Axiology

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Outline

Introduction

Axions play role in

- the solution for the strong CP problem
  [PRL **38** (1977) 1440, PRL **40** (1978) 223, ...]
- candidates for dark matter
  [PLB **120** (1983) 127, PLB **120** (1983) 133, ...]
- the formulation of natural inflation
  [PRL **65** (1990) 3233, PRD **74** (2006) 123511, ...]
- candidates for quintessence
  [PRD **62** (2000) 043509, JHEP **0006** (2000) 016, ...]

Introduction

Axions play role in

- the solution for the strong CP problem

- the formulation of natural inflation

[JCAP **0501** (2005) 005]

Introduction

Axions play role in

- the solution for the strong CP problem

- candidates for quintessence

[PLB **553** (2003) 1-6]

Formalism

The explicit breaking of a shift symmetry gives rise to a potential of the form

$$V(\varphi) = \Lambda^4 \left[1 - \cos\left(\frac{\varphi}{f_a}\right) \right]$$

$$10^9\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$$

$$10^{-6}\,\mathrm{eV} \lesssim m_a \lesssim 10^{-3}\,\mathrm{eV}$$

[Rev. Mod. Phys. **82** (2010) 557]

$$f_{\mathrm{infl}} \gtrsim 4\,M_{\mathrm{P}}$$

Three-axion model

Three fields: θ, ρ, a

$$\begin{aligned} V(\theta,\,\rho,\,a) &= \Lambda_1^4\left[1-\cos\left(\frac{\theta}{f_1}+\frac{\rho}{g_1}+\frac{a}{h}\right)\right] \\ &+ \Lambda_2^4\left[1-\cos\left(\frac{\theta}{f_2}+\frac{\rho}{g_2}+\frac{a}{h}\right)\right] \\ &+ \Lambda_3^4\left[1-\cos\left(\frac{\theta}{f_3}+\frac{\rho}{g_3}+\frac{a}{h}\right)\right] \end{aligned}$$

Mass matrix:

$$M^2 = \begin{pmatrix} \frac{\Lambda_1^4}{f_1^2}+\frac{\Lambda_2^4}{f_2^2}+\frac{\Lambda_3^4}{f_3^2} & \frac{\Lambda_1^4}{f_1 g_1}+\frac{\Lambda_2^4}{f_2 g_2}+\frac{\Lambda_3^4}{f_3 g_3} & \frac{1}{h}\left(\frac{\Lambda_1^4}{f_1}+\frac{\Lambda_2^4}{f_2}+\frac{\Lambda_3^4}{f_3}\right) \\ \frac{\Lambda_1^4}{f_1 g_1}+\frac{\Lambda_2^4}{f_2 g_2}+\frac{\Lambda_3^4}{f_3 g_3} & \frac{\Lambda_1^4}{g_1^2}+\frac{\Lambda_2^4}{g_2^2}+\frac{\Lambda_3^4}{g_3^2} & \frac{1}{h}\left(\frac{\Lambda_1^4}{g_1}+\frac{\Lambda_2^4}{g_2}+\frac{\Lambda_3^4}{g_3}\right) \\ \frac{1}{h}\left(\frac{\Lambda_1^4}{f_1}+\frac{\Lambda_2^4}{f_2}+\frac{\Lambda_3^4}{f_3}\right) & \frac{1}{h}\left(\frac{\Lambda_1^4}{g_1}+\frac{\Lambda_2^4}{g_2}+\frac{\Lambda_3^4}{g_3}\right) & \frac{\Lambda_1^4+\Lambda_2^4+\Lambda_3^4}{h^2} \end{pmatrix}$$

$$\det M^2 = \frac{\Lambda_1^4\Lambda_2^4\Lambda_3^4}{(f_1 f_2 f_3 g_1 g_2 g_3 h)^2}\left[f_1 f_2 g_3(g_1-g_2)+f_2 f_3 g_1(g_2-g_3)+f_1 f_3 g_2(g_3-g_1)\right]^2$$

Flat directions ($\det M^2 = 0$):

- $f_1 = f_2 = f_3$
- $g_1 = g_2 = g_3$
- $\dfrac{f_1}{g_1} = \dfrac{f_2}{g_2} = \dfrac{f_3}{g_3}$

For simplicity: $f_1 = f_3$ and $g_2 = g_3$

Assumption: $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$

$$\theta,\ \rho,\ a \quad \xrightarrow{\text{step 1}} \quad \varphi,\ \chi,\ \psi$$

$$V_1 = \Lambda_1^4 \left[1 - \cos\left(P_1 \varphi\right)\right]$$
$$V_2 = \Lambda_2^4 \left[1 - \cos\left(P_2 \varphi + Q_2 \chi + R_2 \psi\right)\right]$$
$$V_3 = \Lambda_3^4 \left[1 - \cos\left(P_3 \varphi + Q_3 \chi + R_3 \psi\right)\right]$$

Flat directions ($\det M^2 = 0$):

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For simplicity: $f_1 = f_3$ and $g_2 = g_3$

Assumption: $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$

$$\theta,\ \rho,\ a \quad \xrightarrow{\text{step 1}} \quad \varphi,\ \chi,\ \psi \quad \xrightarrow{\text{step 2}} \quad \varphi,\ \chi_1,\ \psi_1$$

$$V_1 = \Lambda_1^4\left[1 - \cos\left(P_1\varphi\right)\right]$$

$$V_2 = \Lambda_2^4\left[1 - \cos\left(P_2\varphi + \tilde{Q}_2\chi_1\right)\right]$$

$$V_3 = \Lambda_3^4\left[1 - \cos\left(P_3\varphi + \tilde{Q}_3\chi_1 + \tilde{R}_3\psi_1\right)\right]$$

Masses

$$m_\varphi^2 \simeq \left(\frac{1}{f_1^2}+\frac{1}{g_1^2}+\frac{1}{h^2}\right)\Lambda_1^4$$

$$m_{\chi_1}^2 \simeq \frac{A}{f_2^2 g_2^2 (g_1^2 h^2+f_1^2(g_1^2+h^2))^2}\ \Lambda_2^4$$

$$m_{\psi_1}^2 \simeq \frac{(f_1-f_2)^2(g_2-g_1)^2(f_1^2+g_1^2)(f_1^2+h^2)}{A}\Lambda_3^4$$

Decay constants: $m=\Lambda^2/f$

$$f_\varphi \simeq \frac{f_1 g_1 h}{\sqrt{g_1^2 h^2+f_1^2(g_1^2+h^2)}}$$

$$f_{\chi_1} \simeq \frac{f_2 g_2 (g_1^2 h^2+f_1^2(g_1^2+h^2))}{\sqrt{A}}$$

$$f_{\psi_1} \simeq \frac{\sqrt{A}}{|f_1-f_2||g_2-g_1|\sqrt{f_1^2+g_1^2}\sqrt{f_1^2+h^2}}$$

Masses

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$$m_{\chi_1}^2 \simeq \frac{A}{f_2^2 g_2^2 (g_1^2 h^2+f_1^2(g_1^2+h^2))^2}\ \Lambda_2^4$$

$$m_{\psi_1}^2 \simeq \frac{(f_1-f_2)^2(g_2-g_1)^2(f_1^2+g_1^2)(f_1^2+h^2)}{A}\Lambda_3^4$$

Decay constants: $m=\Lambda^2/f$

$$f_{\varphi} \simeq \frac{f_1 g_1 h}{\sqrt{g_1^2 h^2+f_1^2(g_1^2+h^2)}}$$

$$f_{\chi_1} \simeq \frac{f_2 g_2 (g_1^2 h^2+f_1^2(g_1^2+h^2))}{\sqrt{A}}$$

$$f_{\psi_1} \simeq \frac{\sqrt{A}}{|f_1-f_2||g_2-g_1|\sqrt{f_1^2+g_1^2}\sqrt{f_1^2+h^2}}$$

Parameter	Value
h	M_{P}
$f_1 = f_3$	$0.15\ M_{\mathrm{P}}$
$g_2 = g_3$	$0.125\ M_{\mathrm{P}}$
f_2	$0.125\ M_{\mathrm{P}}$
g_1	$0.15\ M_{\mathrm{P}}$

Masses

$$m_\varphi^2 \simeq \left(\frac{1}{f_1^2} + \frac{1}{g_1^2} + \frac{1}{h^2}\right) \Lambda_1^4$$

$$m_{\chi_1}^2 \simeq \frac{A}{f_2^2 g_2^2 (g_1^2 h^2 + f_1^2 (g_1^2 + h^2))^2} \Lambda_2^4$$

$$m_{\psi_1}^2 \simeq \frac{(f_1 - f_2)^2 (g_2 - g_1)^2 (f_1^2 + g_1^2)(f_1^2 + h^2)}{A} \Lambda_3^4$$

Decay constants: $m = \Lambda^2/f$

$$f_\varphi \simeq \frac{f_1 g_1 h}{\sqrt{g_1^2 h^2 + f_1^2 (g_1^2 + h^2)}}$$

$$f_{\chi_1} \simeq \frac{f_2 g_2 (g_1^2 h^2 + f_1^2 (g_1^2 + h^2))}{\sqrt{A}}$$

$$f_{\psi_1} \simeq \frac{\sqrt{A}}{|f_1 - f_2||g_2 - g_1| \sqrt{f_1^2 + g_1^2} \sqrt{f_1^2 + h^2}}$$

$$m_\varphi^2 \simeq 10^2 \Lambda_1^4$$

$$m_{\chi_1}^2 \simeq 0.04 \Lambda_2^4 \xrightarrow{\Lambda_2 \simeq 10^{16}\,\mathrm{GeV}} m_{\mathrm{infl}} \simeq 10^{12}\,\mathrm{GeV}$$

$$m_{\psi_1}^2 \simeq 0.83 \Lambda_3^4 \xrightarrow{\Lambda_3 \simeq 0.003\,\mathrm{eV}} m_{\mathrm{QA}} \simeq 10^{-32}\,\mathrm{eV}$$

$$f_\varphi \simeq 0.1\, M_{\mathrm{P}}$$

$$f_{\chi_1} \simeq 5.0\, M_{\mathrm{P}}$$

$$f_{\psi_1} \simeq 1.1\, M_{\mathrm{P}}$$

Four-axion model

$$V = \sum_{i=1}^{4} V_i = \sum_{i=1}^{4} \Lambda_i^4 \left[1 - \cos\left(\frac{\theta}{f_i} + \frac{\rho}{g_i} + \frac{\phi}{h_i} + \frac{a}{h} \right) \right]$$

Assumptions: $f_1 = f_2 = f_4$, $g_2 = g_3 = g_4$ and $h_1 = h_2 = h_3$

$$\det M^2 = \frac{\Lambda_1^4 \Lambda_2^4 \Lambda_3^4 \Lambda_4^4}{(f_1 f_3 g_1 g_2 h_1 h_4 h)^2} (f_1 - f_3)^2 (g_1 - g_2)^2 (h_1 - h_4)^2$$

Flat direction: $f_1 = f_3$, $g_1 = g_2$ and $h_1 = h_4$

$$(\theta,\, \rho,\, \phi,\, a) \quad \longrightarrow \quad (\varphi,\, \chi,\, \psi,\, \omega)$$

$$V_1 = \Lambda_1^4 \left[1 - \cos\left(A_1 \varphi \right) \right]$$

$$V_2 = \Lambda_2^4 \left[1 - \cos\left(A_2 \varphi + B_2 \chi \right) \right]$$

$$V_3 = \Lambda_3^4 \left[1 - \cos\left(A_3 \varphi + B_3 \chi + C_3 \psi \right) \right]$$

$$V_4 = \Lambda_4^4 \left[1 - \cos\left(A_4 \varphi + B_4 \chi + C_4 \psi + D_4 \omega \right) \right]$$

Values of scales, masses and decay constants

$$\begin{array}{ccccccc} \Lambda_1 & \gg & \Lambda_2 & \gg & \Lambda_3 & \gg & \Lambda_4 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ M_{\rm P} & & M_{\rm GUT} & & \Lambda_{\rm QCD} & & E_{\rm vac} \end{array}$$

Masses

$m_\varphi \simeq 10^{18}\,\mathrm{GeV}$

$m_\chi \simeq 10^{12}\,\mathrm{GeV}$

$m_\psi \simeq 10^{-4}\,\mathrm{eV}$

$m_\omega \simeq 10^{-32}\,\mathrm{eV}$

Decay constants

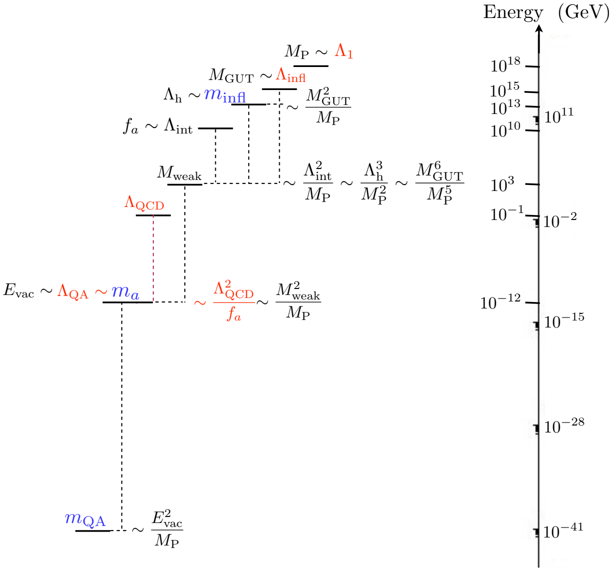
$f_\varphi \simeq 0.4\,M_{\rm P}$

$f_\chi \simeq 4.9\,M_{\rm P}$

$f_\psi \simeq 0.5 \times 10^{-6}\,M_{\rm P}$

$f_\omega \simeq 1.0\,M_{\rm P}$

Axionic see-saw



$$M_P \sim m_\varphi$$

$$\downarrow R \sim \frac{\Lambda_h}{M_P}$$

$$\Lambda_h \sim m_{infl}$$

$$\downarrow R^2$$

$$M_{weak}$$

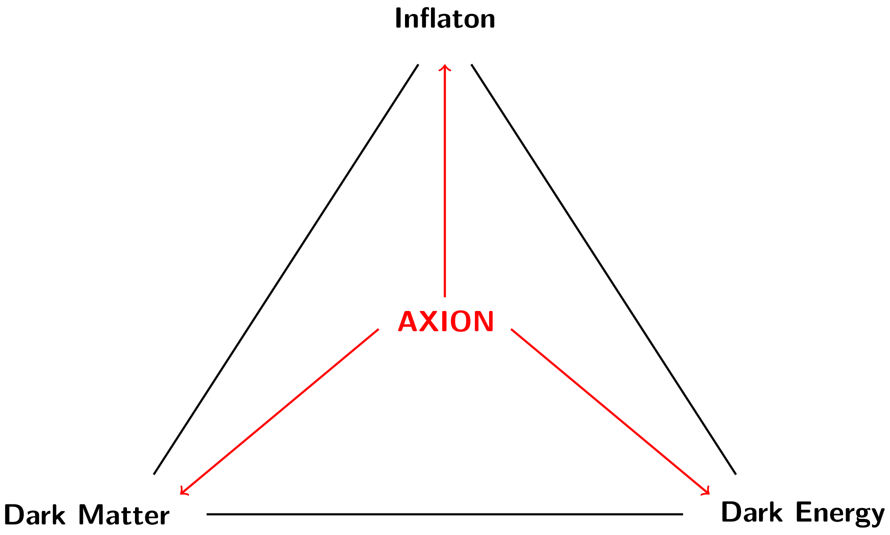
$$\downarrow R^3$$

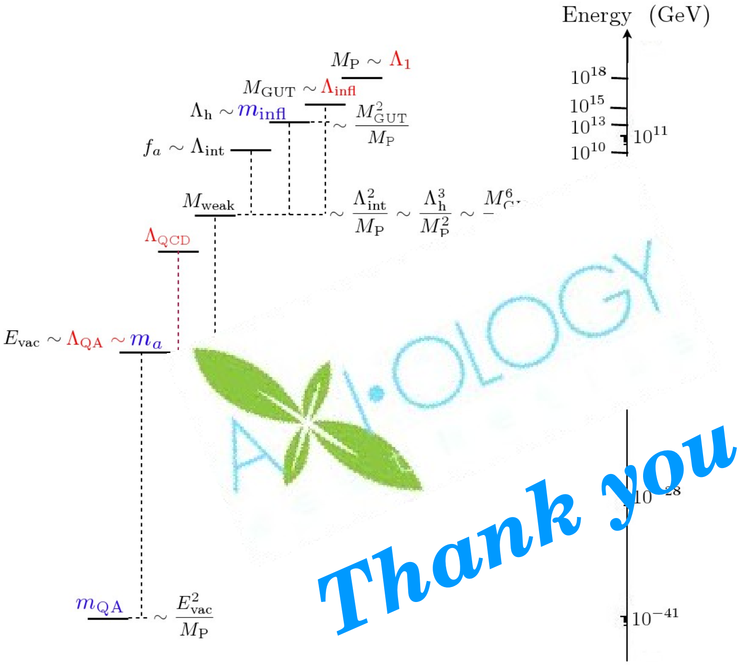
$$E_{vac} \sim m_a$$

$$\downarrow R^6$$

$$m_{QA}$$

Conclusions




Energy (GeV)
10^{18}
10^{15}
10^{13}
10^{11}
10^{10}
10^{-28}
10^{-41}
$M_P \sim \Lambda_1$
$M_{GUT} \sim \Lambda_{infl}$
$\Lambda_h \sim m_{infl}$
$\sim \frac{M_{GUT}^2}{M_P}$
$f_a \sim \Lambda_{int}$
M_{weak}
$\sim \frac{\Lambda_{int}^2}{M_P} \sim \frac{\Lambda_h^3}{M_P^2} \sim$
Λ_{QCD}
$E_{vac} \sim \Lambda_{QA} \sim m_a$
m_{QA}
$\sim \frac{E_{vac}^2}{M_P}$
AXIOLOGY
Thank you