

UNDERSTANDING B-INITIATED PROCESSES AT THE LHC

FABIO MALTONI

CENTRE FOR COSMOLOGY, PARTICLE PHYSICS AND PHENOMENOLOGY (CP3), LOUVAIN

WORK IN COLLABORATION WITH
GIOVANNI RIDOLFI AND MARIA UBIALI

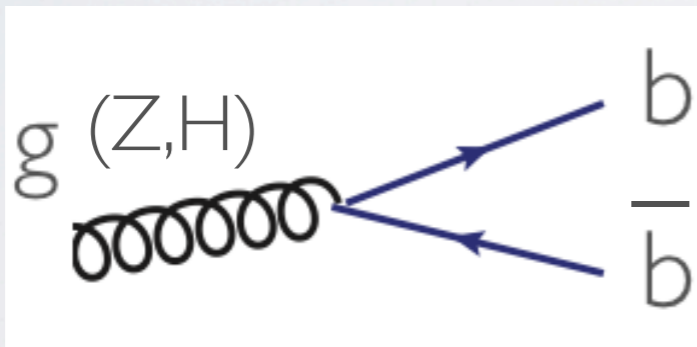
[arXiv: 1203.6393[hep-ph]]

PLAN

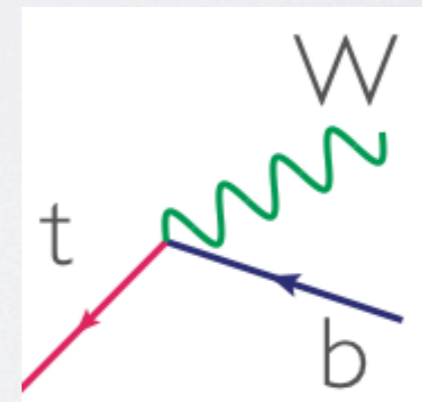
- The rich phenomenology of b-initiated processes Introduction
- Flavor schemes in QCD Problem statement and strategy
- The behaviour of b-pdf Study and results 1
- One-b-initiated processes Study and results 2
- The emergence of a consistent picture Conclusions
- Optimized use of the best available predictions Outlook

BOTTOM QUARKS AT THE LHC

- b quark phenomenology plays a key role at the LHC, from flavor (B mesons) to Higgs searches and measurements, and as a window to New Physics.
- The b is the only quark for which $\Lambda_{\text{QCD}} < m_Q \ll v$ ($=m_W, m_Z, m_h, m_t$).
- Understanding their production is a necessary ingredient to make accurate predictions for signals and backgrounds.
- Bottom quarks can enter in processes at the LHC in two main ways:



$$\Delta B = 0$$

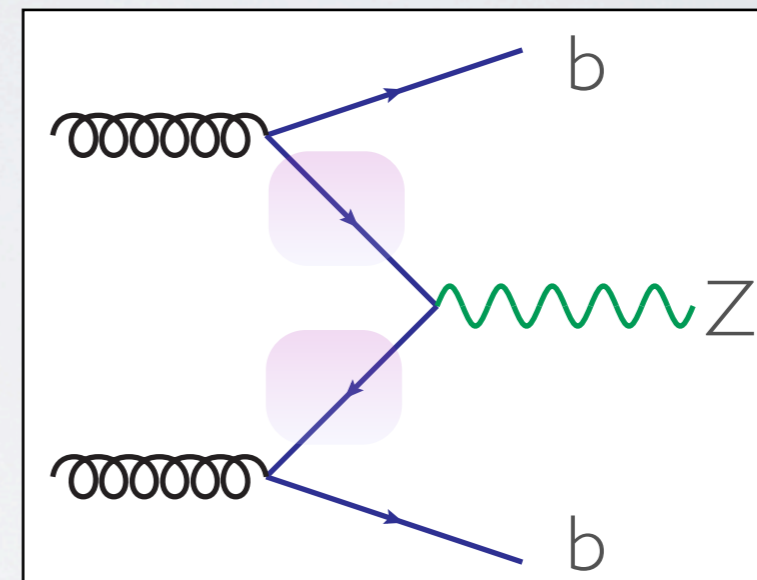
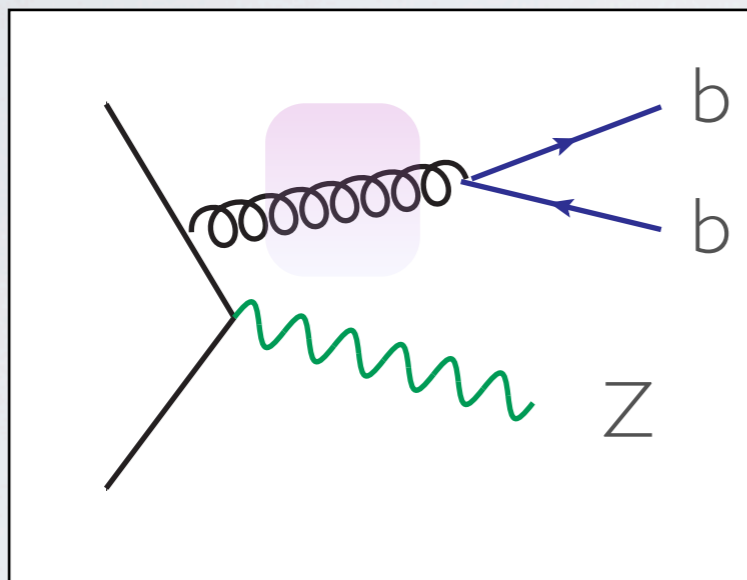


$$\Delta B = \pm 1$$

strong production (let us call it “gluon splitting”) being the dominant one.

BOTTOM QUARKS AT THE LHC

Now, gluon splitting can take place in a s-channel kinematics (in the final state) or in a t-channel kinematics (initial state). So take, for example, $pp \rightarrow Zbb$ associated production:



Both possibilities are affected by the same theoretical worries, which are related to the fact that $m_b \ll s$ -partonic, and therefore one expects:

$$\sigma \sim \alpha_S^2 \log \frac{\hat{s}}{m_b^2}$$

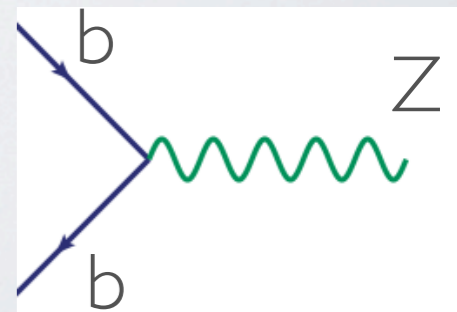
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BOTTOM QUARKS AT THE LHC

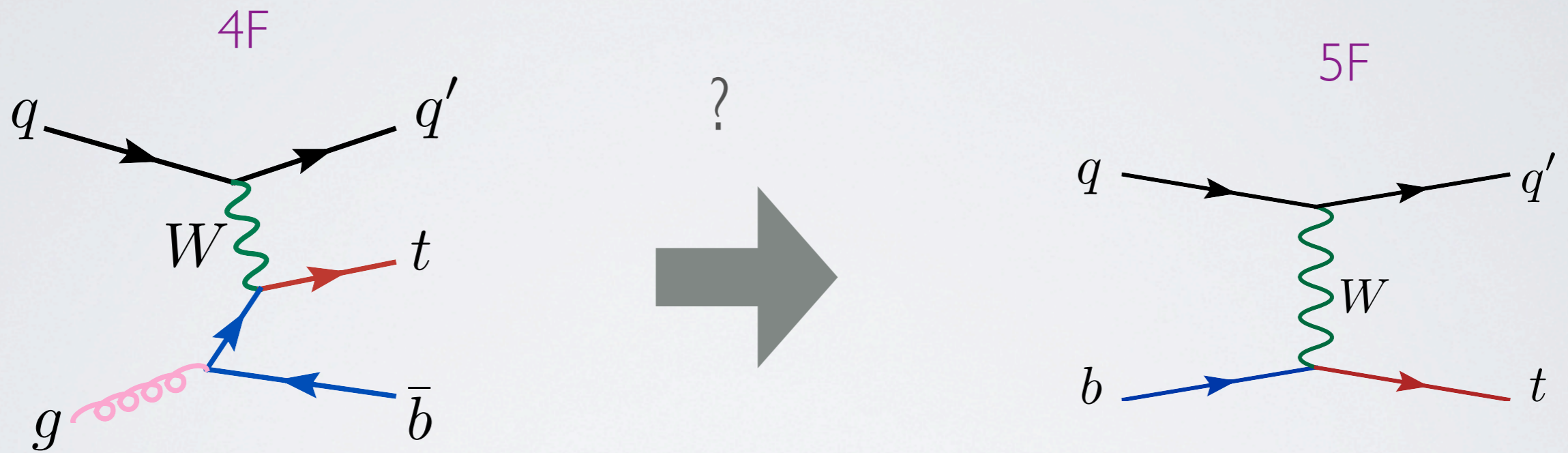
- At face value these logs might be large, possibly spoiling perturbation theory

$$\alpha_S(\hat{s}) \log \frac{\hat{s}}{m_b^2} \sim \frac{1}{\log \frac{\hat{s}}{\Lambda_{QCD}^2}} \log \frac{\hat{s}}{m_b^2} \simeq 1$$

- Such worries can be lifted, by defining an 5 flavor QCD effective field theory where the effects of such logs are resummed using DGLAP equations into fragmentation function and b-pdf's, in the final and initial state respectively.
- This leads to the widely employed, yet sometimes confusing, concept of initial-state b's, i.e. b's considered as partons in the protons, on the same ground as u,d,s,(and c).
- So now $pp \rightarrow Z(bb)$ is seen as $2 \rightarrow 1$ process, much SIMPLER to calculate and also (apparently) more accurate than the $2 \rightarrow 3$ process in the 4F.



HOW DOES THAT WORK?



4F AND 5F SCHEMES

- This results from integrating over a t-channel propagator

$$\frac{1}{t - m_b^2} \sim \frac{1}{p_T^2 + m_b^2}$$

$$t = (p_{\bar{b}} - p_g)^2, \quad p_T^2 = p_{T,\bar{b}}^2$$

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✻ Contribution to the cross section:

$$\int_0^{p_{T,\max}^2} \frac{dp_T^2}{p_T^2 + m_b^2} = \log \left(\frac{p_{T,\max}^2}{m_b^2} \right) + \dots$$

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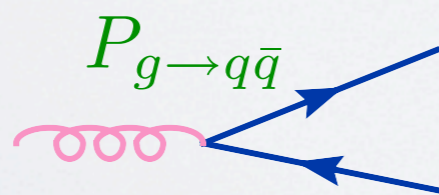
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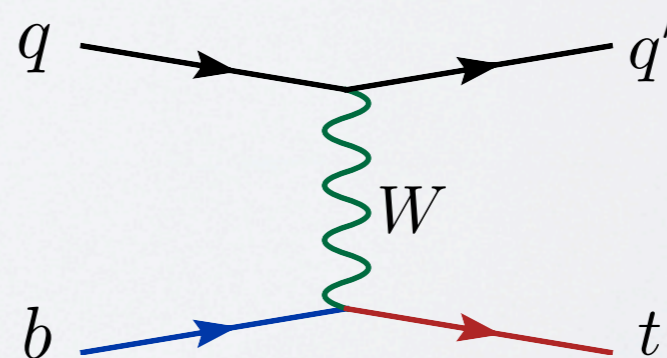
$$\int_0^{p_{T,\max}^2} \frac{dp_T^2}{p_T^2 + m_b^2} = \log \left(\frac{p_{T,\max}^2}{m_b^2} \right) + \dots$$

- Coefficient of the logarithm is:

AP splitting function



times



matrix elements with splitting removed

4F AND 5F SCHEMES

- Putting it together:
$$\frac{d\sigma(qg \rightarrow q't\bar{b})}{d\log p_{T,\max}^2} \sim \left(\frac{\alpha_s}{2\pi}\right) \left[\int \frac{dx}{x} P_{g \rightarrow q\bar{q}} f_g \right] \times \hat{\sigma}(qb \rightarrow q't)$$

- But the first part resembles the evolution equation for a quark:

$$\frac{df_q}{d\log q^2} \sim \left(\frac{\alpha_s}{2\pi}\right) \int \frac{dx}{x} [P_{g \rightarrow q\bar{q}} f_g + P_{q \rightarrow qg} f_q]$$

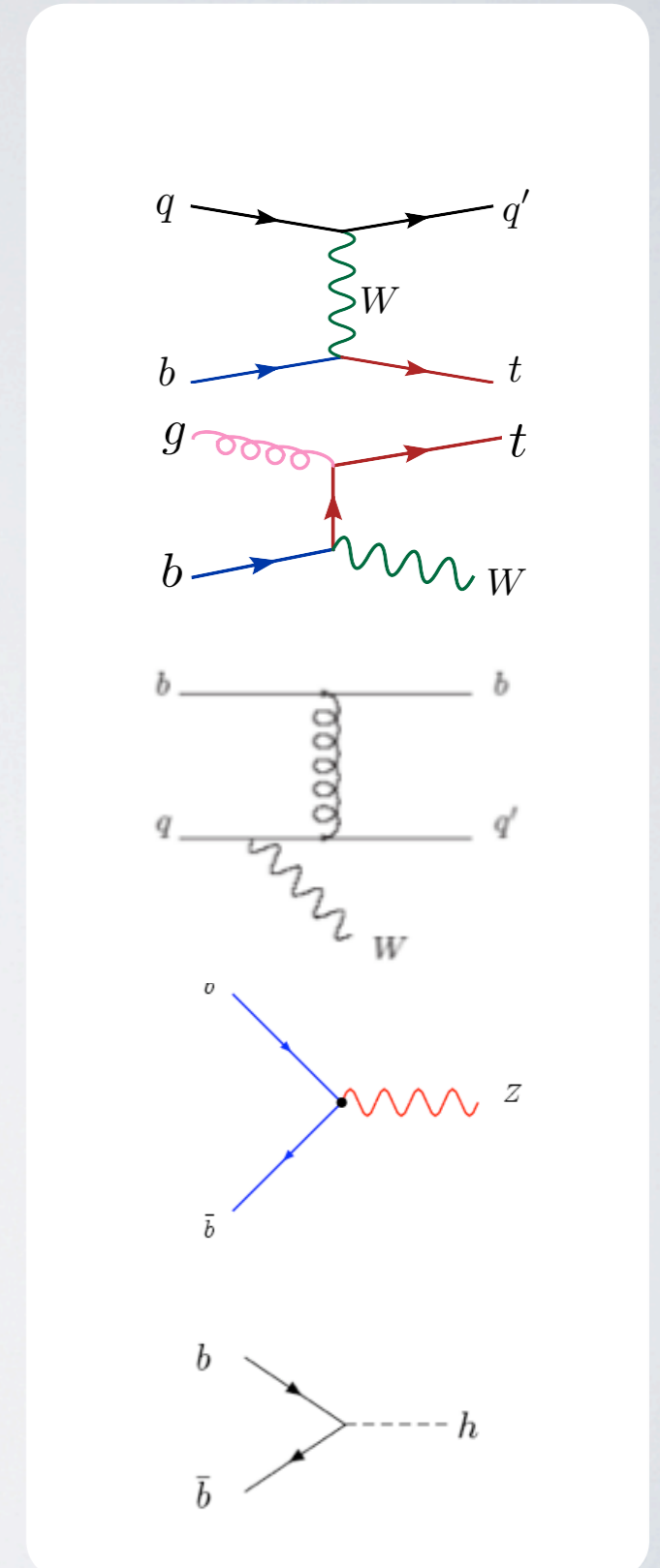
- So when the logarithms really dominate, we can replace this description by

$$\sigma(qg \rightarrow q't\bar{b}) \approx \sigma(q\tilde{b} \rightarrow q't) \quad \tilde{b}(x) \sim \frac{\alpha_s}{2\pi} \log \frac{\mu^2}{m_b^2} \left[\int \frac{dz}{z} P_{g \rightarrow q\bar{q}} f_g \right]$$

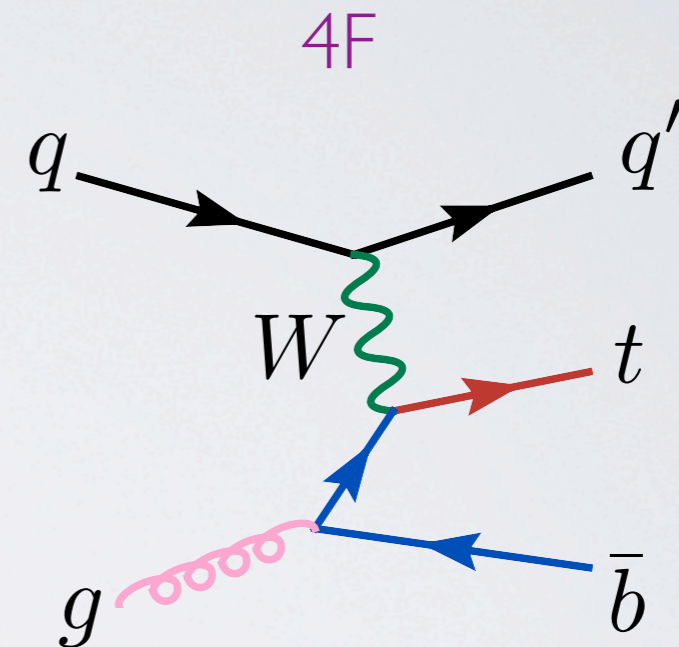
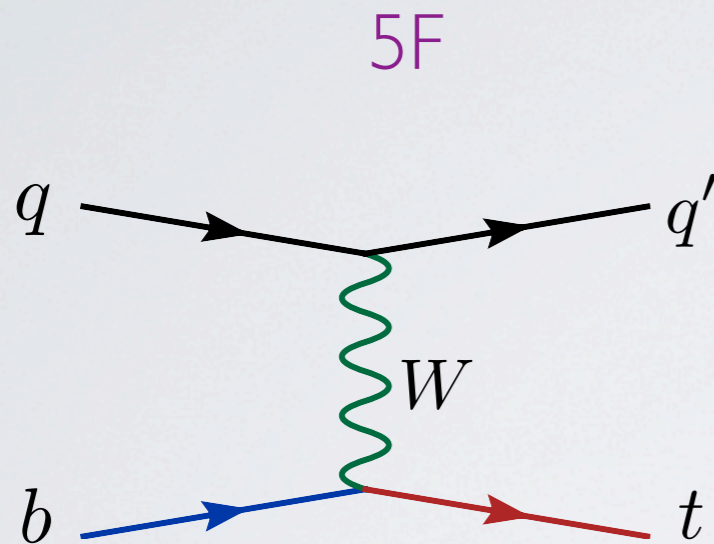
- At all orders 5F and 4F descriptions should agree; order by order they differ:
 - DGLAP evolution of b-PDF = resummation
 - The exact range of integration is not relevant at LL.
 - Only the large logarithm at LO goes to the PDF, differences moved to NLO.

B-INITIATED PROCESSES AT THE LHC

Class	Process	Interest
Top	$qb \rightarrow tq$ (t-channel)	SM, top EW couplings and polarization, V_{tb} . Anomalous couplings. H^+ : SUSY, 2HDM
	$gb \rightarrow t(W, H^+)$	
Vector Bosons	$pp \rightarrow Wb$ $pp \rightarrow Wbj$	SM, bkg to single top
	$bb \rightarrow Z$ $gb \rightarrow Zb$ $pp \rightarrow Zbj$	Standard candle: SM BSM bkg, b-pdf
	$gb \rightarrow \text{gamma} + b$	
Higgs	$bb \rightarrow (h, A)$ $gb \rightarrow (h, A) + b$	SUSY discovery/ measurements at large $\tan(\beta)$



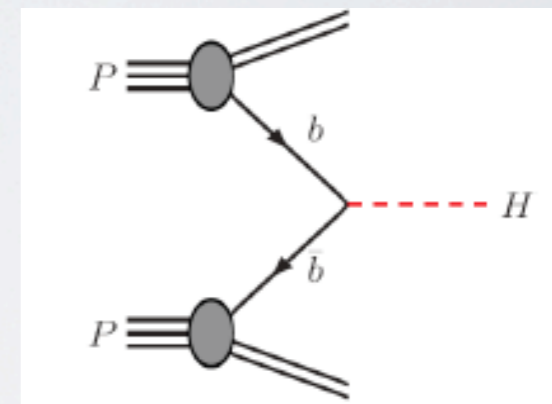
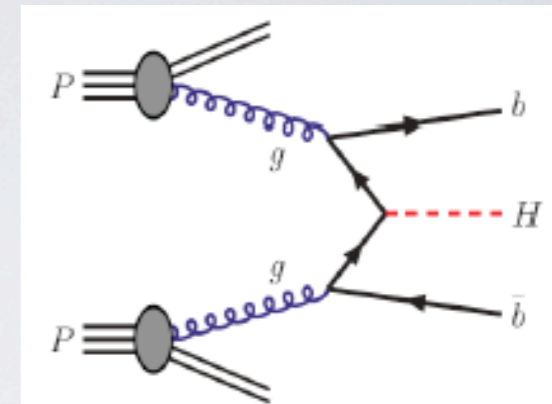
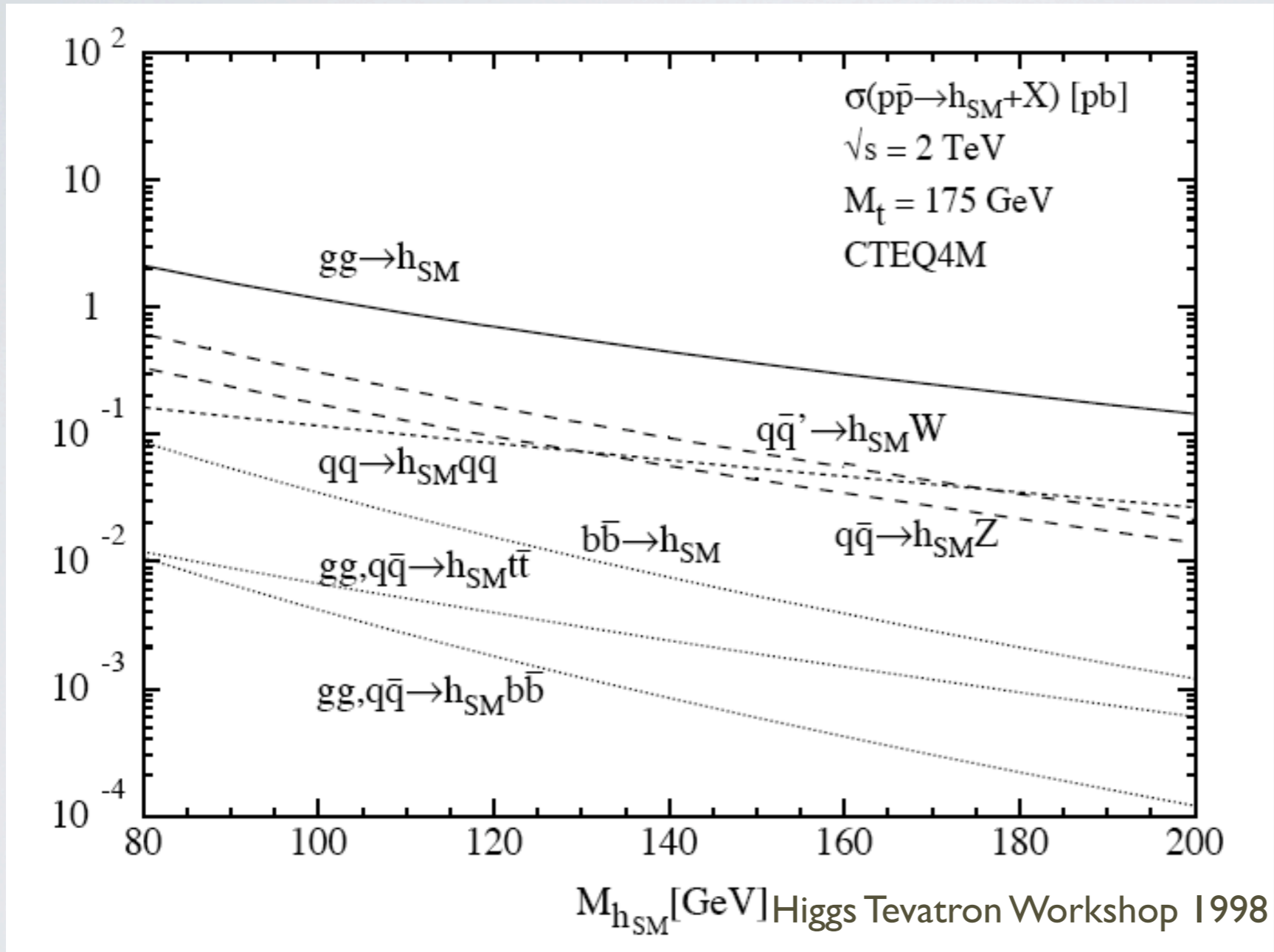
5F VS 4F : SUMMARY



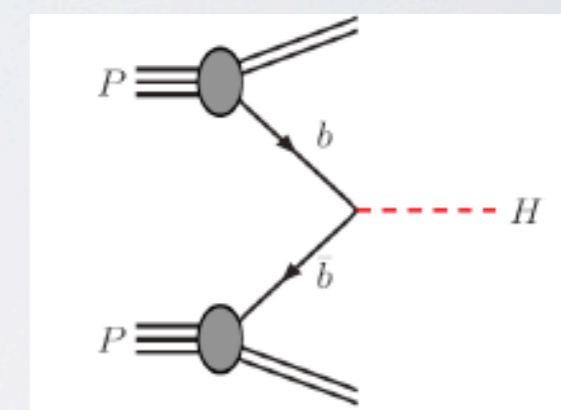
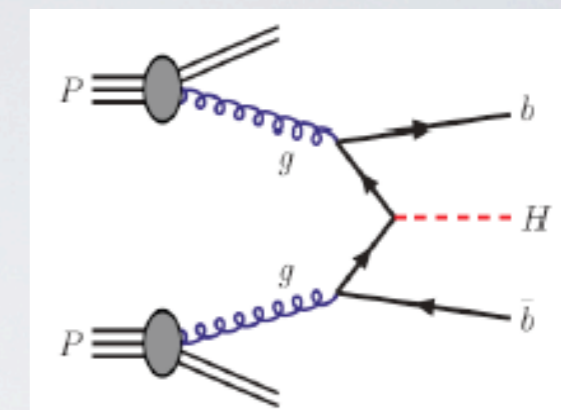
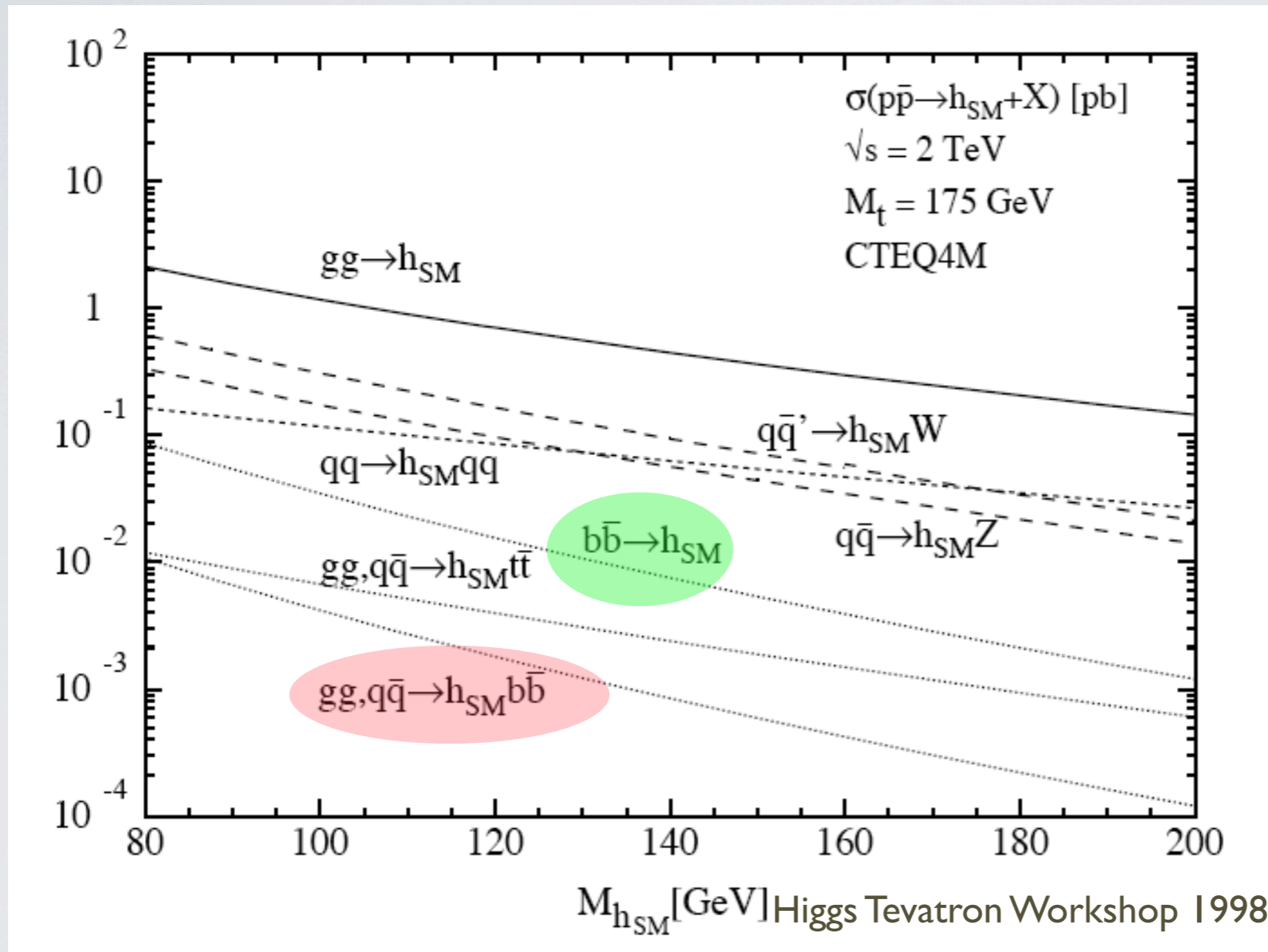
1. It resums initial state large logs in the b pdf, leading to more stable predictions
2. Going NLO (and NNLO) “easy”.
3. Mass effects enter at higher orders.
4. Implementation in MC depends on the gluon splitting model in the PS.

1. It does not resum (possibly) large logs, yet it handles them explicitly.
2. Going NLO WAS difficult.
3. Mass effects are there at any order in PT.
4. MC at LO and NLO no problem.

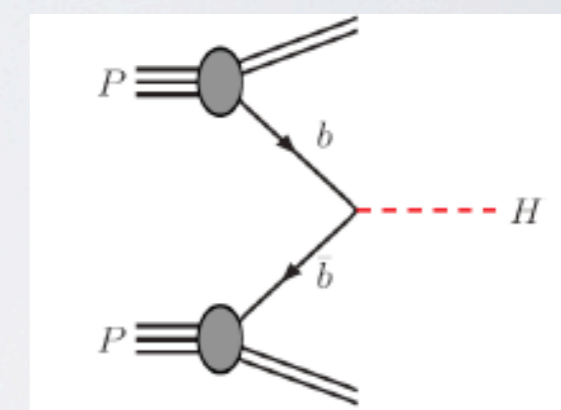
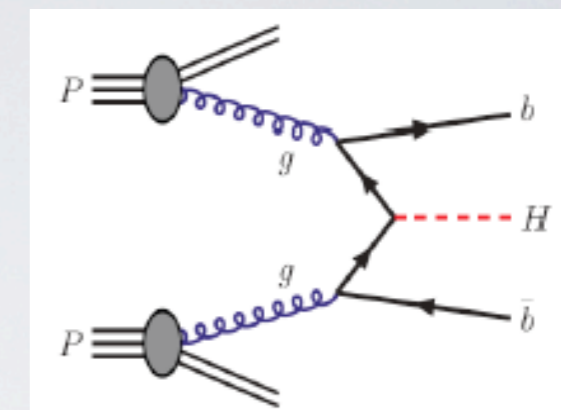
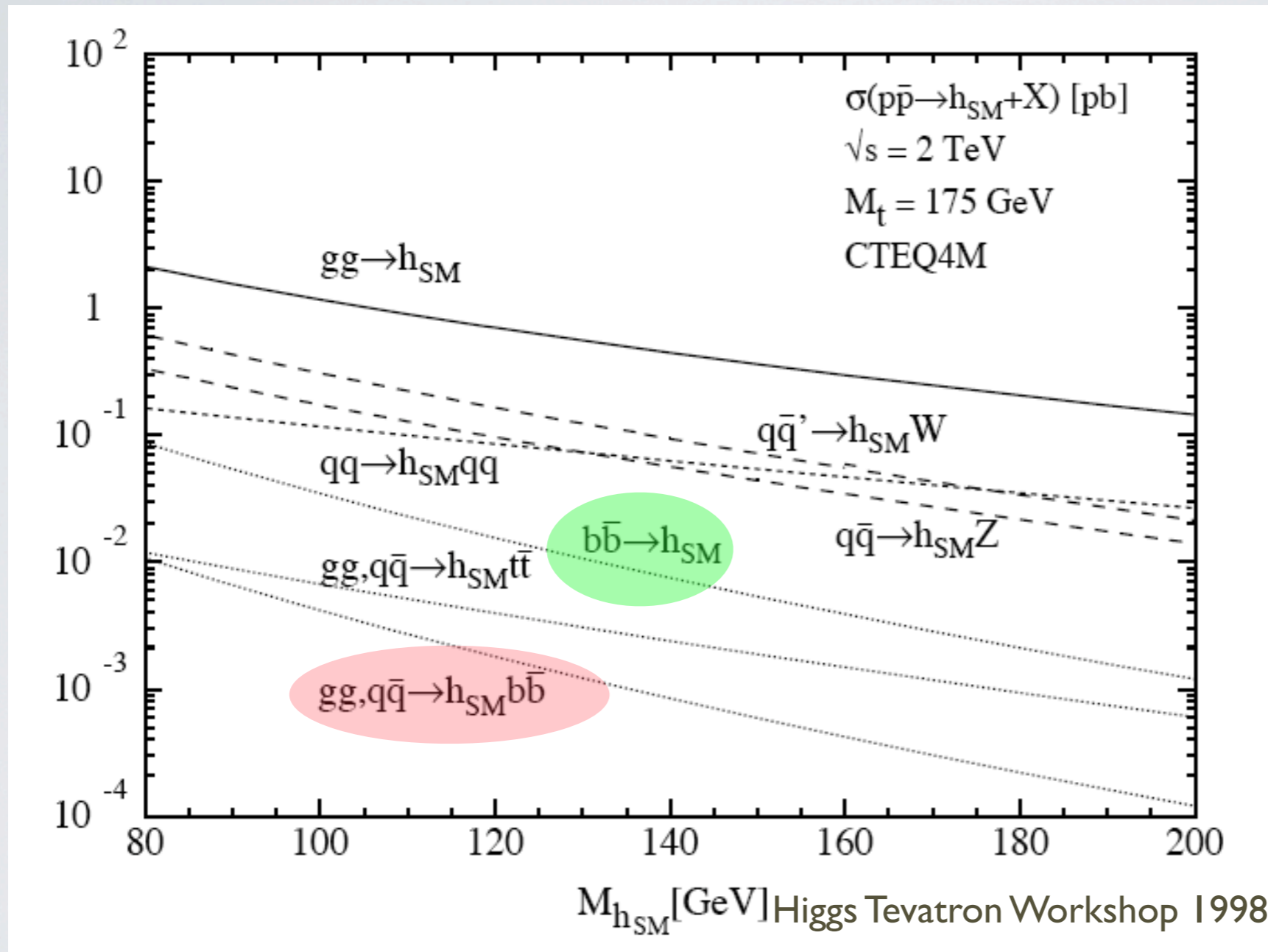
QUESTIONS AND PUZZLES: LEVEL 1



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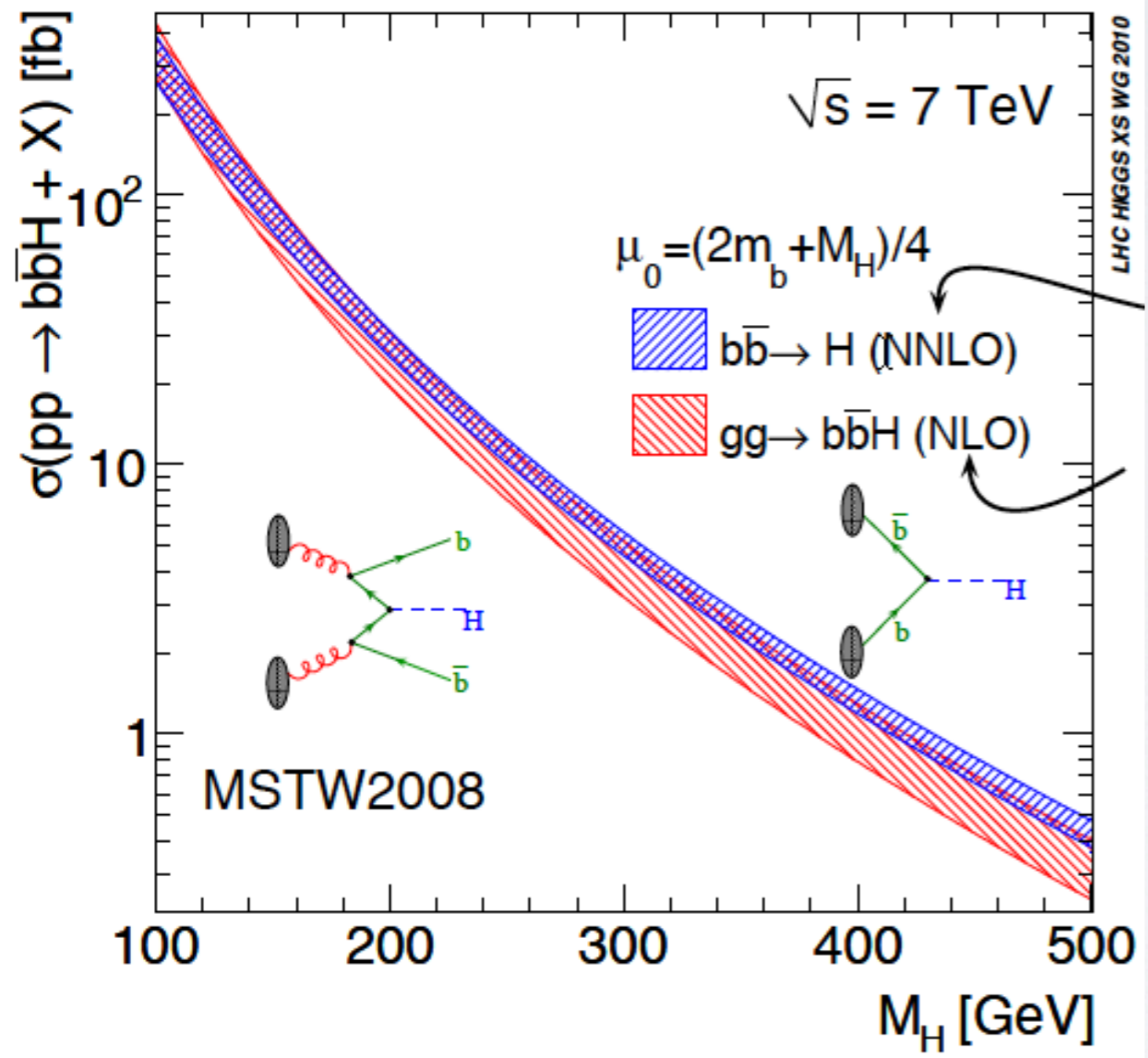


QUESTIONS AND PUZZLES: LEVEL 1



Factor of ten difference??? Is this the effects of the logs? How can that be?

QUESTIONS AND PUZZLES: LEVEL 1



Two important ingredients helped in “solve” this puzzle:
 1. Inclusion of higher order corrections

[Harlander and Kilgore, 2003]

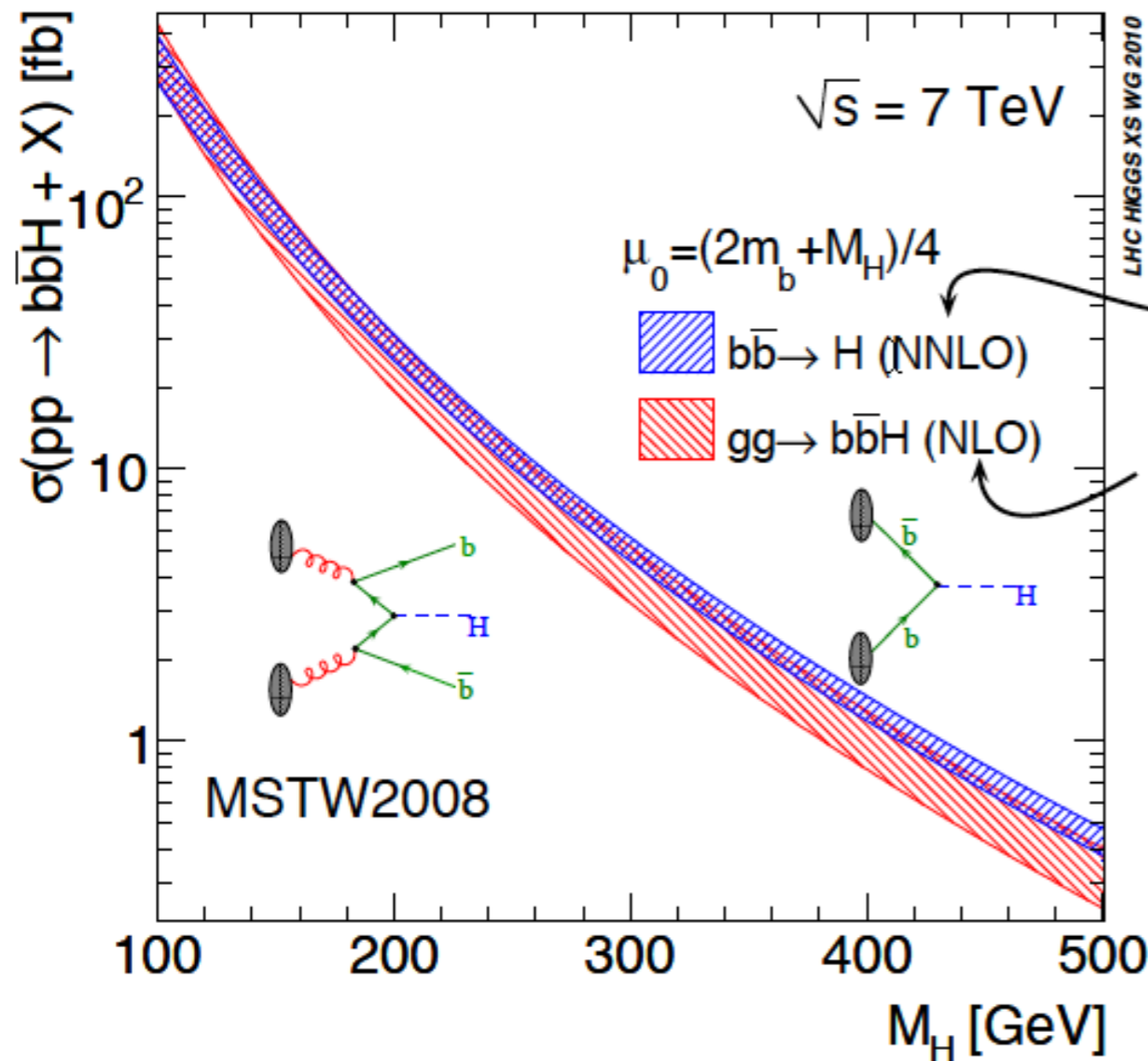
[Dittmaier, Krämer, Spira '04]

[Dawson, Jackson, Reina, Wackerroth '04]

[Hirschi et al. [1103.0621](#)]

2. Scale choices : better agreement when smaller than naive choices M_H .

QUESTIONS AND PUZZLES: LEVEL 2

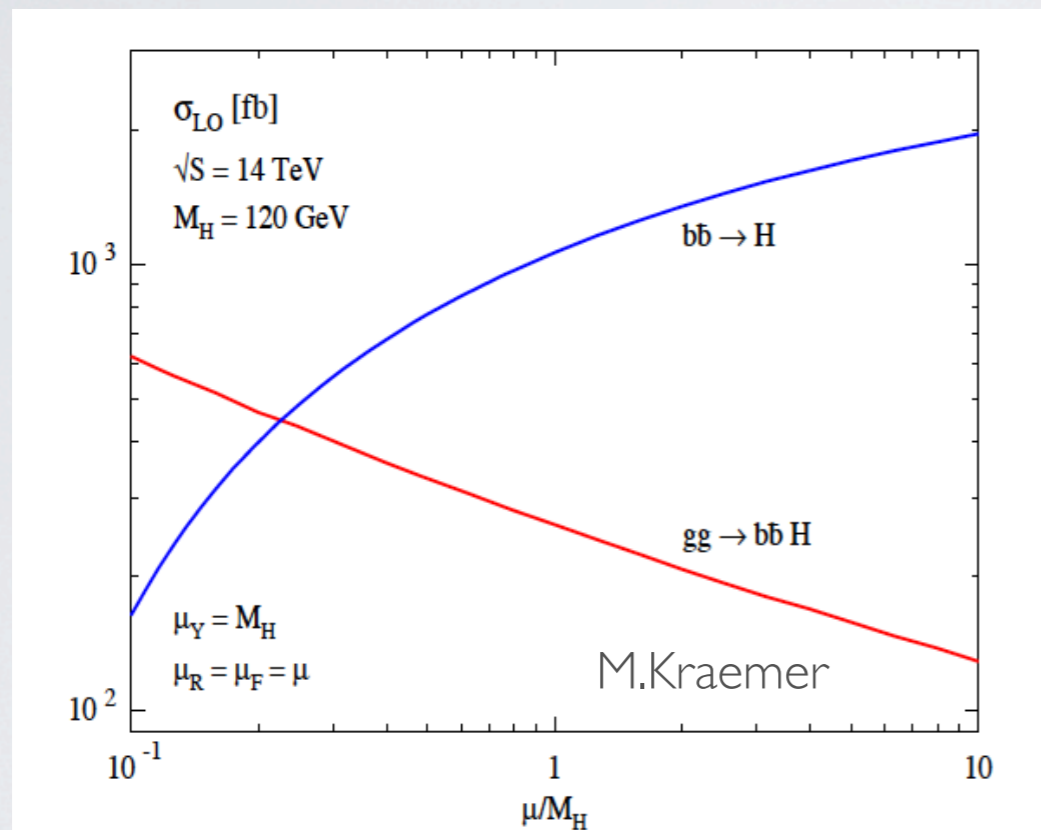


However, this plot now raises new burning questions:

1. Why the agreement is so good around $m_H = 100 \text{ GeV}$ and the uncertainty band comparable?
2. It looks like one needs a 500 GeV Higgs to really see the effects of the large logs? Is this the reason?
3. How is the smaller scale choice $m_H/4$ justified? Agreement seems ad hoc.
4. Is this behavior only proper of $bb \rightarrow \text{Higgs}$ or it is general?

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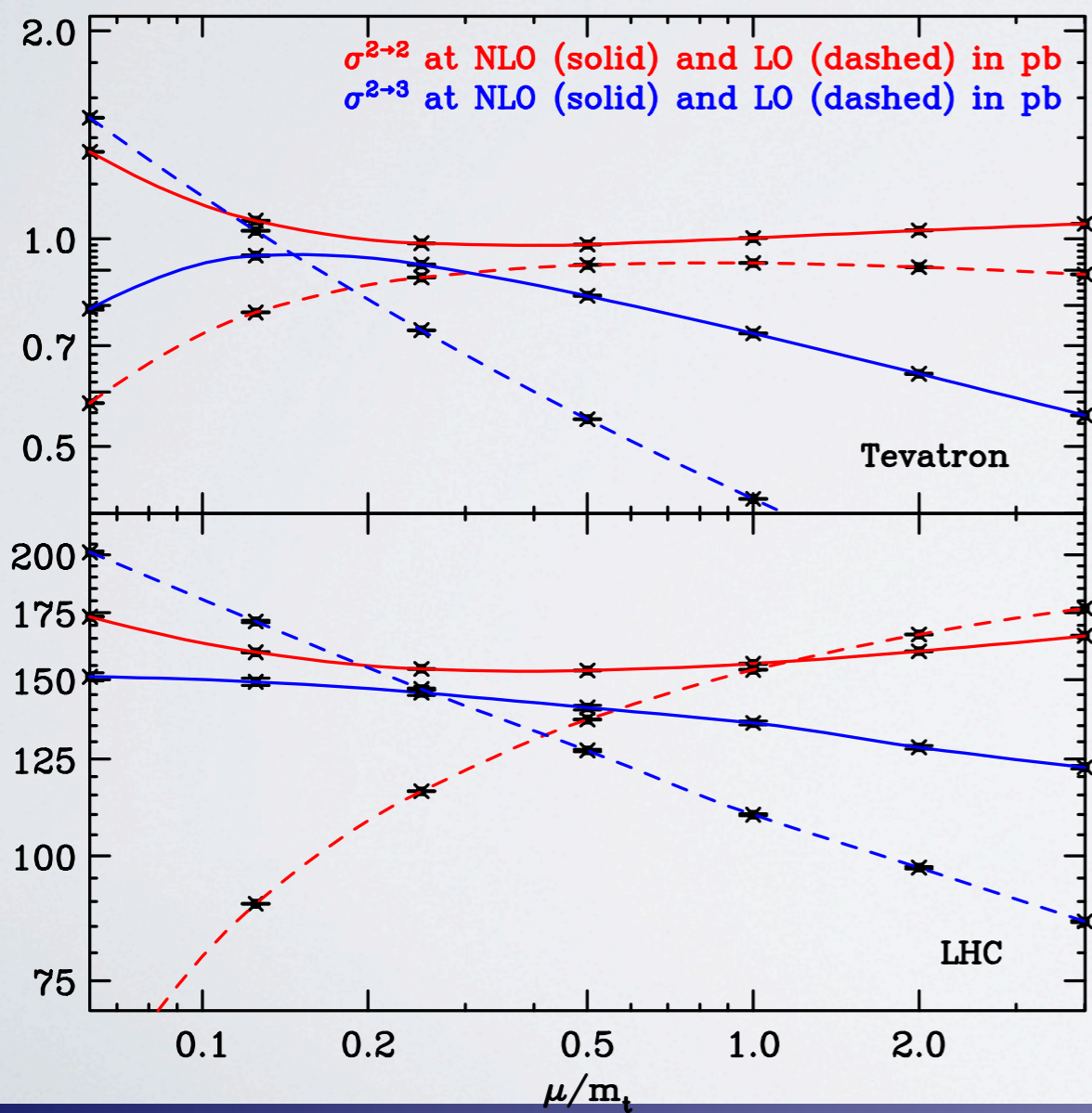
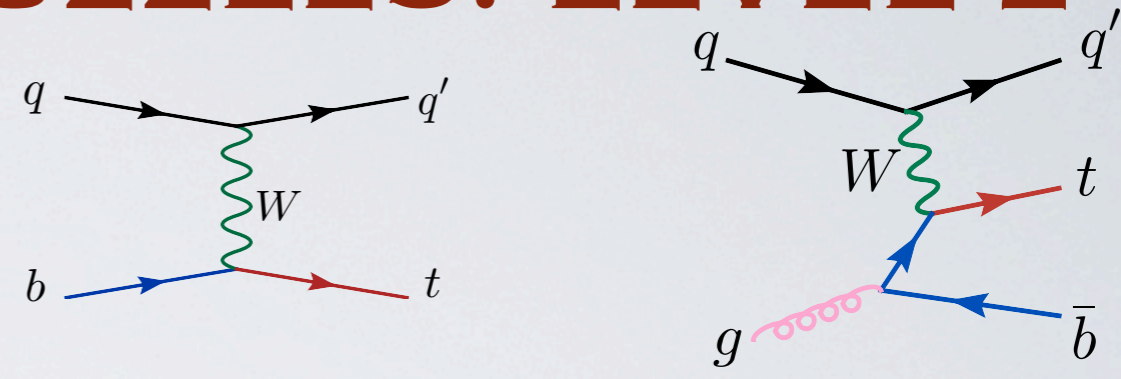
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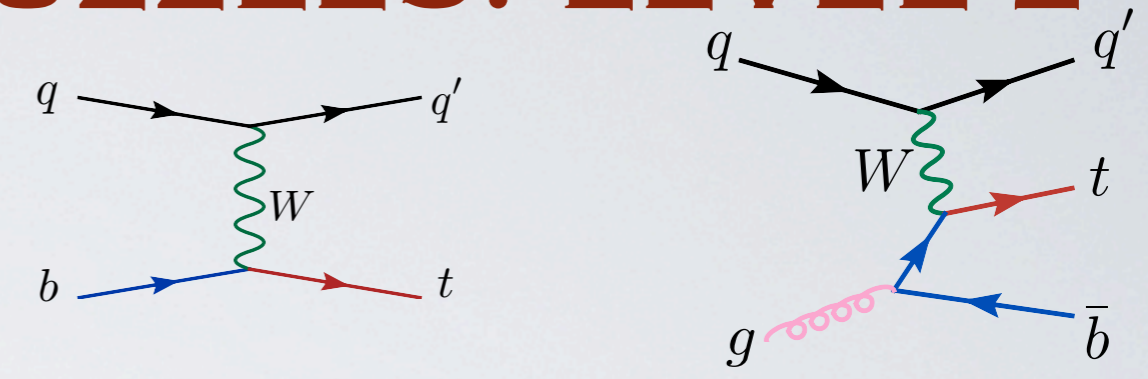
QUESTIONS AND PUZZLES: LEVEL 2

t-channel single top:



1. Differences at natural scales m_t become smaller at lower scales, $\mu \sim m_t/4$. Why?
2. At LHC both scale dependences are rather mild. 4F is as good as 5F. Where is the need for resummation?
3. Differences are smaller at the LHC than Tevatron. Why? The logs should have more space to develop at the LHC...
3. What happens for a heavier top?

QUESTIONS AND PUZZLES: LEVEL 2

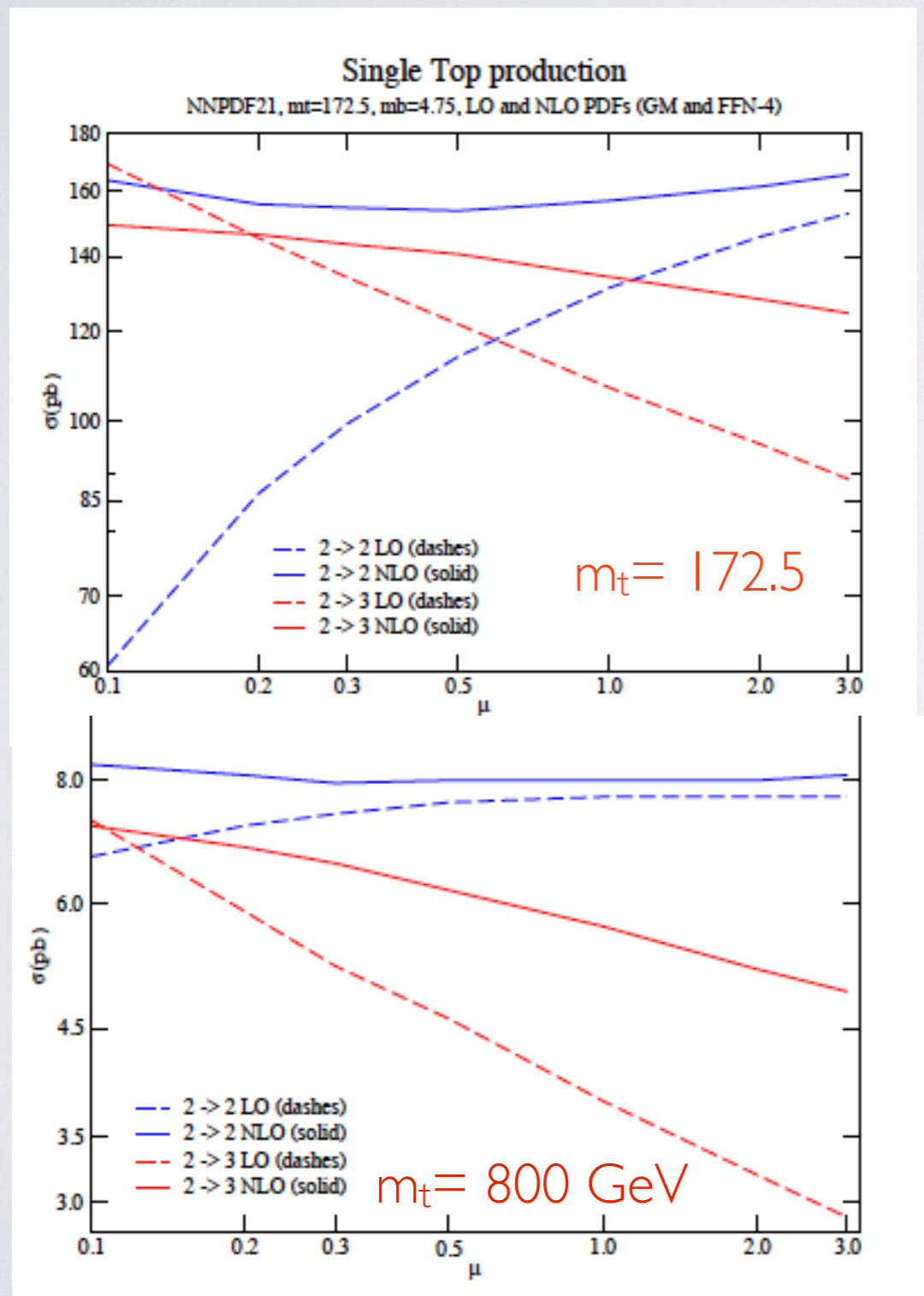


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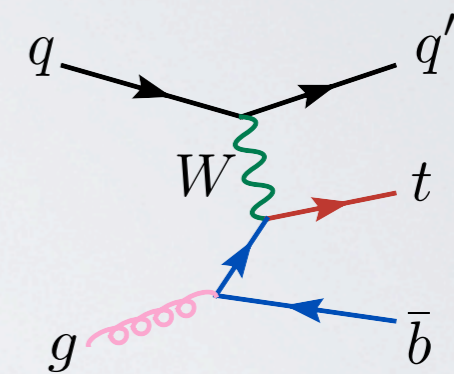
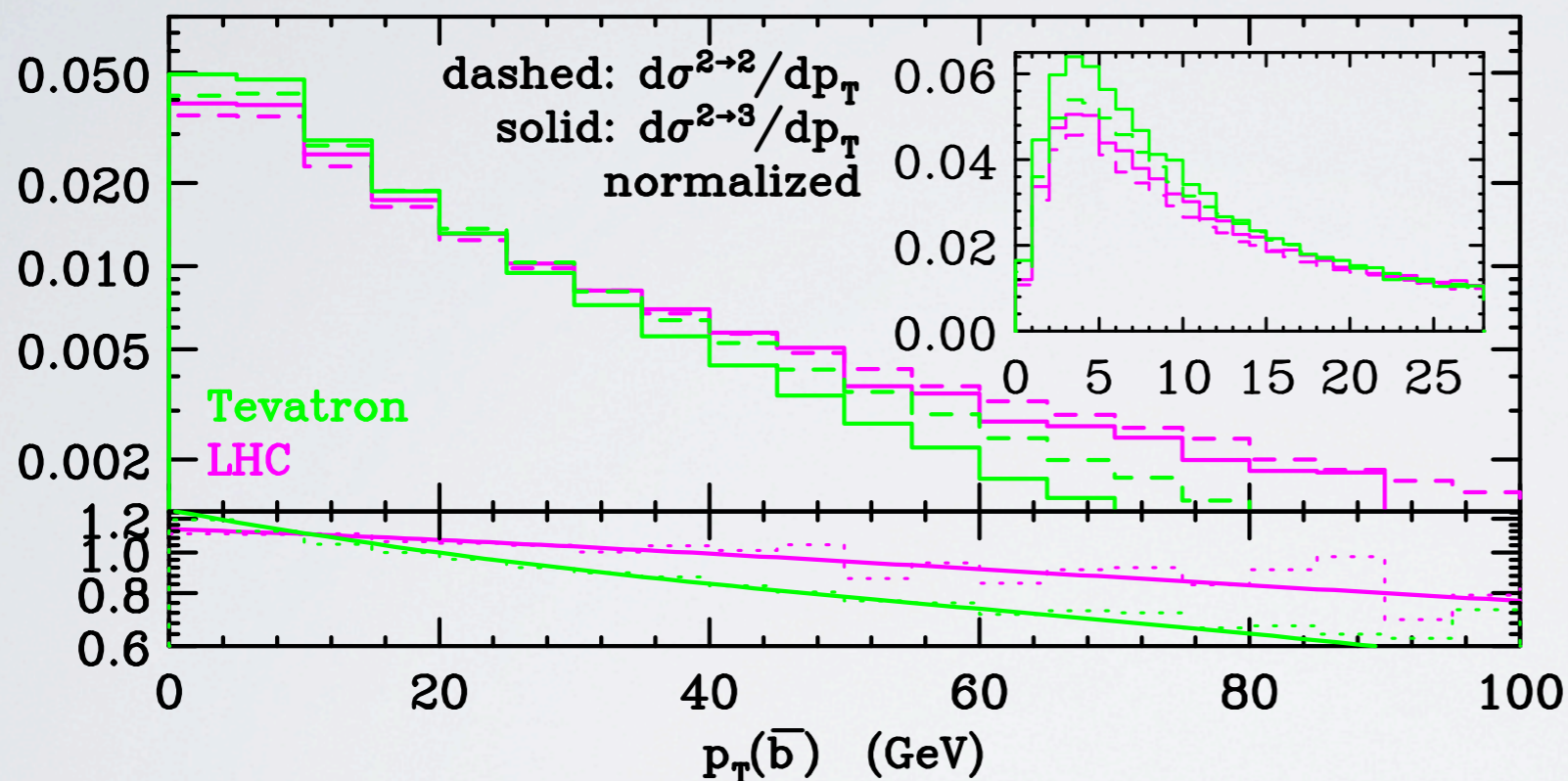
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QUESTIONS AND PUZZLES: LEVEL 3

- What about other more exclusive observables?

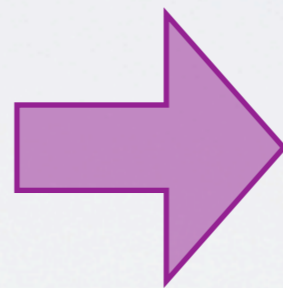
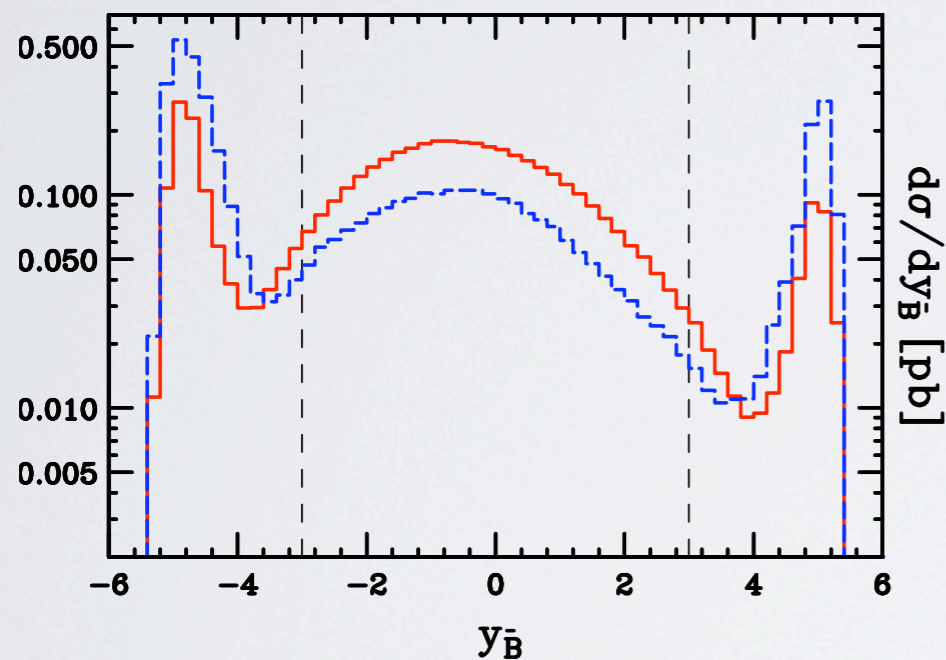


- This observable is NLO only in the 4F calculation.
- Slightly softer in 4F ($2 \rightarrow 3$), particularly at the Tevatron
- Deviations up to $\sim 20\%$: stable perturbative expansion, no large corrections
- A 4F calculation is much more EXP handy and useful in actual analyses.

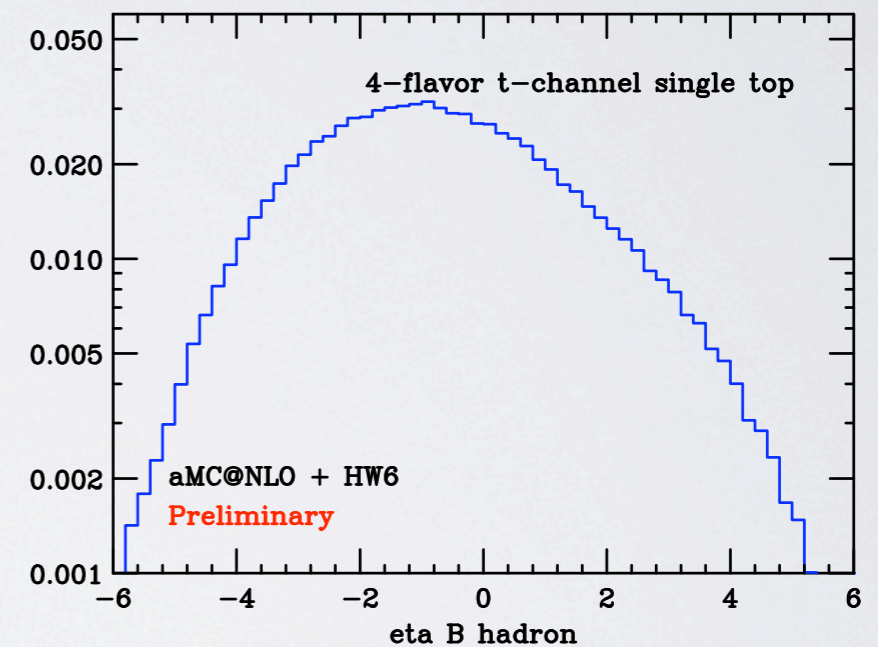
QUESTIONS AND PUZZLES: LEVEL 3

- What about other more exclusive observables?

5-flavor scheme with Fortran Herwig



4-flavor scheme



- Not first example where leaving the shower to do gluon splitting is not exactly a good idea.
- 4F calculations are easily interfaced to MC@NLO or POWHEG.
- Many available automatically from aMC@NLO.

QUESTIONS AND PUZZLES: SUMMARY

- At the level of total cross section 5F predictions are in general better behaved than 4F ones.
- However, a substantial and unexpected agreement between 4F and 5F is found when scales smaller than a naive choice is made.
- Agreement is found even in regions where the logs should be large. Only exception seems to take place for very heavy object production.
- Independently of 5F results: No sign of breakdown of the perturbative expansion for 4F in total cross sections as well as for more exclusive observables is found.

THE INVESTIGATION



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I will now argue that all the (apparent) puzzles and odd findings listed above can be easily merged in a simple and consistent picture, by simply taking into account the following two main results:

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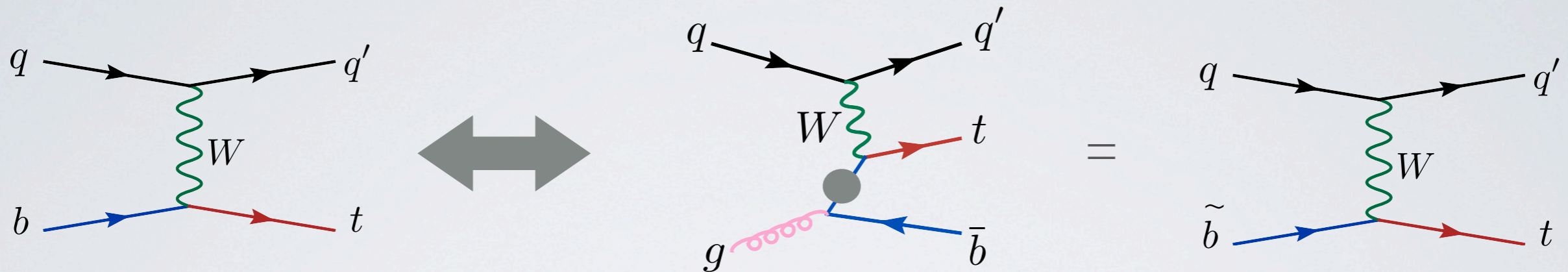
THE INVESTIGATION



I will now argue that all the (apparent) puzzles and odd findings listed above can be easily merged in a simple and consistent picture, by simply taking into account the following two main results:

1. The resummation effects of the initial state logarithms in to the b-PDF is important only at large Bjorken-x.
2. The possibly large ratios Q^2/m_b^2 are always accompanied by universal phase space factors that at hadron colliders lead to their suppression.

IMPACT OF RESUMMATION



b-pdf has all the logs resummed

\tilde{b} -pdf has only the leading log :

$$\int dx_1 dx_2 q(x_1, \mu_F^2) b(x_2, \mu_F^2) \hat{\sigma}(qb \rightarrow q't) \quad \int dx_1 dx_2 q(x_1, \mu_F^2) \tilde{b}(x_2, \mu_F^2) \hat{\sigma}(qb \rightarrow q't)$$

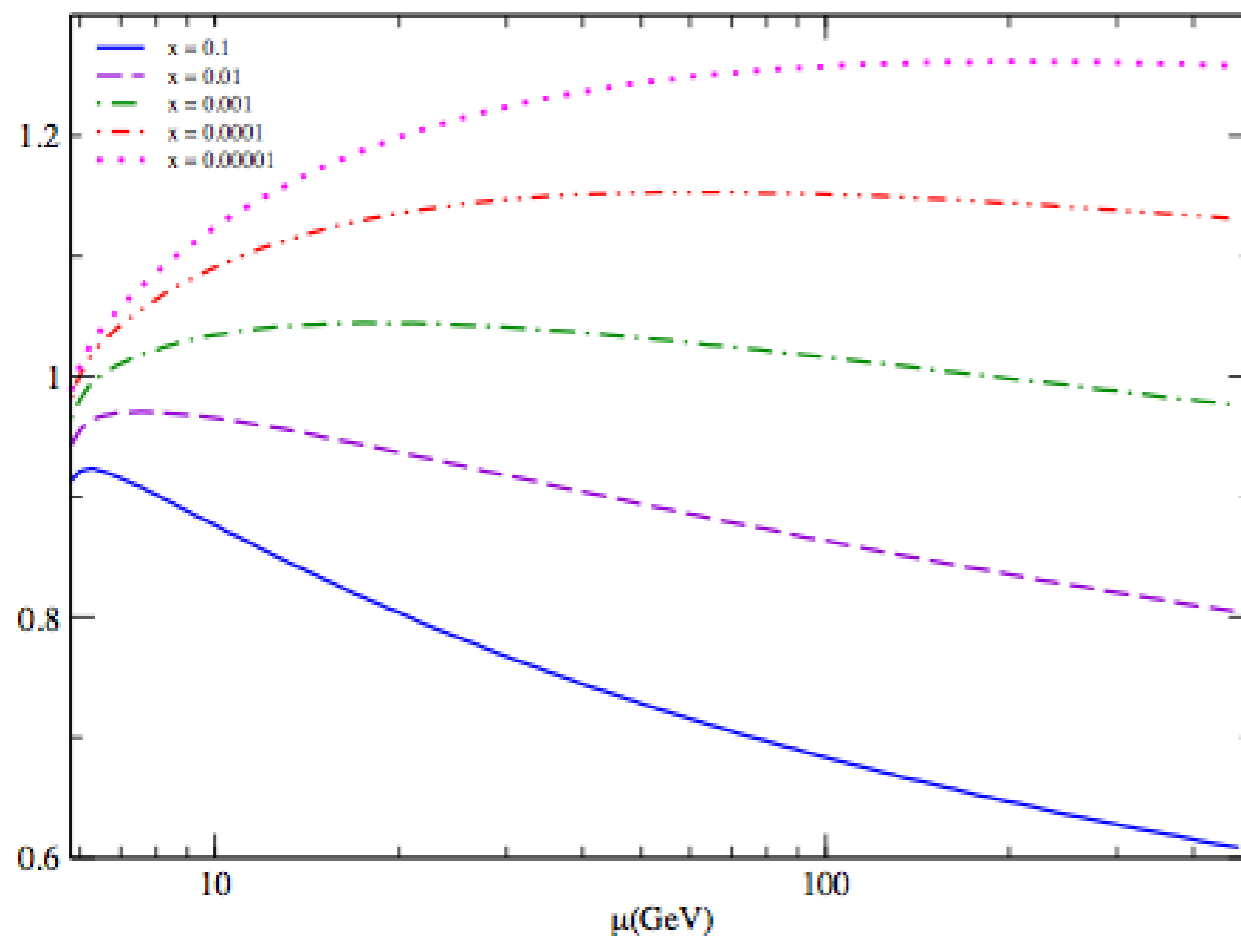
$$\tilde{b}(x, \mu_F) \sim \frac{\alpha_s}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dy}{y} P_{qg} \left(\frac{x}{y} \right) g(y, \mu_f)$$

\tilde{b} is just the first log that one gets from a LO 4F calculation. The b-pdf resums the full tower of such logs that come from higher orders in the 4F calculation.

IMPACT OF RESUMMATION

$$\tilde{b}^{(1)}(x, \mu^2) = \frac{\alpha_S}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dz}{z} P_{qg}(z) g\left(\frac{x}{z}, \mu^2\right) / b^{(1)}(x, \mu^2)$$

Ratio between \tilde{b} and b at LO (MSTW08)



Comparison between the first log which the one included in the LO 4F calculation of single-top, and the full resummed result given by the AP equations.

The various curves correspond to different Bjorken x 's.

At small x the effect is positive, in other words \tilde{b} is a kind of bad overestimate.

At large x resummation effects are manifest.

LO approximation does not look good enough.

IMPACT OF RESUMMATION

$$\tilde{b}^{(2)}(x, \mu^2) = \int_x^1 \frac{dz}{z} \Sigma^{4F,(2)}\left(\frac{x}{z}, \mu^2\right) \left(\frac{\alpha_S}{4\pi}\right)^2 a_{\Sigma,b}^{(2)}(z, \mu^2/m_b^2)$$

$$+ \int_x^1 \frac{dz}{z} g^{4F,(2)}\left(\frac{x}{z}, \mu^2\right) \left[\left(\frac{\alpha_S}{4\pi}\right) a_{g,b}^{(1)}(z, \mu^2/m_b^2) + \left(\frac{\alpha_S}{4\pi}\right)^2 a_{g,b}^{(2)}(z, \mu^2/m_b^2) \right] / b^{(2)}(x, \mu^2)$$

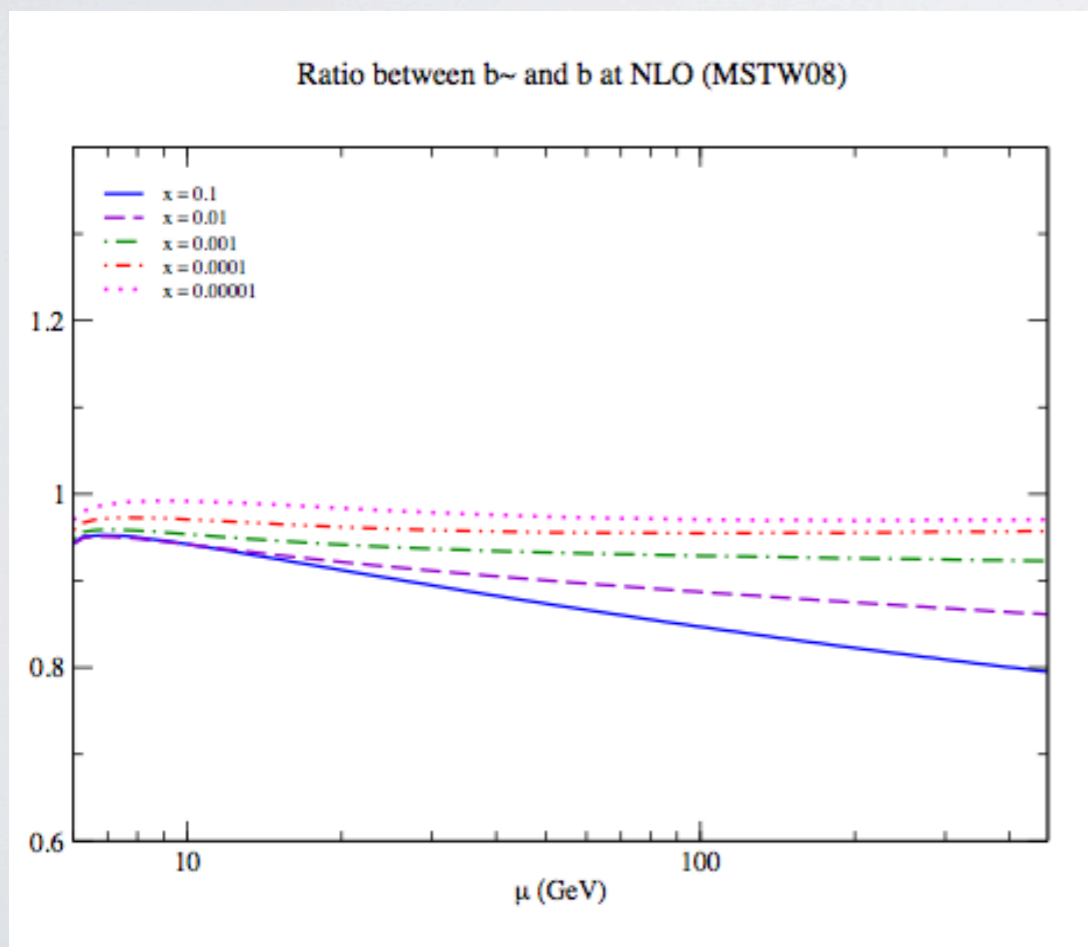
Comparison between the first $\log^2 + \log$ which are included in the NLO 4F calculation of single-top, and the full resummed result given by the AP equations at NLO.

The various curves correspond to different Bjorken x 's.

At small x also now the resummation is visible yet is very small.

At large x resummation effects are manifest.

NLO approximation does look reasonably behaved.



IMPACT OF RESUMMATION

- Can I understand this behaviour (at least roughly)?

I write the DGLAP equation for the b-pdf:

$$\left\{ \begin{array}{l} \frac{d}{d \log \mu^2} b(N, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \left[\gamma_{qq}^{(0)}(N) b(N, \mu^2) + \gamma_{qg}^{(0)}(N) g(N, \mu^2) \right], \\ b(N, m_b^2) = 0 \quad \text{boundary condition} \end{array} \right.$$

whose solution at LO can be easily written as:

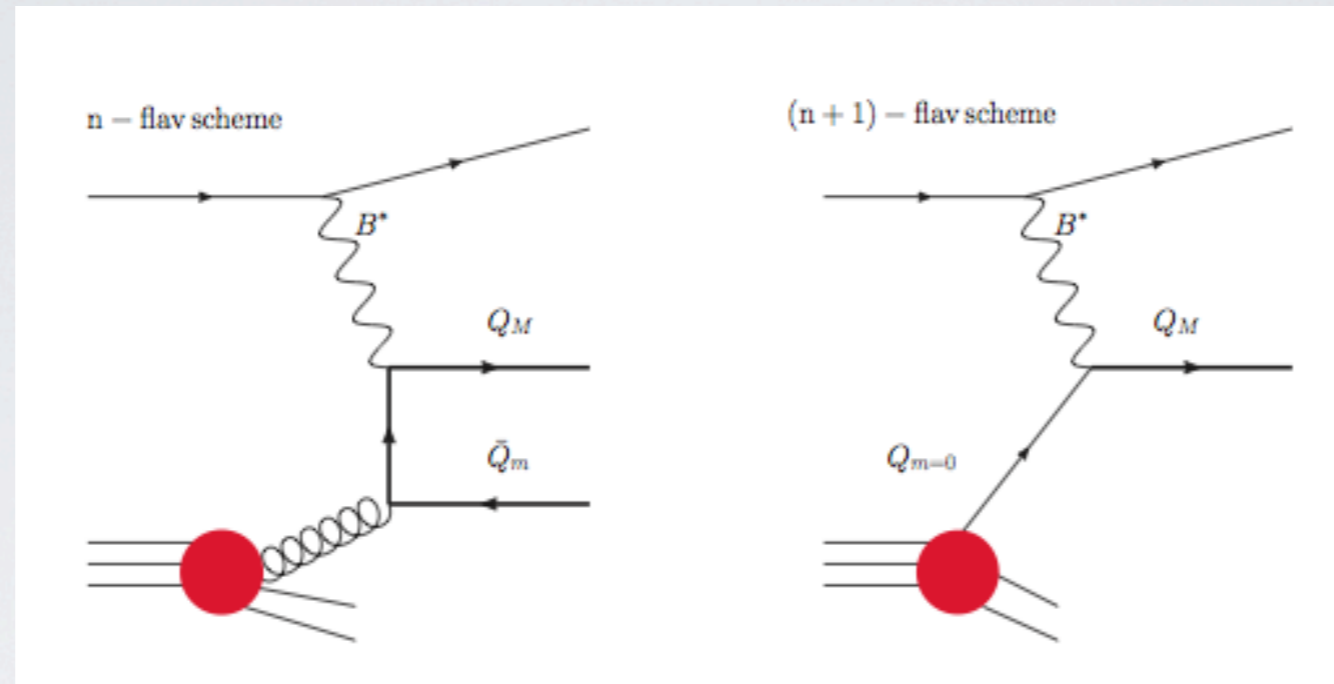
$$b(N, \mu^2) = \gamma_{qg}^{(0)}(N) g(N, m_b^2) \left\{ \frac{\alpha_S(m_b^2)}{2\pi} \log \frac{\mu^2}{m_b^2} + \sum_{k=2}^{\infty} A_k(N) \frac{1}{k!} \left[\frac{\alpha_S(m_b^2)}{2\pi} \log \frac{\mu^2}{m_b^2} \right]^k \right\},$$

$$\text{with } A_k(N) = \left[\gamma_{qq}^{(0)}(N) - \beta_0 \right] \left[\gamma_{qq}^{(0)}(N) - 2\beta_0 \right] \dots \left[\gamma_{qq}^{(0)}(N) - (k-1)\beta_0 \right].$$

The logarithms resummed in the b-PDF are larger:

1. as μ gets larger with respect to m_b
2. at large $N \Leftrightarrow$ large x

THE UNIVERSAL LOGS : DIS



$$\sigma_b(\mu^2) = \int_{y_{\min}}^{y_{\max}} dy \int_{Q_{\min}^2}^{Q_{\max}^2} dQ^2 \frac{2\pi\alpha_l\alpha_h}{y(M^2 + Q^2)^2} \left\{ [1 + (1 - y)^2] F_2^b(x, Q^2, m_b^2) \right. \\ \left. - y^2 F_L^b(x, Q^2, m_b^2) + [1 - (1 - y)^2] F_3^b(x, Q^2, m_b^2) \right.$$

$$y = \frac{Q^2}{xS}$$

THE UNIVERSAL LOGS : DIS

Let's take the expression for the 4F process $\gamma^* + g \rightarrow b + \bar{b}$ at small t :

$$\frac{d\hat{\sigma}_2}{dt} = \frac{\pi\alpha_e e_b^2 \alpha_S C_F}{16} \left[-\frac{4z}{Q^2(t - m_b^2)} \frac{z^2 + (1-z)^2}{2} \right] + \text{non-singular terms}$$

Integrating over t gives:

$$\int_{t_-}^{t_+} dt \frac{d\hat{\sigma}_2}{dt} = \frac{\pi\alpha_e e_b^2 \alpha_S C_F}{4Q^2} z P_{qg}(z) \log \frac{1+\beta}{1-\beta} \quad t_{\pm} = m_b^2 - \frac{s+Q^2}{2}(1 \pm \beta); \quad \beta = \sqrt{1 - \frac{4m_b^2}{s}}$$

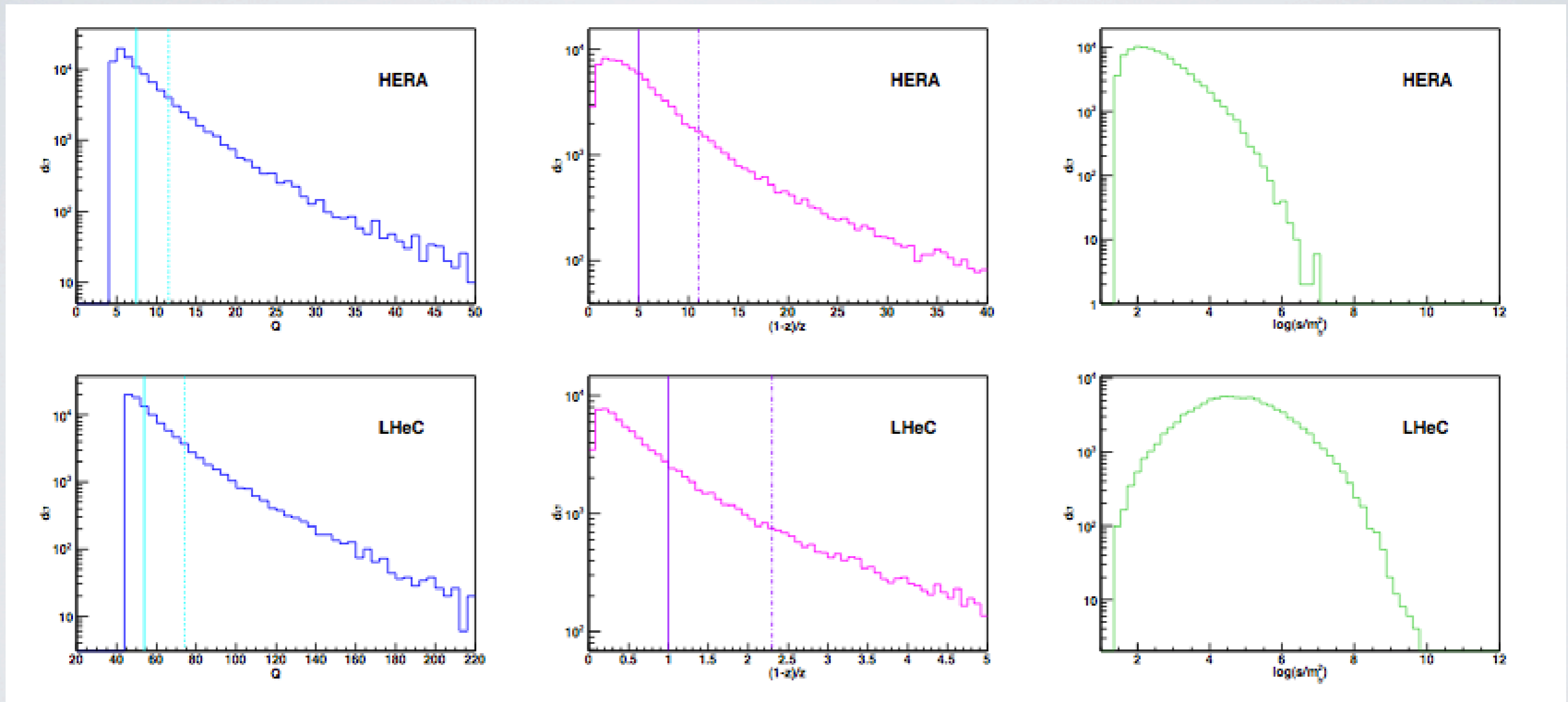
$$= \left(\frac{\pi^2 \alpha_e e_b^2 C_F}{2Q^2} \right) \frac{\alpha_S}{2\pi} z P_{qg}(z) \left[\log \frac{m_b^2}{s} + O\left(\frac{m_b^2}{s}\right) \right],$$

i.e., doing it properly, one sees that the naively expected $\log Q^2/m_b^2$ is actually:

$$L_{\text{DIS}} \equiv \log \left[\frac{Q^2}{m_b^2} \frac{1-z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2}$$

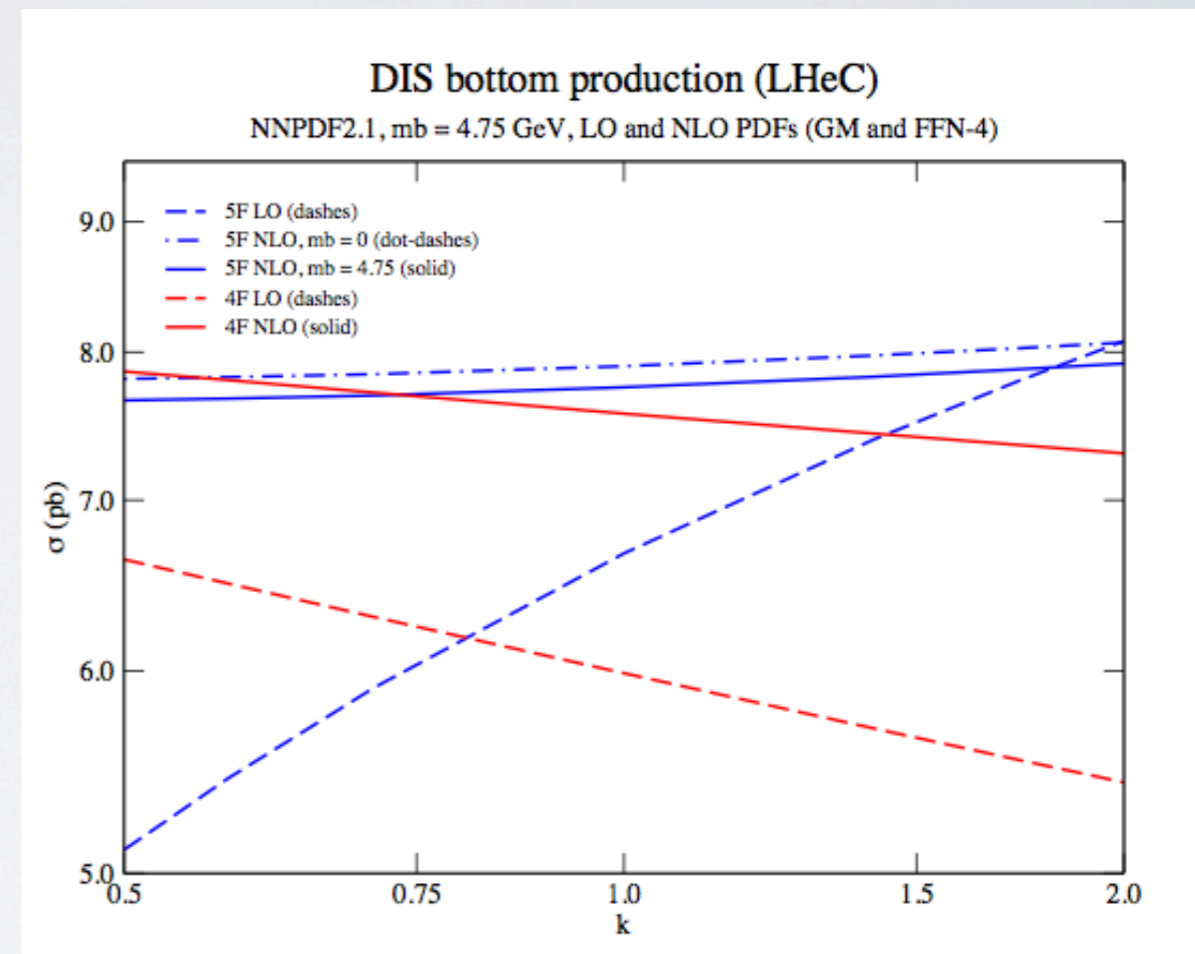
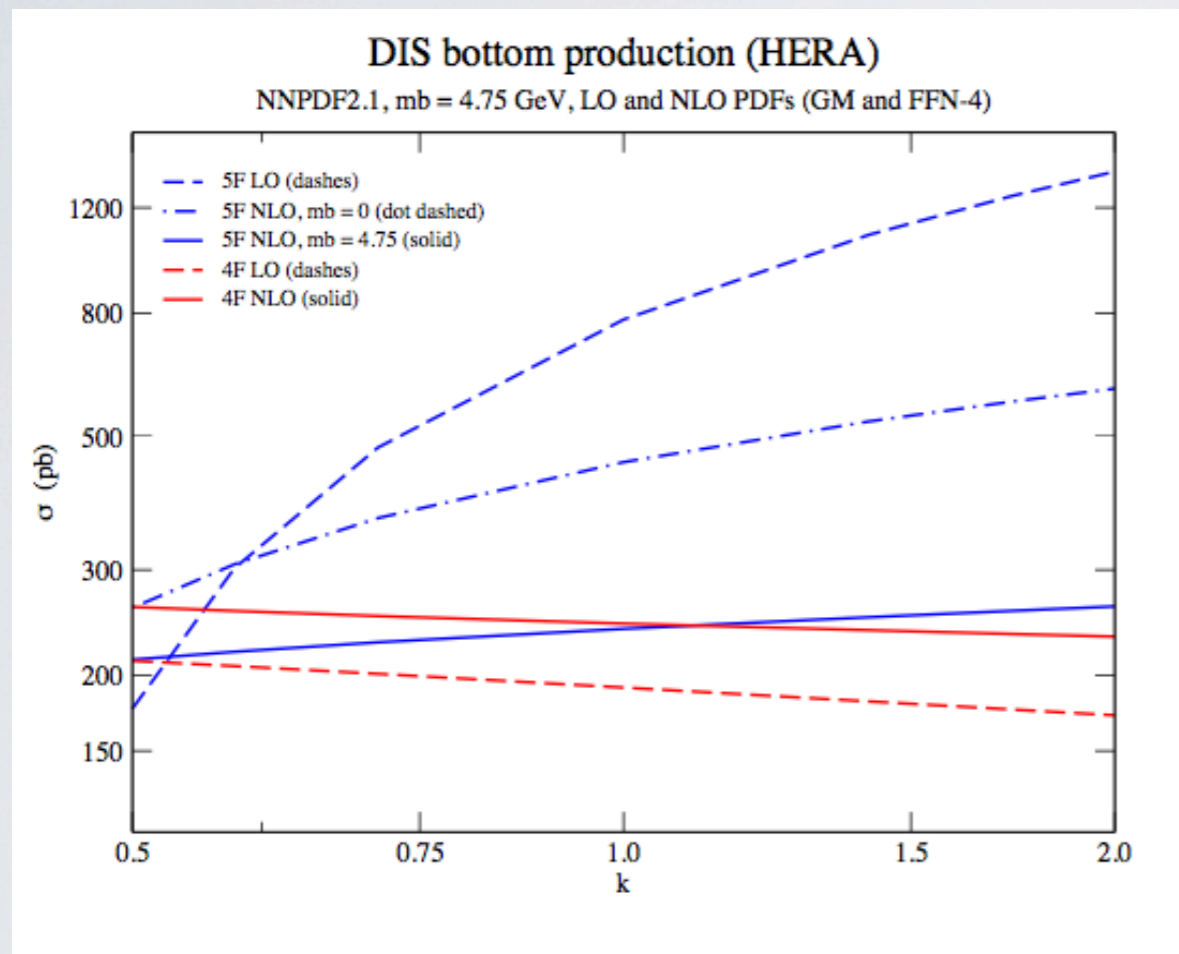
THE UNIVERSAL LOGS : DIS

$$L_{\text{DIS}} \equiv \log \left[\frac{Q^2}{m_b^2} \frac{1-z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2}$$

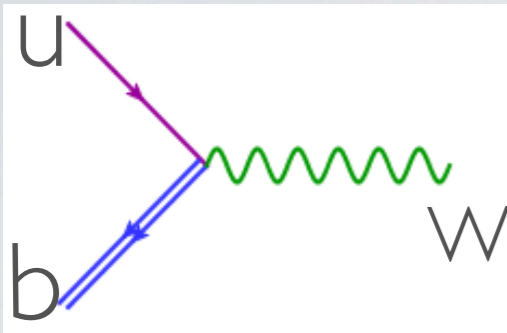


The typical values for $(1-z)/z$ lead to an enhancement of the log at HERA and ~ 1 at the LHeC

THE UNIVERSAL LOGS : DIS

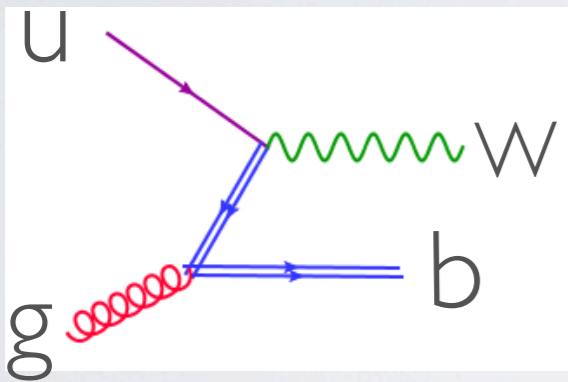


THE UNIVERSAL LOGS : DY



$$b(k_1) + u(k_2) \longrightarrow W(k)$$

$$\sigma^{5F}(\tau) = \left(\frac{\pi\sqrt{2}}{3} G_F \tau \right) \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z}, \mu_F^2 \right) \frac{\alpha_S}{2\pi} P_{qg}(z) \log \frac{\mu_F^2}{m_b^2} + \dots$$



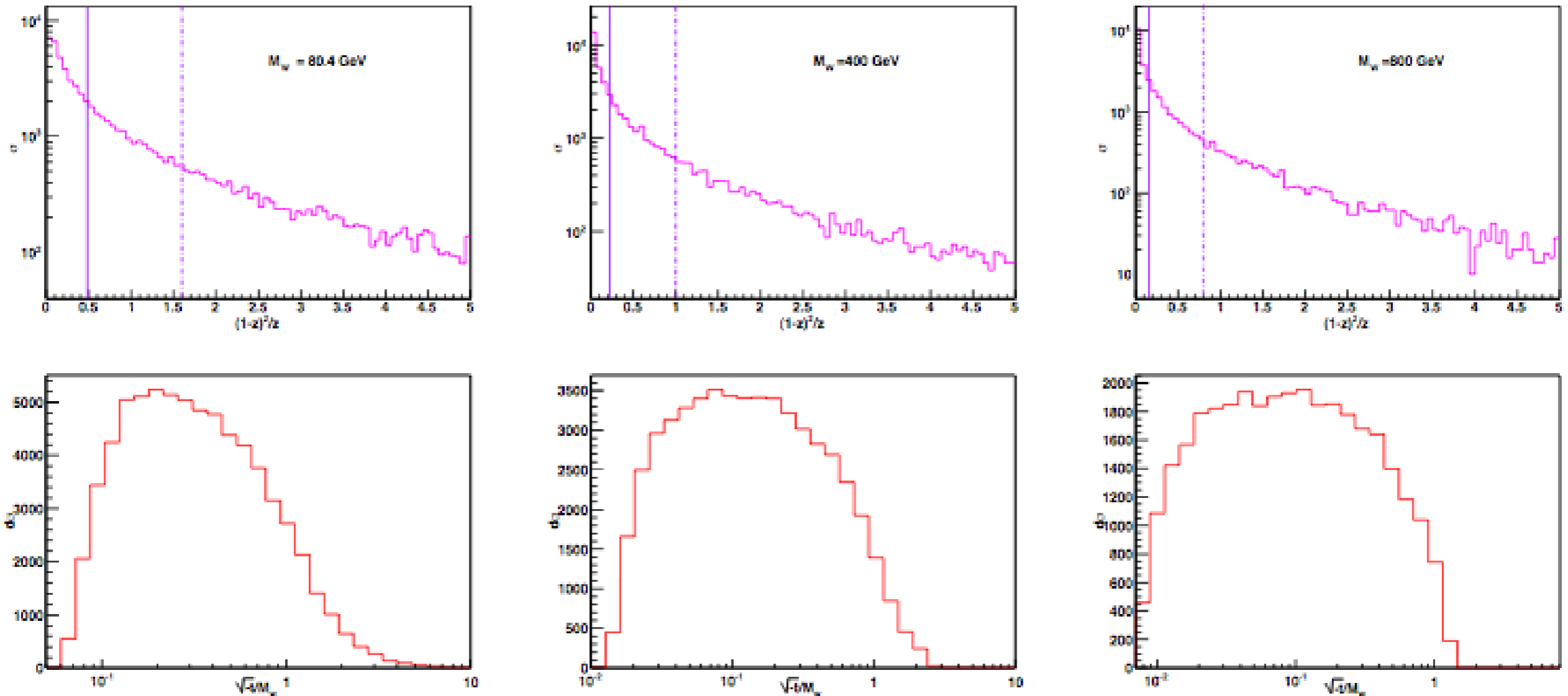
$$g(p_1) + u(p_2) \longrightarrow b(p_3) + W(p_4)$$

$$\hat{\sigma}^{4F}(z) = \int_{t_-}^{t_+} dt \frac{d\hat{\sigma}}{dt}(s, t, \alpha_S) = \frac{\alpha_s}{2\pi} \left(\pi \frac{\sqrt{2}}{3} G_F \right) z \frac{z^2 + (1-z)^2}{2} \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right] + \mathcal{O}(m_b^0)$$

$$\sigma^{4F}(\tau) = \left(\pi \frac{\sqrt{2}}{3} G_F \tau \right) \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) \frac{\alpha_S}{2\pi} P_{qg}(z) L_{DY} + \mathcal{O}(m_b^0)$$

$$L_{DY} \equiv \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right].$$

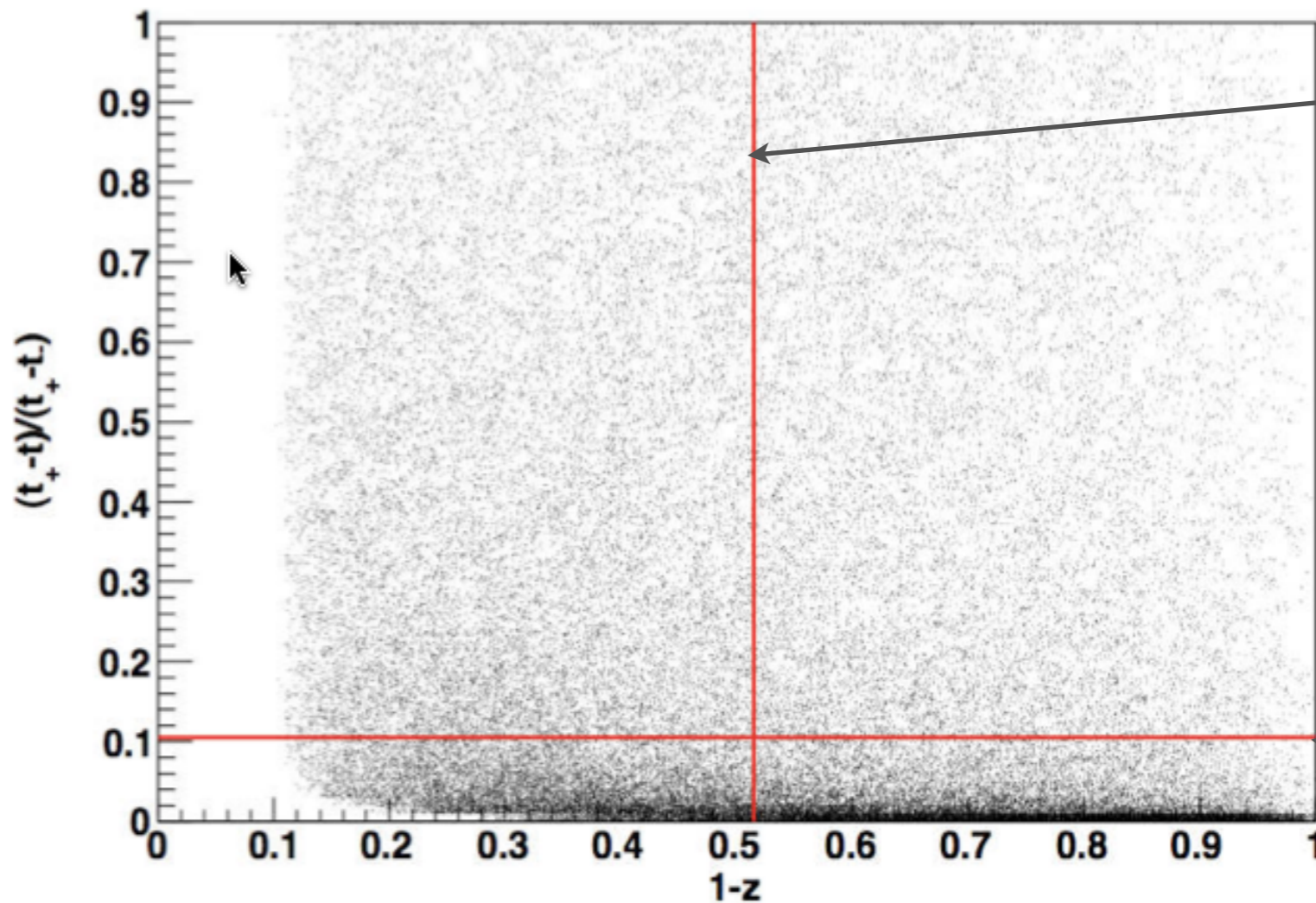
THE UNIVERSAL LOGS : DY



The typical values for $(1-z)^2/z$ and t lead to a suppressed L_{DY}

THE UNIVERSAL LOGS : DY

Are these logs “soft” or “collinear”?



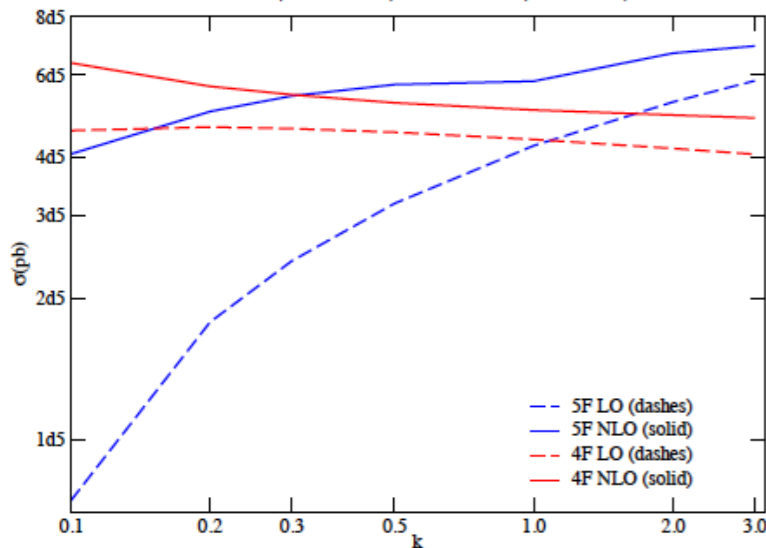
Soft Median

Collinear Median

THE UNIVERSAL LOGS : DY

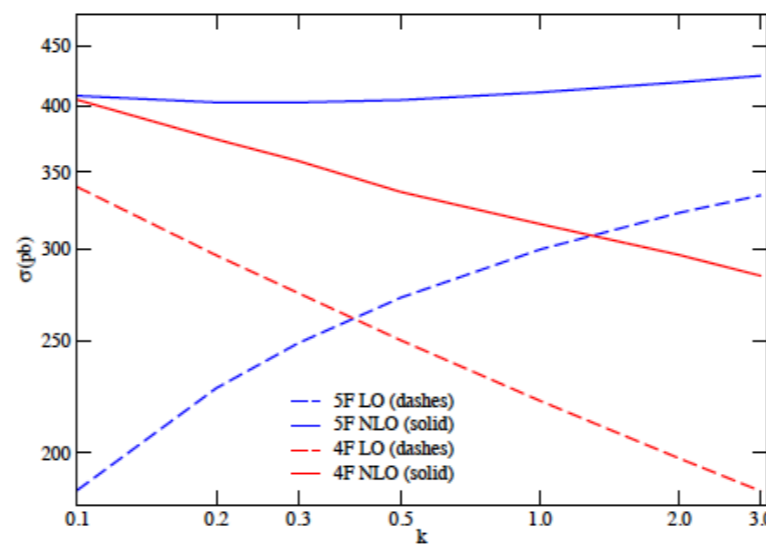
Bottom-initiated W production

NNPDF2.1, $M_W = 80.4$ GeV, LO and NLO PDFs (GM and FFN-4)



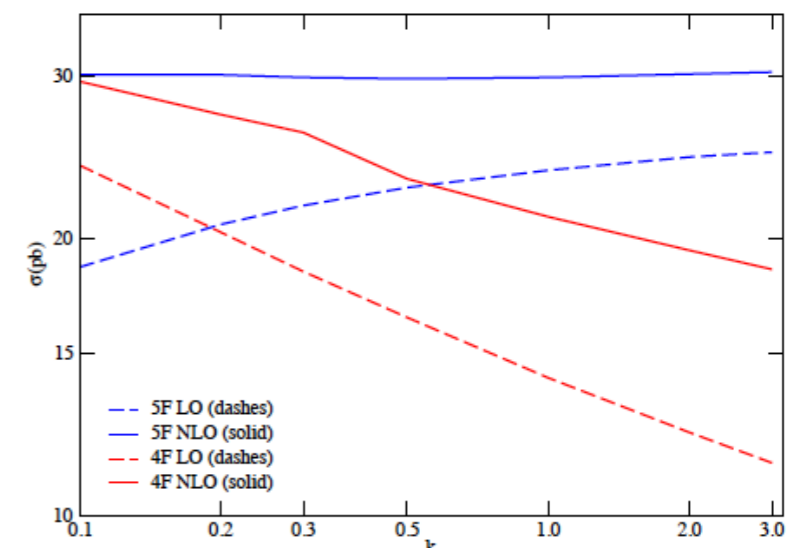
Bottom-initiated W production

NNPDF2.1, $M_W = 400$ GeV, LO and NLO PDFs (GM and FFN-4)



Bottom-initiated W production

NNPDF2.1, $M_W = 800$ GeV, LO and NLO PDFs (GM and FFN-4)



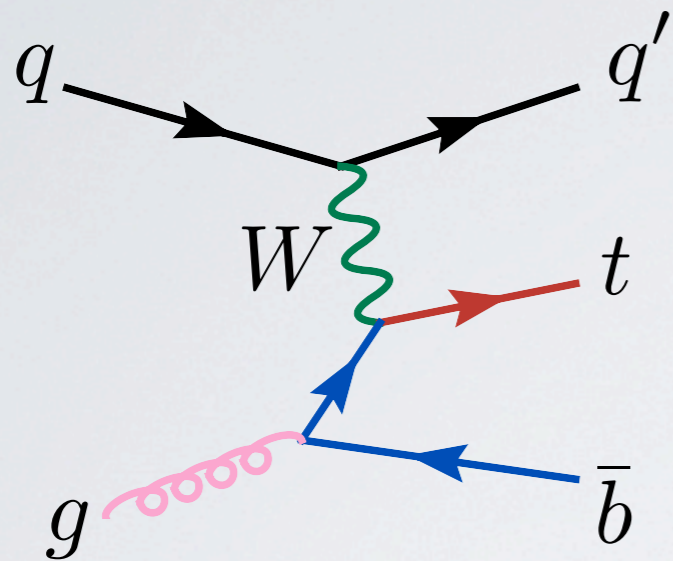
$$\log \frac{\tilde{\mu}_F^2}{m_b^2} = \frac{\int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) P_{qg}(z) \log \left[\frac{M_W^2}{m_b^2} \frac{(1-z)^2}{z} \right]}{\int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ug} \left(\frac{\tau}{z} \right) P_{qg}(z)}$$

$$M_W = 80 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.4, 0.5] M_W$$

$$M_W = 400 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.3, 0.4] M_W$$

$$M_W = 800 \text{ GeV} \quad , \quad \tilde{\mu}_F \simeq [0.25, 0.35] M_W$$

THE UNIVERSAL LOGS : SINGLE-TOP



$$Q^2 \rightarrow 0 \Rightarrow \text{Drell-Yan}$$

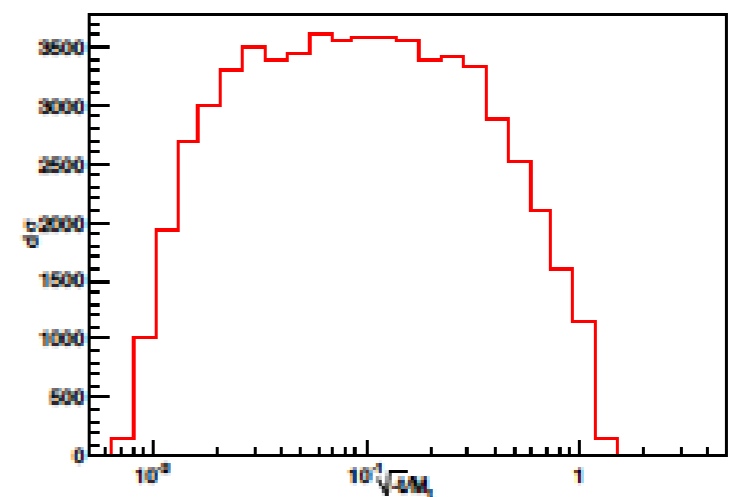
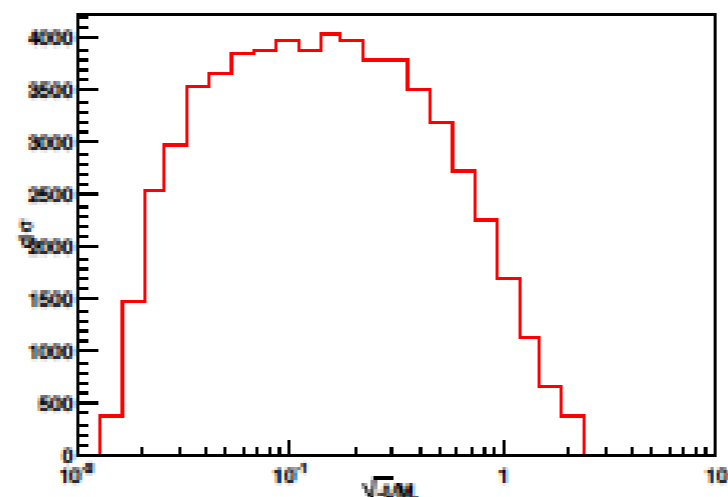
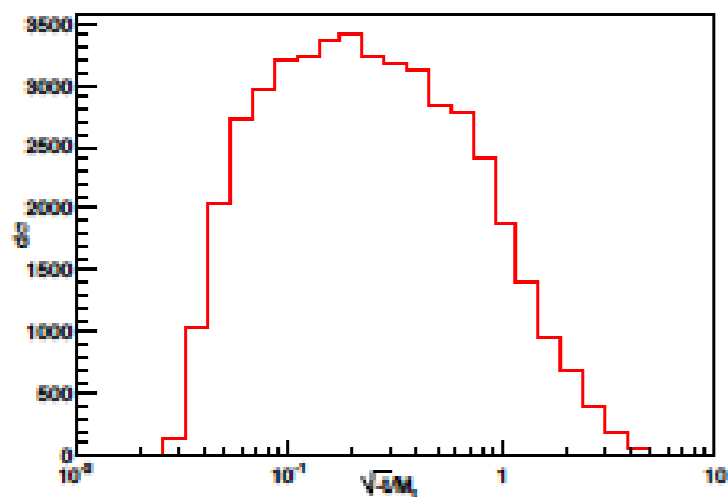
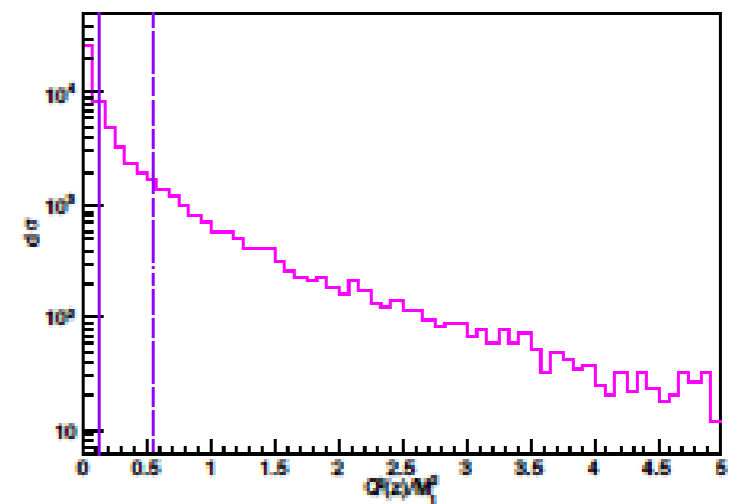
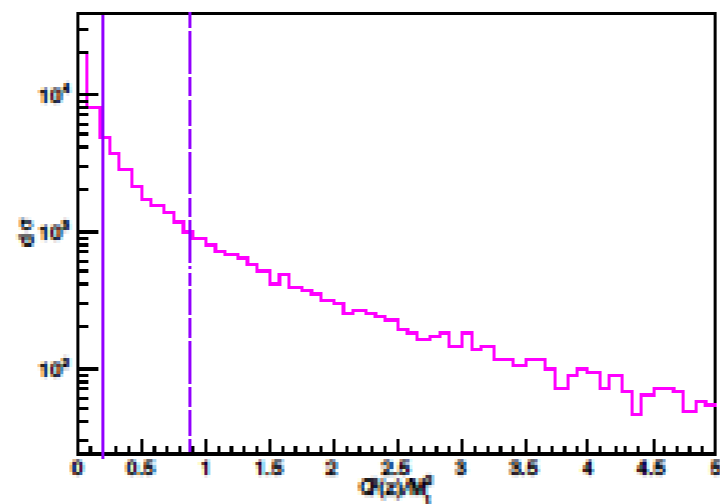
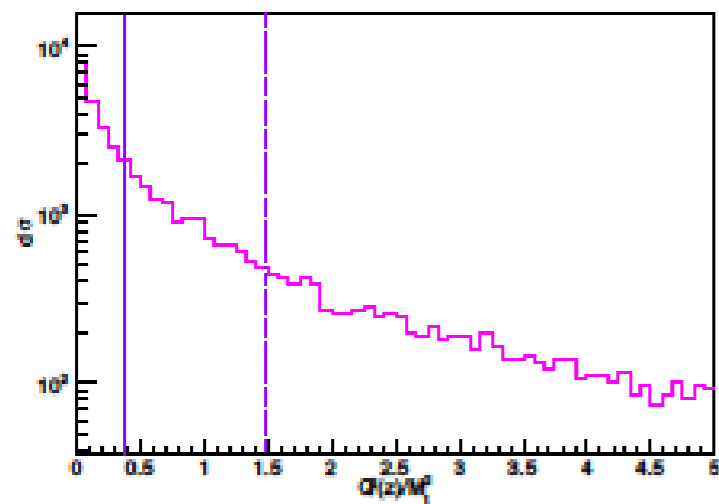
$$M_t \rightarrow m_b \Rightarrow \text{DIS}$$

The same procedure followed before leads to:

$$\int_{t_{\min}}^{t_{\max}} dt \frac{d\hat{\sigma}_2^{4F}}{dt} = \frac{3\alpha_S g_W^2 C_F}{64(s+Q^2)} \frac{z^2 + (1-z)^2}{2} \log \frac{Q^2(z)}{m_b^2}, \quad z = \frac{M^2 + Q^2}{s + Q^2}$$

$$Q^2(z) = (M^2 + Q^2) \frac{(1-z)^2}{z} \frac{1}{1 - \frac{zQ^2}{M^2 + Q^2}}$$

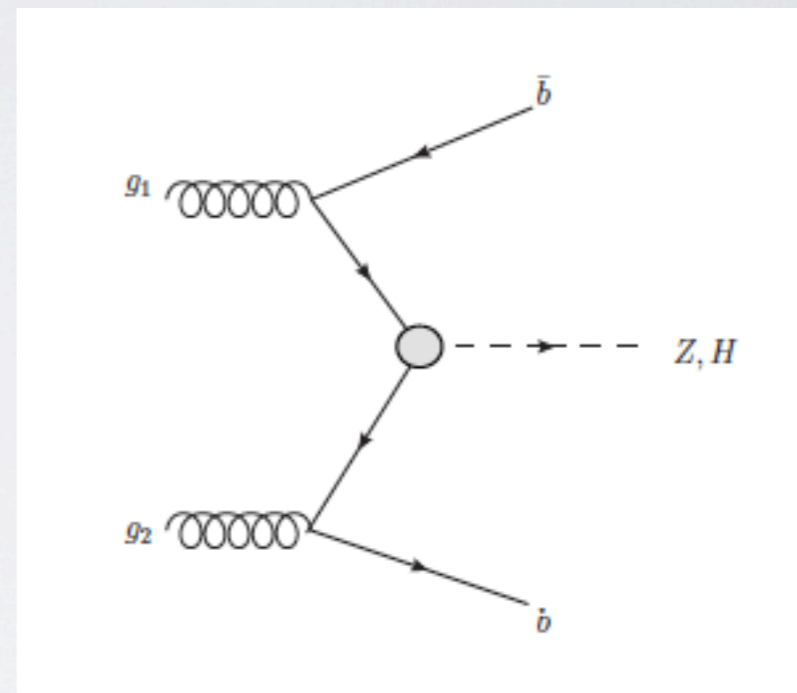
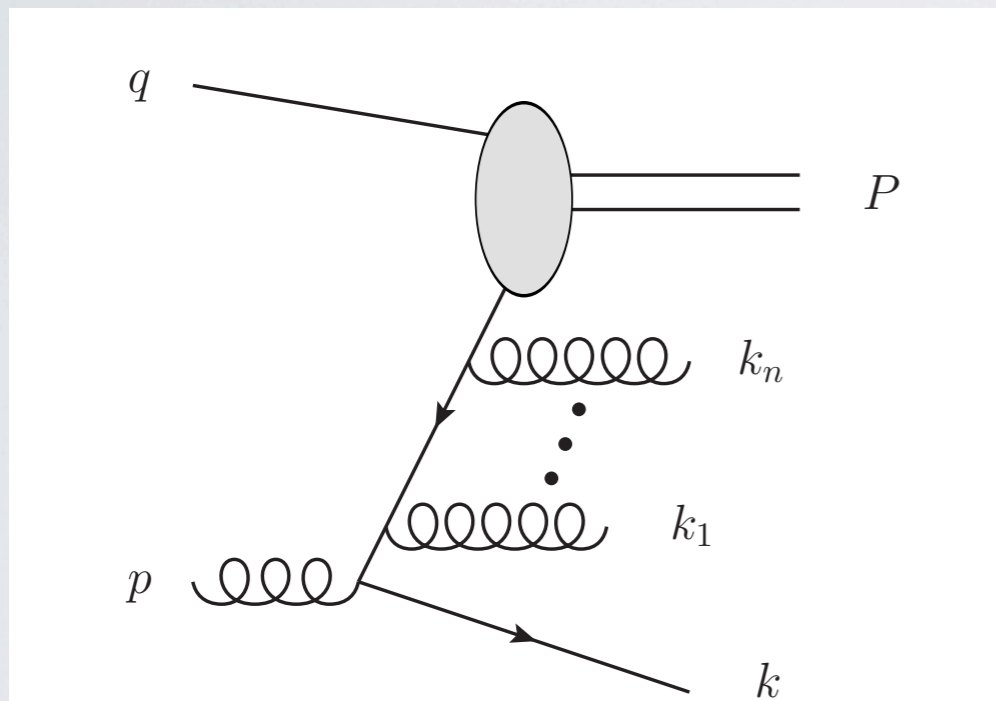
THE UNIVERSAL LOGS : SINGLE-TOP



The typical values for Q^2 and t lead to a suppressed Logarithm

THE UNIVERSAL LOGS

$$I(q) + g(p) \rightarrow b(k) + g(k_1) + \dots + g(k_n) + X(P)$$



$$L_{\text{UNIV}} = \log \frac{Q^2(z)}{m_b^2}$$

$$L_{\text{UNIV}}^2 = \log^2 \frac{Q^2(z)}{m_b^2}$$

CONCLUSIONS & OUTLOOK

We have shown that a large set of TH results obtained in the 4F and 5F schemes at higher order can be consistently understood, taking into account that:

1. The resummation effects of the initial state logarithms in to the b-PDF is important only at large Bjorken- x .
2. The initial state $L = \log Q^2(z)/m_b^2$ associated to a generic one-b-initiated process at the LHC (single top encompassing all other cases) can be written in terms of

$$Q^2(z) = (M^2 + Q^2) \frac{(1-z)^2}{z} \frac{1}{1 - \frac{zQ^2}{M^2+Q^2}}$$

$$z = \frac{M^2 + Q^2}{s + Q^2}$$

$$Q^2 \rightarrow 0 \Rightarrow L_{\text{DY}} = \log \left[\frac{M^2 (1-z)^2}{m_b^2 z} \right]$$

$$M \rightarrow 0 \Rightarrow L_{\text{DIS}} = \log \left[\frac{Q^2 (1-z)}{m_b^2 z} \right]$$

QUESTIONS AND PUZZLES: ANSWERS

- At the level of total cross section 5F predictions are in general better behaved than 4F.
 - However, a substantial and unexpected agreement between 4F and 5F is found when scales smaller than a naive choice is made.
 - Agreement is found even in regions where the logs should be large. Only exception seems to take place for very heavy object production.
 - Independently of 5F results: No sign of breakdown of the perturbative expansion for 4F in total cross sections as well as for more exclusive observables is found.
- ➔ 5F predictions formally always start at one power in α_s less, do resum logs (large or small) and therefore display a rather milder scale dependence. Finally for some procs we have NNLO calculations available.
 - ➔ The agreement is found for scales that are indicated by the 4F calculations themselves. No artificial tuning is necessary. This is due to the same kinematical mechanism that suppresses the possibly large logs. An average scale of the order of the p_T of the spectator b also falls in the same ball park.
 - ➔ The effect of resummation is in general small so a 4F calculation at NLO catches already the main logs. The logs are anyway smaller than what one would guess. Very heavy objects compared to the total energy available production demands large Bjorken- x and here the resummation effects are the largest.
 - ➔ The logs are small, so there is no clear call for resummation from the calculation itself.

CONCLUSIONS & OUTLOOK

- We have also argued that our findings generalized to the case of two b's in the initial state, yet a detailed 4F/5F comparison in this context has still to come.
- A substantial and justified agreement between 4F and 5F calculations for a given process means that both calculations can be used in different contexts.
- When available 5F calculations at NNLO clearly give very useful predictions for total cross sections and should be certainly used in that case. 5F calculations should also be used for processes involving very high Bjorken-x.
- For exclusive final states 4F calculations are (currently) easily and more reliably obtained at NLO in the form of an event generator and can provide a wider spectrum of observables at NLO accuracy.
- An analogous study for final state gluon splitting at high p_T , and in particular 4F vs 5F approaches in the context of ME+PS merging and NLOwPS not yet available (to be done!).

ACKNOWLEDGMENTS

Applications of the b-pdf framework to phenomenology was introduced to me for the first time by Scott Willenbrock more than ten years ago in the context of single-top and Higgs.

Over the years, I have enjoyed an uncountable number of discussions on this topics with him and with many of the Les Houches enthusiasts and in particular with Francesco Tramontano, Fred Olness, John Campbell, Michael Kraemer, Michael Spira, Michelangelo Mangano, Paolo Nason, Robert Harlander and Stefano Forte.

A big thanks to Maria Ubiali and Giovanni Ridolfi for the fantastic collaboration: without their fresh look, new insights and hard work, I would still be wondering about incomprehensible patterns...