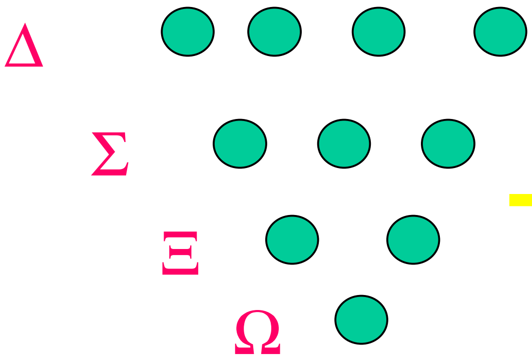
The state Δ^{++} with spin 3/2 = u↑u↑u↑

is symmetric in space and spin but antisymm. in colour

and for explaining the observed spectrum

For example:

the “decuplet”



ddd ddu duu uuu

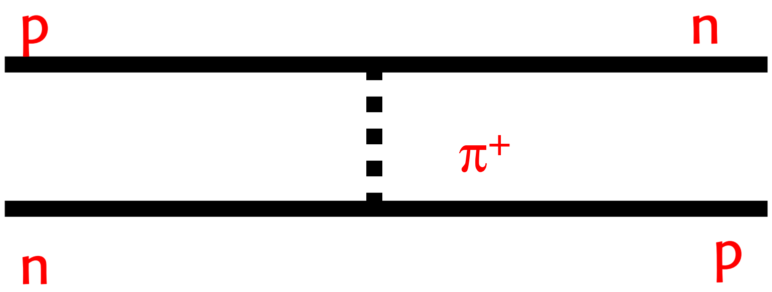
dds dus uus

dss uss

sss

Confinement explains why the nuclear forces are short range while massless gluon exchange would be long range:

Nucleons are colour singlets:
they can only exchange colour singlets (pions not gluons)



$V \sim \exp(-m_\pi r)/r$

The range of nuclear forces is determined by the pion mass:
$r \sim m_\pi^{-1} \sim 10^{-13}$ cm

SU(N_C): many processes measure N_C

Examples:

- $R = \sigma(e^+ e^- \to \text{hadrons}) / \sigma(e^+ e^- \to \mu^+ \mu^-)$

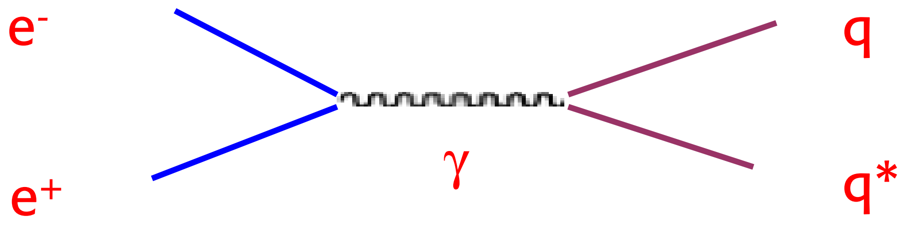
Above bb* thresh. and below m_Z

$$R = N_C \cdot \left[3 \cdot \frac{1}{9} + 2 \cdot \frac{4}{9} \right] = \left(N_C \cdot \frac{11}{9} \right)$$

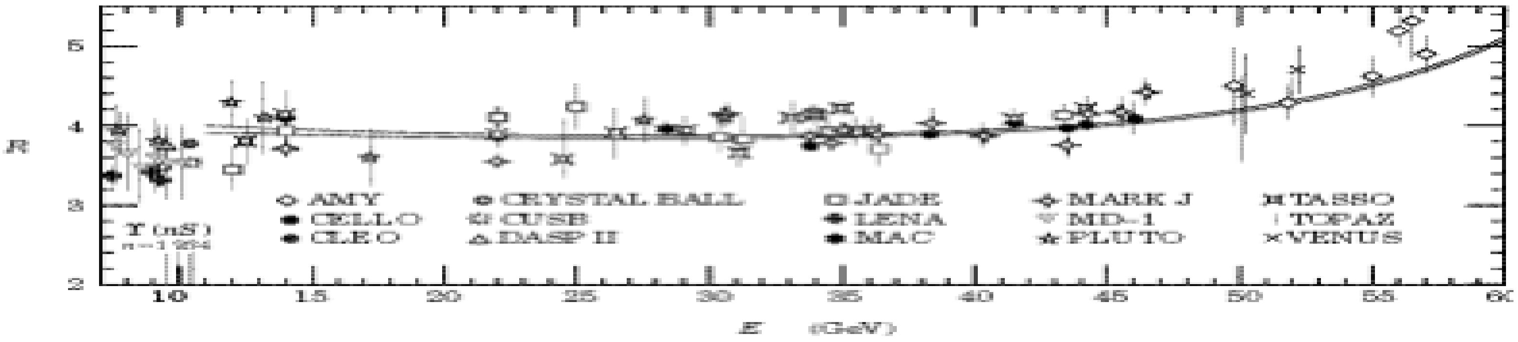
d,s,b

u,c

(Computable small rad corr's neglected)



q=d,s,b,u,c

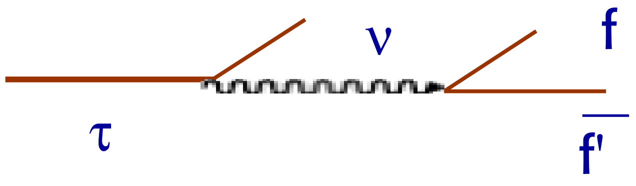


11/3

- B(τ ->e$\nu\nu$) ~ $\frac{1}{2+N_C}$

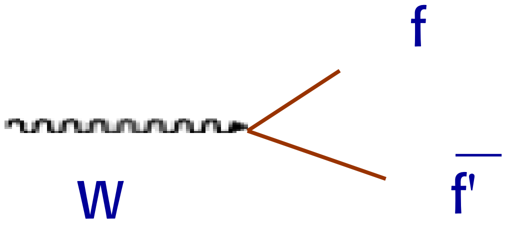
N_C=3 -> B~ 20%
Exp. : B~ 18%

Tree level!



f=e, μ, u

- B(W ->eν) ~ $\frac{1}{3+2\cdot N_C}$



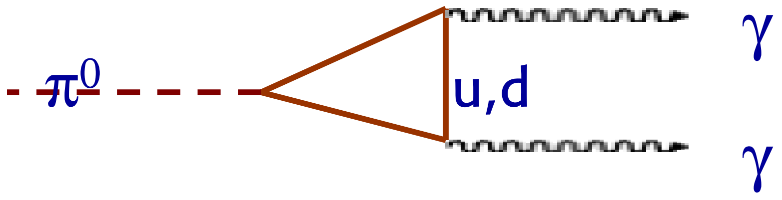
N_C=3 -> B~ 11%
Exp. : B~ 10.7%

f=e, μ, τ, u, c

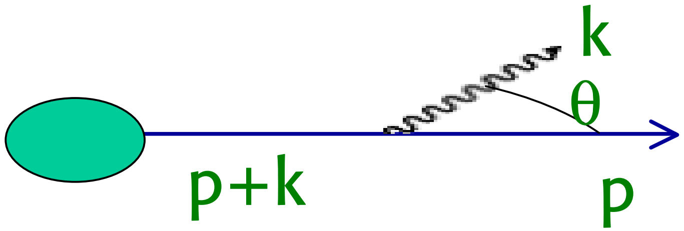
- $\Gamma(\pi^0$ ->$\gamma\gamma$) ~ $\left(\frac{N_C}{3}\right)^2 \frac{\alpha^2 m_{\pi^0}^3}{32\pi^3 f_\pi^2} = (7.73 \pm 0.04)\left(\frac{N_C}{3}\right)^2 eV$

Exp. : (7.7±0.5) eV

f_π=(130.7±0.37)MeV
(PDG2000)



Infrared and collinear safety



$$Propagator = \frac{1}{(p+k)^2 - m^2} = \frac{1}{2(pk)} =$$

$$= \frac{1}{2E_kE_p} \cdot \frac{1}{1-\beta_p\cos\theta}$$

- E_k -> 0 ➡ infrared singularity.

- For m -> 0, $\beta_p = \sqrt{1 - \frac{m^2}{E_p^2}}$ -> 1 and

(1-$\beta_p\cos\theta$) vanishes at cos $\theta = 1$ ➡

➡ collinear (mass) singularity

Summarising: we started from the massless classical theory and we ended up with QCD where an energy scale $\Lambda = \Lambda_{QCD}$ appears.

Λ depends on the def. of α_s (i.e. the reg. procedure, the ren. scheme…) and on the number of excited flavours n_f.

Definition of α_s

We have introduced the ren. coupling α_s in terms of the 3-g ren. vertex at $p^2 = -\mu^2$ (momentum subtraction). The value of α_s (hence Λ) in this scheme depends on μ.

But the most common def. of α_s is in the framework of dimensional reg.

Dim. reg. is a gauge and Lorentz inv. reg. that is most simply implemented in calculations. It consists in formulating the theory in $d<4$ space-time dimensions.

Dimensional Regularisation (DR)

Rewrite the theory in d (integer) dim. Expression of diagrams also OK for any d.

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & . & 0 \\ 0 & -1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & -1 \end{bmatrix} \quad (\mathrm{d \times d})$$

$$k^{\mu} = (k^0, k^1, \ldots\ldots, \mathrm{k}^{d-1})$$

Dirac γ^{μ} f(d)xf(d)

Tr $\gamma^{\mu}\gamma^{\nu}$=f(d)$g^{\mu\nu}$

…

For d<4 loop integrals less divergent.

e.g. $$I = \int \frac{1}{k^2(p-k)^2} \, d^d k$$

The coupling carries dimensions: $e_d=\mu^{\varepsilon}e$

(d=4-2ε; this is how a mass scale enters!)

$$\psi \propto m^{\frac{d-1}{2}}$$

$$\phi, A_{\nu} \propto m^{\frac{d-2}{2}}$$

$$e \propto m^{\frac{4-d}{2}}$$

$$Action = \int \begin{bmatrix} m\bar{\psi}\psi \\ m^2\phi^{\dagger}\phi \\ e\bar{\psi}\gamma^{\nu}\psi A_{\nu} \end{bmatrix} d^d x \sim 1$$

The formal expression of loop integrals can be written for all d. For example:

$$\int \frac{1}{\left(k^2 - m^2\right)^2} \frac{d^d k}{(2\pi)^d} = \frac{\Gamma\left(2 - \frac{d}{2}\right)\left(-m^2\right)^{\frac{d}{2} - 2}}{(4\pi)^{\frac{d}{2}}}$$

For d=4-2ε we can expand, using:

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + 0(\varepsilon) \qquad \gamma_E = 0.5772\ldots$$

For some quantity we obtain from diagrams

$$G = 1 + \alpha\left(\frac{\mu^2}{-p^2}\right)^{\varepsilon}\left[B\left(\frac{1}{\varepsilon} + \log(4\pi) - \gamma_E\right) + A + 0(\varepsilon)\right]$$

In $\overline{\text{MS}}$ we write this as (diagram by diagram):

$$Z = 1 + \alpha\left[B\left(\frac{1}{\varepsilon} + \log(4\pi) - \gamma_E\right)\right]$$

$$G = Z G_R$$

$$G_R = 1 + \alpha\left[B \log\frac{\mu^2}{-p^2} + A\right]$$

Consider first the case $\gamma(\alpha)=0$.

This is not unphysical: it occurs for R_{e+e-}

Recall that $\gamma(\alpha)=d\log Z/d\log\mu^2$. It is zero because QCD corr's cannot renormalise the electric charge (or the proton and positron charges would be different)

$$\left[-\frac{\partial}{\partial t}+\beta(\alpha)\frac{\partial}{\partial\alpha}\right]\cdot F(t,\alpha) = 0$$

The solution is $F[0,\alpha(t)]$, where the "running coupling" $\alpha(t)$ is defined by:

$$t = \int_{\alpha}^{\alpha(t)} \frac{1}{\beta(\alpha')}d\alpha'$$

Take d/dt and d/dα of both sides:

$$(d/dt) \qquad 1 = \frac{1}{\beta(\alpha(t))}\frac{\partial}{\partial t}\alpha(t)$$

$$(d/d\alpha) \qquad 0 = -\frac{1}{\beta(\alpha)}+\frac{1}{\beta(\alpha(t))}\frac{\partial}{\partial\alpha}\alpha(t)$$

We have found

$$\frac{\partial}{\partial t}\alpha(t) = \beta(\alpha(t)) \qquad ; \qquad \frac{\partial}{\partial \alpha}\alpha(t) = \frac{\beta(\alpha(t))}{\beta(\alpha)}$$

Using these eqs. we check that

$$F(t,\alpha) = F[0,\alpha(t)]$$

is the solution (note that $\alpha(0)=\alpha$, so that the boundary cond. is satisfied)

$$\left[-\frac{\partial}{\partial t} + \beta(\alpha)\frac{\partial}{\partial \alpha}\right] \cdot F(0, \alpha(t)) = 0$$

With $F' = dF(0,\alpha)/d\alpha$, we have:

$$\left[-\frac{\partial}{\partial t}\alpha(t) + \beta(\alpha)\frac{\partial}{\partial \alpha}\alpha(t)\right]F' =$$

$$= \left[-\beta(\alpha(t)) + \beta(\alpha) \cdot \frac{\beta(\alpha(t))}{\beta(\alpha)}\right]F' = 0$$

Similarly for the more general equation:

$$\left[-\frac{\partial}{\partial t}+\beta(\alpha)\frac{\partial}{\partial \alpha}+\gamma(\alpha)\right]\cdot F(t,\alpha)\ =\ 0$$

The solution is:

$$F[0,\ \alpha(t)]\exp\int_{\alpha}^{\alpha(t)}\frac{\gamma(\alpha')}{\beta(\alpha')}d\alpha'$$

as can be easily checked given that:

- the differential operator applied to $F[0,\alpha(t)]$ vanishes
- the exponential is by itself a solution of the complete equation.

Summary: The important point is the appearance of the running coupling that determines the asympt. behaviour.

γ-N cross-section

$W_i = W_i(Q^2,\nu)$

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2 E'}{Q^4 E}\left[2\left(\sin\frac{\theta}{2}\right)^2 W_1 + \left(\cos\frac{\theta}{2}\right)^2 W_2\right]$$

ν-N (ν-$\bar{\text{N}}$) cross-section

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dQ^2 d\nu} = \frac{G_F{}^2 E'}{2\pi E}\left(\frac{m_W^2}{Q^2+m_W^2}\right)^2\left[2\left(\sin\frac{\theta}{2}\right)^2 W_1^{\nu} + \left(\cos\frac{\theta}{2}\right)^2 W_2^{\nu} \mp \frac{E+E'}{m}\left(\sin\frac{\theta}{2}\right)^2 W_3^{\nu}\right]$$

Scaling limit: $Q^2 >> m^2$ x fixed

$mW_1(Q^2,\nu) \to F_1(x)$

$\nu W_2(Q^2,\nu) \to F_2(x)$

$\nu W_3(Q^2,\nu) \to F_3(x)$

Bjorken

In the scaling limit the following relations with the cross sections of the fixed-helicity gauge bosons (γ, $W^{\pm}$…) hold:

σ_L: longitudinal -> helicity = 0
σ_{RH}: right-handed -> helicity = +1
σ_{LH}: left-handed -> helicity = -1
[$\sigma_T = \sigma_{RH} + \sigma_{LH}$: transverse]

$$\sigma_L = \frac{2\pi}{s}\left[\frac{F_2(x)}{2x} - F_1(x)\right]$$

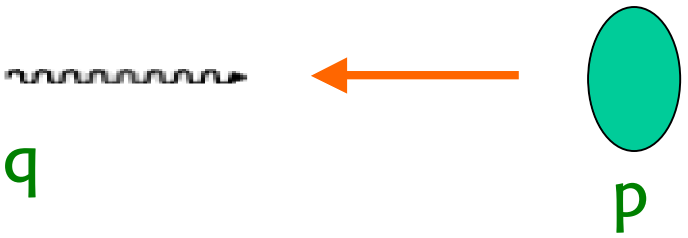
$$\sigma_{RH} = \frac{2\pi}{s}\left[F_1(x) + \frac{1}{2}F_3(x)\right]$$

$$\sigma_{LH} = \frac{2\pi}{s}\left[F_1(x) - \frac{1}{2}F_3(x)\right]$$

"Naïve" parton model

Lorentz contracted proton

Bjorken
Feynman

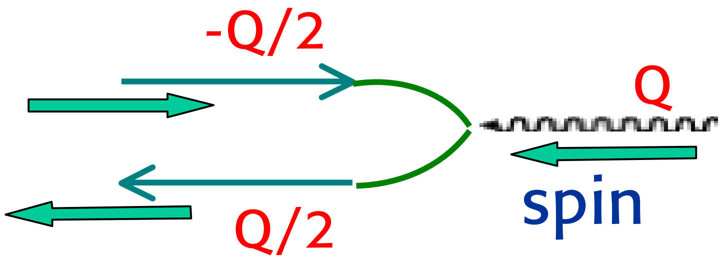


Bjorken & Feynman language: The virtual γ sees the quark partons inside the proton as quasi-free because the QCD interaction time (Lorentz dilated) is much longer than $\tau_\gamma \sim 1/Q$

Breit frame: in this frame $E\gamma=0$:

$$q = (0;0,\ 0,\ Q)$$

$$p = \left(\frac{Q}{2x};0,\ 0,\ -\frac{Q}{2x}\right)$$



$$p_q = \left(\frac{yQ}{2x};0,\ 0,\ -y\frac{Q}{2x}\right)$$

Note: $x=Q^2/2(pq)$

Take a parton with 4-mom $p_q= yp$. Since $E\gamma=0$, the quark momentum is reversed: $y=x$.

$\sigma_{point} \sim e^2\delta(x/y-1)$

$2F_1= e^2q_0(x)$

Spin 1/2 partons: $\sigma_L=0$

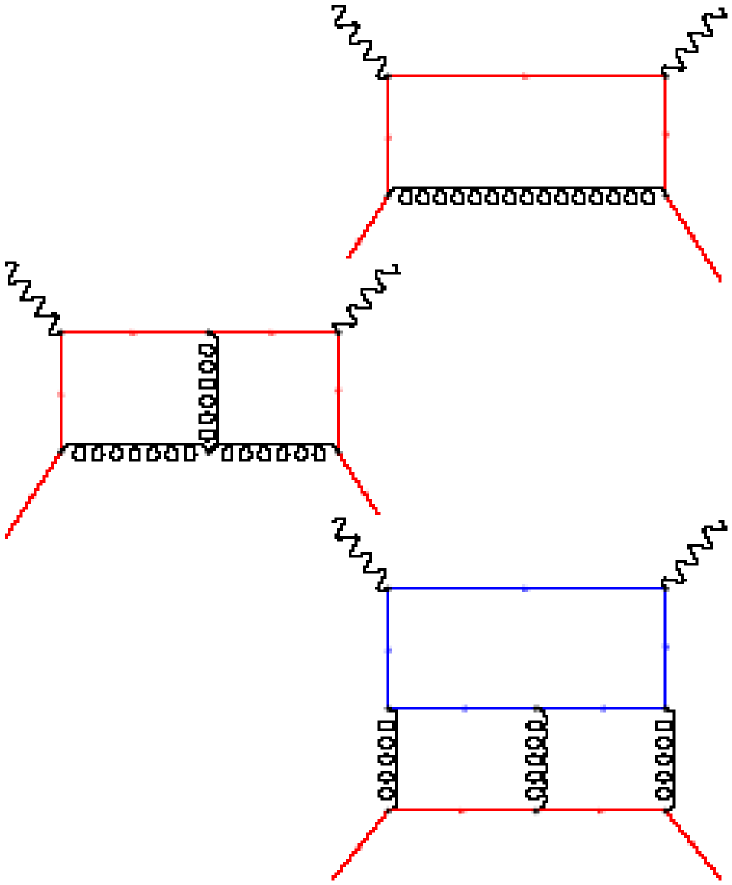
Spin 0 partons: $\sigma_T=0$

The calculation (in a nut shell)

Moch

- Calculate anomalous dimensions (Mellin moments of splitting functions)
  $\longrightarrow$ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

$$\gamma_{\rm ij}^{(n)}(N) = -\int_0^1 dx\, x^{N-1} P_{\rm ij}^{(n)}(x)$$



- **One-loop** Feynman diagrams
  $\longrightarrow$ in total 18 for $\gamma_{\rm ij}^{(0)}$ / $P_{\rm ij}^{(0)}$
  (pencil + paper)

- **Two-loop** Feynman diagrams
  $\longrightarrow$ in total 350 for $\gamma_{\rm ij}^{(1)}$ / $P_{\rm ij}^{(1)}$
  (simple computer algebra)

- **Three-loop** Feynman diagrams
  $\longrightarrow$ in total 9607 for $\gamma_{\rm ij}^{(2)}$ / $P_{\rm ij}^{(2)}$
  (cutting edge technology $\longrightarrow$ computer algebra system FORM Vermaseren '89-'04)

NLO singlet splitting functions

$$\begin{aligned}
P^{(0)}_{\mathrm{ps}}(x) &= 0 \\
P^{(0)}_{\mathrm{qg}}(x) &= 2n_f\, p_{\mathrm{qg}}(x) \\
P^{(0)}_{\mathrm{gq}}(x) &= 2C_F\, p_{\mathrm{gq}}(x) \\
P^{(0)}_{\mathrm{gg}}(x) &= C_A\Big(4p_{\mathrm{gg}}(x) + \frac{11}{3}\delta(1-x)\Big) - \frac{2}{3}n_f\,\delta(1-x)
\end{aligned}$$

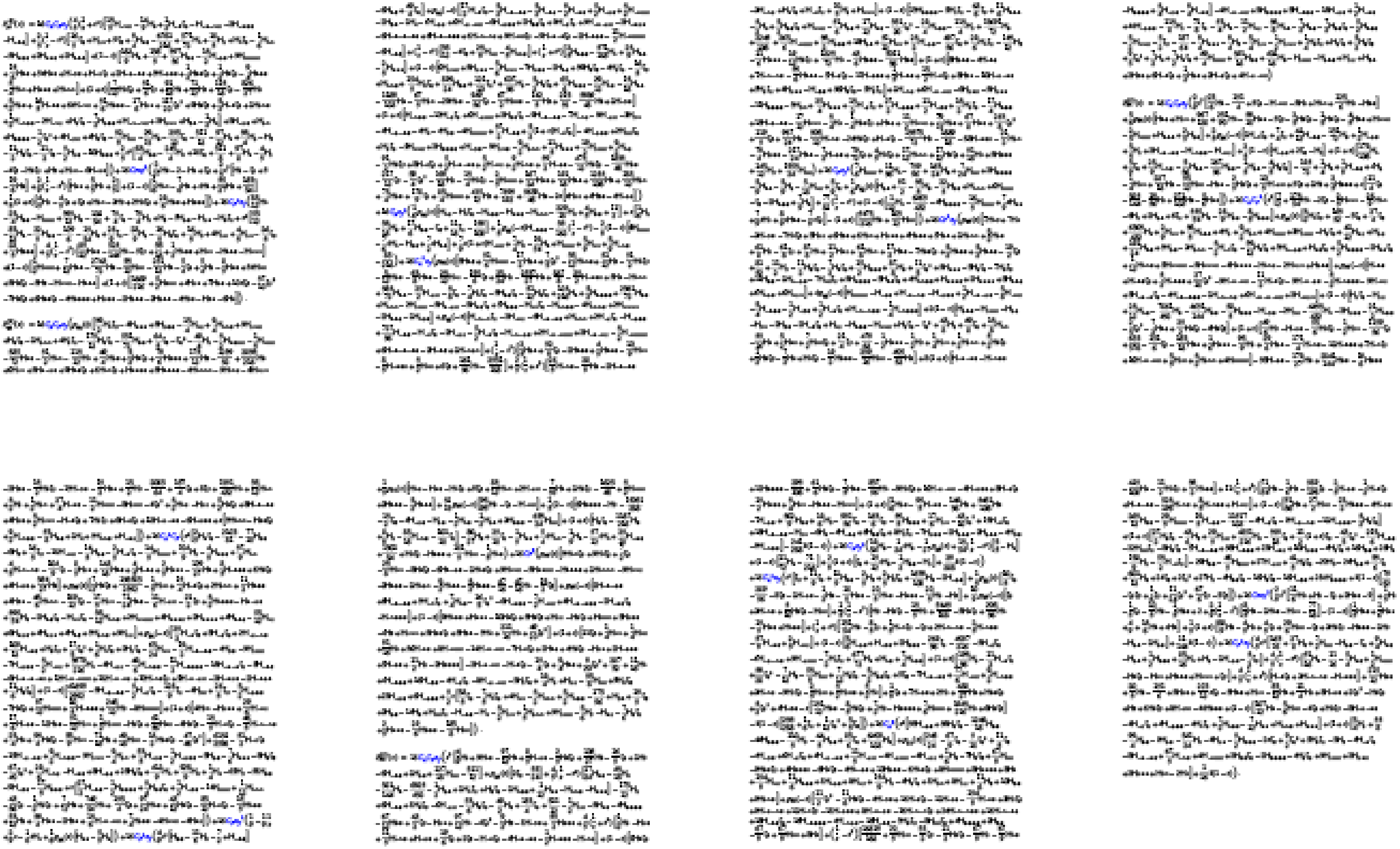
$$\begin{aligned}
P^{(1)}_{\mathrm{ps}}(x) \;&=\; 4C_F n_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4\mathrm{H}_0 + x^2\Big[\frac{8}{3}\mathrm{H}_0 - \frac{56}{9}\Big] + (1+x)\Big[5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\Big]\Big)
\end{aligned}$$

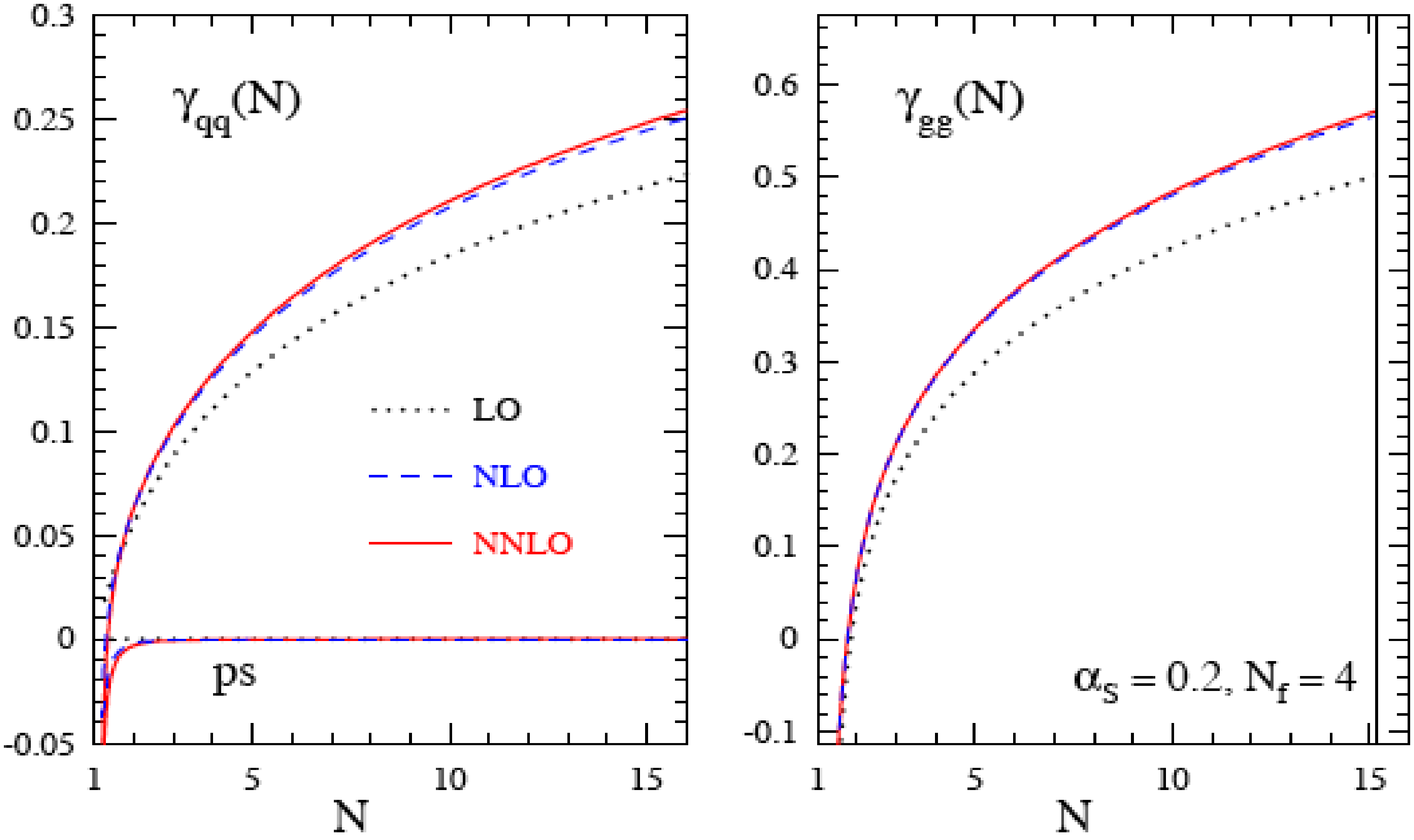
$$\begin{aligned}
P^{(1)}_{\mathrm{qg}}(x) \;&=\; 4C_A n_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\mathrm{qg}}(-x)\mathrm{H}_{-1,0} - 2p_{\mathrm{qg}}(x)\mathrm{H}_{1,1} + x^2\Big[\frac{44}{3}\mathrm{H}_0 - \frac{218}{9}\Big] \\
&+4(1-x)\Big[\mathrm{H}_{0,0} - 2\mathrm{H}_0 + x\mathrm{H}_1\Big] - 4\zeta_2 x - 6\mathrm{H}_{0,0} + 9\mathrm{H}_0\Big) + 4C_F n_f\Big(2p_{\mathrm{qg}}(x)\Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 \\
&-\zeta_2\Big] + 4x^2\Big[\mathrm{H}_0 + \mathrm{H}_{0,0} + \frac{5}{2}\Big] + 2(1-x)\Big[\mathrm{H}_0 + \mathrm{H}_{0,0} - 2x\mathrm{H}_1 + \frac{29}{4}\Big] - \frac{15}{2} - \mathrm{H}_{0,0} - \frac{1}{2}\mathrm{H}_0\Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\mathrm{gq}}(x) \;&=\; 4C_A C_F\Big(\frac{1}{x} + 2p_{\mathrm{gq}}(x)\Big[\mathrm{H}_{1,0} + \mathrm{H}_{1,1} + \mathrm{H}_2 - \frac{11}{6}\mathrm{H}_1\Big] - x^2\Big[\frac{8}{3}\mathrm{H}_0 - \frac{44}{9}\Big] + 4\zeta_2 - 2 \\
&-7\mathrm{H}_0 + 2\mathrm{H}_{0,0} - 2\mathrm{H}_1 x + (1+x)\Big[2\mathrm{H}_{0,0} - 5\mathrm{H}_0 + \frac{37}{9}\Big] - 2p_{\mathrm{gq}}(-x)\mathrm{H}_{-1,0}\Big) - 4C_F n_f\Big(\frac{2}{3}x \\
&-p_{\mathrm{gq}}(x)\Big[\frac{2}{3}\mathrm{H}_1 - \frac{10}{9}\Big]\Big) + 4{C_F}^2\Big(p_{\mathrm{gq}}(x)\Big[3\mathrm{H}_1 - 2\mathrm{H}_{1,1}\Big] + (1+x)\Big[\mathrm{H}_{0,0} - \frac{7}{2} + \frac{7}{2}\mathrm{H}_0\Big] - 3\mathrm{H}_{0,0} \\
&+1 - \frac{3}{2}\mathrm{H}_0 + 2\mathrm{H}_1 x\Big)
\end{aligned}$$

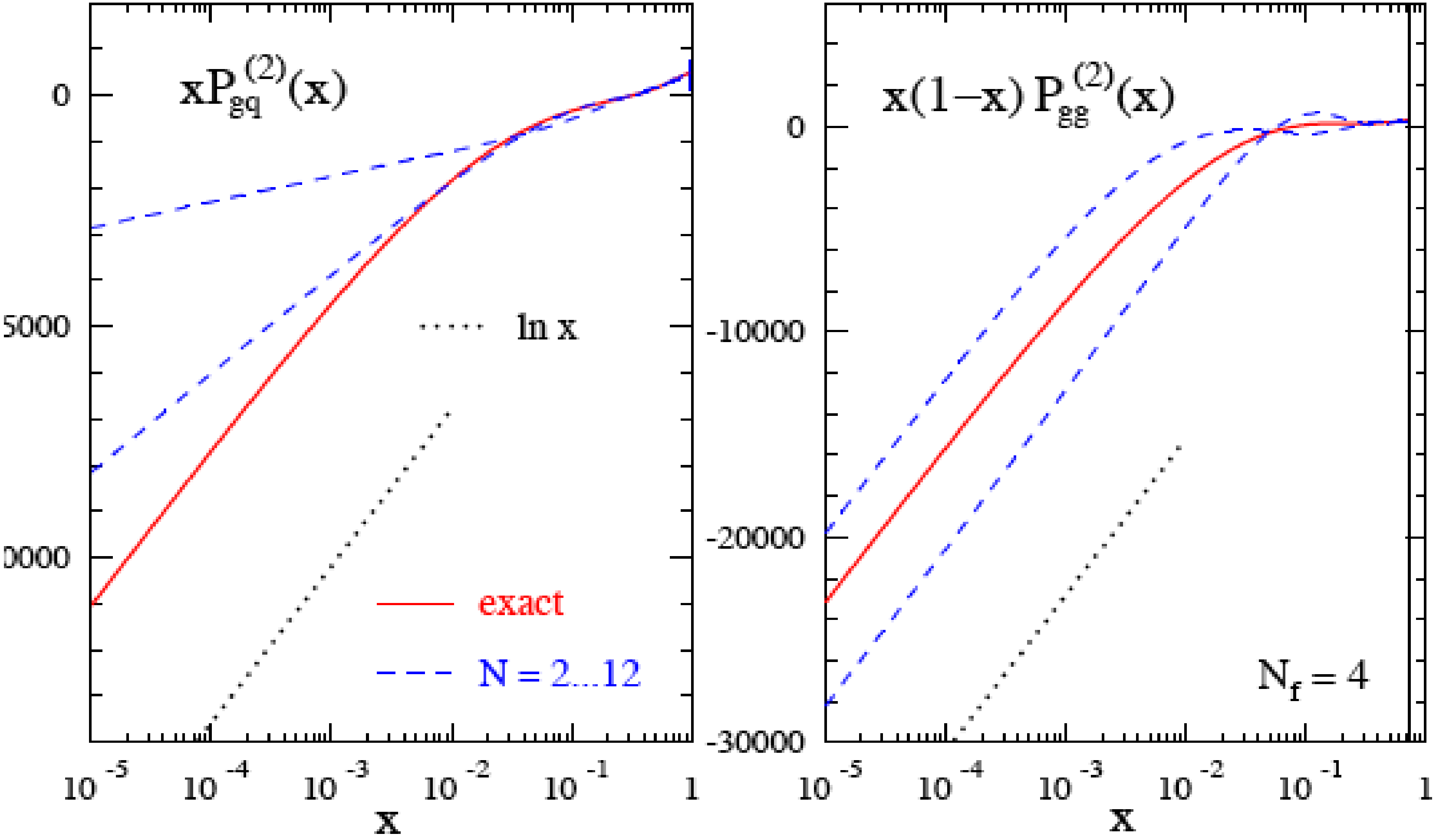
$$\begin{aligned}
P^{(1)}_{\mathrm{gg}}(x) \;&=\; 4C_A n_f\Big(1 - x - \frac{10}{9}p_{\mathrm{gg}}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2\Big) - \frac{2}{3}(1+x)\mathrm{H}_0 - \frac{2}{3}\delta(1-x)\Big) + 4{C_A}^2\Big(27 \\
&+(1+x)\Big[\frac{11}{3}\mathrm{H}_0 + 8\mathrm{H}_{0,0} - \frac{27}{2}\Big] + 2p_{\mathrm{gg}}(-x)\Big[\mathrm{H}_{0,0} - 2\mathrm{H}_{-1,0} - \zeta_2\Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2\Big) - 12\mathrm{H}_0 \\
&-\frac{44}{3}x^2\mathrm{H}_0 + 2p_{\mathrm{gg}}(x)\Big[\frac{67}{18} - \zeta_2 + \mathrm{H}_{0,0} + 2\mathrm{H}_{1,0} + 2\mathrm{H}_2\Big] + \delta(1-x)\Big[\frac{8}{3} + 3\zeta_3\Big]\Big) + 4C_F n_f\Big(2\mathrm{H}_0 \\
&+\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\Big[4 - 5\mathrm{H}_0 - 2\mathrm{H}_{0,0}\Big] - \frac{1}{2}\delta(1-x)\Big)\ .
\end{aligned}$$

NNLO singlet splitting functions





$\gamma_{qq}(N)$
$\gamma_{gg}(N)$
LO
NLO
NNLO
ps
$\alpha_S = 0.2, N_f = 4$
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
1
5
10
15
N



- Exact result, estimates from fixed moments and leading small-x term
- Splitting function $P_{gq}^{(2)}$ (left) and $P_{gg}^{(2)}$ (right)

$$\frac{d}{dt}q(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 dy\frac{q(y,t)}{y}P\left(\frac{x}{y}\right) = \frac{\alpha_s(t)}{2\pi}[q\otimes P](x,t)$$

(Mellin) Moments:

$$q_n = \int_0^1 q(x)x^{n-1}dx \qquad\qquad P_n = \int_0^1 P(x)x^{n-1}dx$$

Taking moments of both sides

$$\frac{d}{dt}q_n(t) = \frac{\alpha_s(t)}{2\pi}\cdot P_n\cdot q_n(t)$$

A much simpler equation!

Proof: $\int_0^1 dx x^{n-1}\int_x^1 dy\frac{q(y,t)}{y}P\left(\frac{x}{y}\right) = \int_0^1 dy\frac{q(y,t)}{y}y^n\int_0^1 dz z^{n-1}P(z)$

PDF or structure function moments $M_n(t,\alpha_s)$ obey RGE (q_n is a particular case).

$$\left[-\frac{\partial}{\partial t}+\beta(\alpha)\frac{\partial}{\partial\alpha}+\gamma_n(\alpha)\right]\cdot M_n(t,\alpha) = 0$$

RGE general solution:

$$M_n(t,\alpha_s) = c_n(0,\alpha_s(t))\exp\int_{\alpha_s}^{\alpha_s(t)}\frac{\gamma_n(\alpha')}{\beta(\alpha')}d\alpha'\cdot O_n(\alpha_s)$$

In lowest order, applied to q_n , we have:

$$\gamma_n(\alpha)\approx\frac{P_n}{2\pi}\alpha+\ldots \qquad \beta(\alpha)\approx -b\alpha^2+\ldots$$

$$q_n(t) = q_n(0)\exp\int_{\alpha_s}^{\alpha_s(t)}\frac{\gamma_n(\alpha')}{\beta(\alpha')}d\alpha'\approx\left[\frac{\alpha_s}{\alpha_s(t)}\right]^{\frac{P_n}{2\pi b}}q_n(0)$$

This is exactly the solution of

$$\frac{d}{dt}q_n(t) = \frac{\alpha_s(t)}{2\pi}\cdot P_n\cdot q_n(t)$$

with boundary cond.
at t=0: $q_n(0)$

Gross,Wilczek; Politzer

Scaling violations in DIS

The scaling violations are clearly observed and the (N)NLO QCD fits are remarkably good.

These fits provide

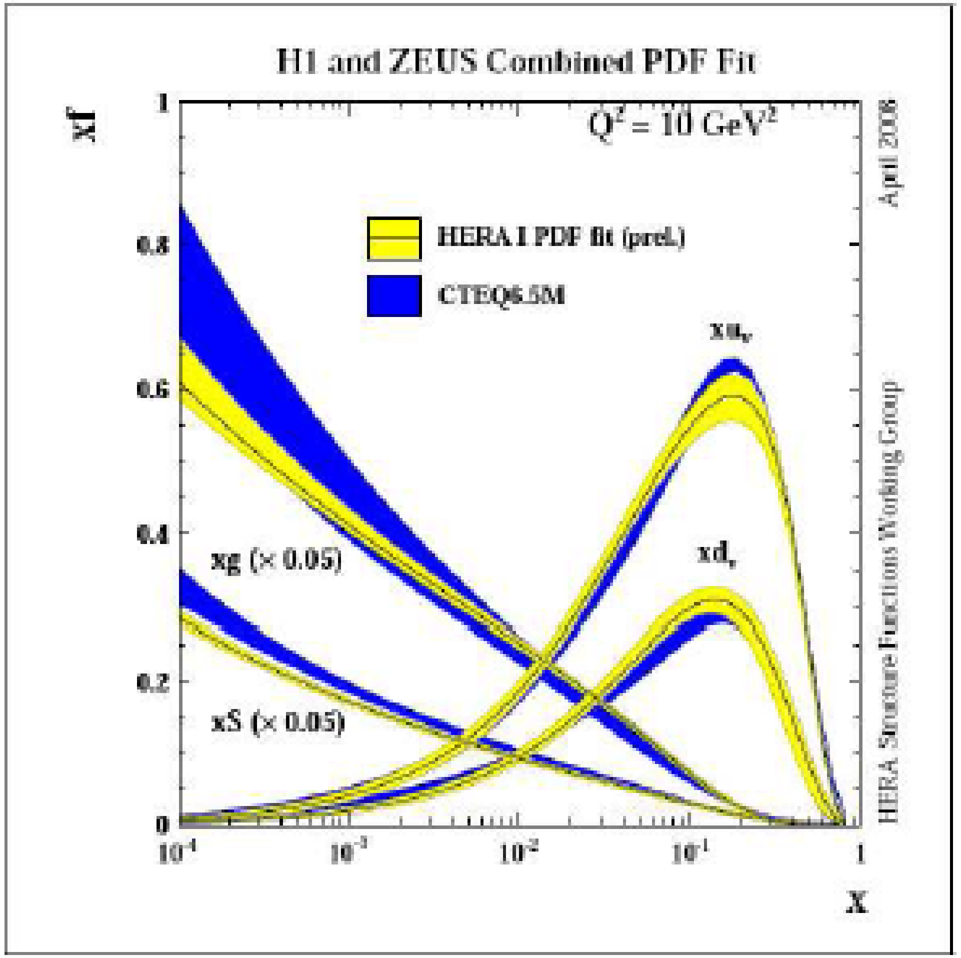
- an impressive set of QCD tests
- measurements of $q(x,Q^2)$, $g(x,Q^2)$
- measurements of $\alpha_s(Q^2)$

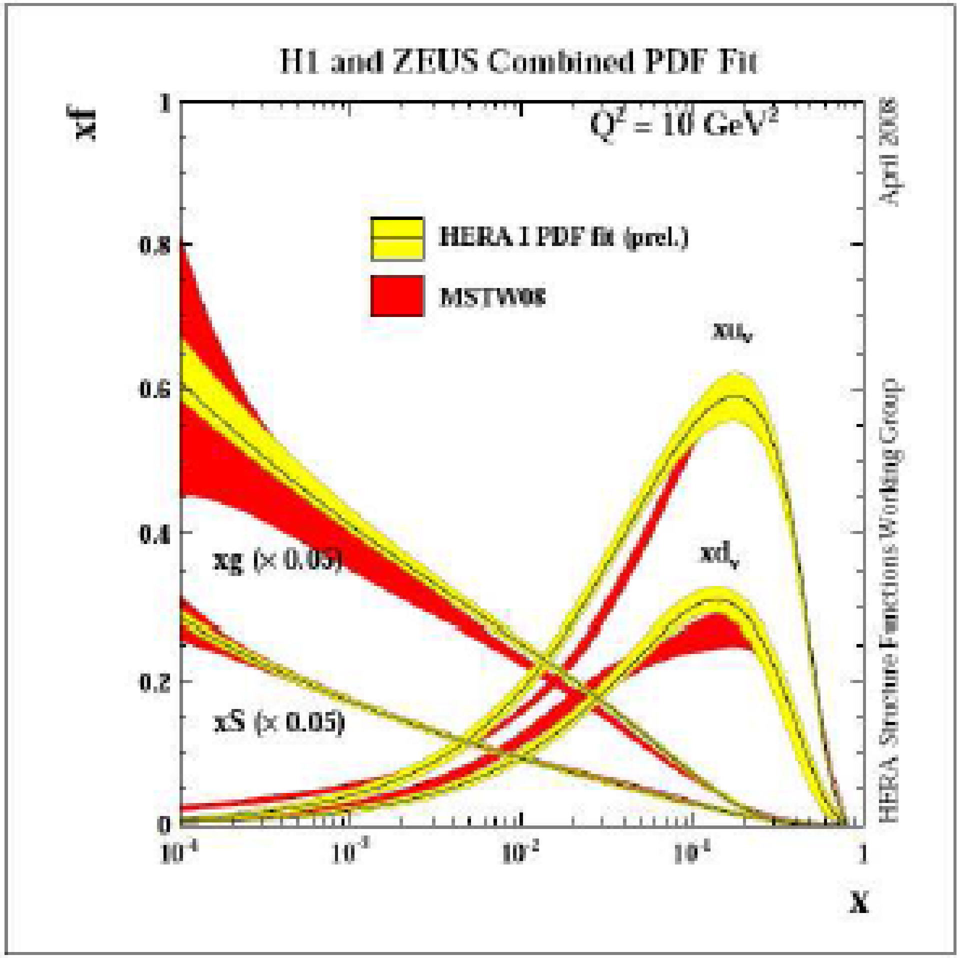
$$\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{q_iq_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2) \; + \; P_{q_ig}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{gq_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2) \; + \; P_{gg}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

GLAP

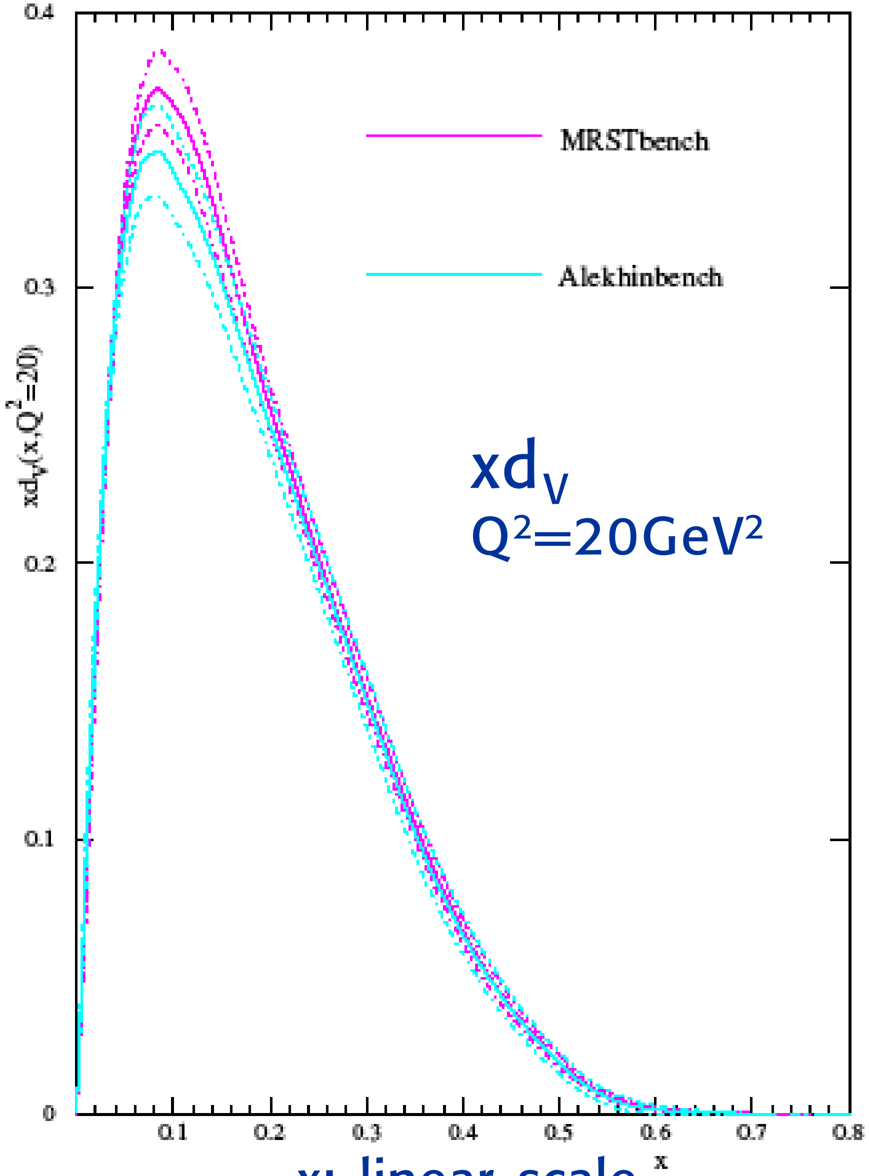
HERA is a main source of information on pdf’s for LHC





Different fits to same DIS data are comparable

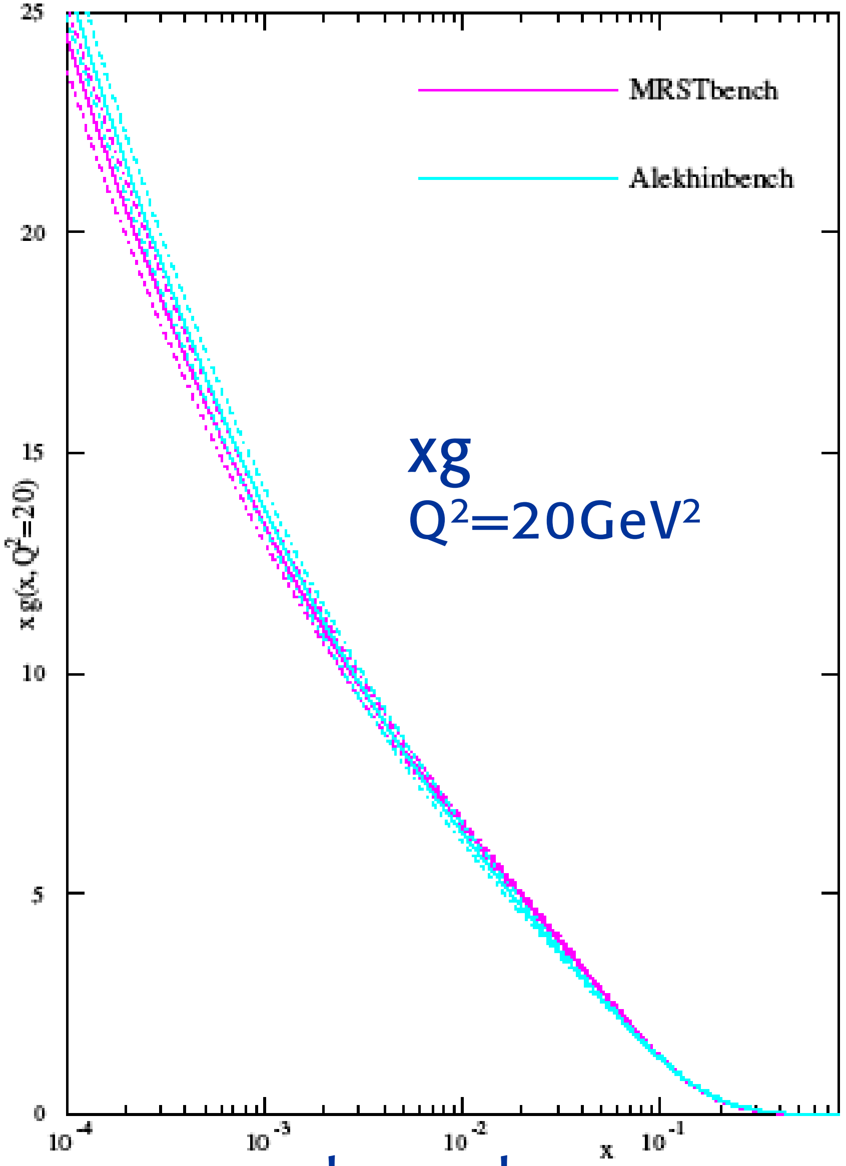
HERA LHC Workshop ‘06



xd_V

Q^2=20GeV^2

x: linear scale

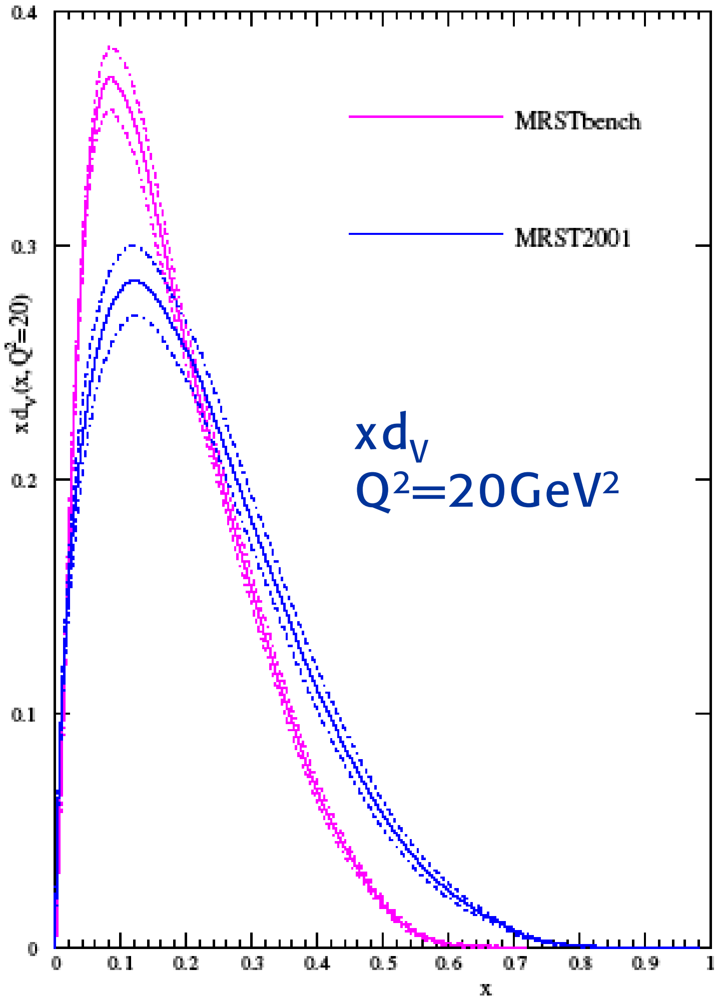


xg

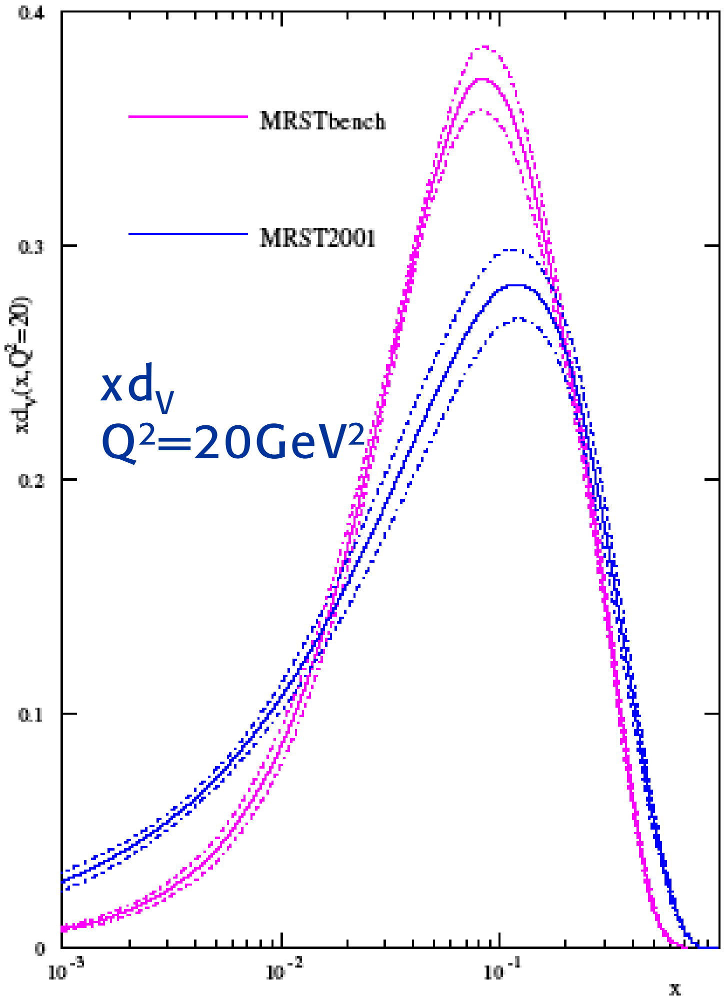
Q^2=20GeV^2

x: log scale

But differ from those obtained from all the data

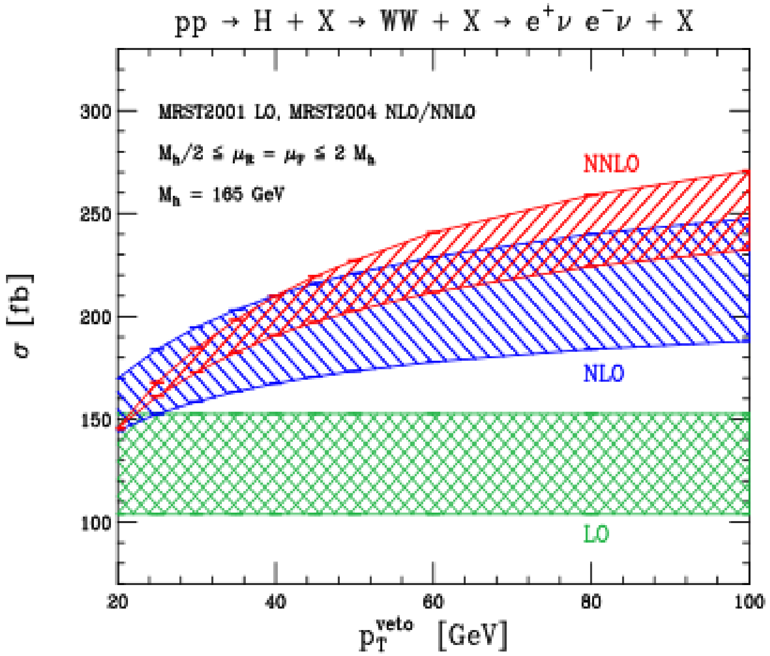


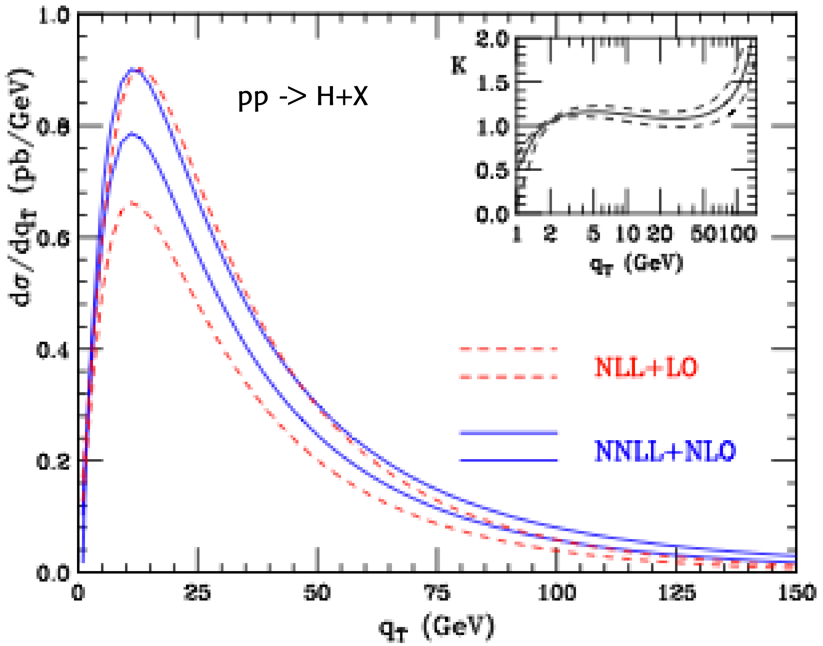
x: linear scale

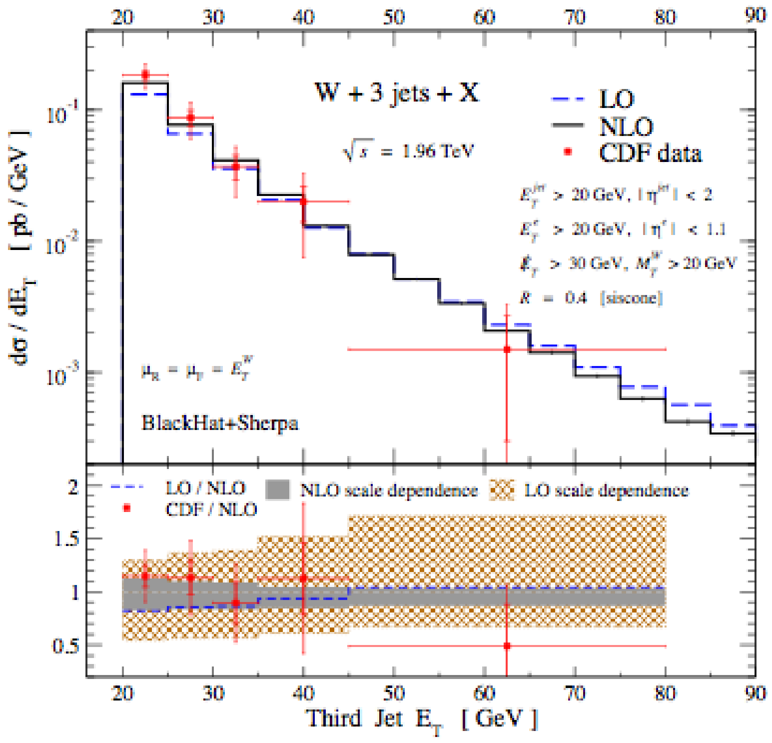


x: log scale

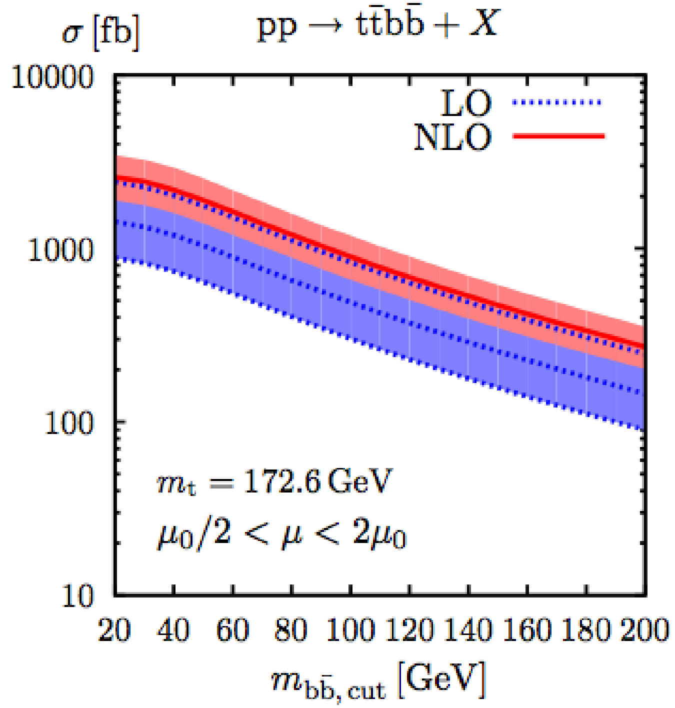
- This shows that extrapolation from one data set to another is dangerous







Fantastic technical skill!!



Essential for the LHC