Scattering Amplitudes in Three Dimensions

Arthur Lipstein
University of Oxford

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3d Gauge Theories

- Yang-Mills:

$$\frac{1}{g^2} tr \left(F_{\mu\nu} F^{\mu\nu} \right)$$

- Chern-Simons:

$$\frac{k}{4\pi} tr \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right)$$

N=8 sYM vs ABJM

N=8 sYM:

- SO(7) R-symmetry
- gauge group U(N)
- adjoint matter

ABJM:

- SU(4) R-symmetry
- gauge group $U(N)_k x U(N)_{-k}$ (twisted CS)
- bifundamental matter

Conjecture:

- $U(N)_1 x U(N)_{-1}$ ABJM = IR limit of N=8 sYM

4d to 3d

- 4d null momentum: $p_{\alpha\dot{\beta}} = \lambda_\alpha \tilde{\lambda}_{\dot{\beta}}$
- 3d null momentum: $p_{\alpha\beta} = \lambda_\alpha \lambda_\beta$
- To reduce 4d amplitudes to 3d, take $\tilde{\lambda}_{\dot{\alpha}} \to \lambda_\alpha$
- MHV amplitudes of N=8 sYM:

$$\mathcal{A}_n^{MHV} = \frac{\delta^3(P)\,\delta^8(Q)}{\langle 12\rangle \langle 23\rangle \ldots \langle n1\rangle}$$

where $P^{\alpha\beta} = \sum_{i=1}^{n} \lambda_i^\alpha \lambda_i^\beta$ and $Q^{I\alpha} = \sum_{i=1}^{n} \lambda_i^\alpha \eta_i^I$, $I = 1, \ldots, 4$

3d Twistors

- Twistor: $$\xi^m = \begin{pmatrix} \lambda^\alpha \\ \mu_\beta \end{pmatrix} \qquad \mu^\alpha = x^{\alpha\beta}\lambda_\beta$$

- Minitwistor: $$\xi^m = \begin{pmatrix} \lambda^\alpha \\ \mu \end{pmatrix} \qquad \mu = x^{\alpha\beta}\lambda_\alpha\lambda_\beta$$

R-Symmetry

- N=8 sYM amplitudes have manifest SU(4) R-symmetry:

$$r^I{}_J = \sum_{i=1}^{n} \left(-\eta_i^I \frac{\partial}{\partial \eta_i^J} + \frac{1}{4} \delta^I_J \eta_i^K \frac{\partial}{\partial \eta_i^K} \right)$$

- 4-point amplitude has SO(8) R-symmetry:

$$r^{IJ} = \sum_{i=1}^{n} \eta_i^I \eta_i^J, \quad r_{IJ} = \sum_{i=1}^{n} \frac{\partial}{\partial \eta_i^I} \frac{\partial}{\partial \eta_i^J}, \quad r = \sum_{i=1}^{n} \eta_i^I \frac{\partial}{\partial \eta_i^I}$$

Agarwal, Young

- For n>4, amplitudes have SO(7) R-symmetry:

$$\tilde{r}^{IJ} = r^{IJ} + \tfrac{1}{2} \epsilon^{IJKL} r_{KL}$$

- $\tilde{r}^{IJ}$ mixes amplitudes of different helicity

Dual Conformal Covariance

- Dual space:

$$(x_i - x_{i+1})^{\alpha\beta} = p_i^{\alpha\beta} = \lambda_i^{\alpha}\lambda_i^{\beta}$$

$$(\theta_i - \theta_{i+1})^{A\alpha} = q_i^{A\alpha} = \lambda_i^{\alpha}\eta_i^{A}$$

- Dual inversion:

$$I\left[x_i^{\alpha\beta}\right] = \frac{x_i^{\alpha\beta}}{x_i^2},\ I\left[\theta_i^{I\alpha}\right] = \frac{x_i^{\alpha\beta}\theta_{i\beta}^{I}}{x_i^2}$$

- MHV amplitudes transform covariantly under dual inversions:

$$I\left[\frac{\delta^3(P)\delta^8(Q)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\right] = \frac{1}{x_1^2}\left(x_1^2 x_2^2 \ldots x_n^2\right)\frac{\delta^3(P)\delta^8(Q)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

Dual Conformal Covariance

- L-loop n-point integrand also transforms covariantly:

$$I\left[\mathcal{I}_n^L\right] = \Pi_{i=1}^n x_i^2 \Pi_{j=1}^L \left(x_j^2\right)^4 \mathcal{I}_n^L$$

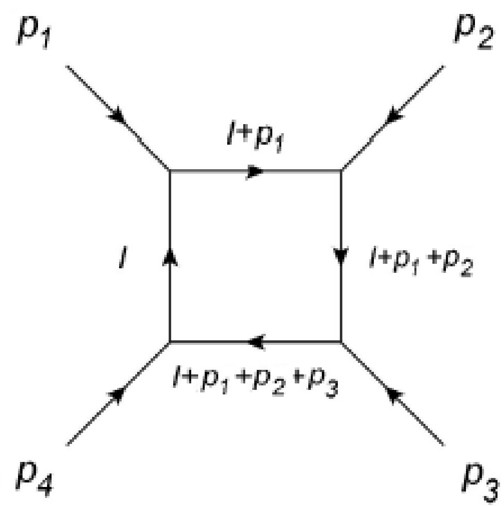
- In D dimensions, the loop integration measure transforms as

$$I\left[d^D x\right] = \left(x^2\right)^{-D}$$

- N=8 sYM loop integrand transforms like 4d loop integrand
- 3d loop integrals are NOT dual conformal invariant

Loop Amplitudes

- 1-loop amplitudes of 4d N=4 sYM can be reduced to scalar box diagrams:



- The corresponding loop integral is

$$\int \frac{d^d l}{(l^2 + i\epsilon)\left((l+p_1)^2 + i\epsilon\right)\left((l+p_1+p_2)^2 + i\epsilon\right)\left((l+p_1+p_2+p_2)^2 + i\epsilon\right)}$$

Loop Amplitudes

- Evaluating box diagrams in $d=3+2\varepsilon$ gives

1-loop MHV = 0

1-loop non-MHV = finite

- Resemble loop corrections in ABJM
- Expect IR divergences at 2 loops

Summary

N=8 sYM amplitudes have many interesting properties:

- Helicity structure
- SO(8) R-symmetry for n=4
- SO(7) R-symmetry for n>4
- Dual conformal covariance (tree amps and loop integrands)
- Finite at 1-loop

Future directions:

- 3d MHV formalism
- Grassmannian integral formula
- Higher loops
- mass-deformed theories