Lepton mass and flavour violation in Randall Sundrum Models

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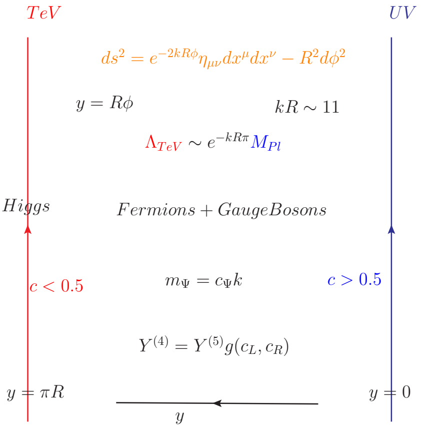
Outline

Randall Sundrum Model Randall, Sundrum '99

One extradimension compactified on S_1/Z_2



TeV UV

$$ds^2 = e^{-2kR\phi}\eta_{\mu\nu}dx^\mu dx^\nu - R^2 d\phi^2$$

$y = R\phi$ $kR \sim 11$

$$\Lambda_{TeV} \sim e^{-kR\pi} M_{Pl}$$

$Higgs$ $Fermions + GaugeBosons$

$c < 0.5$ $m_\Psi = c_\Psi k$ $c > 0.5$

$$Y^{(4)} = Y^{(5)} g(c_L, c_R)$$

$y = \pi R$ $y = 0$

y

Leptonic sector

- Leptonic sector in RS has been subject of intense study in the past Huber and Shafi '01-'04, Sundrum et al. '09,
- Offers numerous possibilities corresponding to Dirac or Majorana nature of neutrinos.
- Flavour consideration places very strict constraints on the model.
- Minimal RS incapable of satisfying mixing data and flavour constraints simulatneously. Mu-chun chen, Perez et al, Csaki

Goal of the study

- Shed light on the finer details of two most important quantities involved in fitting in leptonic sector:
  a) c parameters and
  b)$\mathcal{O}(1)$ Yukawa parameters, both of which are varied to arrive at the best fit region for the bulk mass parameters
- While the $\mathcal{O}(1)$ Yukawa are varied in the interval $[.08, 4]$, the c parameters should ideally lie in the interval $[-1, 1]$
- Constrain the parameter space by flavour considerations.

χ^2 minimization

- Fit the bulk RS parameters to the leptonic masses and mixing angles
- Results presented for normal hierarchy of neutrino masses.
- Perform the analysis for different models of neutrino masses

  a) LHLH case

  b) Dirac Case

  c) Bulk 'Majorana' mass terms

LLHH Case

LLHH case

No Right Handed neutrinos

9+6+6=21 parameters

The $\mathcal{O}(1)$ Yukawa parameters are defined as

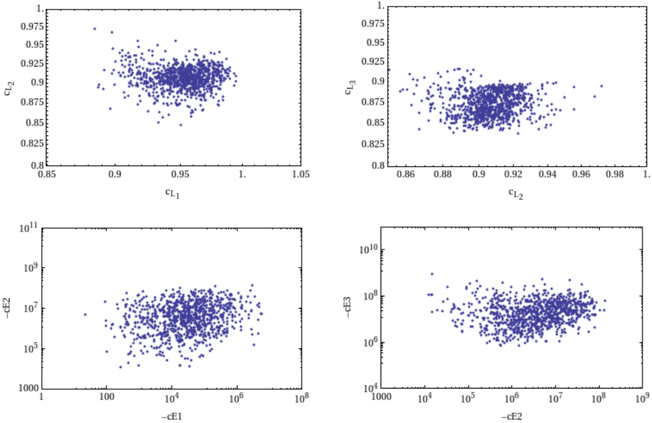
$$Y'_E = 2kY_E \quad ; \quad \kappa' = 2k\kappa$$

and the mass matrices are given as

$$(\mathcal{M}_e^{(0,0)})_{ij} = \frac{v}{\sqrt{2}}(Y'_E)_{ij}e^{(1-c_{L_i}-c_{E_j})kR\pi}N^{(0)}(c_{L_i})N^{(0)}(c_{E_j})$$

$$(\mathcal{M}_\nu^{(0,0)})_{ij} = \frac{v^2}{2\Lambda^{(5)}}(\kappa')_{ij}e^{(2-c_{L_i}-c_{L_j})kR\pi}N^{(0)}(c_{L_i})N^{(0)}(c_{L_j})$$

Parameter space for bulk masses in LLHH case



Summary of LLHH case

Allowed range for the bulk parameters with minimum χ^2. Neutrino masses have normal hierarchy. Range of first KK scale $M^{(1)}$ corresponding to the bulk mass parameter is also given.

parameter	range	range of $M^{(1)}$ (TeV)
c_{L_1}	0.87-0.995	1.49-1.59
c_{L_2}	0.86-0.98	1.48-1.58
c_{L_3}	0.84-0.92	1.47-1.53
c_{E_1}	-10.0 - -5.0×10^6	7.9-3.9 $\times 10^6$
c_{E_2}	-1.0×10^4 - -1.2×10^8	7.9×10^3-9.5×10^7
c_{E_3}	-7.0×10^5- -1×10^9	5.5×10^5 7.9×10^8

Implications of large negative c parameters

- The bulk c parameters have a dual 4D description by means of the holographic duality Contino,Pomarol
- The doublets in the LLHH case are completely elementary from the 4D point of view.
- However, the charged singlets are completely composite objects of the CFT.
- Composite singlets has 'interesting' flavour implications.

Dirac Case

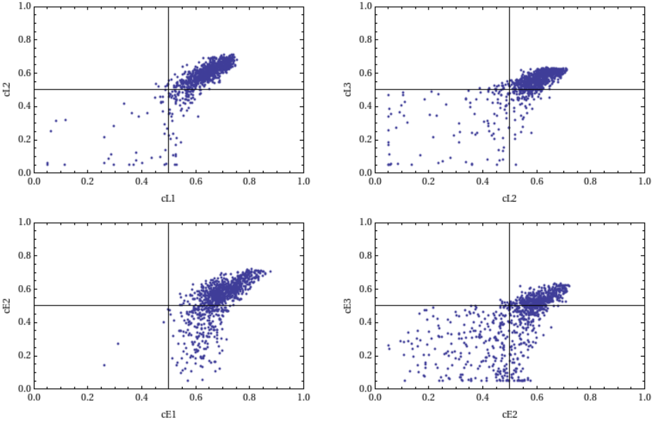
Dirac Neutrinos

Add three right Handed neutrinos

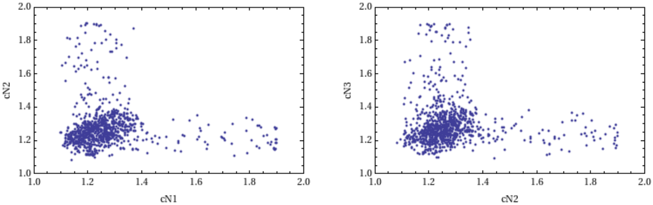
9+9+9=27 parameters and the effective four dimensional Yukawas for both the charged lepton and neutrinos read as

$$Y_{ij}^{(4)} = \frac{Y_{ij}^{\prime(5)}}{N_{0L}N_{0R}} e^{(1-c_{iL}-c_{iR})}$$

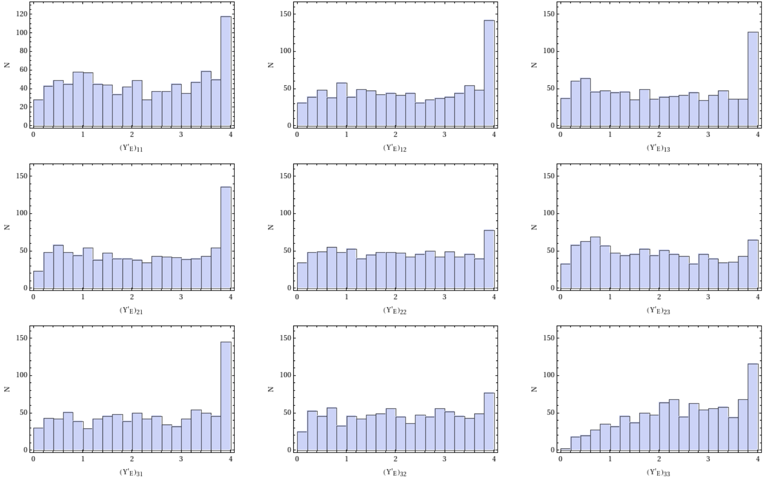
Parameter space for the bulk masses of doublets and charged singlets.



Parameter space for the bulk masses of neutral singlets.



Distribution of charged $\mathcal{O}(1)$ Yukawa parameters.



Summary for Dirac Case

Table: Allowed ranges of bulk parameters with normal hierarchy of neutrino masses. The range of first KK scale corresponding to the range of c values is also given.

parameter	range	$M^{(1)}$ TeV
c_{L_1}	0.05-0.76	0.839-1.4
c_{L_2}	0.05-0.72	0.839-1.37
c_{L_3}	0.05-0.64	0.839-1.31
c_{E_1}	0.2-0.88	0.959-1.5
c_{E_2}	0.05-0.73	0.839-1.38
c_{E_3}	0.05-0.64	0.839-1.31
c_{N_1}	1.1-1.9	1.67-2.31
c_{N_2}	1.1-1.9	1.67-2.31
c_{N_3}	1.1-1.9	1.67-2.31

Bulk Majorana case

Bulk Majorana mass term

Analysis with UV localized Majorana mass term has been considered earlier Huber&Shafi '04: Perez & Randall '09

Similar terms can be added in the bulk of RS but 'Majorana' mass terms in 5D do not have the same interpretation as is 4D.

$$S_N = \int d^4x \int dy \sqrt{-g} \left(m_M \bar{N} N^c + m_D \bar{N} N + \delta(y - \pi R) Y_N \bar{L} \tilde{H} N \right)$$

where $N^c = C_5 \bar{N}^T$and$m_D = c_N k, m_M = c_M k$.

Eigenvalue Equations

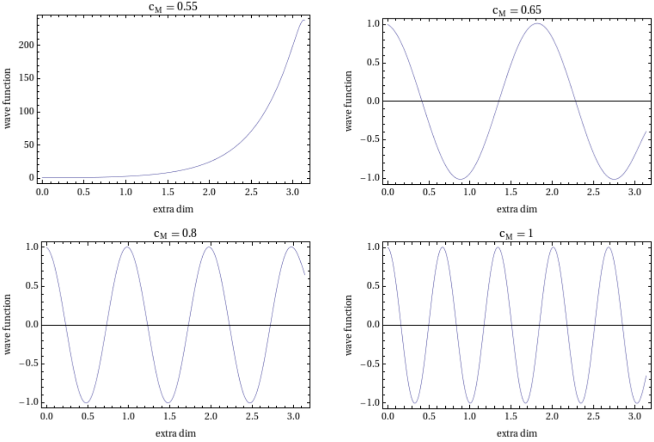
The eigenvalue equations for the left(right) profiles $g_L(g_R)$ of N are

$$(\partial_y + m_D) g_L^{(n)}(y) = m_n e^{\sigma} g_R^{(n)}(y) - m_M g_R^{(n)}(y)$$
$$(-\partial_y + m_D) g_R^{(n)}(y) = m_n e^{\sigma} g_L^{(n)}(y) - m_M g_L^{(n)}(y)$$

We assume $g_L(y)$ to be Z_2 even. Zero mode solutions not consistent with boundary conditions.

The decoupled second order equations are difficult to solve generally and we solve them numerically.

Profile plots



$c_N = 0.58$.The profile becomes oscillatory as c_M becomes greater than c_N.

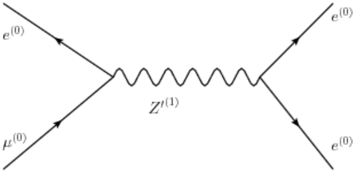
Sample Point

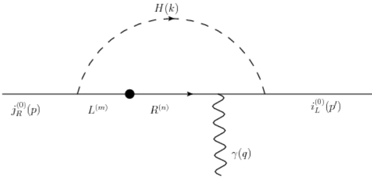
Table: Sample points with corresponding fits of observables for Normal and Inverted Hierarchy schemes in Bulk Majorana case with O(1) Yukawas. The masses are in GeV

Parameter	Normal	Inverted
M_{KK}	161.4	161.4
c_{M_i}	0.55	0.55
$g_L^{(1)}(\pi R)$	3×10^{-13}	1.2×10^{-12}
c_{L_1}	0.58	0.59
c_{L_2}	0.56	0.57
c_{L_3}	0.55	0.55
c_{E_1}	0.735	0.735
c_{E_2}	0.5755	0.575
c_{E_3}	0.501	0.501
c_{N_i}	0.58	0.58
m_e	5.09×10^{-4}	5.08×10^{-4}
m_μ	0.1055	0.1055
m_τ	1.77	1.774
θ_{12}	0.58	0.58
θ_{23}	0.80	0.8
θ_{13}	0.13	0.13
Δm^2_{sol}	7.8×10^{-23}	7.8×10^{-23}
Δm^2_{atm}	2.4×10^{-21}	2.4×10^{-21}

Flavour

$l_i \to l_j l_k l_k$ and $l_i \to l_j \gamma$





Present Experimental Bounds on LFV Processes

Process	Experiment	Present upper bound
BR($\mu \to e\,\gamma$)	MEG	2.4×10^{-12}
BR($\mu \to e\,e\,e$)	MEG	1.0×10^{-12}
CR($\mu \to e\,\mathrm{in}\,\mathbf{Ti}$)	SINDRUM-II	6.1×10^{-13}
BR($\tau \to \mu\,\gamma$)	BABAR/Belle	4.4×10^{-8}
BR($\tau \to e\,\gamma$)	BABAR/Belle	3.3×10^{-8}
BR($\tau \to \mu\,\mu\,\mu$)	BABAR/Belle	2.0×10^{-8}
BR($\tau \to e\,e\,e$)	BABAR/Belle	2.6×10^{-8}

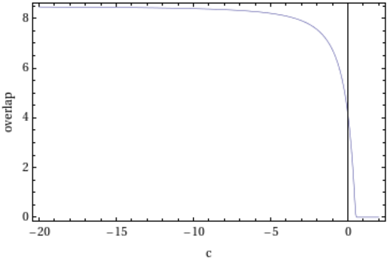


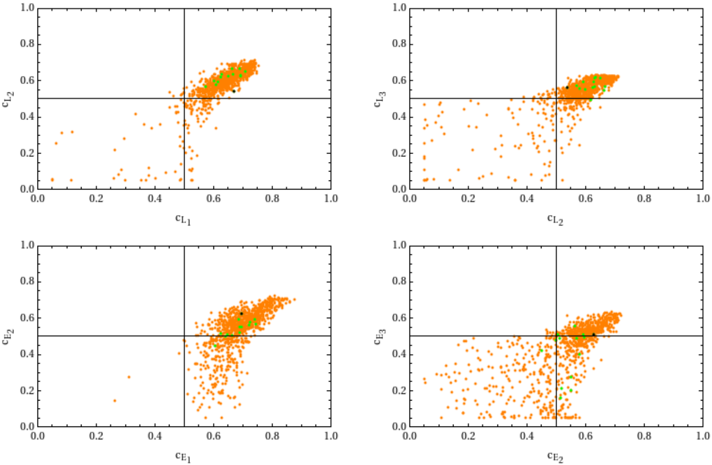
Figure: Coupling of two zero mode fermions to Z_1 as a function of bulk mass parameter

Constraints on LHLH case

- Doublets and the charged singlets couple universally to the KK gauge boson, thus leading order effects are highly suppressed.
- The large effective Yukawa coupling of the zero mode singlet to the KK mode $\propto \sqrt{0.5 - c}$
- The dipole processes due to gauge boson contribution is suppressed due to heavy KK scales.
- The large universal shift in the gauge coupling can be suppressed by either a very high KK gauge boson scale or by invoking custodial symmetry.

Constraints on Dirac Case

NO Point survives the $\mu \to e\gamma$ constraint-Requires fermionic KK scale $\mathcal{O}(10)$TeV



Bulk Majorana case

For the tree-level decays, constraints obtained on the bulk masses apply in this case as well.

The dominant contribution to dipole decays in this case is due to Higgs exchange diagram.

Table: BR for dipole decays for the case with bulk Majorana mass

Hierarchy	BR($\mu \to e\gamma$)	BR($\tau \to \mu\gamma$)	BR($\tau \to e\gamma$)
Inverted	2.4×10^{-5}	1.9×10^{-5}	7.6×10^{-6}
Normal	1.4×10^{-5}	3.4×10^{-5}	1.3×10^{-5}

branching fractions are evaluated for $M_{KK} \sim 1250$ GeV

Need for Flavour symmetries

- The constraints from dipole processes are far more severe. No point survives the $\mu \to e\gamma$ constraint for low fermion KK scale.
- Large contributions to FCNC are due to the 'large' misalignment between the flavor structure of the diagram and the zero mode mass matrix.
- The mass square matrix in the charge lepton sector goes as $Y_E F_E F_E^\dagger Y_E^\dagger$ while the mixing is controlled by $Y_E Y_E^\dagger$

Flavour symmetries

- The parameter space of Dirac and the bulk Majorana case are not consistent with flavour constraints.
- Turn to the ansatz of Minimal Flavour Violation (MFV) Perez & Randall, Mu-chun Chen & Hai-Bo Yu
- Dipole Constriaints can be satisfied for KK fermion scales as low as 3 TeV
- We are looking at various definitions of MFV applicable to the bulk Majorana case.

Summary and Conclusions

- The LLHH case is not very favourable in the RS scenario owing to the extreme choice of bulk mass parameters required to fit the data.
- The Dirac and the bulk Majorana cases offers a very viable alternative.
- The constraints from flavour considerations are severe and one is forced to invoke flavour symmetries.
- Future work involves exploring various schemes of MFV in the Majorana case.

Minimal Flavour Violation in Dirac Case

Cirigliano et al. '08

- In the presence of right handed neutrinos the flavour group is $SU(3)_L \times SU(3)_E \times SU(3)_N$
- $Y_E \rightarrow (3, \bar{3}, 1)$ $Y_N \rightarrow (3, 1, \bar{3})$
- The Yukawa couplings are aligned with the five dimensional bulk mass matrices

$$c_L = a_1 I + a_2 Y'_E Y'^{\dagger}_E + a_3 Y'_N Y'^{\dagger}_N \;\; ; c_E = b Y'^{\dagger}_E Y'_E \;\; ; c_N = c Y'^{\dagger}_N Y'_N$$

- Owing to the flavor symmetry we work in a basis in which Y'_E is diagonal. In this basis $Y'_N \rightarrow V_{PMNS}\text{Diag}(Y'_N)$
- Flavor violating part

$$\Delta = Y'_N Y'^{\dagger}_N$$

- Lowering of fermion KK scale required to satisfy all constraints from dipole processes to as low as 3 TeV.

MFV with bulk Majorana Neutrinos

- We choose the flavour group $SU(3)_L \times SU(3)_E \times O(3)_N$
- $Y_E \rightarrow (3, \bar{3}, 1)$ and $Y_N \rightarrow (3, 1, 3)$
- The bulk Majorana term $\bar{N}^c N$ transforms as $(1, 1, 6)$
- The bulk mass parameters can be expressed as

$$c_L = a_1 I + a_2 {Y'}_E Y'^{\dagger}_E + a_3 Y'_N Y'^{T}_N$$

$$c_E = 1 + b Y'^{\dagger}_E Y'_E \quad c_N = 1 + c Y'^{T}_N Y'_N \quad c_M = d I_{3\times3}$$