Holographic Thermalization

of Mutual and Tripartite Information in 2d CFTs

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Based on: arXiv 1110.0488

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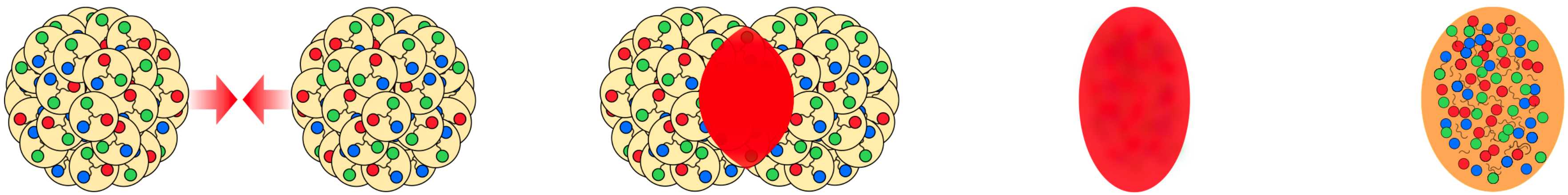
Motivations

Heavy-ion collisions at RHIC and LHC:



ultra relativistic heavy ions | collision | out-equilibrium QGP | locally equilibrated QGP

- Deconfined phase of QCD: quark gluon plasma (QGP)
- Rapid thermalization (≤ 1 fm/c) followed by an almost ideal hydrodynamic regime
- Strongly coupled QGP

Holographic thermalization

Difficult to describe with conventional methods
(perturbative and lattice QCD...)

Holographic approach: AdS/CFT correspondence

Equilibrated QGP $\longleftrightarrow$ AdS black hole

Thermalization $\longleftrightarrow$ Black hole formation in AdS

Probes

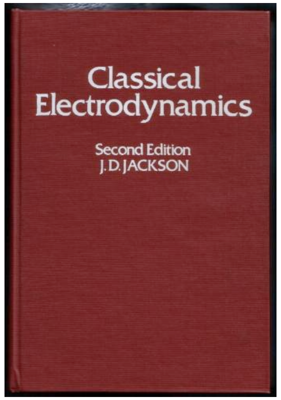
Approach to thermality: non-local probes

- two-point functions $\langle \mathcal{O}\mathcal{O} \rangle_{vacuum} \longrightarrow \langle \mathcal{O}\mathcal{O} \rangle_{thermal}$ (see next talk)
- mutual information
- tripartite information

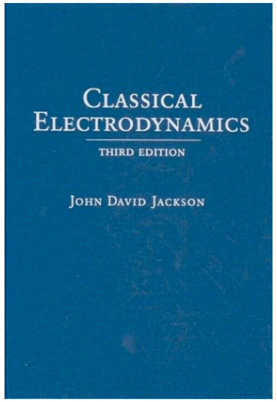
$\longrightarrow$ comparison with FT results

Mutual information $I(A,B)$

Measures how much information A and B share: what we learn about A by looking at B



A



B

Probes

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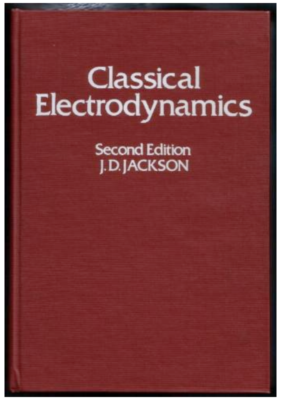

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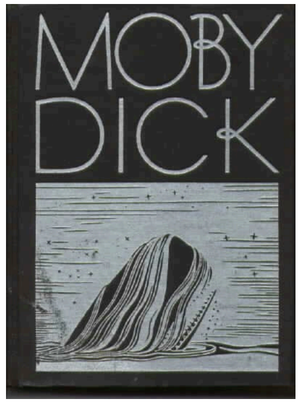


B

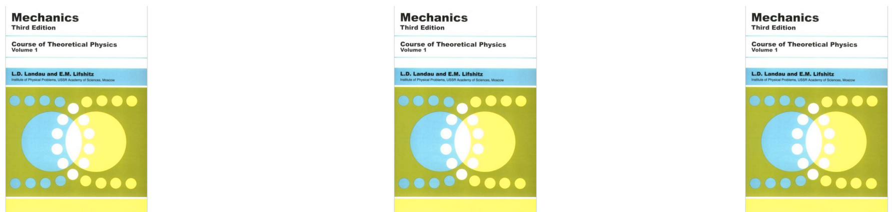
Probes

Tripartite information

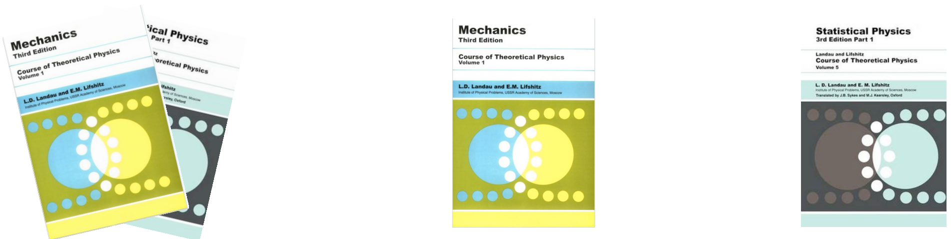
$$I_3(A,B,C) = I(A,B) + I(A,C) - I(A,B \cup C)$$

What you learn about A looking at B and C separately, with respect to what you learn looking at B ∪ C

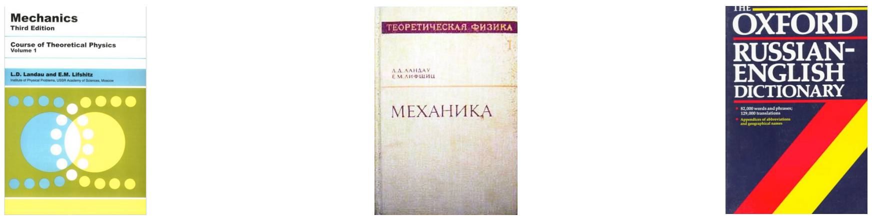
A B C



$I_3 > 0$



$I_3 = 0$



$I_3 < 0$

Setup: thin infalling shell

Homogeneous injection of energy in 2d QFT at $t = 0$	$\longleftrightarrow$	Infalling shell of null dust in 3d AdS spacetime

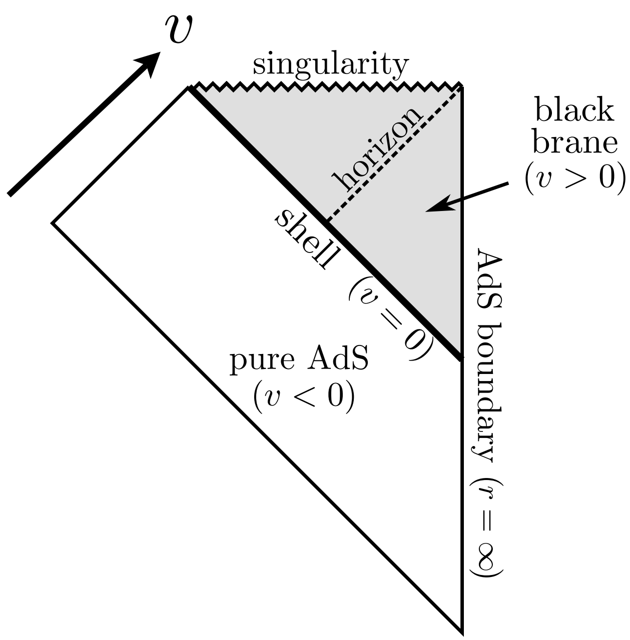
[Hubeny, Rangamani, Takayanagi 2007]
[Lin, Shuryak 2008]
[Bhattacharyya, Minwalla 2009]

AdS-Vaidya geometry

$$ds^2 = -[r^2 - r_H^2\theta(v)]dv^2 + 2drdv + r^2dx^2$$

Thin shell limit

- $v < 0$ pure AdS
- $v > 0$ black brane with $T_H = \dfrac{r_H}{2\pi}$



Probes of thermalization

Entanglement entropy

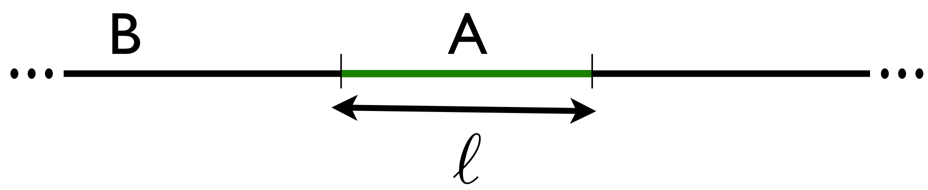
t_0

Total system: ρ

Reduced density matrix: $\rho_A = \mathrm{Tr}_B(\rho)$

$$\boxed{S(A) = -\mathrm{Tr}_A(\rho_A \log \rho_A)}$$

Measures to what extent the d.o.f. in A are entangled with those in B



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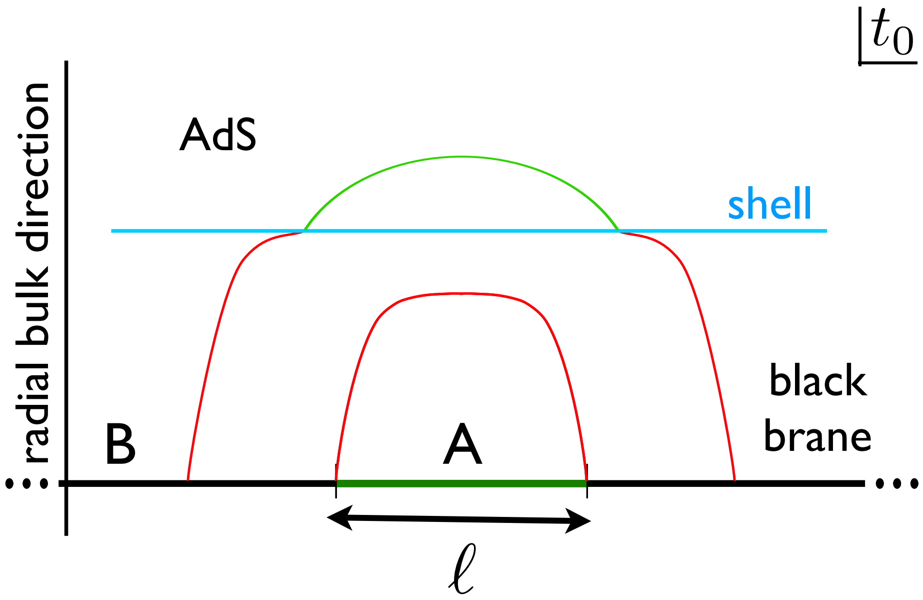


Holographic proposal

[Ryu,Takayanagi 2006]
[Hubeny,Rangamani,Takayanagi 2007]

$$\boxed{S(A) = \frac{\mathrm{Length}(\gamma_A)}{4G_N}}$$

γ_A geodesic connecting endpoints of A

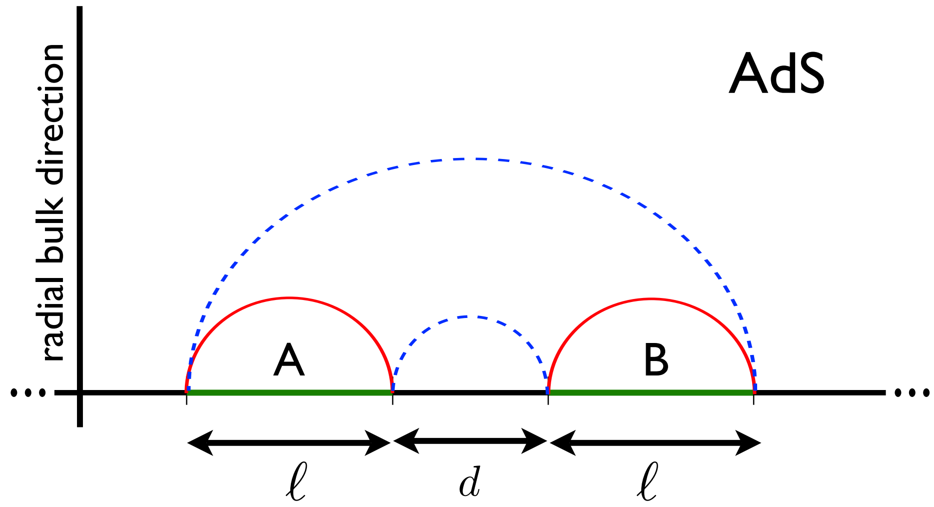
Probes of thermalization

Mutual information

$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

- Measure of the amount of correlation between A and B
- $I(A,B) \geq 0$ with $I(A,B) = 0$ iff no correlation ($\rho = \rho_A \otimes \rho_B$)

• $S(A \cup B)$: collection of geodesics with minimal length, joining the endpoints of A and B



Probes of thermalization

Tripartite information

$$I_3(A,B,C) = I(A,B) + I(A,C) - I(A,B\cup C)$$

- What you learn about A looking at B and C separately, with respect to what you learn looking at B ∪ C
- $I_3(A,B,C)\begin{cases} >0 \\ =0 \\ <0 \end{cases}$

• Perturbative QFT: $I_3(A,B,C) \geq 0$ [Balasubramanian,McDermott,VanRaamsdonk 2011]

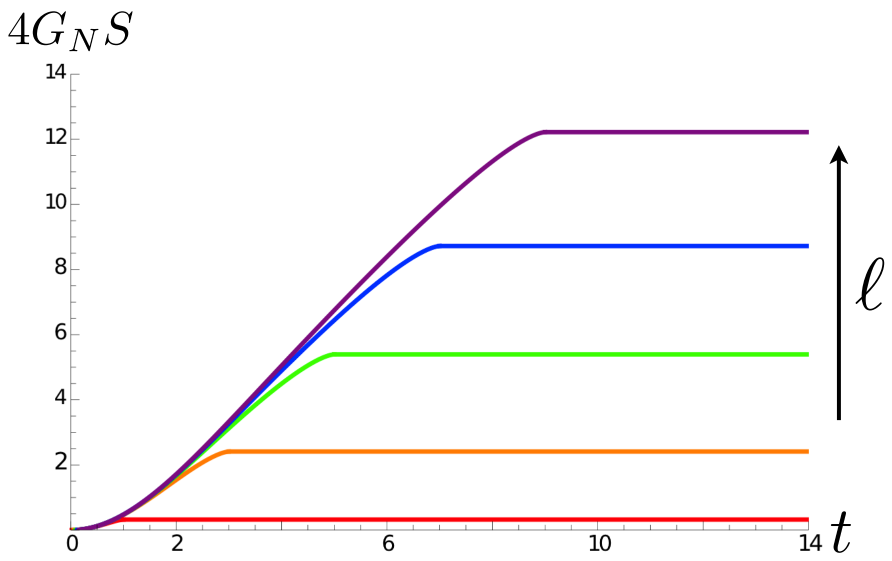
• Holographic static setup: $I_3(A,B,C) \leq 0$ [Hayden,Headrick,Maloney 2011]

Mutual information is “monogamous”: the amount of information that can be shared is bounded $I(A,B) + I(A,C) \leq I(A,B\cup C)$

Entanglement entropy

[Abajo-Arrastia,Aparicio, Lopez 2010]

[Balasubramanian,Bernamonti,deBoer et al. 2010-11]



- Almost linear growth from vacuum to thermal value
- Thermalization time $t_{th} = \dfrac{\ell}{2}$
- Top-down thermalization: thermalization proceeds from UV to IR

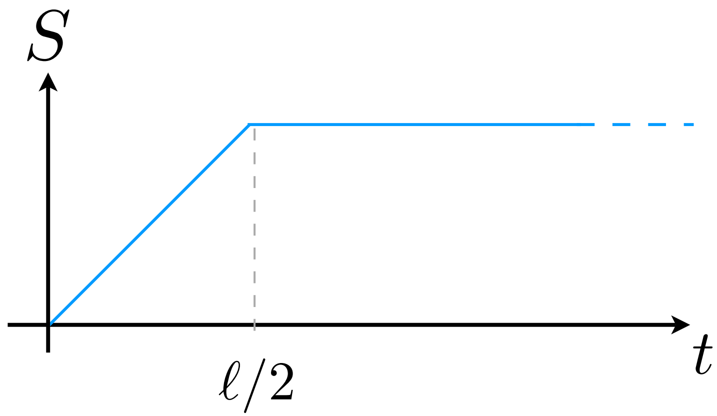
Causality argument reproduces the intermediate linear growth and explains the thermalization time

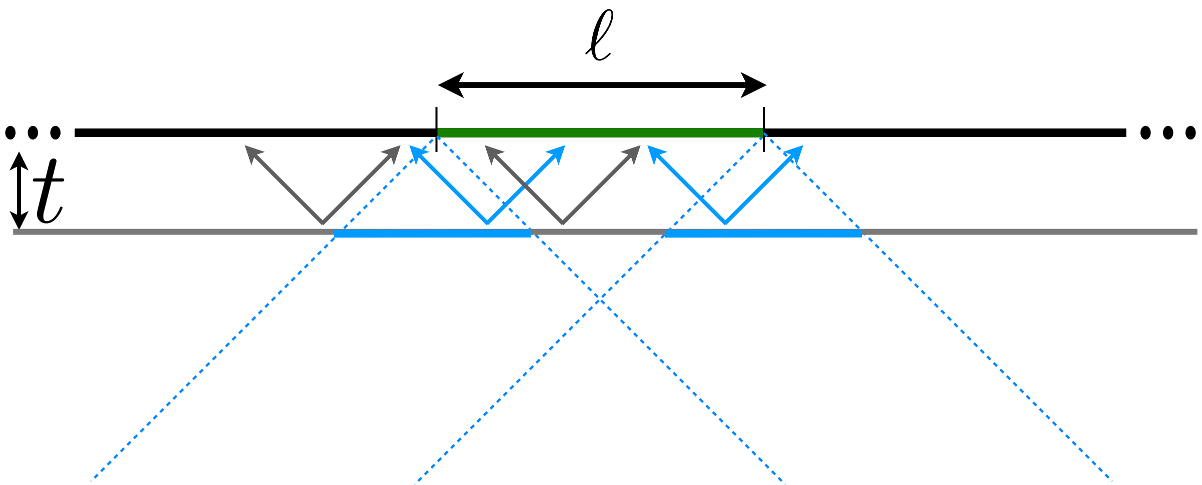
Entanglement entropy

Global quench to a CFT in 2d

[Calabrese,Cardy 2005,2009]

- t<0 mass gap: short range correlation
- t =0 quench to CFT: mass gap removed
- t>0 CFT in excited state, with short range correlation





sea of quasi particle excitations with short range correlation

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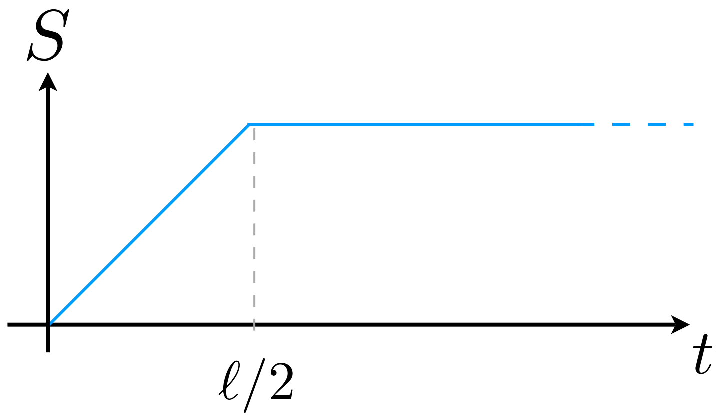

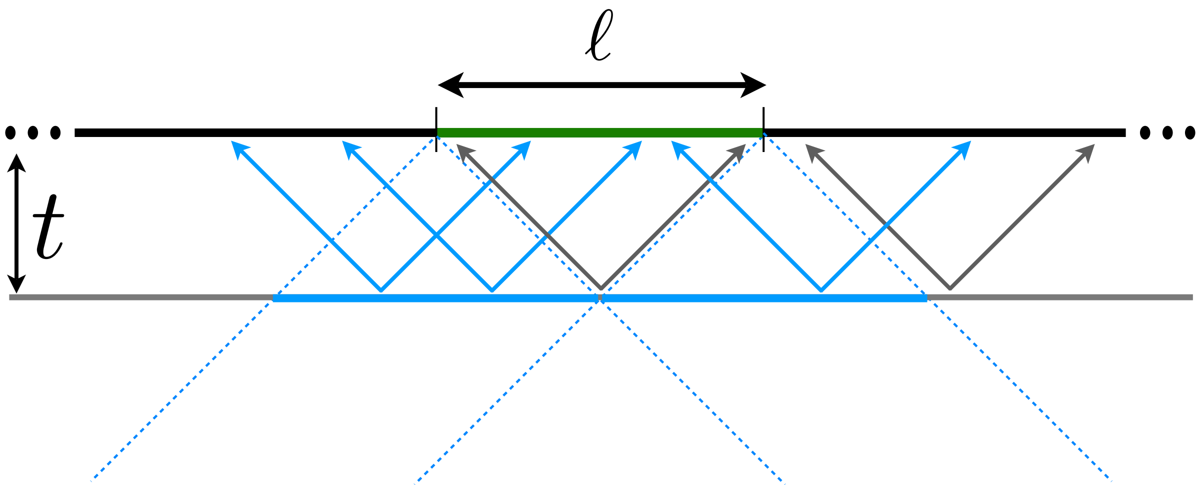

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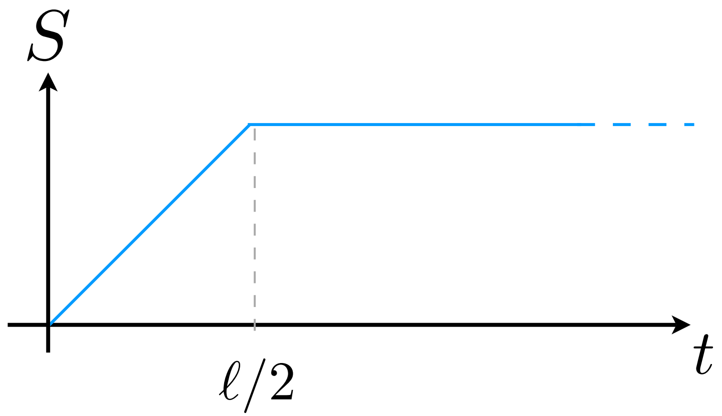

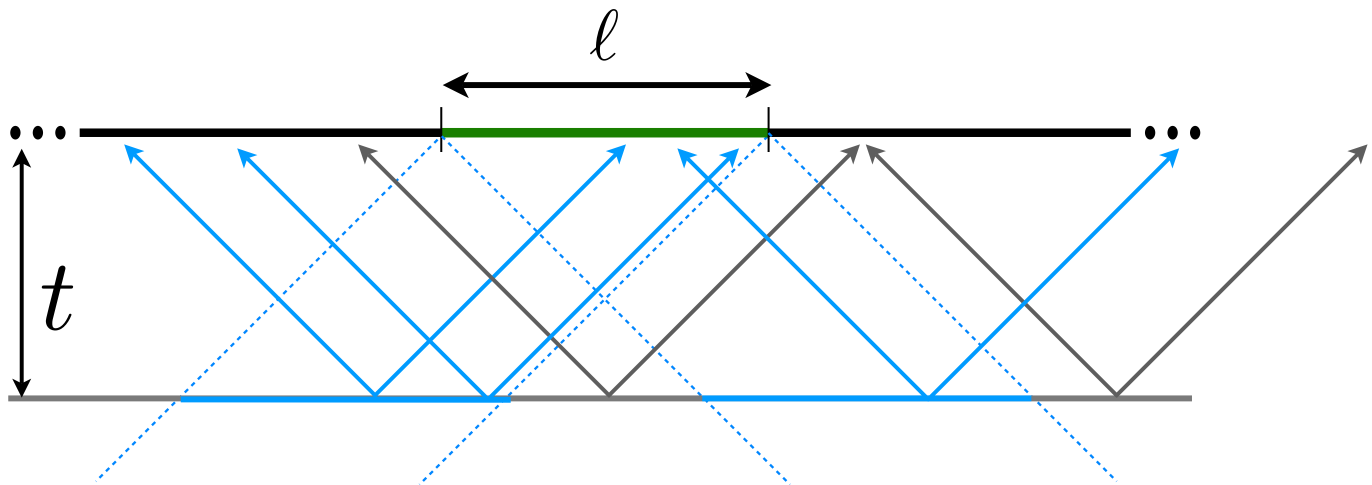

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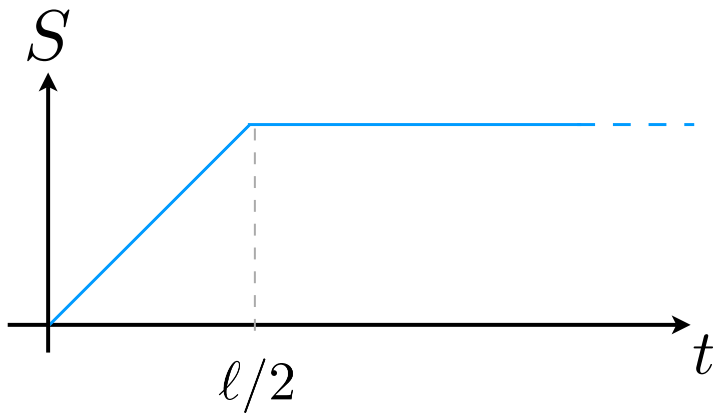
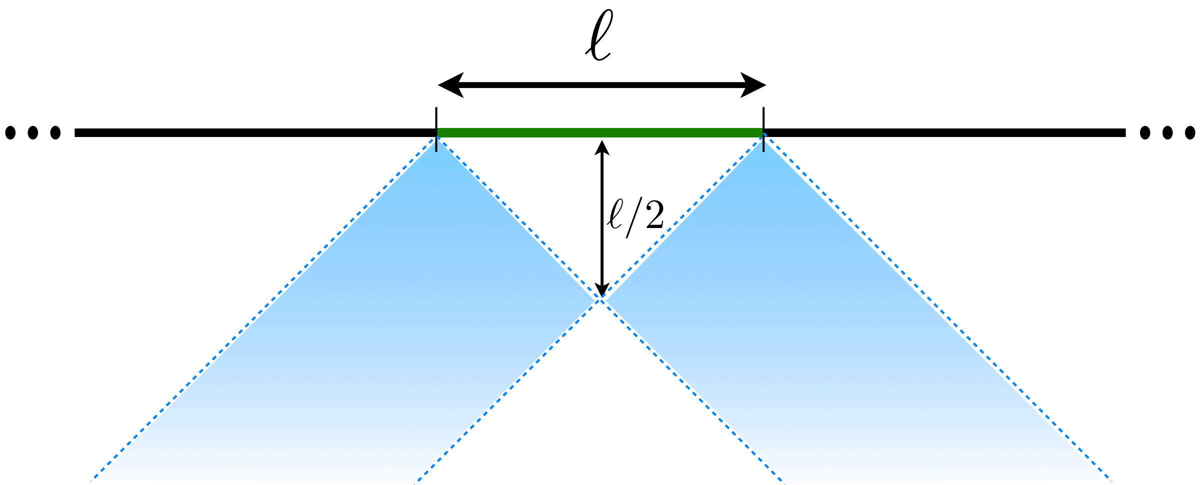
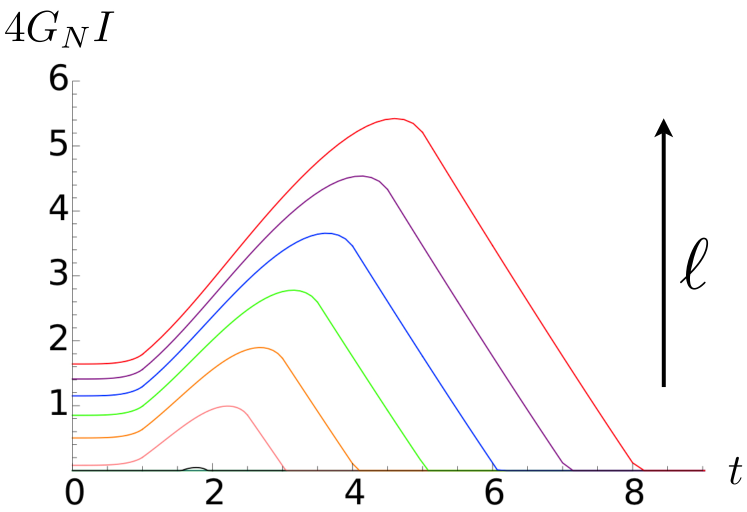




Mutual information

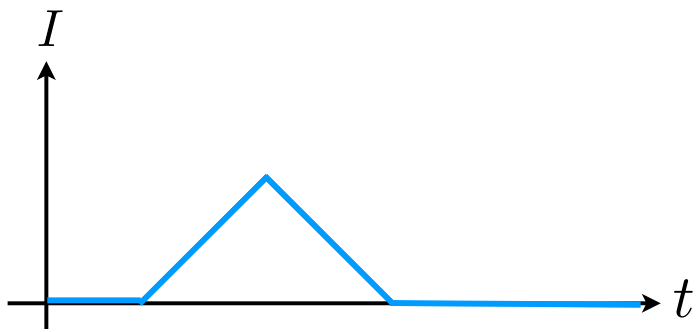
[Balasubramanian,Bernamonti, Copland,Craps,FG 2011]

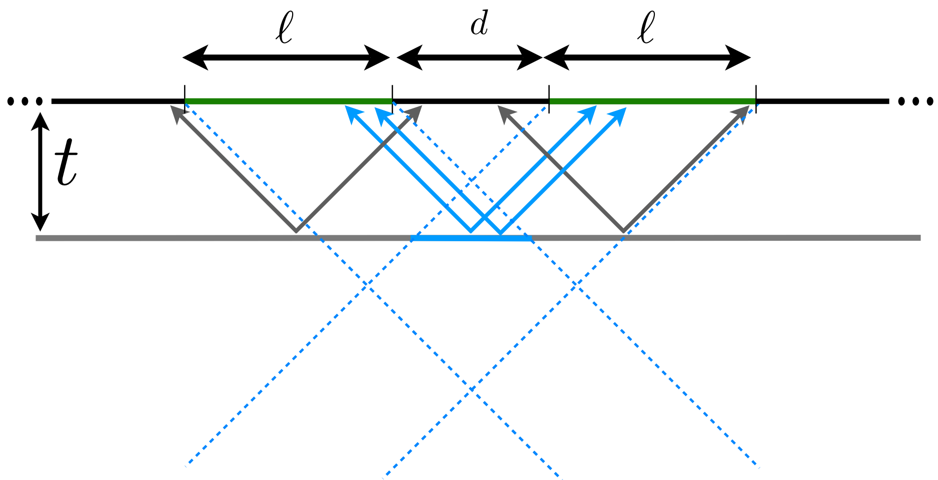


- Equal length intervals, separated by a distance d

- Sharp peak at intermediate times

Same causality argument explains the main features





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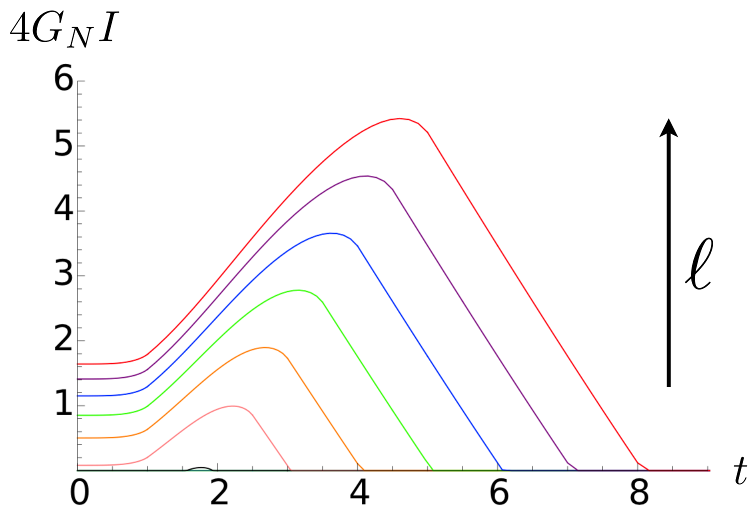

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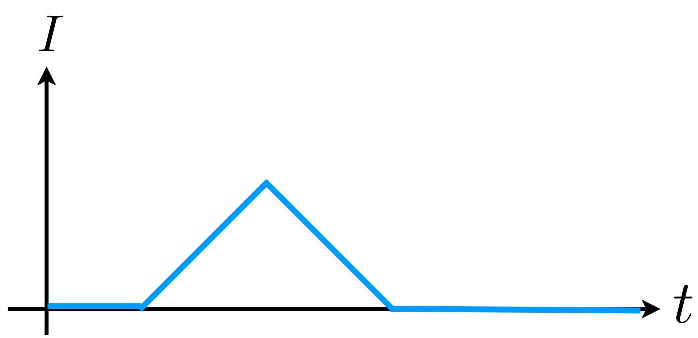

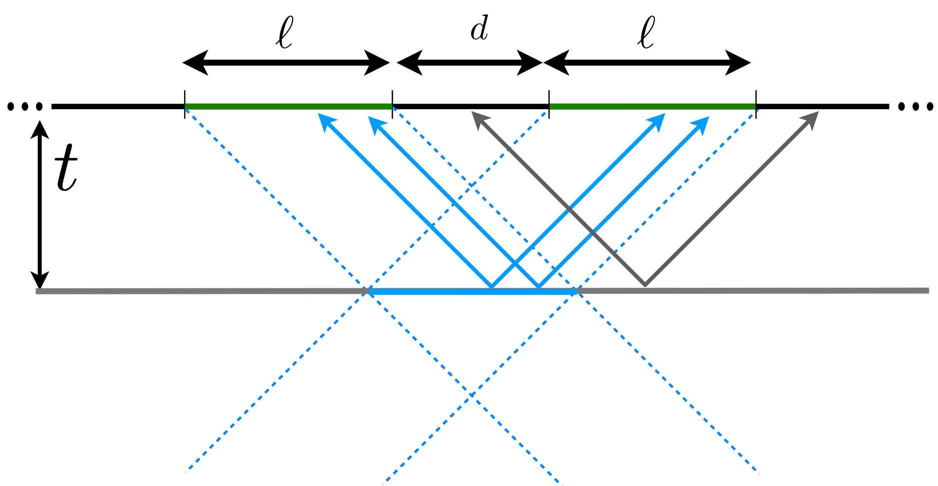

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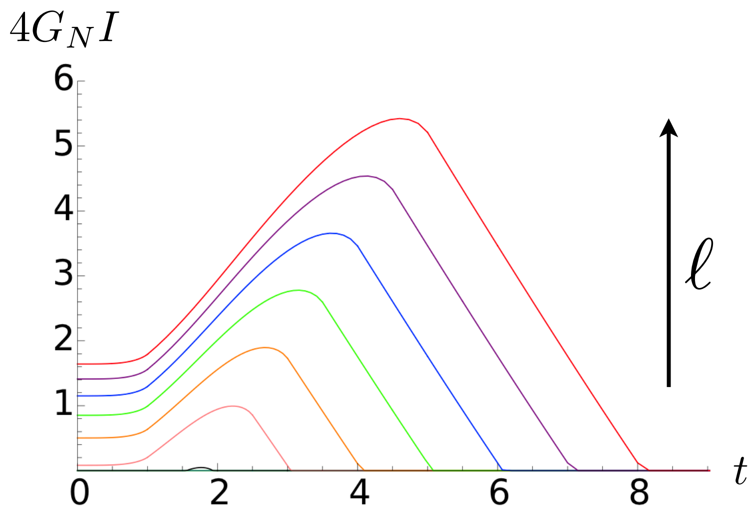

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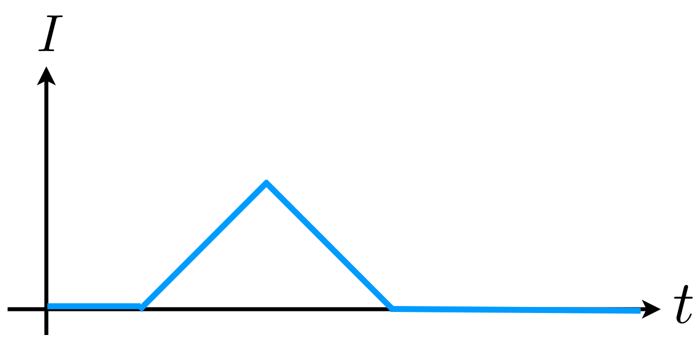

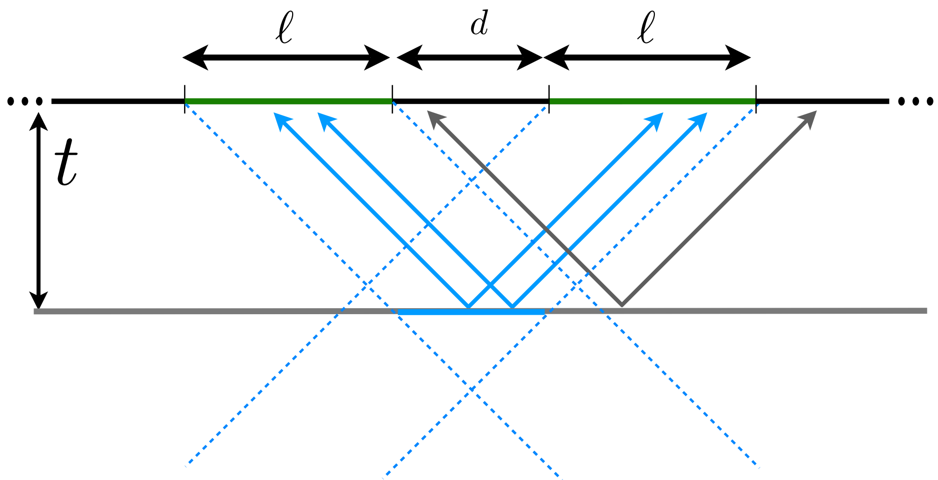

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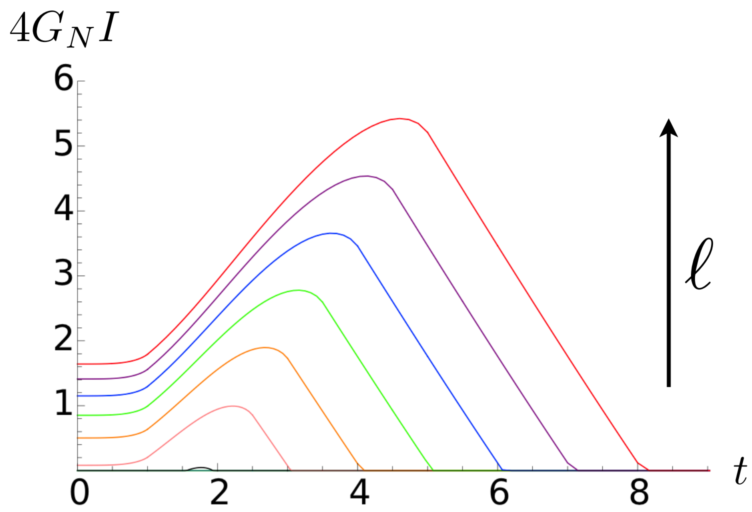

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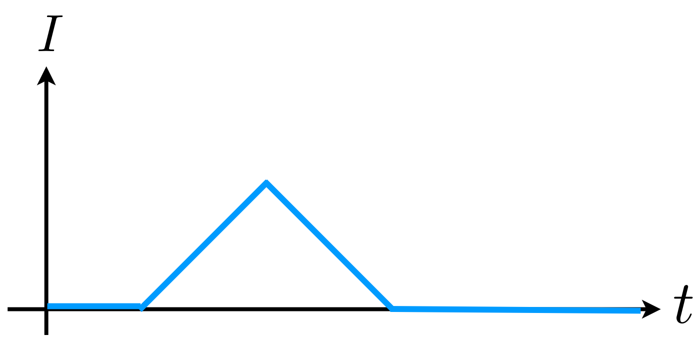

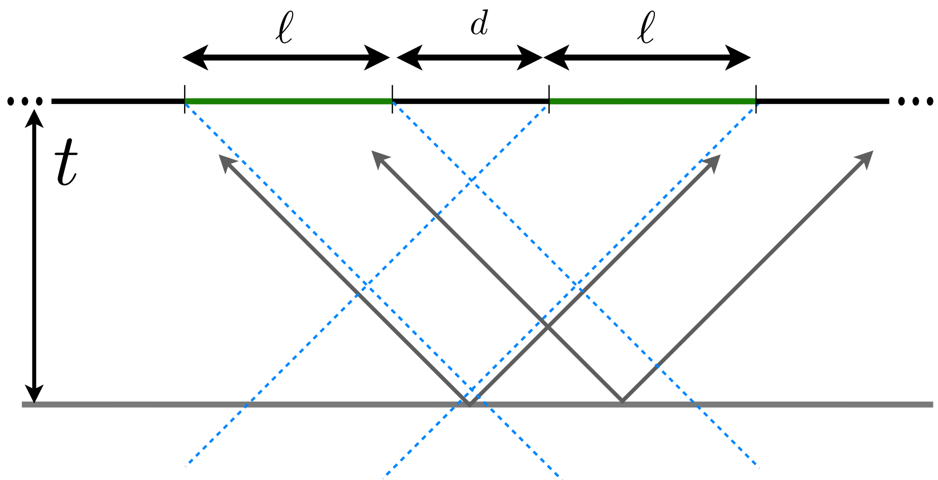

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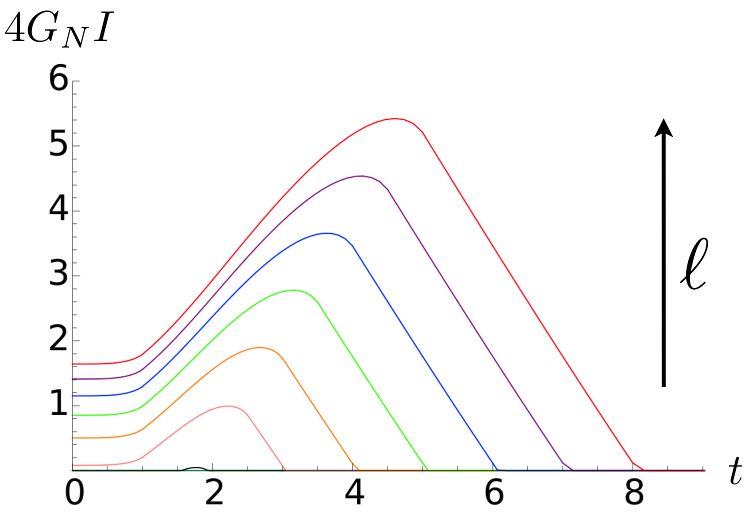

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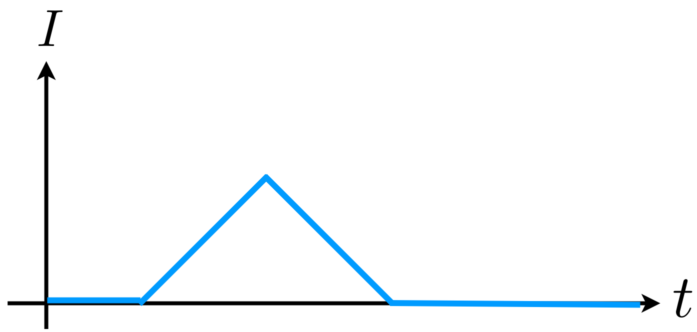

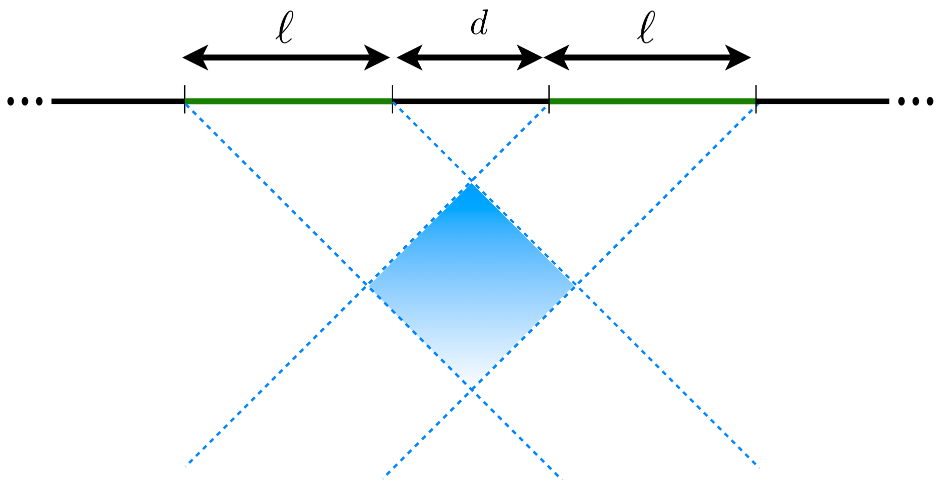

Tripartite information

[Balasubramanian,Bernamonti,
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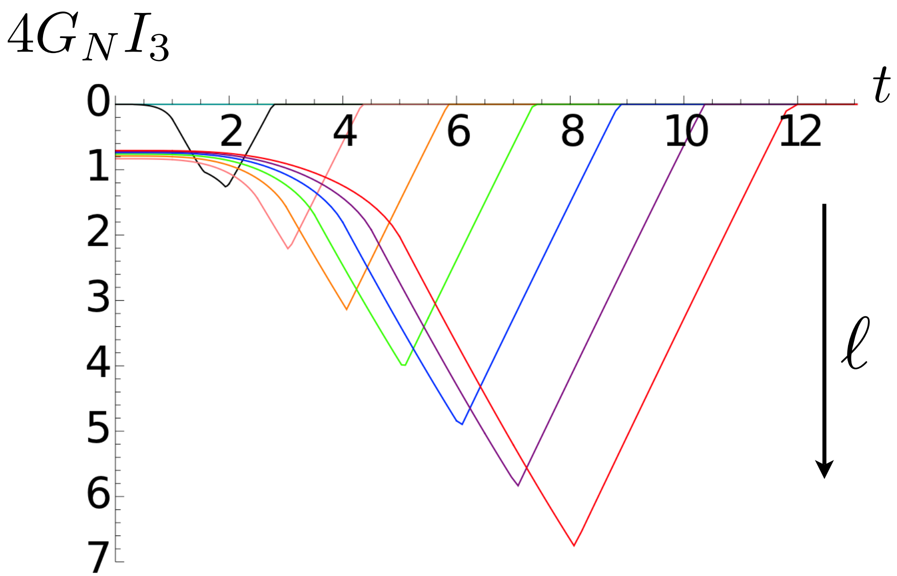
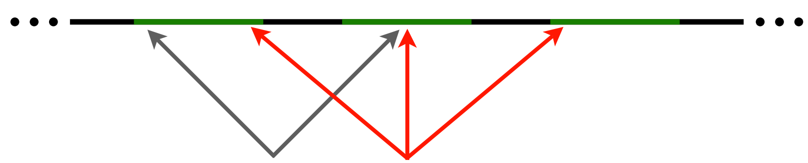


- Time dependent $I_3 \leq 0$ out-of-equilibrium at strong coupling
- Monogamy as in static setup (vs. perturbative QFT)

Causality argument: I_3 constant in time

➜ need to include interactions to populate three intervals



Summary & Outlook

- Toy model for thermalization: thin infalling shell
- Out-of-equilibrium mutual and tripartite information for strongly coupled 2d CFT
- Causality argument captures the evolution of the entanglement entropy and of the mutual information
- Time dependent tripartite information and monogamy of the mutual information

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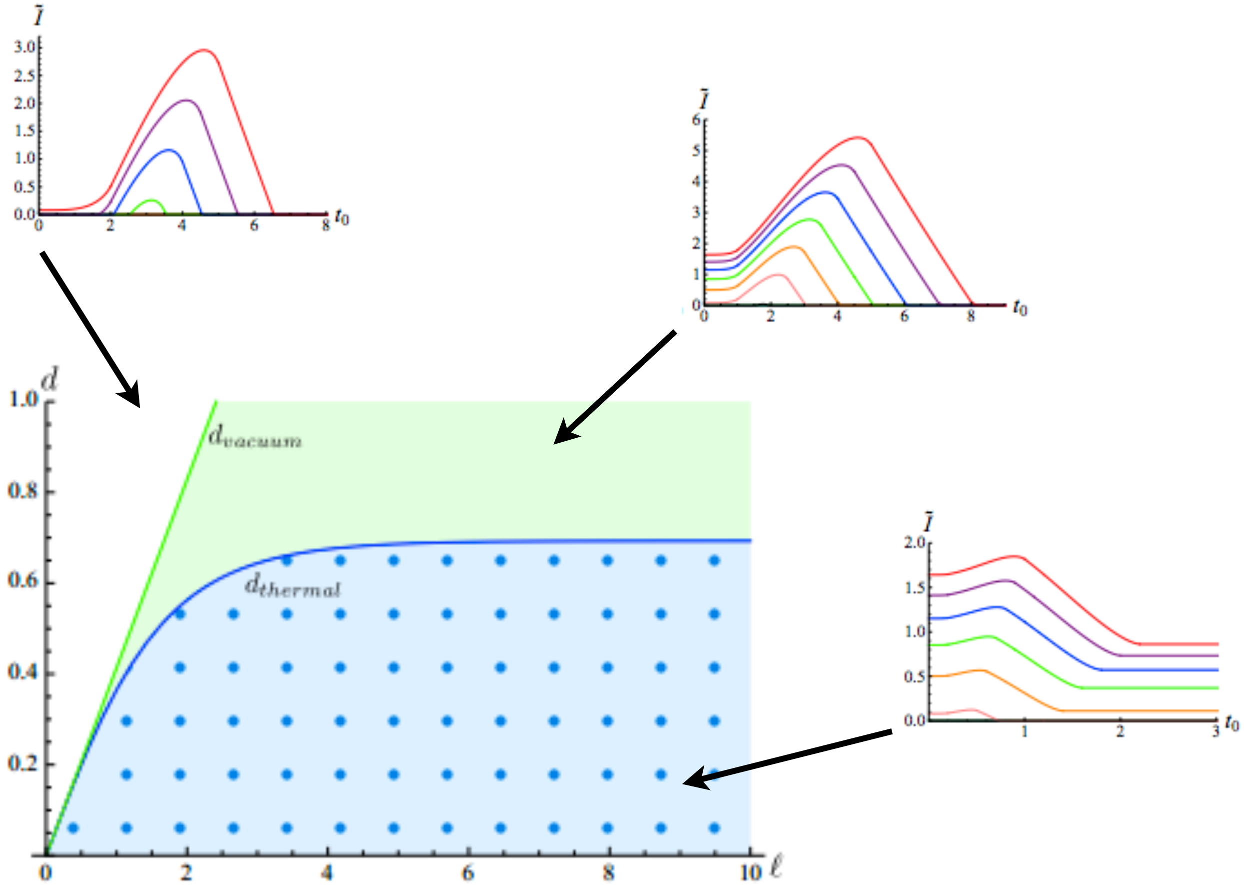
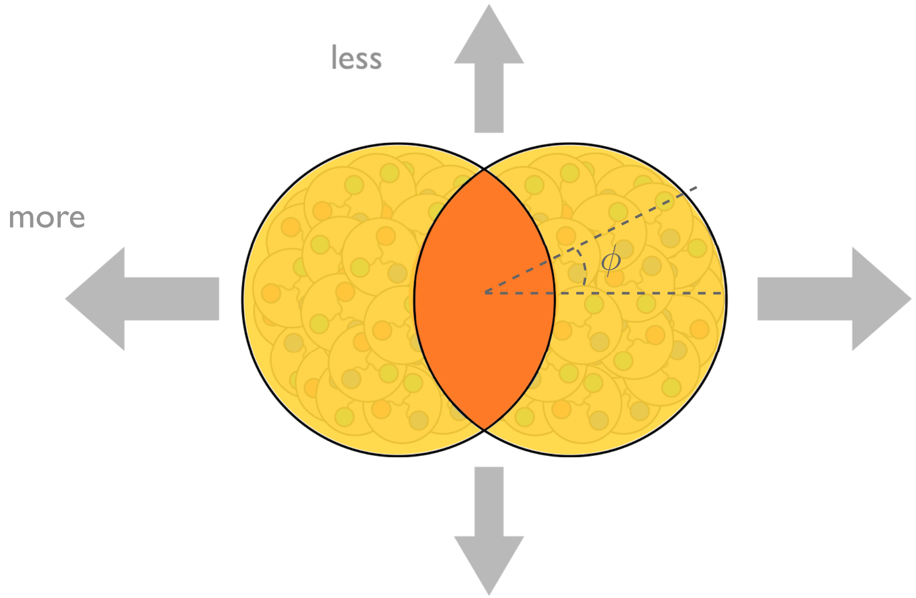
- Better understand mutual and tripartite information
- More realistic holographic model: include inhomogeneities (work in progress)

Thank you!

Phases of mutual information for two disjoint intervals of equal length



Collective flow of hadrons



asymmetry in
hadrons multiplicity

$$\frac{dN}{d^2p_T} = \frac{dN}{\pi dp_T^2}\left(1 + \sum_{n=1} v_n \cos(n\phi)\right)$$

Elliptic flow v_2

- important for hydrodynamical properties of the QGP
- compatible with small shear viscosity - entropy density ratio: $\eta/s \sim (1 \div 2.5)/4\pi$

Higher order flow coefficients are relevant! $\longrightarrow$ Study the relation with the initial deposition of energy in the collision