# Kaluza-Klein inspired actions for self-dual tensors

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21/09/2012

arXiv:1206.1600 [hep-th] FB, T. W. Grimm, S. Hohenegger

arXiv:1209.3017 [hep-th] FB, T. W. Grimm, S. Hohenegger

# Outline

Introduction

A 5d superconformal action for massive tensor towers

One-loop effect: anomalies

Conclusions

# Introduction

# Motivation: non-Abelian self-dual tensors in 6d

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- Our approach: Kaluza-Klein inspired 5d Lagrangians

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- Obstacles to a 6d effective action for (2,0) non-Abelian theories:
  1. self-duality is hard to get from Lorentz covariant action
     $\rightarrow$ auxiliary fields [Pasti, Sorokin, Tonin 96]
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- 6d difficulties can be circumvented:
  1. no self-duality in 5d
     $\rightarrow$ standard Lorentz covariant actions
  2. the zero mode sector contains 5d vectors
     $\rightarrow$ possibility to construct gauge theories
  3. circle radius $r$
     $\rightarrow$ tunable coupling constant $g^2 = r$

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Robust quantities (anomalies, large $N$ scaling, ...) should be accessible

# A 5d superconformal action for non-Abelian tensor towers

# Warm-up: 5d action for one self-dual tensor on a circle

- Compactify one direction on a circle: $\hat{x}^{\hat{\mu}} = (x^\mu, y)$, $\hat{\mu} = 0, \ldots, 5$, $\mu = 0, \ldots, 4$

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} + r^2 A^0_\mu A^0_\nu & -r^2 A^0_\mu \\ -r^2 A^0_\nu & r^2 \end{pmatrix} , \quad \hat{B}_{\hat{\mu}\hat{\nu}} = \sum_{n\in\mathbb{Z}} e^{iny} \begin{pmatrix} B_{n\,\mu\nu} - 2A_{n[\mu}A^0_{\nu]} & A_{n\,\mu} \\ -A_{n\,\nu} & 0 \end{pmatrix}$$

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$$\mathcal{D}_\mu = \partial_\mu + inA^0_\mu \qquad \mathrm{U}(1)_{\mathrm{KK}} \text{ covariant derivative on } n\text{th KK level}$$

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- 6d dynamics of $\hat{H}$:

$$\hat{H} = \hat{*}\hat{H} \ , \quad \hat{H} = d\hat{B}$$

- 5d dynamics of $B_n, A_n$ coupled to $r, A^0$:

$$S_{5\text{d}} = \int -\tfrac{1}{2} r^{-1} F \wedge *F - \tfrac{1}{2} A^0 \wedge F \wedge F + \sum_{n=1}^{\infty} \left( \tfrac{i}{n} \bar{F}_n \wedge \mathcal{D} F_n - r^{-1} \bar{F}_n \wedge *F_n \right)$$

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- Stückelberg mechanism: $B_n$ 'eats' $A_n$ and yields a tensor with mass $m_n = nr^{-1}$

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$$\begin{array}{rcrcl} \text{zero mode } A & \rightarrow & \text{gauge connection } A^I & \rightarrow & \delta A^I = d\alpha^I + f_{JK}{}^I A^J \alpha^K \\ \text{excited modes } F_n & \rightarrow & \text{adjoint matter } F_n^I & \rightarrow & \delta F_n^I = f_{JK}{}^I F_n^J \alpha^K \end{array}$$

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- Possible interpretation: residual local symmetry after gauge-fixing of the 5d reduction of the non-Abelian generalization of $\delta \hat{B} = d\hat{\Lambda}$

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- No known non-Abelian gauging of tensors in 5d $\mathcal{N} = 4$ [Schoen, Weidner 06]

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- We use 5d $\mathcal{N} = 2$ superconformal formalism [Bergshoeff et al. 02]
- We include all dof's of $\mathcal{N} = 4$ multiplets in terms of $\mathcal{N} = 2$ multiplets

$$\text{6d (2,0) TM} \quad \begin{cases} n = 0: \quad \text{5d } \mathcal{N} = 4 \text{ VM} & \begin{cases} \text{5d } \mathcal{N} = 2 \text{ VM} \\ \text{5d } \mathcal{N} = 2 \text{ HM} \end{cases} \\ n > 0: \quad \text{5d } \mathcal{N} = 4 \text{ TM} & \begin{cases} \text{5d } \mathcal{N} = 2 \text{ TM} \\ \text{5d } \mathcal{N} = 2 \text{ HM} \end{cases} \end{cases}$$

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| $\mathcal{N}=2$ vector multiplets | $A^I,\ \phi^I,\ \chi^I,\ Y^I$ | $n=0$ |
| $\mathcal{N}=2$ tensor multiplets | $F^I_n,\ \phi^I_n,\ \chi^I_n,\ Y^I_n$ | $n>0$ |
| $\mathcal{N}=2$ hypermultiplets | $4q^I_n,\ \zeta^I$ | $n\geq 0$ |

- $r$ and $A^0$ are packed into an additional

| | | |
|---|---|---|
| $\mathcal{N}=2$ vector multiplet | $A^0,\ \phi^0\equiv r^{-1},\ \chi^0,\ Y^{0ab}$ | $n=0$ |

# 5d $\mathcal{N} = 2$ superconformal theory: Lagrangian

$$\mathcal{L} = \phi^\Theta C_{\Theta\Lambda\Sigma}\left(-\tfrac{1}{4}F^\Lambda_{\mu\nu}F^{\Sigma\,\mu\nu} - \tfrac{1}{2}\mathcal{D}_\mu\phi^\Lambda\mathcal{D}^\mu\phi^\Sigma + Y^\Lambda_{ab}Y^{\Sigma\,ab}\right) - \tfrac{1}{2}C_{\mathcal{I}\mathcal{J}}\mathcal{D}_\mu q^{\mathcal{I}\,a\dot{b}}\mathcal{D}^\mu q^{\mathcal{J}}{}_{a\dot{b}}$$
$$+ \tfrac{1}{16}\epsilon^{\mu\nu\lambda\rho\sigma}\Omega_{MN}F^M_{\mu\nu}\mathcal{D}_\lambda F^N_{\rho\sigma} - \tfrac{1}{24}\epsilon^{\mu\nu\lambda\rho\sigma}C_{\widehat{I}\widehat{J}\widehat{K}}A^{\widehat{I}}_\mu F^{\widehat{J}}_{\nu\lambda}F^{\widehat{K}}_{\rho\sigma}$$
$$- \tfrac{1}{2}\phi^{\widehat{K}}C_{\widehat{K}MN}t_{\widehat{I}P}{}^M t_{\widehat{J}Q}{}^N\phi^{\widehat{I}}\phi^{\widehat{J}}\phi^P\phi^Q$$
$$+ C_{\mathcal{I}\mathcal{J}}\left(t_{\widehat{K}\mathcal{L}}{}^{\mathcal{J}}q^{\mathcal{L}\,a\dot{c}}q^{\mathcal{I}\,b}{}_{\dot{c}}Y^{\widehat{K}}_{ab} - \tfrac{1}{2}t_{\widehat{I}\mathcal{K}}{}^{\mathcal{I}}t_{\widehat{J}\mathcal{L}}{}^{\mathcal{J}}\phi^{\widehat{I}}\phi^{\widehat{J}}q^{\mathcal{K}a\dot{b}}q^{\mathcal{L}}{}_{a\dot{b}}\right) + \text{fermions}$$

- Real notation for complex multiplets ($n > 0$): $\mathrm{U}(1)_{\mathrm{KK}} \cong \mathrm{SO}(2)_{\mathrm{KK}}$

$$X^M = X^{I\alpha}_n = (\mathrm{Re}\,X^I_n, \mathrm{Im}\,X^I_n)\ , \quad \alpha \equiv \mathrm{SO}(2)_{\mathrm{KK}}\ \text{index}\ , \quad M \equiv \{I\alpha n\}$$

- Collective indices $\widehat{I} = (0, I)$ , $\Lambda = (0, I, M)$ , $\mathcal{I} = (I, M)$
- All couplings are specified in terms of

$$C_{(\Lambda\Sigma\Theta)}\ ,\ \Omega_{[MN]}\ ,\ t_{\widehat{I}\Lambda}{}^{\Sigma}\ ,\ C_{(\mathcal{I}\mathcal{J})}$$

- Full covariant derivative $\mathcal{D}_\mu X^\Sigma = \partial_\mu X^\Sigma + t_{\widehat{I}\Lambda}{}^{\Sigma}A^{\widehat{I}}_\mu X^\Lambda$

## 5d $\mathcal{N} = 2$ superconformal theory: specifying the couplings

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  1. group invariants $f_{JK}{}^{I}$, $d_{IJ} = \mathrm{tr}(t_I t_J)$
  2. KK levels $n > 0$
  3. $SO(2)_{KK}$ invariants $\delta_{\alpha\beta}$, $\epsilon_{\alpha\beta}$

- Vector, tensor, hypermultiplets are gauged democratically
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$$C_{0IJ} = d_{IJ}\ ,\quad C_{0MN} = d_{IJ}\delta_{\alpha\beta}\delta_{n,m}\ ,\quad C_{MNK} = n^{-1} f_{IJK}\epsilon_{\alpha\beta}\delta_{n,m}$$
$$\Omega_{MN} = -2n^{-1} d_{IJ}\epsilon_{\alpha\beta}\delta_{n,m}$$
$$t_{KI}{}^{J} = f_{KI}{}^{J}\ ,\quad t_{KM}{}^{N} = f_{KI}{}^{J}\delta_{\alpha}^{\beta}\delta_{n,m}\ ,\quad t_{0M}{}^{N} = n\delta_I^J\epsilon_{\alpha\gamma}\delta^{\gamma\beta}\delta_{n,m}$$
$$C_{IJ} = d_{IJ}\ ,\quad C_{MN} = d_{IJ}\delta_{\alpha\beta}\delta_{n,m}$$

where $M = \{I\alpha n\}$, $N = \{J\beta m\}$

## Gauge-fixing of superconformal symmetry

- Invariance under the full 5d $\mathcal{N} = 2$ superconformal algebra
- Weyl weights are compatible with 6d conformal symmetry, in particular

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- Gauge-fixing to Poincaré superalgebra:

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- Couplings to $\phi^0$ generate
  1. YM coupling constant
  2. mass terms for excited modes

# Two special cases

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- the 6d interpretation of the 5d Lagrangian is straightforward

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1. Zero mode sector

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2. Abelian gauge group

$$\left.\begin{array}{l}\text{gauge-fix to Poincaré susy}\\ \text{set } f_{JK}{}^{I} = 0\\ \text{integrate out } Y_n^{Iab},\ n \geq 0\end{array}\right\} \longrightarrow \begin{array}{l}\mathcal{N} = 4\\ \text{Abelian tensor tower}\end{array}$$

6d interpretation: Abelian $(2,0)$ theory on a circle of arbitrary $r$

# One-loop effect: anomalies

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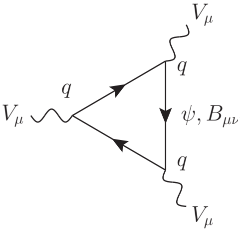
[Witten 96, Intriligator, Morrison, Seiberg 97]




$$\rightarrow \quad q^3 \cdot \mathrm{sgn}(m) \cdot \epsilon^{\mu\nu\rho\sigma\tau} V_\mu \, \partial_\nu V_\rho \, \partial_\sigma V_\tau$$

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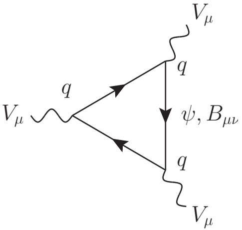


$$\rightarrow \quad q^3 \cdot \mathrm{sgn}(m) \cdot \epsilon^{\mu\nu\rho\sigma\tau} V_\mu \, \partial_\nu V_\rho \, \partial_\sigma V_\tau$$

- Parity anomalies depend only on $\mathrm{sgn}(m)$: no suppression for $m \rightarrow \infty$
- KK setup: parity anomalies in Wilsonian effective actions for zero modes by integrating out excited modes

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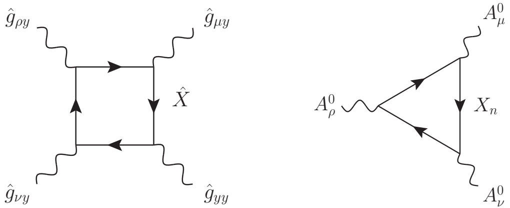


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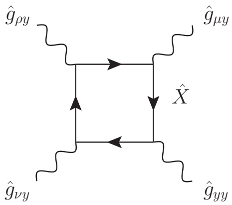


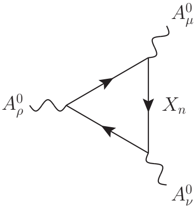


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- Results of 1-loop computation

| | | |
|---|---|---|
| $k_{1\text{-loop}}\ A^0 \wedge dA^0 \wedge dA^0$ | massive spin-1/2 | $k_{1\text{-loop}} \propto n^3 \cdot \mathrm{sgn}(m)$ |
| | massive spin-3/2 | $k_{1\text{-loop}} \propto 5 \cdot n^3 \cdot \mathrm{sgn}(m)$ |
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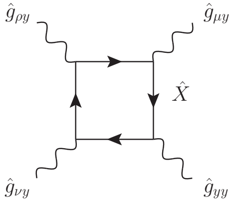


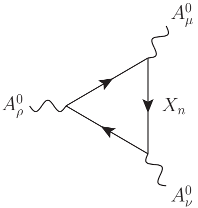


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- Tested against F-theory (1,0) effective actions [FB, Grimm 11]

# Further developments: conformal anomaly of M5 branes

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Compute 5d parity anomalies in a theory of non-Abelian tensor towers coupled to background R-symmetry gauge field

- Preliminary test: large $N$-scaling is not spoiled by 5d approach

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  supersymmetry is enhanced to 16 supercharges and the connection to 6d (2,0) theories is straightforward
- 6d anomalies can be probed using 5d parity anomalies for background gauge fields
  - gravitational anomalies $\longrightarrow$ KK vector $A^0_\mu$
  - conformal anomalies $\stackrel{?}{\longrightarrow}$ R-symmetry gauge field

Thank you
for your attention

# Further developments: conformal anomaly of M5 branes

- Can we access 6d conformal anomaly of M5 branes via 5d parity anomalies?
- 16 supercharges $\rightarrow$ stress-energy tensor and R-symmetry current belong to the same supermultiplet
- Consider (2,0) theories coupled to background R-symmetry gauge field
- 6d anomalies can be matched between the unbroken phase with gauge group $SU(N)$ and the broken phase with gauge group $SU(N-1) \times U(1)$ where one M5 brane is moved away from the stack [Intriligator 00]

## Strategy

Compute 5d parity anomalies in a theory of non-Abelian tensor towers coupled to background R-symmetry gauge field

- All kinds of massive fields contribute:
  - excited KK modes of fermions and tensors
  - fields that get massive by gauge symmetry breaking
- Preliminary test: large $N$-scaling should be accessible

## Further development: R-symmetry enhancement

- To write down our superconformal action the $\mathcal{N} = 4$ R-symmetry group is broken

$$USp(4)_R \to SU(2)_R \times SU(2)_H$$

- Inequivalent splits are parameterized by the coset

$$\frac{USp(4)_R}{SU(2)_R \times SU(2)_H} \cong \frac{SO(5)}{SO(4)} \cong S^4$$

- For each split, the full $\mathcal{N} = 2$ spectrum can be rearranged into $\mathcal{N} = 4$ multiplets

### Strategy

Restore full $USp(4)_R$ R-symmetry by integrating over all possible inequivalent splits

- Techniques inspired from harmonic superspace:
  1. parametrize $S^4$ with additional bosonic coordinates $u$
  2. re-write the $\mathcal{N} = 2$ Lagrangian in terms of $\mathcal{N} = 4$ multiplets and $u$-coordinates
  3. integrate over $u$-coordinates
- The correct result is reproduced in the two special cases of zero modes and Abelian gauging

# Full Lagrangian

$$\mathcal{L} = \phi^{\Theta} C_{\Theta\Lambda\Sigma} \Big( -\tfrac{1}{4} F^{\Lambda}_{\mu\nu} F^{\Sigma\,\mu\nu} - \tfrac{1}{2} \bar{\chi}^{\Lambda\,a} \not{\mathcal{D}} \chi^{\Sigma}_{a} - \tfrac{1}{2} \mathcal{D}_{\mu}\phi^{\Lambda} \mathcal{D}^{\mu}\phi^{\Sigma} + Y^{\Lambda}_{ab} Y^{\Sigma\,ab} \Big)$$

$$+ \tfrac{1}{16} \epsilon^{\mu\nu\lambda\rho\sigma} \Omega_{MN} F^{M}_{\mu\nu} \mathcal{D}_{\lambda} F^{N}_{\rho\sigma} - \tfrac{1}{24} \epsilon^{\mu\nu\lambda\rho\sigma} C_{\widehat{I}\widehat{J}\widehat{K}} A^{\widehat{I}}_{\mu} F^{\widehat{J}}_{\nu\lambda} F^{\widehat{K}}_{\rho\sigma}$$

$$- \tfrac{i}{8} C_{\Lambda\Sigma\Theta} \Big( \bar{\chi}^{\Lambda\,a} \gamma^{\mu\nu} F^{\Sigma}_{\mu\nu} \chi^{\Theta}_{a} + 4 \bar{\chi}^{\Lambda\,a} \chi^{b\Sigma} Y^{\Theta}_{ab} \Big)$$

$$+ \tfrac{i}{4} \phi^{\Theta} C_{\Theta\Lambda\Sigma} \Big( t_{[\Upsilon\Omega]}{}^{\Lambda} \bar{\chi}^{\Upsilon\,a} \chi^{\Omega}_{a} \phi^{\Sigma} - 4 t_{(\Upsilon\Omega)}{}^{\Lambda} \bar{\chi}^{\Upsilon\,a} \chi^{\Sigma}_{a} \phi^{\Omega} \Big)$$

$$- \tfrac{1}{2} \phi^{\widehat{K}} C_{\widehat{K}MN} t_{\widehat{I}P}{}^{M} t_{\widehat{J}Q}{}^{N} \phi^{\widehat{I}} \phi^{\widehat{J}} \phi^{P} \phi^{Q}$$

$$+ C_{\mathcal{I}\mathcal{J}} \Big( -\tfrac{1}{2} \mathcal{D}_{\mu} q^{\mathcal{I}\,a\dot{b}} \mathcal{D}^{\mu} q^{\mathcal{J}}_{a\dot{b}} - \bar{\zeta}^{\mathcal{I}\,\dot{b}} \not{\mathcal{D}} \zeta^{\mathcal{J}}_{\dot{b}} \Big)$$

$$+ C_{\mathcal{I}\mathcal{J}} \Big( 2i t_{\widehat{K}\mathcal{L}}{}^{\mathcal{I}} q^{\mathcal{L}\,a\dot{b}} \bar{\chi}^{\widehat{K}\,a} \zeta^{\mathcal{J}}_{\dot{b}} + i \phi^{\widehat{K}} t_{\widehat{K}\mathcal{L}}{}^{\mathcal{I}} \bar{\zeta}^{\mathcal{J}\,\dot{a}} \zeta^{\mathcal{L}}_{\dot{a}} \Big)$$

$$+ C_{\mathcal{I}\mathcal{J}} \Big( t_{\widehat{K}\mathcal{L}}{}^{\mathcal{J}} q^{\mathcal{L}\,a\dot{c}} q^{\mathcal{I}\,b}{}_{\dot{c}} Y^{\widehat{K}}_{ab} - \tfrac{1}{2} t_{\widehat{I}\mathcal{K}}{}^{\mathcal{I}} t_{\widehat{J}\mathcal{L}}{}^{\mathcal{J}} \phi^{\widehat{I}} \phi^{\widehat{J}} q^{\mathcal{K}a\dot{b}} q^{\mathcal{L}}_{a\dot{b}} \Big) \,.$$

# Supersymmetry transformations

$$\delta\phi^\Lambda = \frac{i}{2}\,\bar{\epsilon}^a\chi_a^\Lambda\ ,$$

$$\delta A_\mu^{\hat{I}} = \frac{1}{2}\,\bar{\epsilon}^a\gamma_\mu\chi_a^{\hat{I}}\ ,$$

$$\delta F_{\mu\nu}^\Lambda = -\bar{\epsilon}^a\gamma_{[\mu}\mathcal{D}_{\nu]}\chi_a^\Lambda + it_{(\Sigma\Theta)}{}^\Lambda\phi^\Sigma\,\bar{\epsilon}^a\gamma_{\mu\nu}\chi_a^\Theta + i\bar{\eta}^a\gamma_{\mu\nu}\chi_a^\Lambda\ ,$$

$$\delta\chi^{\Lambda\,a} = -\frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}^\Lambda\epsilon^a - \frac{i}{2}\not{D}\phi^\Lambda\epsilon^a - Y^{\Lambda\,ab}\epsilon_b + \frac{1}{2}\,t_{(\Sigma\Theta)}{}^\Lambda\phi^\Sigma\phi^\Theta\epsilon^a + \phi^\Lambda\eta^a\ ,$$

$$\delta Y^{\Lambda\,ab} = -\frac{1}{2}\bar{\epsilon}^{(a|}\not{D}\chi^{\Lambda|b)} - \frac{i}{2}\left(t_{[\Sigma\Theta]}{}^\Lambda - 3t_{(\Sigma\Theta)}{}^\Lambda\right)\phi^\Sigma\,\bar{\epsilon}^{(a|}\chi^{\Theta|b)} + \frac{i}{2}\bar{\eta}^{(a}\chi^{\Lambda|b)}\ ,$$

$$\delta q^{\mathcal{I}\,a\dot{b}} = -i\bar{\epsilon}^a\zeta^{\mathcal{I}\,\dot{b}}\ ,$$

$$\delta\zeta^{\mathcal{I}\,\dot{b}} = \frac{i}{2}\not{D}q^{\mathcal{I}\,a\dot{b}}\epsilon_a - \frac{1}{2}\phi^{\hat{K}}t_{\hat{K}\mathcal{J}}{}^{\mathcal{I}}q^{\mathcal{J}\,a\dot{b}}\epsilon_a - \frac{3}{2}q^{\mathcal{I}\,a\dot{b}}\eta_a\ .$$