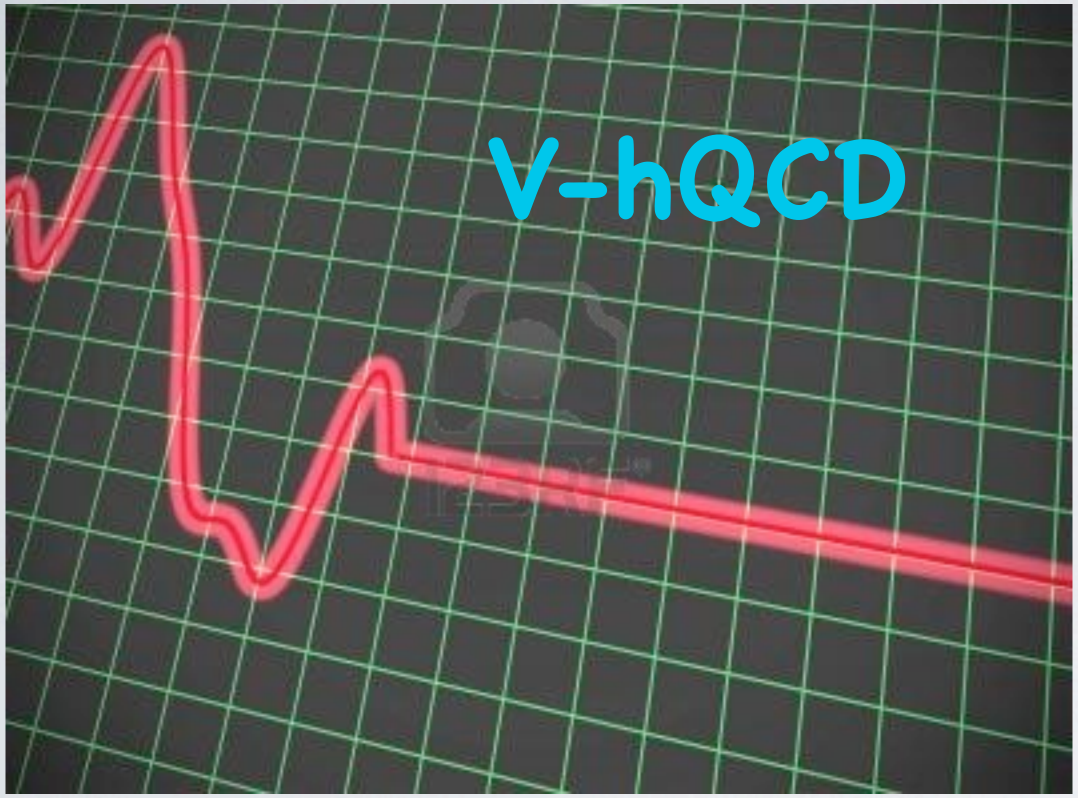
Spectrum of (h)QCD in the Veneziano limit

[work in progress with I. Iatrakis, M. Jarvinen and E. Kiritsis]

Daniel Areán
Corfu, September 2012

FRont Of Galician speaking Scientists

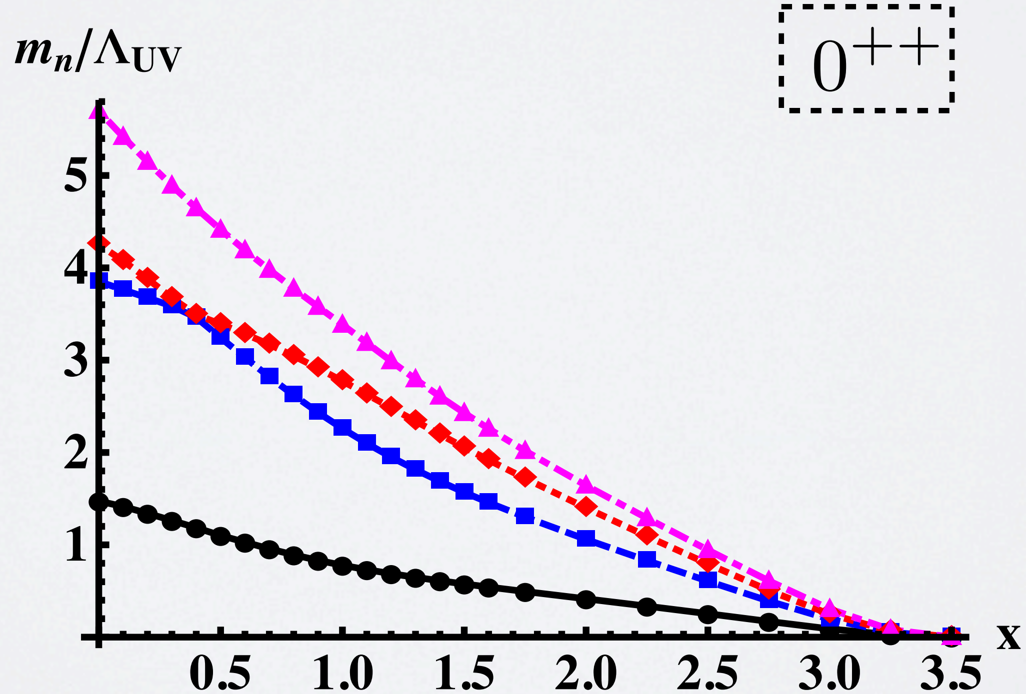


$$\mathcal{S} = \boxed{\text{Einstein} - \text{dilaton}}\,[\text{Glue}] + \boxed{\text{DBI}}\,[\text{Flavor}]$$

(fighting...)

$m_q = 0$

0^{++}



$x = N_f/N_c$

OUTLINE

- V-hQCD: hQCD in the Veneziano limit
  - hQCD (Einstein-dilaton → Glue)
  - Flavor (← DBI), χSB (tachyon condensation)
  - V-hQCD. Phase structure$(x = N_c/N_f)$: χSB, conformal window...
- Computing the Spectrum
  - Action, vacuum solution, DoFs.
  - Sectors (mixing)
  - Some numerical results
- Outlook & To do

★ iHQCD [Gursoy et al'07]

◆ **5d Holographic model $\sim$ large N_c YM (Glue)**

$$S_g = M^3 N_c^2 \int d^5x \sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right)$$

- $\lambda \equiv e^{\Phi} = N_c \, g_{\mathrm{YM}}^2$
- $g_{ab} \xrightarrow{\text{UV}} \mathrm{AdS}_5$

★ iHQCD [Gursoy et al'07]

- **5d Holographic model ∼ large N_c YM (Glue)**

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- $\lambda \equiv e^{\Phi} = N_c \, g_{\mathrm{YM}}^2$
- $g_{ab} \xrightarrow{\text{UV}} \mathrm{AdS}_5$

- **Dilaton Potential V(λ) fixes dynamics:**

$V(\lambda)$:

- $\mathrm{UV}\ (\lambda \to 0)$: $g_{ab} \to AdS_5(l)$ / **2-loop YM β-function** ⟶ $V(\lambda) \sim \frac{12}{l^2}(1 + v_1\,\lambda + v_2\,\lambda^2)$ **does the job ✔**
- $\mathrm{IR}\ (\lambda \to \infty)$: **Confinement (& Linear spectrum)** $m_n^2 \propto n$ ⟵ $V(\lambda) \sim \lambda^{4/3}\,(\log\lambda)^{4/3}$

◆ iHQCD models nicely the glue sector

Fix potential V(λ) → (numerically) Integrate EoMs → 5d geometry:

- V(λ): UV Asymptotic freedom → IR confinement

- Realistic (linear) glueball spectrum. Fits well to lattice data

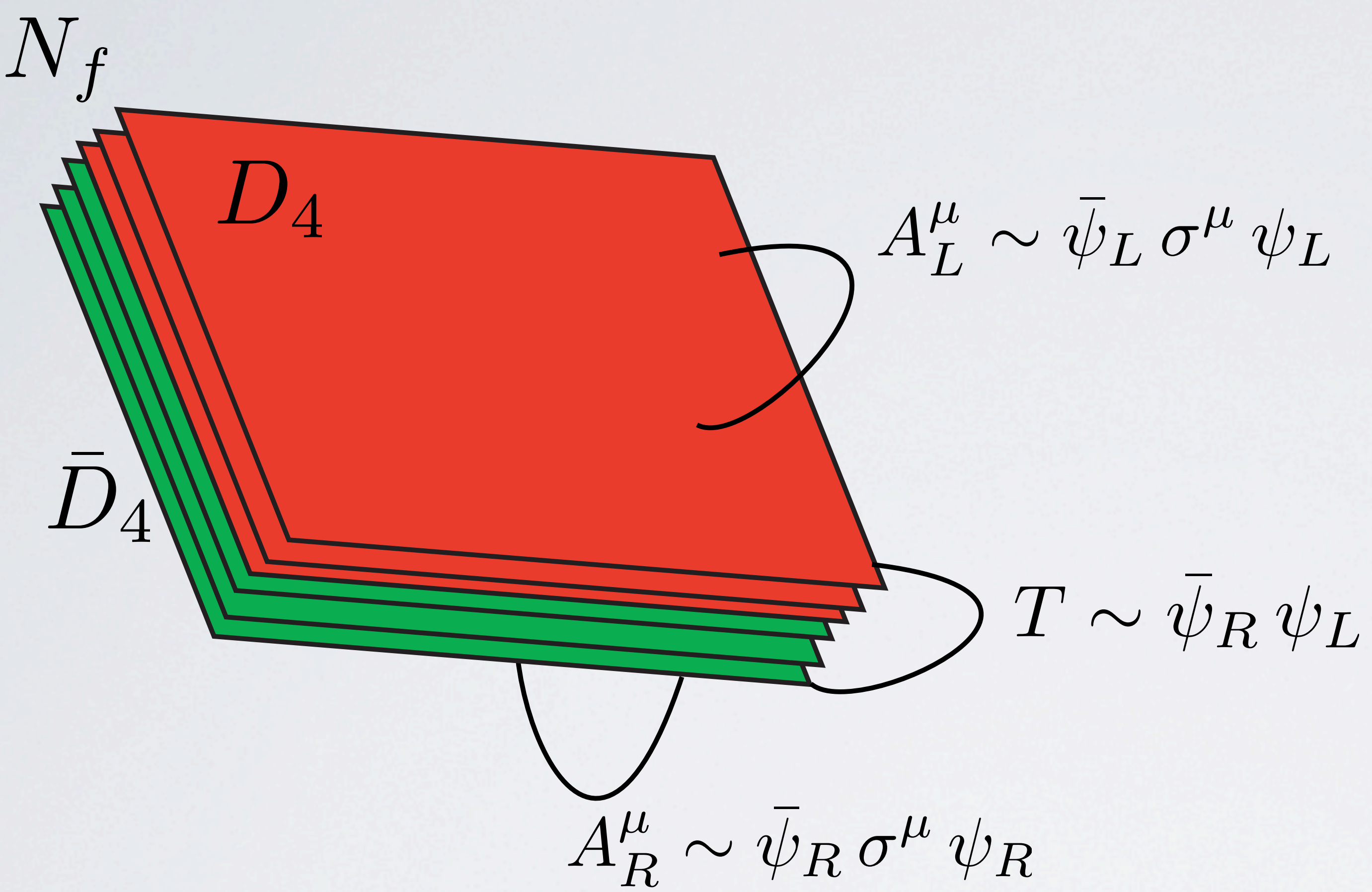
- Generalization to T≠0

  ∃ T_c → deconfinement phase transition

  Good fit to lattice data

. . . what about flavor ?

★ Adding flavor [Casero et al'07]

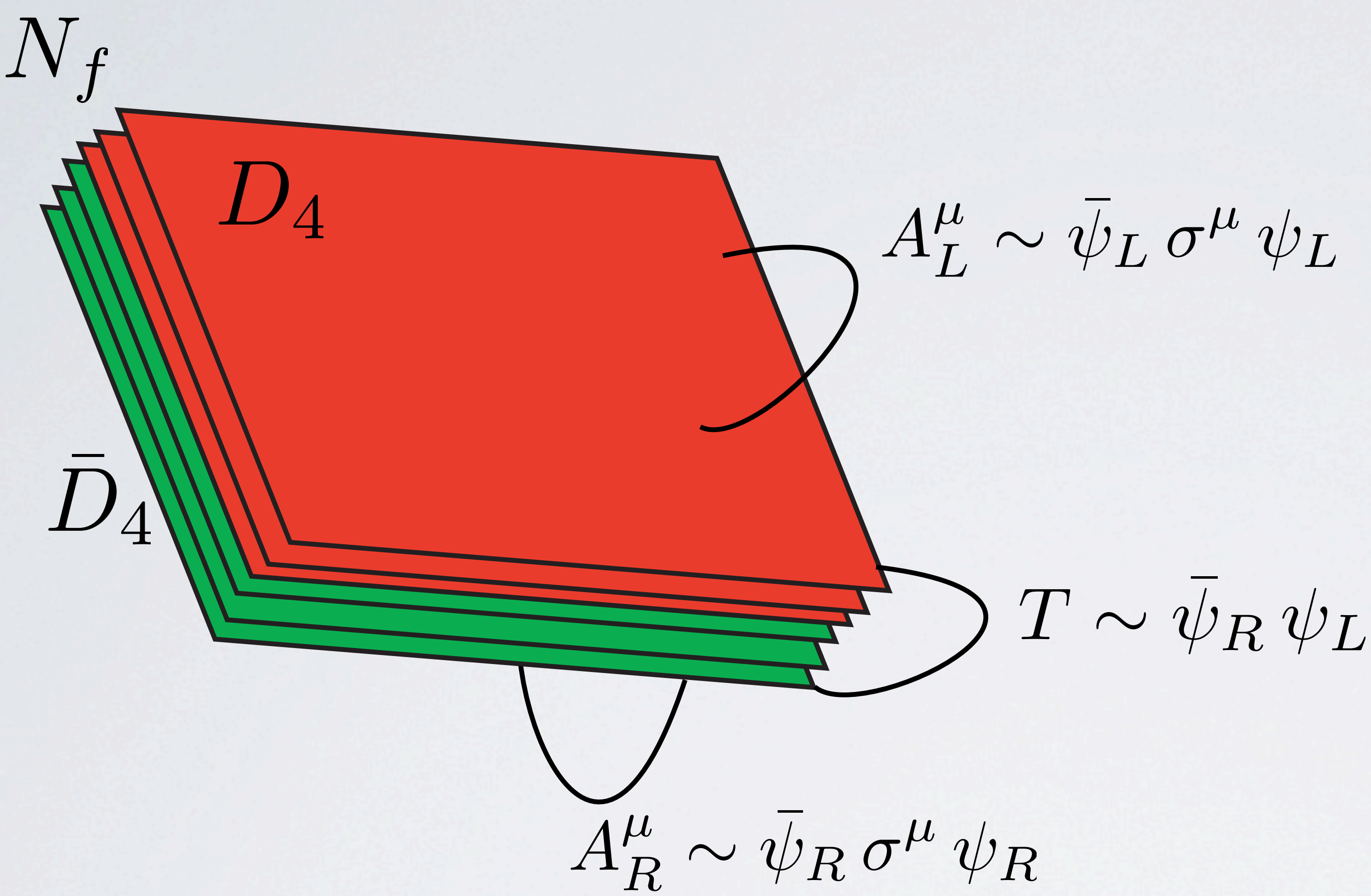


$\longleftrightarrow$

$U(N_f)_L \times U(N_f)_R$

Flavor Sector

★ Adding flavor [Casero et al'07]



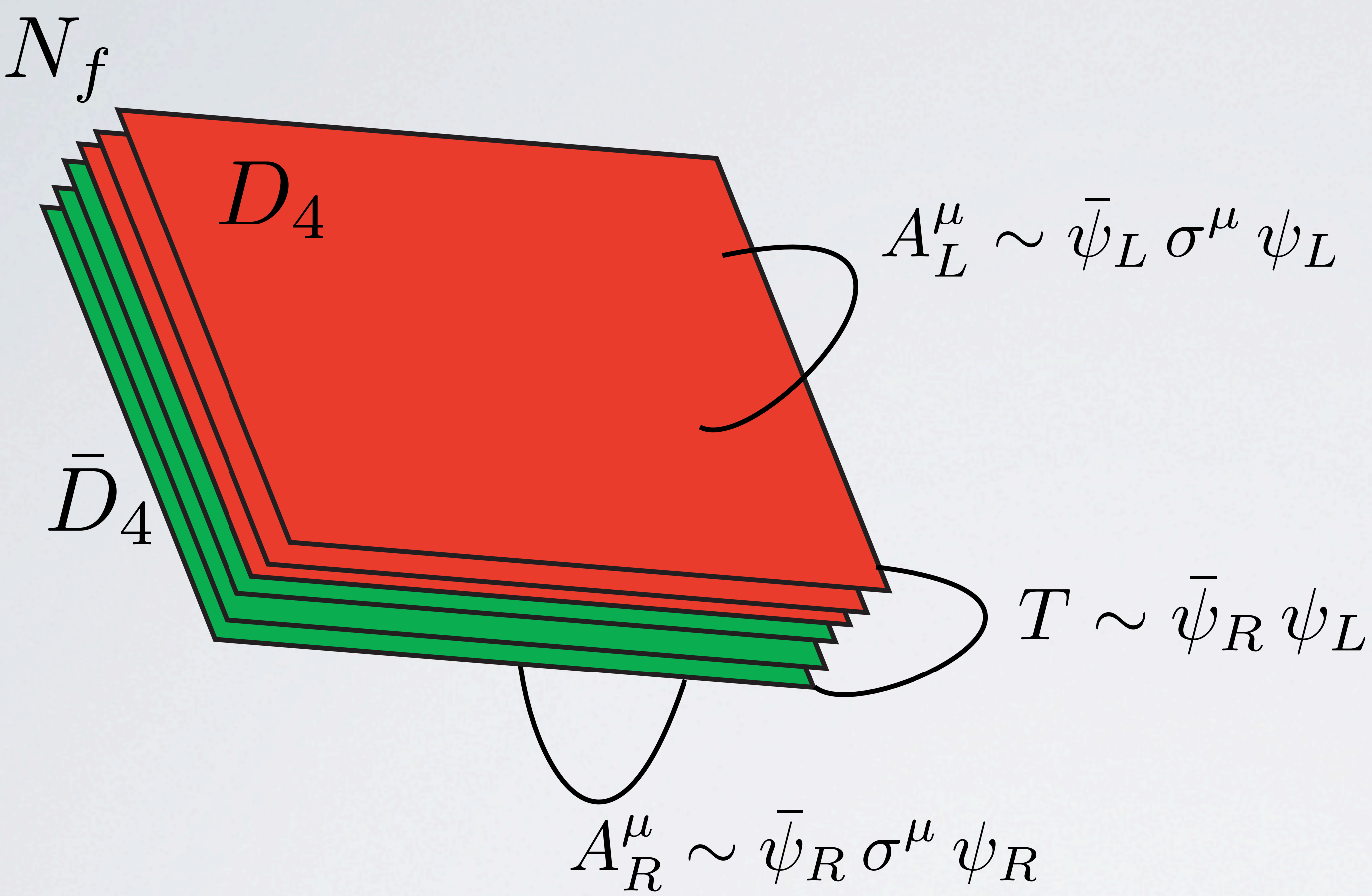
$\longleftrightarrow$

$U(N_f)_L \times U(N_f)_R$

Flavor Sector

FLAVOR SECTOR $\longrightarrow$ $S[T, A^L, A^R] = S_{DBI} + S_{WZ}$

★ Adding flavor [Casero et al'07]



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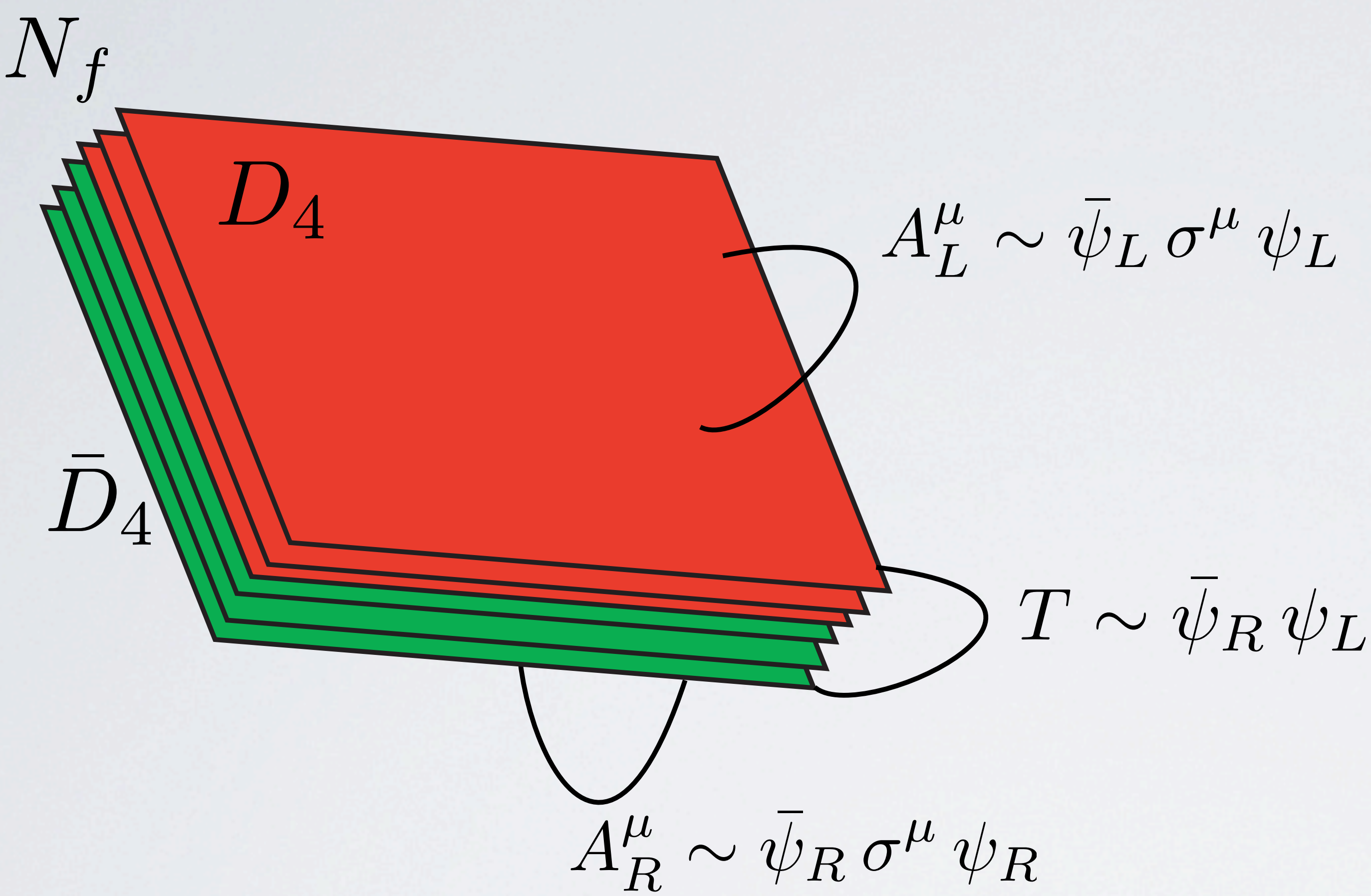


$\longleftrightarrow$

$U(N_f)_L \times U(N_f)_R$

Flavor Sector

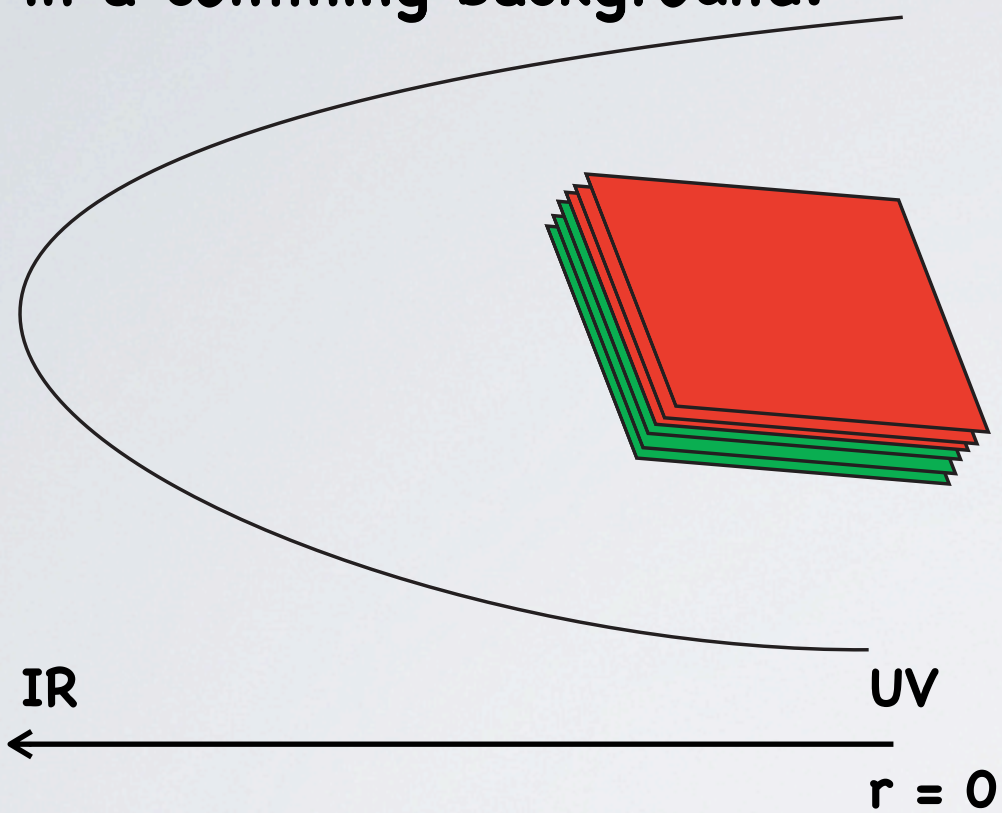
$$S_{DBI} = \int drd^4x \, \frac{N_c}{\lambda} \, \mathbf{Str} \left[V(T) \left(\sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^L_{\mu\nu}\right)} + \sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu} T^\dagger D_{\nu\}} T + F^R_{\mu\nu}\right)} \right) \right]$$

$$V(T) = T_p \, e^{-\frac{1}{N_f} \mathbb{T}r(T\,T^\dagger)} \qquad DT \equiv \partial_\mu T - iTA^L + iA^R T \,, \quad DT^\dagger \equiv \partial_\mu T^\dagger - iA^L T^\dagger + iT^\dagger A^R$$

[Sen'03]

- $N_f \ll N_c$ **Quenched FLAVOR. χSB through TACHYON Condensation**

in a confining background: **with all mq equal** → $T = \tau \, \mathbb{I}_{N_f}$



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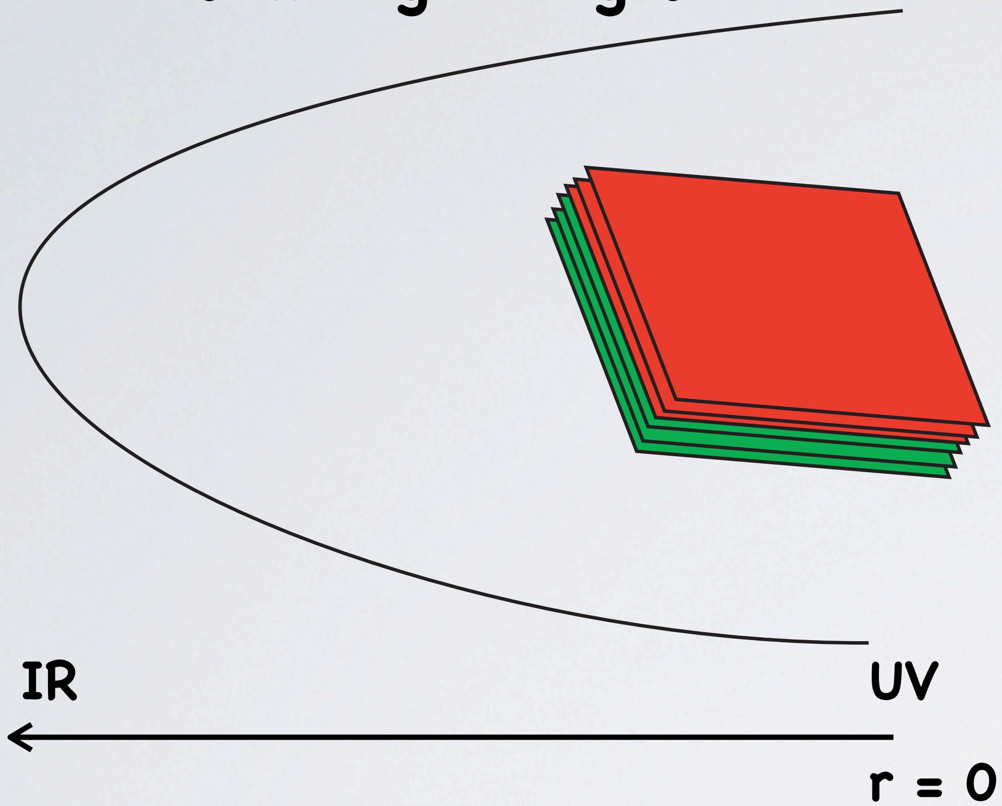


with all mq equal → $T = \tau\, \mathbb{I}_{N_f}$

$$\tau_{UV} \sim m_q\, r + \sigma\, r^3$$
$$\sigma = \sigma(m_q) \neq 0$$

The tachyon condenses ➡ $U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$

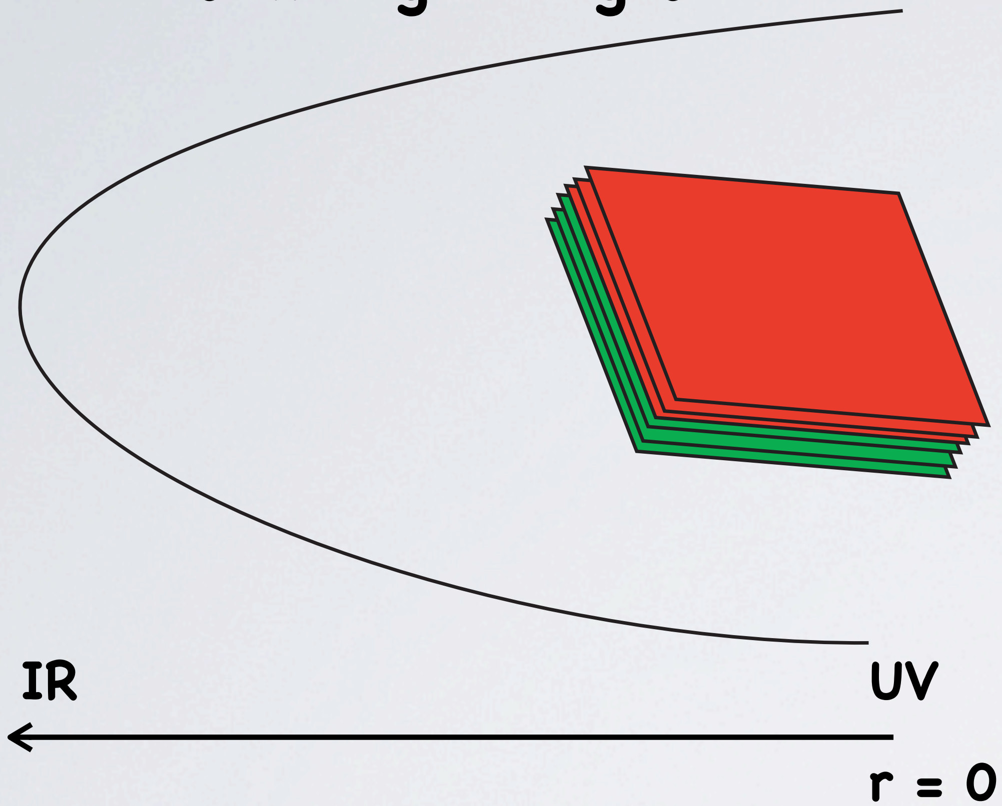
$N_f \ll N_c$

- **$(N_f)^2$ Goldstone Bosons (Pions)**
- **GOR:** $m_\pi^2 = -2\dfrac{m_q}{f_\pi^2}\sigma$
- **Linear spectrum (some sectors)**

- $N_f \ll N_c$ Quenched FLAVOR. χSB through TACHYON Condensation

in a confining background:

with all mq equal → $T = \tau \, \mathbb{I}_{N_f}$



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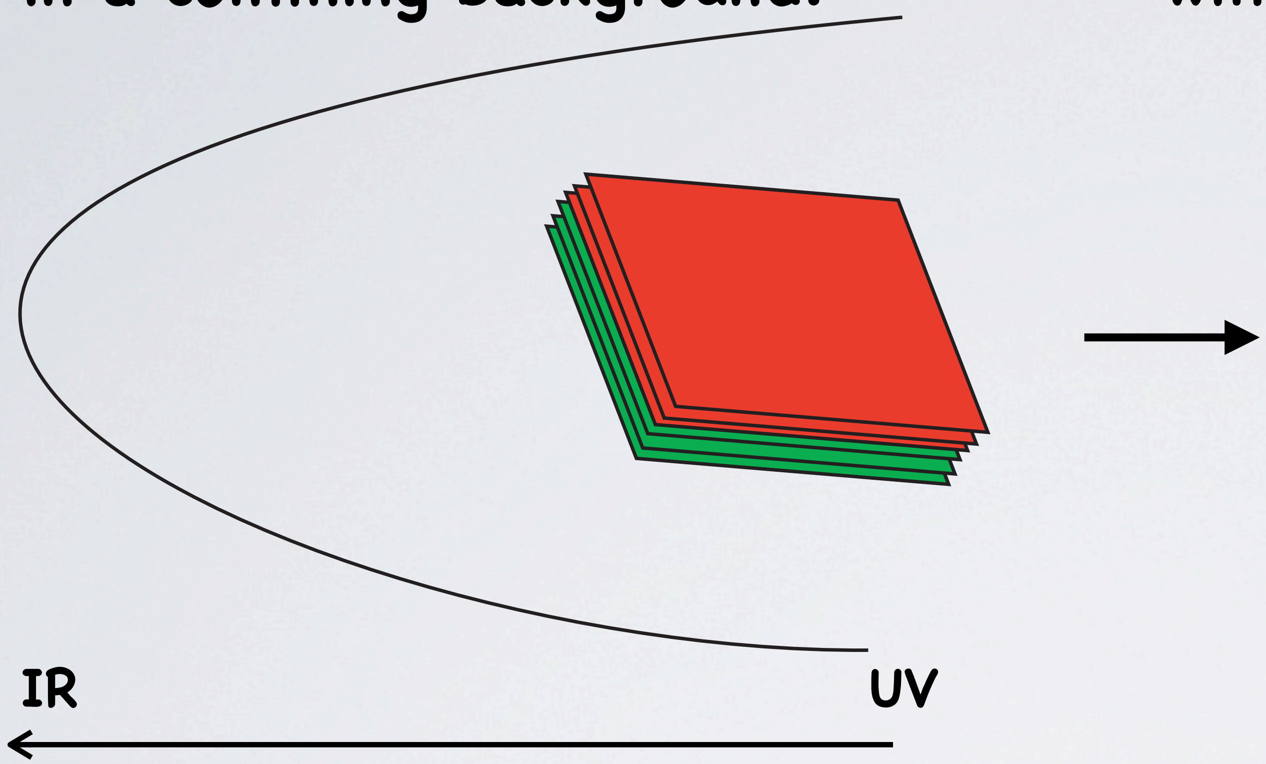
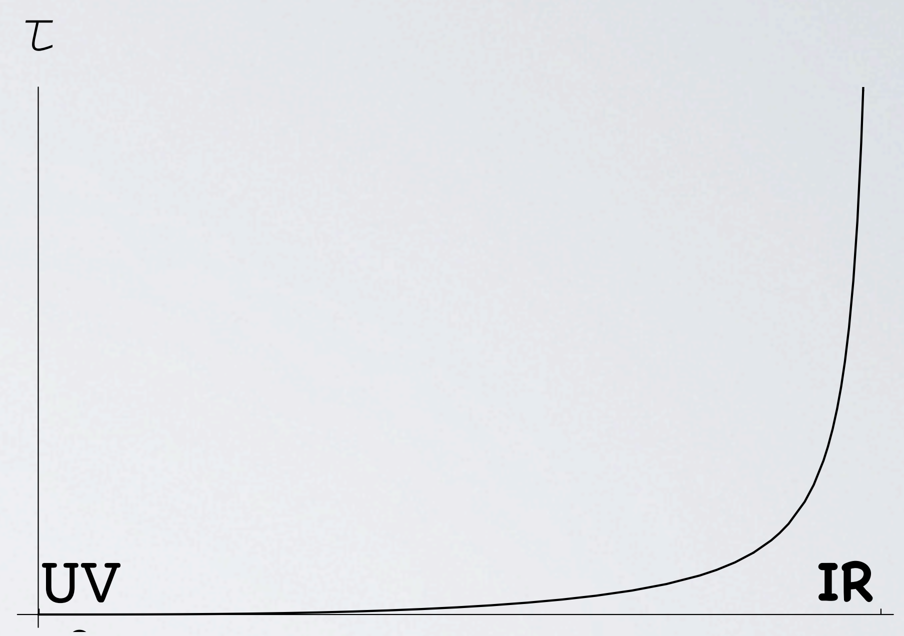


with all mq equal → $T = \tau\, \mathbb{I}_{N_f}$



$$\tau_{UV} \sim m_q\, r + \sigma\, r^3$$

$$\sigma = \sigma(m_q) \neq 0$$

The tachyon condenses ➡ $U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$

$N_f \sim N_c$ ⟶ **Compute backreaction of the tachyon** ⟶ **V-hQCD**

★ iHQCD in the Veneziano limit [Jarvinen & Kiritsis'11]

$N_c \to \infty\,,\ \ N_f \to \infty\,,\ \ x \equiv N_f/N_c\ \ \text{fixed}$ $\xrightarrow{\text{AdS/CFT ?}}$ **Phase structure as function of x**
[Conformal window, ∃ x_c , walking region ?]

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$N_c \to \infty\,,\quad N_f \to \infty\,,\quad x \equiv N_f/N_c$ fixed $\xrightarrow{\text{AdS/CFT ?}}$ **Phase structure as function of x**

[Conformal window, ∃ x_c , walking region ?]

Glue — **Flavor**

V-hQCD: $$\mathcal{L} = (M^3 N_c^2) \int d^4x\, dr \left[\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right) - x\, V_f(\lambda, T)\sqrt{\det\left(g_{ab} + (\lambda, T)\partial_a T\, \partial_b T\right)}\right]$$

★ iHQCD in the Veneziano limit [Jarvinen & Kiritsis'11]

$N_c \to \infty\,, \quad N_f \to \infty\,, \quad x \equiv N_f/N_c \;\; \text{fixed}$ —AdS/CFT ?→ Phase structure as function of x
[Conformal window, ∃ x_c , walking region ?]

Glue | Flavor

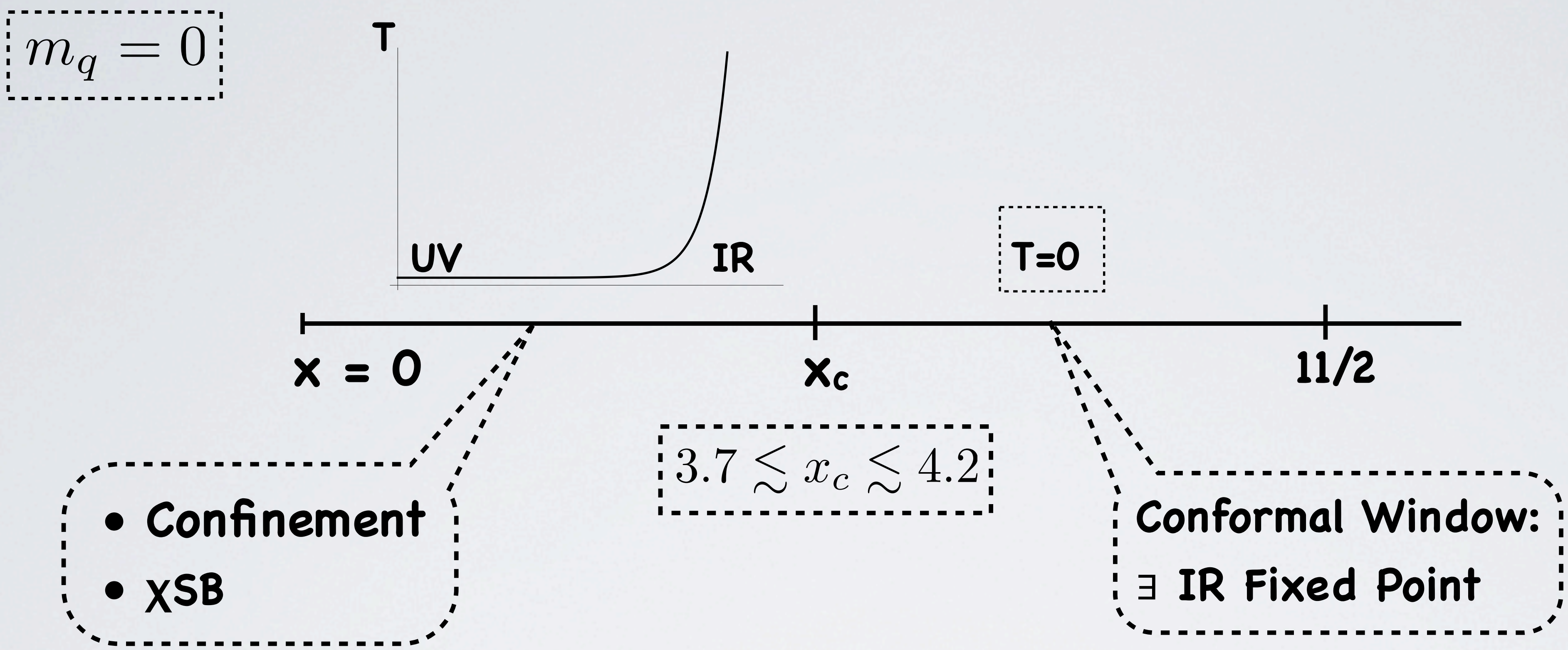
V-hQCD: $$\mathcal{L} = (M^3\,N_c^2) \int d^4x\,dr \left[\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right) - x\,V_f(\lambda,T)\sqrt{\det\left(g_{ab} + (\lambda,T)\partial_a T\,\partial_b T\right)} \right]$$

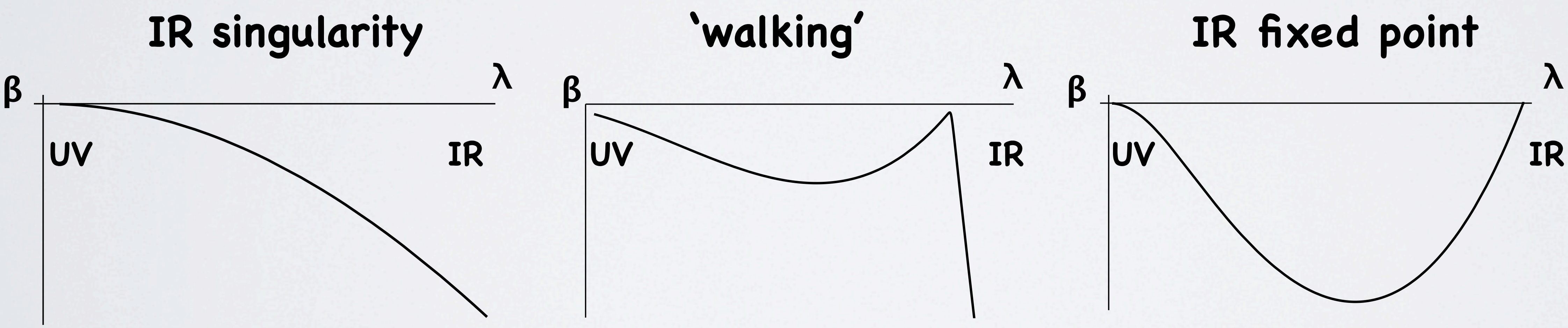
- **Potentials:** $V_g(\lambda)$ as before. $V_f(\lambda,T) = V_{f0}(\lambda)\,e^{-a(\lambda)\,T^2}$

$V_f(\lambda,T) \longleftarrow$

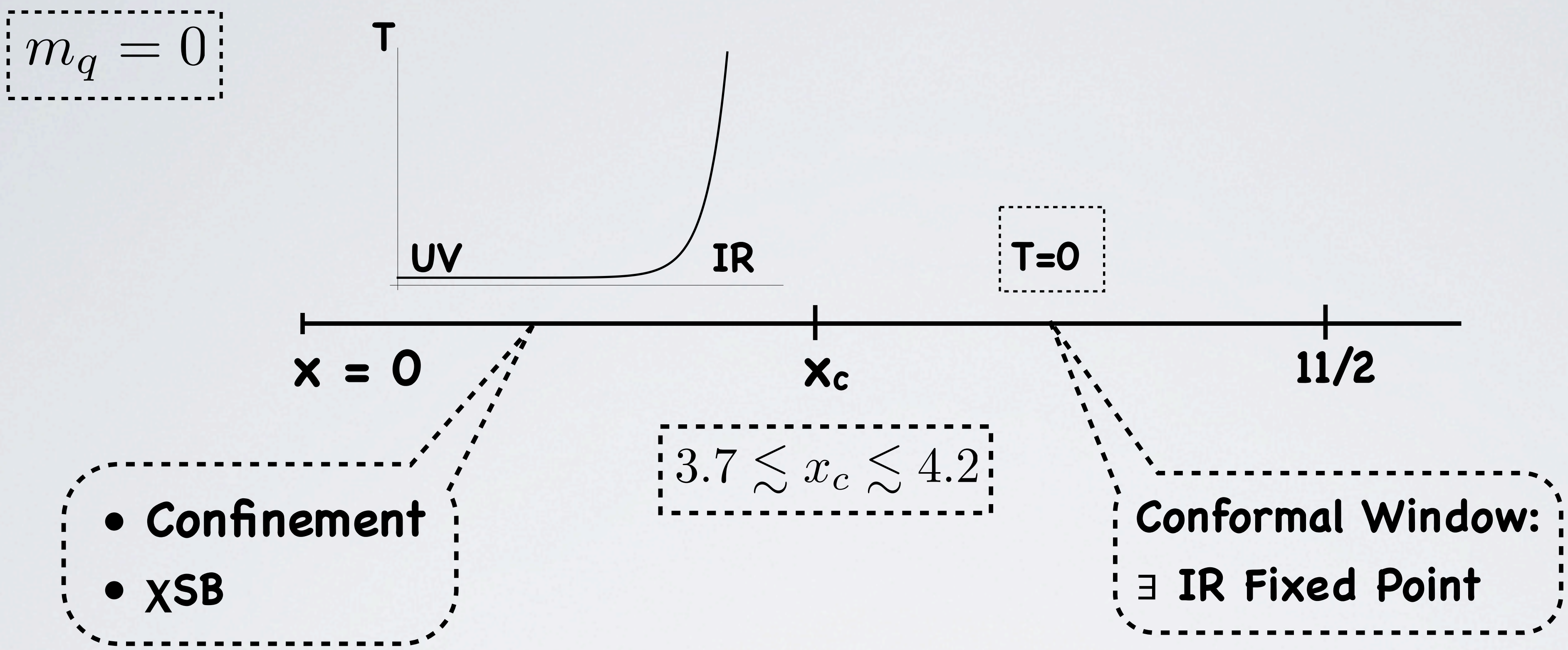
- $\mathrm{UV}\ (\lambda \to 0)$: β-functions, BZ fixed point (x~11/2)
  $\tau_{UV} \sim m_q\,r + \sigma\,r^3$
- $\mathrm{IR}\ (\lambda \to \infty)$: Confinement, T→∞ at IR singularity

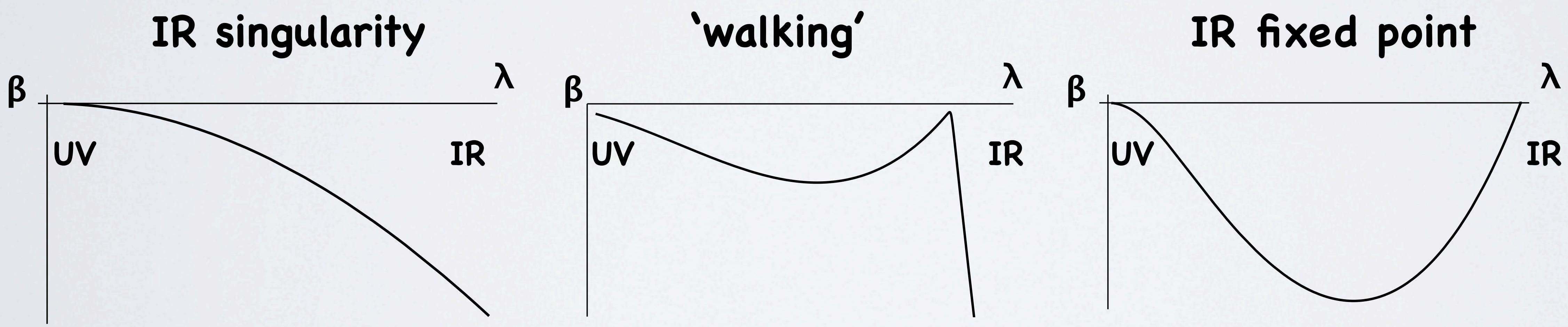
V-hQCD: vacuum solution → Phase structure





V-hQCD: vacuum solution → Phase structure





- Phase structure agrees w/ expectations for QCD
- Find walking region → technicolor?

⟶ Let's compute the spectrum

★ iFluctuations of V-hQCD

- $S = S_g[g_{ab}, \lambda] + S_f[g_{ab}, \lambda, T, A_a^L, A_a^R] + S_{\text{CP}-\text{odd}}$

- **Vacuum:** $T = \tau(r)\,\mathbb{I}_{N_f}\ ,\ g_{ab}(r)\,,\ \lambda(r)\,,\ A_a^L = A_b^R = 0$

★ iFluctuations of V-hQCD

- $S = S_g[g_{ab}, \lambda] + S_f[g_{ab}, \lambda, T, A_a^L, A_a^R] + S_{\mathrm{CP-odd}}$

- **Vacuum:** $T = \tau(r)\,\mathbb{I}_{N_f}\ ,\ g_{ab}(r)\ ,\ \lambda(r)\ ,\ A_a^L = A_b^R = 0$

- **Degrees of freedom:**

GLUEBALLS

J^{PC}
2^{++}
0^{++}
0^{-+}

- **Metric:** $g_{MN} = g_{MN}^{(0)} + \hat{g}_{MN}\ ,\ \hat{g}_{MN}\, d\xi^M d\xi^N = e^{2A_s}\left(2\phi\, dr^2 + 2\hat{A}_\mu\, dr\, dx^\mu + h_{\mu\nu}\, dx^\mu\, dx^\nu\right)$
- **Dilaton (λ):** $\Phi = \Phi_0 + \chi$

MESONS

J^{PC}
1^{--}
1^{++}
0^{++}
0^{-+}

- **Gauge fields:** $A_\mu^L - A_\mu^R\ ,\ \ A_\mu^L + A_\mu^R$
- **Tachyon:** $T = (\tau + s)\, e^{i\theta}$

◆ Mass Spectra

Expand

$$S = M^3 N_c^2 \int d^5x \left[\sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right) - \frac{x}{2} V_f(\lambda, T) \left(\sqrt{-\det \mathbf{A}_L} + \sqrt{-\det \mathbf{A}_R} \right) \right] + S_{\text{CP-odd}}$$

$$\mathbf{A}_{(i)MN} = g_{MN} + \omega(\lambda, T) F^{(i)}_{MN} + \frac{\kappa(\lambda, T)}{2} \left[(D_M T)^* (D_N T) + (D_N T)^* (D_M T) \right]$$

up to O(fluctuation²)

◆ Mass Spectra

Expand

$$S = M^3 N_c^2 \int d^5x \left[\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right) - \frac{x}{2} V_f(\lambda, T)\left(\sqrt{-\det \mathbf{A}_L} + \sqrt{-\det \mathbf{A}_R}\right)\right] + S_{\mathrm{CP-odd}}$$

$$\mathbf{A}_{(i)MN} = g_{MN} + \omega(\lambda, T) F^{(i)}_{MN} + \frac{\kappa(\lambda, T)}{2}\left[(D_M T)^*(D_N T) + (D_N T)^*(D_M T)\right]$$

up to O(fluctuation^2)

◆ SECTORS

- Scalar mesons (0^{++}, SU(N_f)-sector)
- Pseudoscalar mesons (0^{-+}, SU(N_f)-sector) [pions]
- Axial-vector mesons (1^{++})
- Vector mesons (1^{--})
- Spin two glueballs (2^{++})
- Scalar mesons (0^{++}, U(1)-sector) mix with 0^{++} Glueballs
- Pseudoscalar mesons (0^{-+}, U(1)-sector) mix with QCD axion (0^{-+})

- Mixing:
  - Scalar mesons (0^{++}, U(1)-sector) mix with 0^{++} Glueballs
  - Pseudoscalar mesons (0^{-+}, U(1)-sector) mix with QCD axion (0^{-+})

- **Fields** (r, x^μ)

FLAVOR →

Tachyon

$$T = (\tau + s + \mathfrak{s}^a \tau_a)\, e^{i\theta + i\,\pi^a \tau_a}$$

Vector

$$V_\mu = (A^L_\mu + A^R_\mu)/2$$

Axial-Vector

$$A_\mu = (A^L_\mu - A^R_\mu)/2 = A^\perp_\mu + A^\parallel_\mu\,, \qquad A^\parallel_\mu = \hat{A}_\mu \mathbb{I}/\sqrt{N_f} + \tilde{A}^a_\mu \tau^a$$

GLUE →

Metric

$$\hat{g}_{MN}\, d\xi^M d\xi^N = e^{2A_s}\left(2\phi\, dr^2 + 2\hat{A}_\mu\, dr\, dx^\mu + h_{\mu\nu}\, dx^\mu\, dx^\nu\right)$$

$$h_{\mu\nu} = 2\eta_{\mu\nu}\psi + 2\partial_\mu\partial_\nu E + 2\partial_{(\mu}V^T_{\nu)} + h^{TT}_{\mu\nu}$$

$$\hat{A}_\mu = \partial_\mu W + A^\perp_\mu$$

Dilaton

$$\Phi = \Phi_0 + \chi$$

Axion

a

- **Mixing:**

  - **Scalar mesons (0^{++}, U(1)-sector) mix with 0^{++} Glueballs** →

$$\zeta = \psi - \frac{A'_s}{\tau'}\, s$$

$$\xi = \psi - \frac{A'_s}{\Phi'}\, \chi$$

  - **Pseudoscalar mesons (0^{-+}, U(1)-sector) mix with QCD axion (0^{-+})** → $S_{\text{CP-odd}}$

- **Fields** (r, x^μ)

FLAVOR →

Tachyon

$$T = (\tau + s + \mathfrak{s}^a \tau_a)\, e^{i\theta + i\,\pi^a \tau_a}$$

Vector

Axial-Vector

$$A_\mu = (A^L_\mu - A^R_\mu)/2 = A^\perp_\mu + A^\parallel_\mu\,, \qquad A^\parallel_\mu = \hat{A}_\mu \mathbb{I}/\sqrt{N_f} + \tilde{A}^a_\mu\, \tau^a$$

GLUE →

Metric

Dilaton

Axion

a

- **Mixing (DBI):**

  - **Pseudoscalar mesons (0^{-+}, U(1)-sector) mix with QCD axion (0^{-+})** → $S_{\rm CP-odd}$

$$S_{\rm CP-odd} \sim \int d^5x\, \sqrt{g}\, Z(\lambda) \left[da + N_f \left(V(\lambda, T)\, \hat{A} + \theta\, dV(\lambda, T)\right)\right]^2$$

[↔ QCD $U(1)_A$ anomaly]

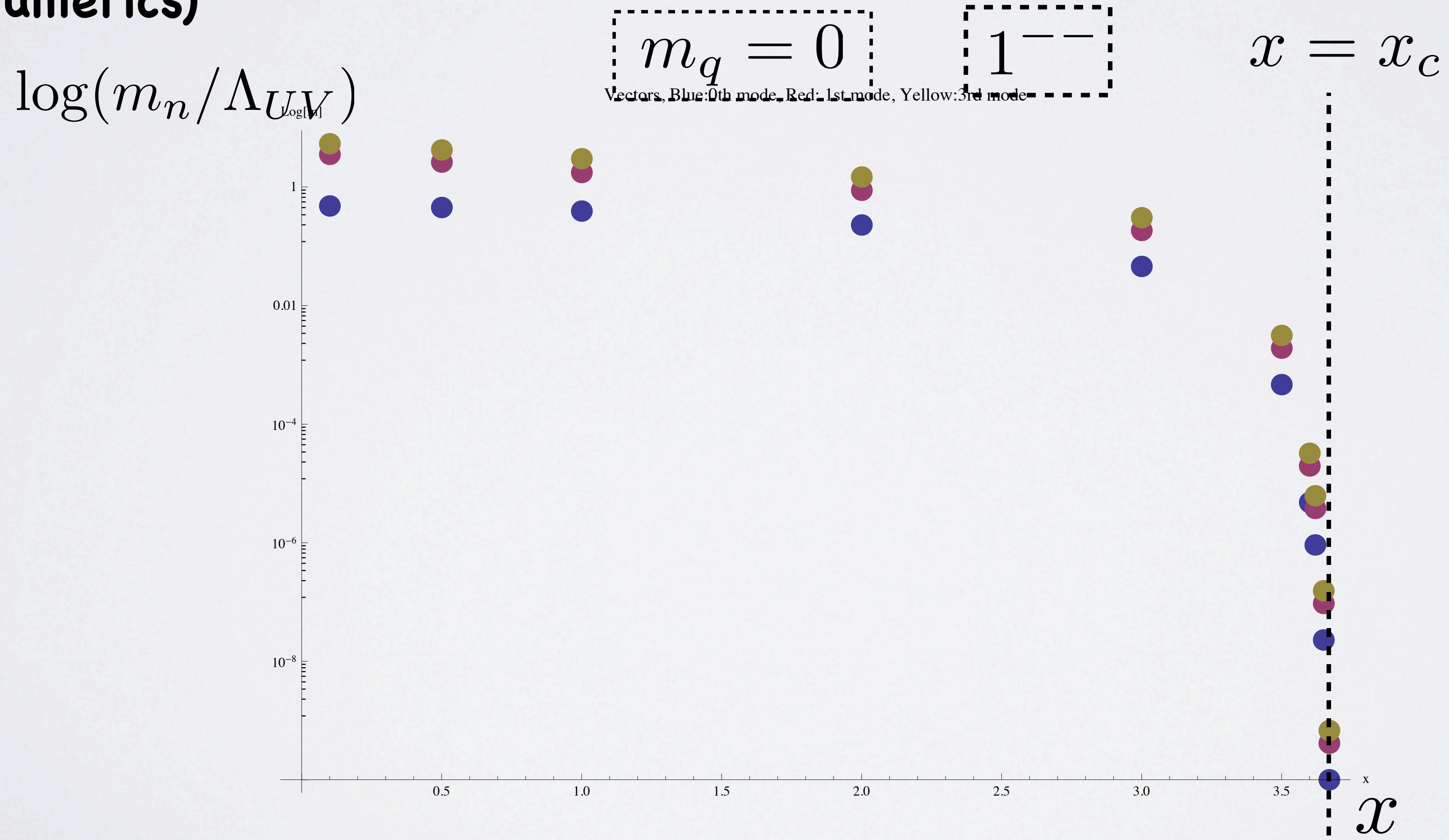
➥ η′ - QCD axion mixing

- Spectrum. Vector mesons

> Tech Specs:

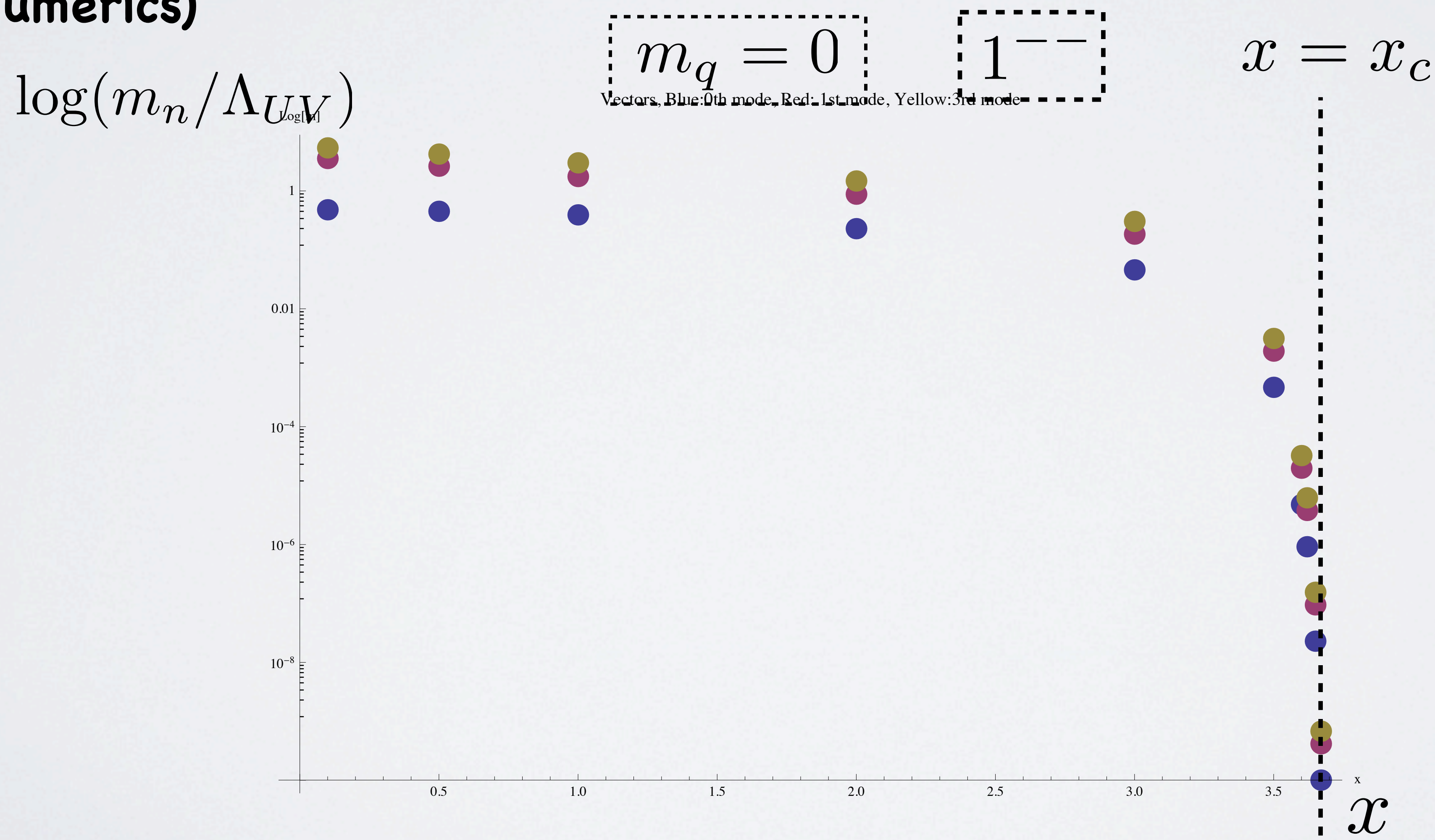
>> Action (vector mesons sector) & potential ('type II')

- Results (numerics)



- **Spectrum. Vector mesons**

- **Results (numerics)**



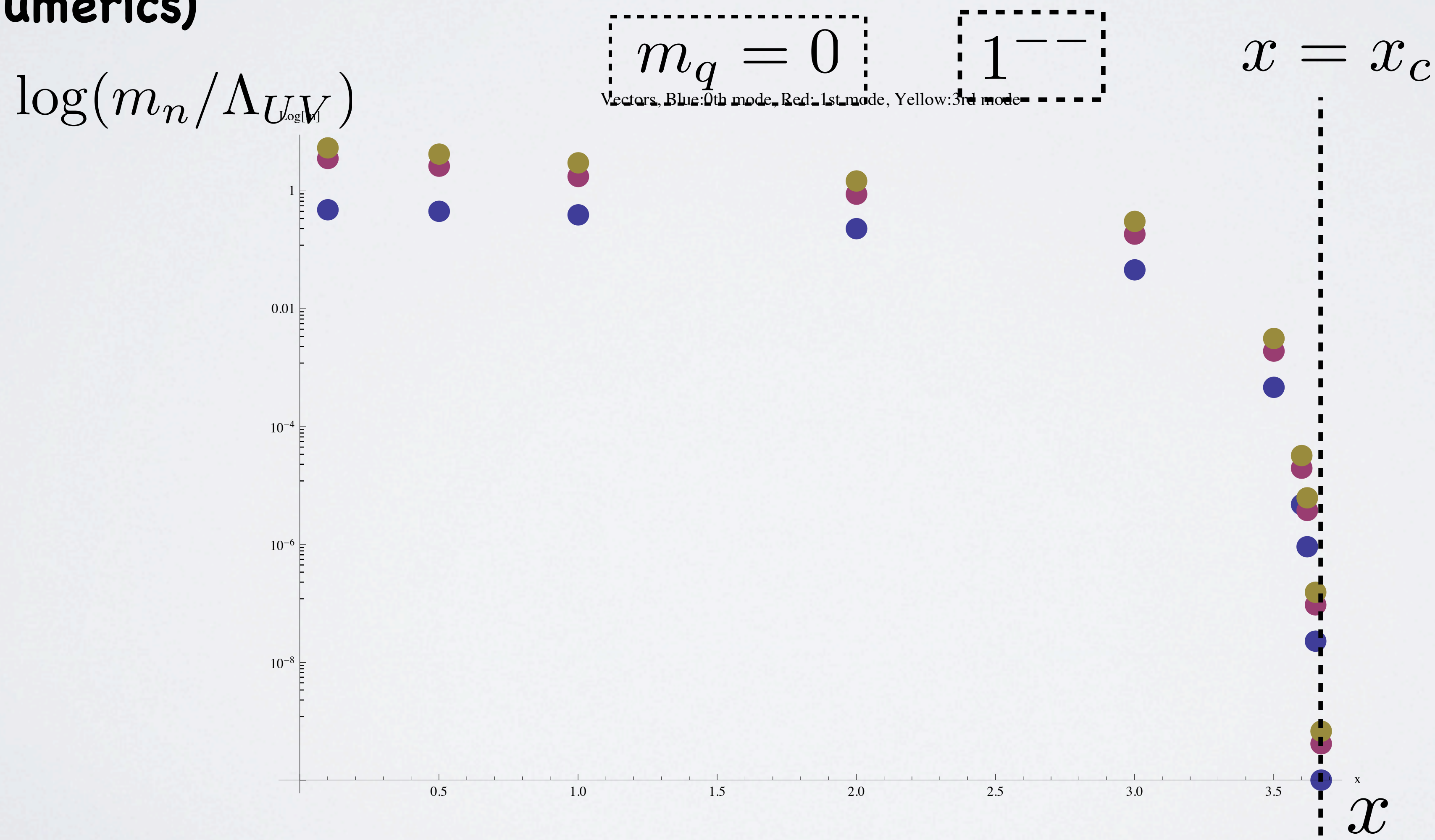
- **Spectrum. Vector mesons**

  - **Action:** $S_V = -xM^3N_c^2\,\mathbb{T}r\int d^4x\,dr V_f(\lambda,T)\omega(\lambda,T)^2\left[\frac{1}{2}\tilde{g}_{rr}^{\frac{1}{2}}VV + e^{2A}\tilde{g}_{rr}^{-\frac{1}{2}}\partial_r V_\mu \partial_r V^\mu\right]$

  - **Potential:** $V_f(\lambda,T) \; = (W_0 + W_1\lambda + W_2\lambda^2)e^{-a(\lambda)T^2}$ $\qquad a(\lambda) = \frac{a_0 + a_1\lambda + a_2\lambda^2}{\left(1+\frac{\lambda}{\lambda_0}\right)^{4/3}}$

$$\omega(\lambda,T) = 1 \qquad \kappa(\lambda) = \frac{\kappa_0}{(1+\frac{\lambda}{\lambda_0})^{4/3}}$$

- **Results (numerics)**

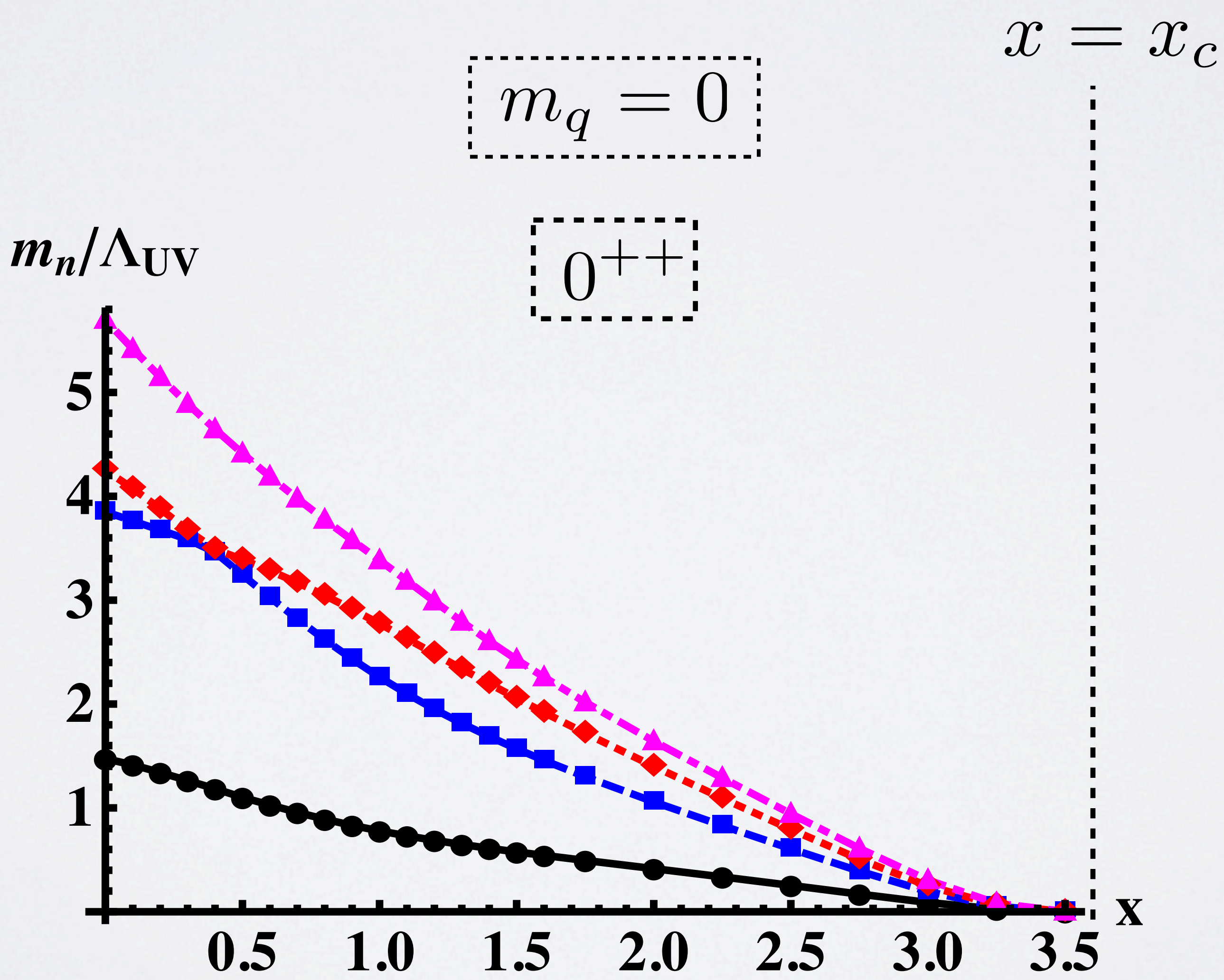


- Spectrum. SCALARS (U(1)-sector, 0^{++})

> Tech Specs:

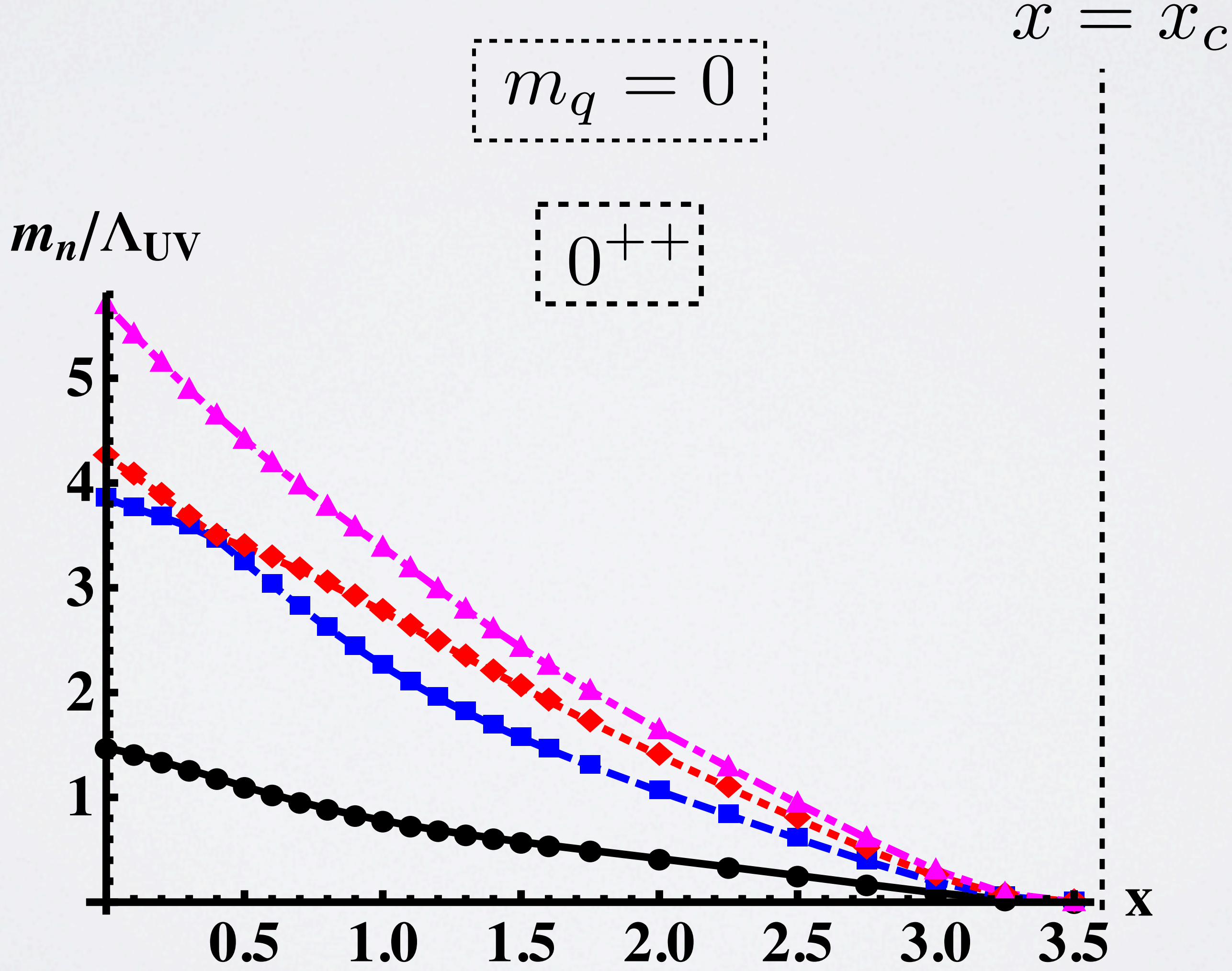
>> 2 coupled ODEs (with page-sized coefficients)

- Results (numerics)



- **Spectrum. SCALARS (U(1)-sector, 0^{++})**

- **Results (numerics)**



◆ **Spectrum.** **SCALARS (U(1)-sector, 0^{++})**

- **EoMs**

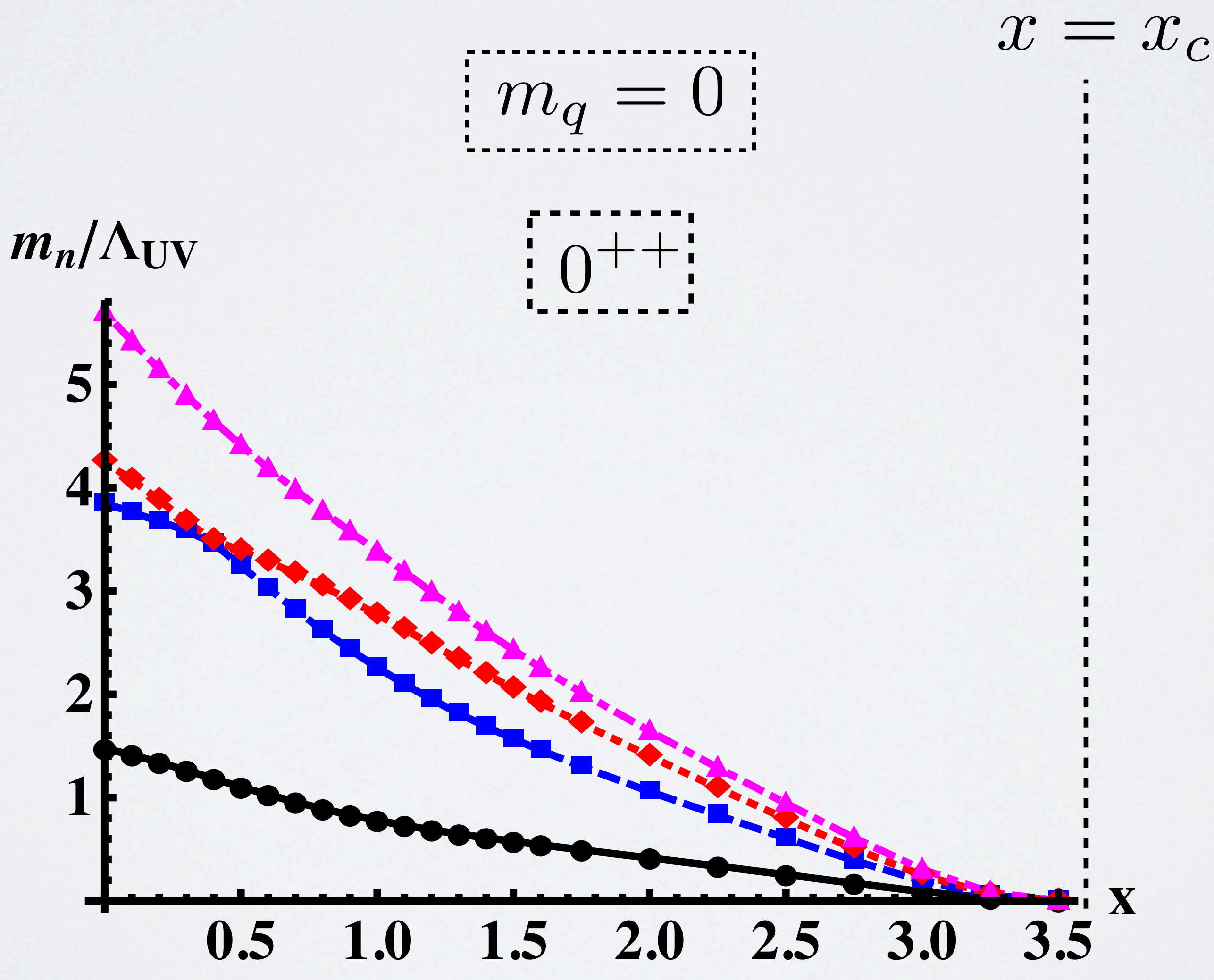
$$\xi'' + m(r)\,\xi' + p(r)\,\zeta' + \Box\xi + N_1(r)\,(\xi - \zeta) = 0$$

$$\zeta'' + q(r)\,\xi' + n(r)\zeta' + t(r)\,\Box\zeta + N_2(r)\,(\zeta - \xi) = 0$$

$$\zeta = \psi - \frac{A_s'}{\tau'}\,s$$

$$\xi = \psi - \frac{A_s'}{\Phi'}\,\chi$$

◆ **Results (numerics)**



- **Walking Technicolor**

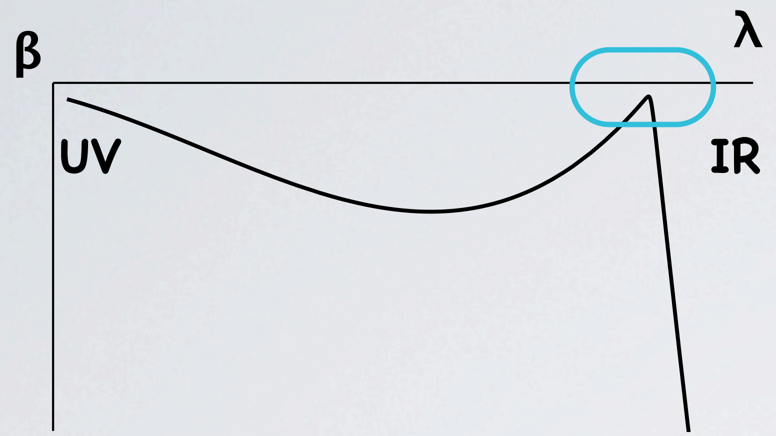
> For $x \sim x_c$ Background shows Walking behaviour

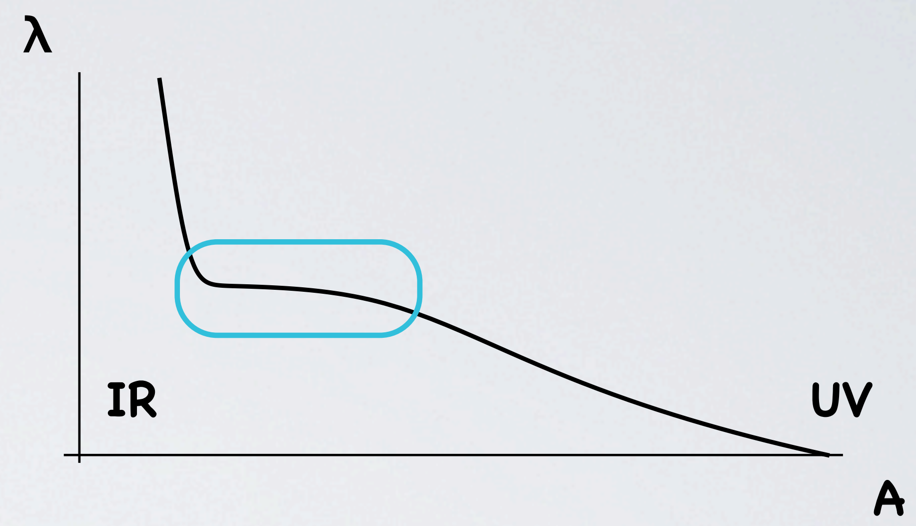
>> Solution approaches IR fixed point but misses it

- **Walking Technicolor**

- **Walking Technicolor**

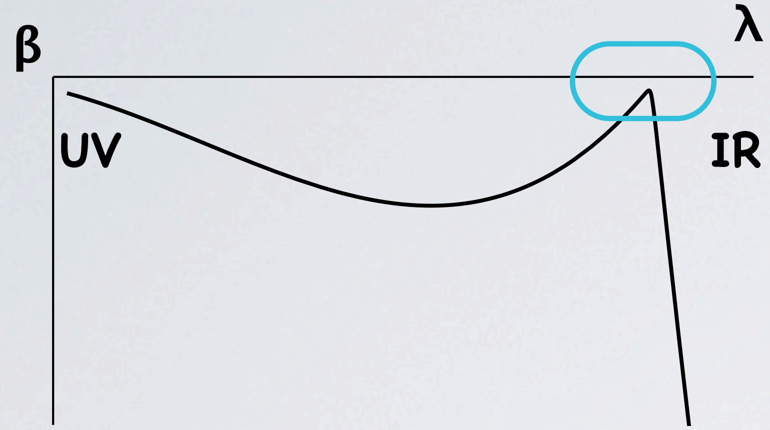
'walking'

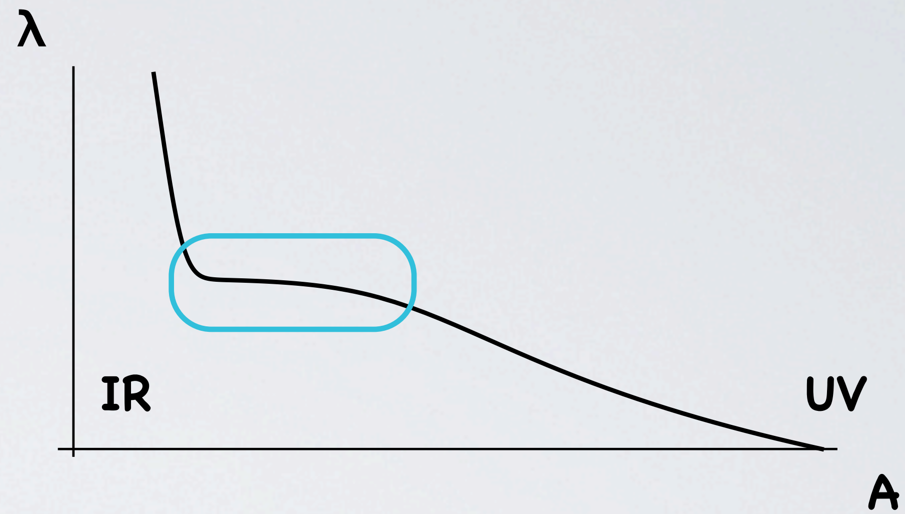




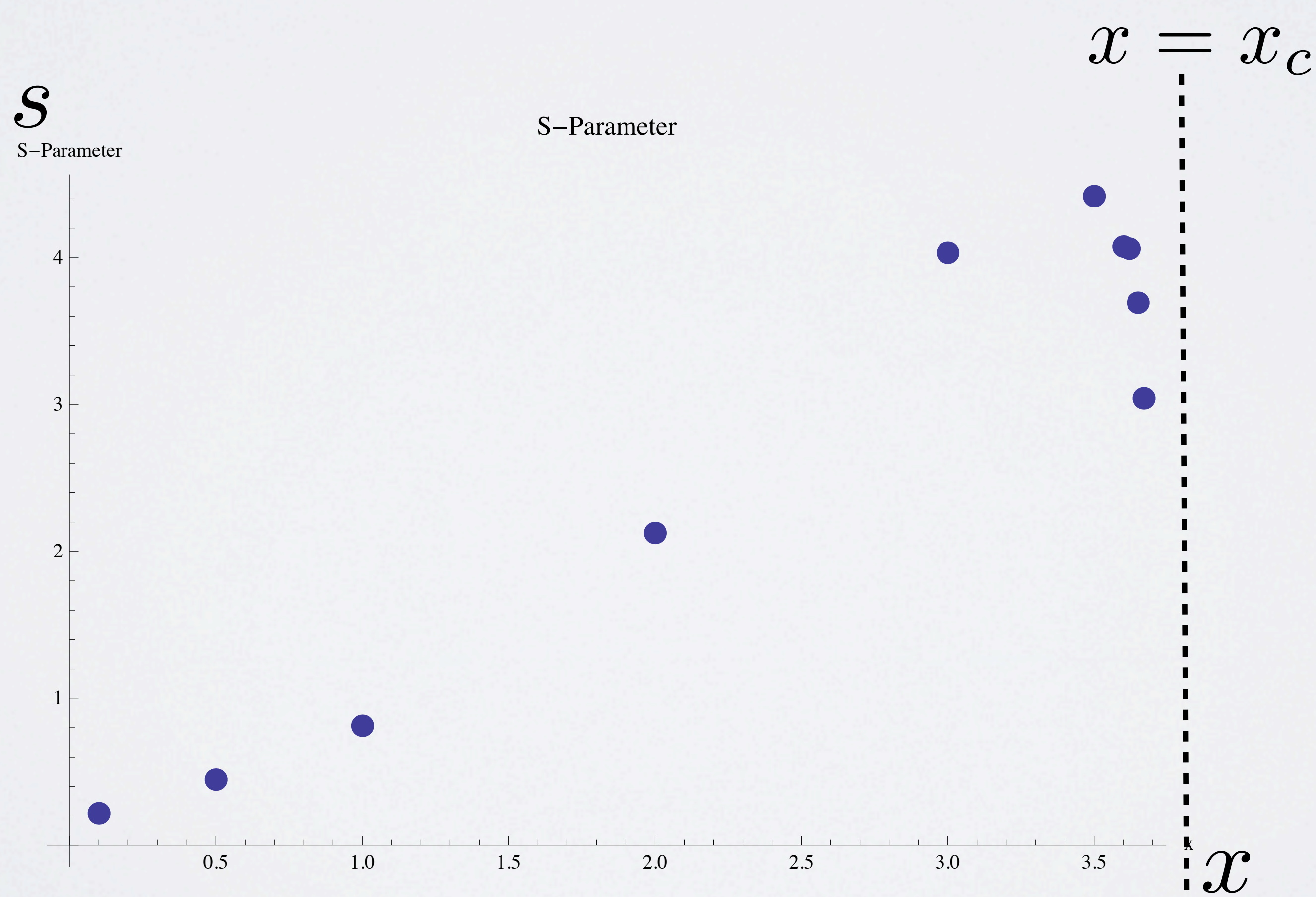
- **Walking Technicolor**

'walking'





- **Technicolor S-parameter** $s = 4\pi \frac{d}{dq^2}(\Pi_V(q) - \Pi_A(q))$



★ OUTLOOK & TO DO

- Fluctuations of V-hQCD ✓
- Spectrum & 2-point functions ✓
- Linear spectrum (type I potentials) ✓
- Technicolor: s-parameter ...