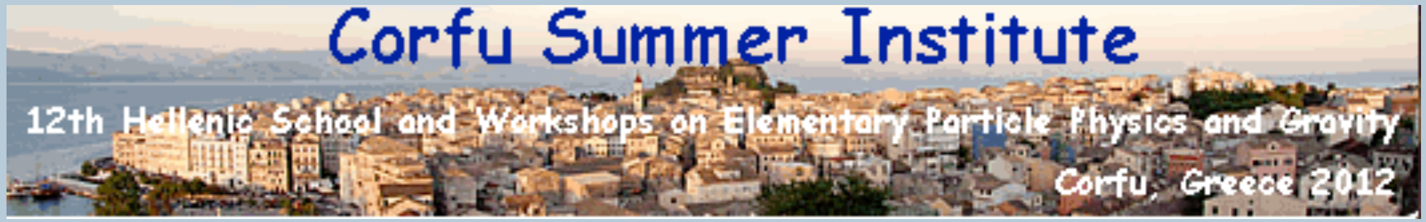
Alessandra Gnecchi

"Galileo Galilei" Physics and Astronomy Department
Padua University - Italy

DUALITY INVARIANCE FOR BLACK HOLES IN N=2 GAUGED SUPERGRAVITY

with G. Dall'Agata, JHEP 1103 (2011) 037
+ work in progress ...



September 20-26, 2012

Intro & Setup

- Supergravity with *gauging*: isometries of the scalar manifolds are made *local*
  - Scalar potential mimics a cosmological constant
  - Solutions now depend on BH charges and parameters of the gauging

$$p^\Lambda = \frac{1}{4\pi}\int_{S^2} F^\Lambda \,, \qquad q_\Lambda = \frac{1}{4\pi}\int_{S^2} G_\Lambda \,, \qquad g_\Lambda$$

→ Beyond asymptotically flat geometries, AdS_4

Cvetic et al., Duff & Liu, Sabra 1998-2003

- Existence of extremal BPS solutions with horizon geometry $AdS_2 \times S^2$ and finite horizon radius, first order equations

Cacciatori & Klemm, 2009

Motivations

and further Applications

- **Gauge/gravity duality**
  - → Near horizon metric $AdS_2 \times S^2$
  - → New solutions with Anti de Sitter asymptotics
- **AdS/CMT correspondence**
- **Possible effects on vacua destabilization?**

SETUP

Electric-Magnetic duality invariance

Gaillard-Zumino construction extends duality invariance of Maxwell field

$$F'^{\mu\nu} = (\cos\alpha + \sin\alpha\, *)F^{\mu\nu} \ , \qquad \alpha \in \mathbb{R}$$

corresponding to invariance of the equations of motion

$$\partial_\mu F^{\mu\nu} = 0$$
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

to a generic Lagrangian

$$\mathcal{L} = \mathcal{L}(F^a, \chi^i, \chi^i_\mu)$$

where the dual Field Strength to $F^a_{\mu\nu}$ is

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma} \equiv 2\frac{\partial L}{\partial F^{a\,\mu\nu}}$$

Setup

Electric-Magnetic duality invariance

Duality transformations act on the fields as:

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} ,$$
$$\delta\chi^i = \xi^i(\chi) ,$$
$$\delta(\partial_\mu \chi^i) = \partial_\mu \xi^i = \partial_\mu \chi^j \frac{\partial \xi^i}{\partial \chi^j} .$$

The most general duality group is restricted to the Symplectic Group

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in sp(2n, \mathbb{R})$$

The field strength and their duals transform as a vector

$$\mathbf{V} \equiv \begin{pmatrix} {}^\star F \\ {}^\star G \end{pmatrix} ,$$

$$\mathbf{V}' = \mathcal{S}\mathbf{V} , \qquad \mathcal{S} \in Sp(2n, \mathbb{R}) .$$

Symplectic Structure of the Action

Extended Supergravity bosonic sector

$$\mathcal{S} = \int \sqrt{-g}\; d^4x \left(-\frac{1}{2} R + \mathrm{Im}\mathcal{N}_{\Lambda\Gamma} F^{\Lambda}_{\mu\nu} F^{\Gamma,\ \mu\nu} + \frac{1}{2\sqrt{-g}} \mathrm{Re}\mathcal{N}_{\Lambda\Gamma} \epsilon^{\mu\nu\rho\sigma} F^{\Lambda}_{\mu\nu} F^{\Gamma}_{\rho\sigma} + \right.$$
$$\left. + \frac{1}{2} g_{rs}(\Phi) \partial_\mu \Phi^r \partial^\mu \Phi^s \right) \ .$$

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Einstein-Hilbert term

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Einstein-Hilbert term

Vector fields kinetic term

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$$\left. + \frac{1}{2} g_{rs}(\Phi) \partial_\mu \Phi^r \partial^\mu \Phi^s \right) \, .$$

Einstein-Hilbert term

Vector fields kinetic term

Axionic coupling

SYMPLECTIC STRUCTURE OF THE ACTION

Extended Supergravity bosonic sector

$$\mathcal{S} = \int \sqrt{-g}\; d^4x \left(-\frac{1}{2} R + \mathrm{Im}\mathcal{N}_{\Lambda\Gamma} F^{\Lambda}_{\mu\nu} F^{\Gamma,\,\mu\nu} + \frac{1}{2\sqrt{-g}} \mathrm{Re}\mathcal{N}_{\Lambda\Gamma} \epsilon^{\mu\nu\rho\sigma} F^{\Lambda}_{\mu\nu} F^{\Gamma}_{\rho\sigma} + \right.$$
$$\left. + \frac{1}{2} g_{rs}(\Phi) \partial_\mu \Phi^r \partial^\mu \Phi^s \right) \; .$$

Einstein-Hilbert term

Vector fields kinetic term

Axionic coupling

Non-linear sigma model

$$\mathcal{M} = \frac{\mathcal{G}}{\mathcal{H}}$$

N=2, U(1)-Gauged Supergravity

Momentum map procedure

[Ceresole-D'Auria-Ferrara]

Gauging isometries of the scalar manifold

Isometries are defined via a prepotential

$$k^i_\Lambda = i g^{i\bar{\jmath}} \partial_{\bar{\jmath}} \mathcal{P}_\Lambda \ , \quad \mathcal{P}^*_\Lambda = \mathcal{P}_\Lambda$$

Fayet-Iliopoulos Gauging

$$\mathcal{P}^x_\Lambda = \xi^x_\Lambda$$

$$V_{FI} = (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma)\xi^x_\Lambda \xi^x_\Sigma$$

Scalar potential mimics a cosmological constant

Scalar fields are still neutral and field strenghts abelian

N=2, U(1)-Gauged Supergravity

New, modified action

$$S = \int d^4x \left(-\frac{R}{2} + g_{i\bar{\jmath}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{\jmath}} + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Lambda\,\mu\nu} + \right.$$
$$\left. + \frac{1}{2\sqrt{-g}} \mathrm{Re}\mathcal{N}_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma}_{\rho\sigma} - V_g \right)$$

Duality covariance recovered

$$V_g = g^{i\bar{\jmath}} D_i \mathcal{L} D_{\bar{\jmath}} \mathcal{L} - 3|\mathcal{L}|^2$$

$$\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle = e^{\mathcal{K}/2} (X^\Lambda g_\Lambda - F_\Lambda \tilde{g}^\Lambda)$$

➡ analogous to the central charge that defines the black hole potential

$$\mathcal{Z} \equiv \langle Q, \mathcal{V} \rangle \qquad V_{BH} = |D\mathcal{Z}|^2 + |\mathcal{Z}|^2$$

Second Order equations

Metric ansatz

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left[dr^2 + e^{2\psi(r)}(d\theta^2 + \sin\theta^2 d\phi^2)\right]$$

New conformal parameter:

$$\tau(r) = -\int_r^\infty \exp(-2\psi(s))\; ds$$

From the variation of the action:

$$\begin{aligned} -\frac{d^2}{d\tau^2}U(\tau) &= e^{2U}V_{BH}(p^\Lambda, q_\Lambda, z, \bar{z}) \;+ e^{4\psi-2U}V_g(z,\bar{z}) \\ \frac{d^2}{d\tau^2}\psi(\tau) &= e^{2\psi} - 2e^{4\psi-2U}V_g(z,\bar{z}) \\ e^{-U}\frac{d^2}{d\tau^2}e^{U(\tau)} - e^{-\psi}\frac{d^2}{d\tau^2}e^{\psi(\tau)} &= g_{i\bar{\jmath}}\partial_\tau z^i \partial_\tau \bar{z}^{\bar{\jmath}} \end{aligned}$$

Cfr Breitelhoner-Maison-Gibbons: General base space

Static black holes
metric ansatz

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}(dr^2 + e^{2\psi(r)}d\Omega^2)$$

$AdS_2 \times S^2$ ⟷ AdS_4

$\partial_i \mathcal{W} = 0$ $\qquad$ $D_i \mathcal{L} = 0$

Double attractor condition

- More constraints on the fields
- Less solutions less easily found

Duality Covariant BPS flow

Fields equations of motion

$$
\begin{aligned}
U' &= -e^{U-2\psi}\mathrm{Re}(e^{-i\alpha}\mathcal{Z}) + e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\
\psi' &= 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L}) \\
\dot{z}^i &= -e^{i\alpha}g^{i\bar{\jmath}}(e^{U-2\psi}\bar{D}_{\bar{\jmath}}\bar{\mathcal{Z}} + ie^{-U}\bar{D}_{\bar{\jmath}}\bar{\mathcal{L}}) \\
\alpha' + \mathcal{A}_r &= -2e^{-U}\mathrm{Re}(e^{-i\alpha}\mathcal{L})
\end{aligned}
$$

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\alpha' + \mathcal{A}_r &= -2e^{-U}\mathrm{Re}(e^{-i\alpha}\mathcal{L})
\end{aligned}
$$

- Identify the phase from $e^{2U-2\psi}\mathrm{Im}(e^{-i\alpha}\mathcal{Z}) = \mathrm{Re}(e^{-i\alpha}\mathcal{L})$

Duality Covariant BPS flow

Fully symplectic vector of gauge couplings $\mathcal{G} = (g^\Lambda, g_\Lambda)$

$$\mathcal{L} = \langle \mathcal{G} \,, \mathcal{V} \rangle \qquad \mathcal{Z} \equiv \langle Q, \mathcal{V} \rangle$$

BPS first order flow driven by the superpotential

$$\mathcal{W} = e^U |\mathcal{Z} - i e^{2(\psi - U)} \mathcal{L}|$$

BPS constraints ~ Charge quantization condition

$$\langle \mathcal{G}, Q \rangle + 1 = 0$$

cfr. Romans, '92

1/4 - BPS condition

$$\begin{aligned} \gamma^0 \epsilon_A &= i\, e^{i\alpha}\, \varepsilon_{AB} \epsilon^B \\ \gamma^1 \epsilon_A &= e^{i\alpha}\, \delta_{AB} \epsilon^B \end{aligned}$$

Attractor Equations

At the attractor point $\partial_i \mathcal{W}|_h = 0 \, , \qquad \mathcal{W}|_h = 0$

$$Q + e^{2A}\,\Omega\mathcal{M}\mathcal{G} = -2\mathrm{Im}(\overline{\mathcal{Z}}\mathcal{V}) + 2\,e^{2A}\,\mathrm{Re}(\overline{\mathcal{L}}\mathcal{V})$$

$$e^{2A} = -i\,\frac{\mathcal{Z}}{\mathcal{L}} = R_S^2$$

Any geometric interpretation?

Interesting relation for the black hole entropy

$$e^{-2A} = 2\left(|D_i\mathcal{L}|^2 - |\mathcal{L}|^2\right)$$

$$\rightarrow \quad e^{2A} = 2\left(|D_i\mathcal{Z}|^2 - |\mathcal{Z}|^2\right)$$

Black Hole Solutions

One modulus case

- Quadratic model $F = iX^0X^1$

  with Kähler metric $K = -\log 2(z+\bar{z})$ → $\mathrm{Re} z > 0$

  - AdS vacuum fixes the asymptotic modulus at

$$z = \frac{g_0 g_1 + g^0 g^1 + i\left(g_0 g^0 - g_1 g^1\right)}{(g_1)^2 + (g^0)^2}$$

  - Attractor equations are

$$I_2(\mathcal{G}) = |\mathcal{G}|^2 - |D_i\mathcal{G}|^2 = g_0 g_1 + g^0 g^1$$

$$e^{-2A} = -I_2(\mathcal{G})$$

thus requiring → $g_0 g_1 + g^0 g^1 < 0$

Black Hole Solutions

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$$e^{-2A} = -I_2(\mathcal{G})$$

thus requiring $\longrightarrow$ $g_0 g_1 + g^0 g^1 < 0$

Inconsistent!!

Black Hole Solutions

The stu model

$$F = -i\sqrt{X^0 X^1 X^2 X^3}$$

- Regular solutions with spherical horizon in STU model for *magnetic* charges

[Cacciatori-Klemm 0911.4926]

→ Exploit duality covariant formulation

$$\mathcal{V}_{CK} = e^{K/2}\,(1, -tu, -su, -st, -stu, s, t, u)^T$$

$$\mathcal{V} = e^{K/2}\,(1, s, t, u, -stu, tu, su, st)^T$$

$$S = \begin{pmatrix} 1 & & & & & & & \\ & & & & & -1 & & \\ & & & & & & -1 & \\ & & & & & & & -1 \\ & & & & 1 & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \end{pmatrix}$$

$$\mathcal{V}_{CK} = S\mathcal{V}$$

$$\mathcal{G} = S^{-1}\mathcal{G}_{CK}$$

$$\mathcal{Q} = S^{-1}\mathcal{Q}_{CK}$$

Entropy of The Solution

The stu model

$$e^{2A} = -i\,\frac{\mathcal{Z}_h}{\mathcal{L}_h} = R_S^2$$

thus the entropy is proportional to $S \sim \frac{\mathcal{Z}_h}{\mathcal{L}_h}$

The explicit expression has the form

$$e^{2A} = \frac{1}{4}\sqrt{\frac{1+2(1-4gq)\sqrt{1-16gq+48g^2q^2}-3(1-4gq)^2}{g_0 g^3}}$$

Integer values??

Different attempts

- Apply ADM prescription [Abbott, Deser, 1982]

  Need for a reference background

  [Brown, York, 1993]

- Holographic renormalization [Liu, Sabra et al., 2005]

  Weird dependence on the gauging charges

- BPS algebra [Hristov, Toldo, Vandoren, 2011]

  Only valid when a Killing spinor exists

Anti De Sitter coordinates

$$-X_0^2 - X_5^2 + X_1^2 + X_2^2 + X_3^2 = -R^2$$

- **Extensions to multicenter BHS?**

Is it possible to have a parametrization with a flat 3d slicing???

Global coordinates

$$\frac{ds^2}{R^2} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2$$

Cosmological coordinates

$$\frac{ds^2}{R^2} = -d\tau^2 + \cos^2\tau(d\rho^2 + \sinh^2\rho \; d\Omega^2)$$

More insights...

- **First order equations for rotating solutions**

introduce $\mathbf{G} = \mathcal{G} e^i dx^i \qquad \mathbf{L} \equiv \langle \mathbf{G}, \mathcal{V} \rangle$

$$\frac{1}{2} e^{V-U} \star_0 d\omega + \langle \mathcal{I}, d\mathcal{I} \rangle - e^{-2U} \mathrm{Re}(e^{-i\alpha - V} \mathbf{L}) = 0$$

→ $\mathcal{P}_I^x = \xi_I e^x \delta_x^I$ SU(2)~SO(3)

More insights...

- **First order equations for rotating solutions**

introduce $\mathbf{G} = \mathcal{G}e^i dx^i \qquad \mathbf{L} \equiv \langle \mathbf{G}, \mathcal{V} \rangle$

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..yet to come!

Thank You