university of groningen

faculty of mathematics and natural sciences

Andrea Borghese

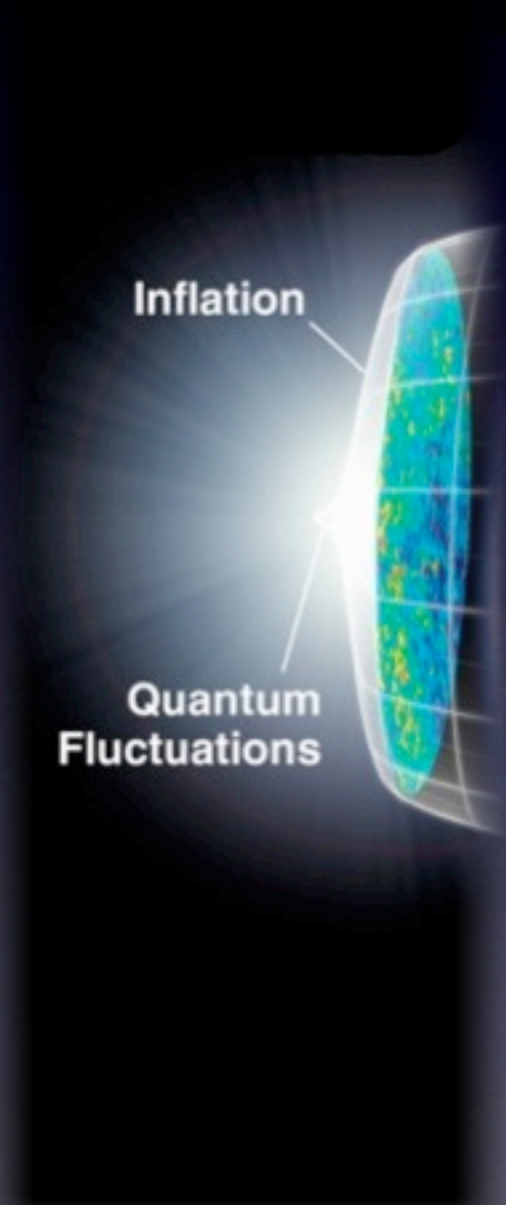
# A geometric bound on F-term inflation

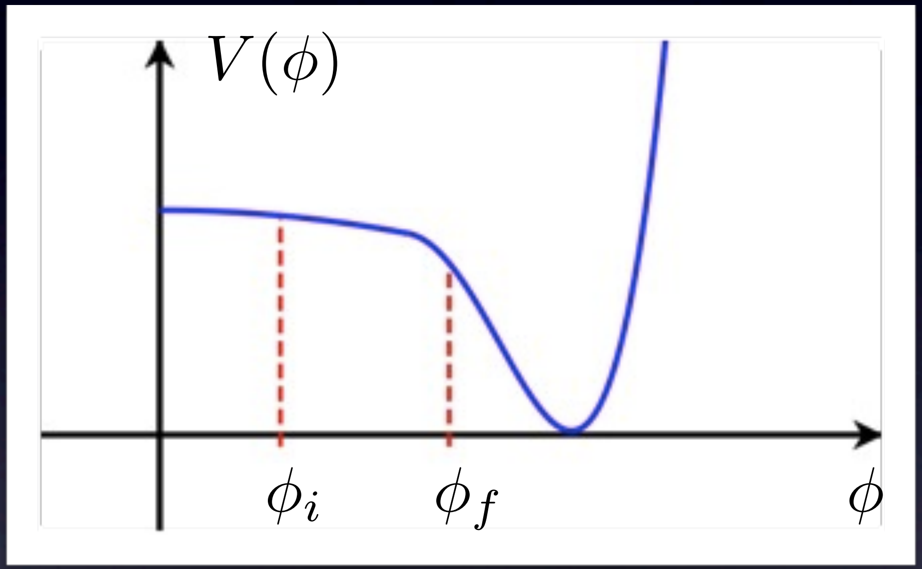
Based on: A.B., D.Roest, I.Zavala, [1203.2909]

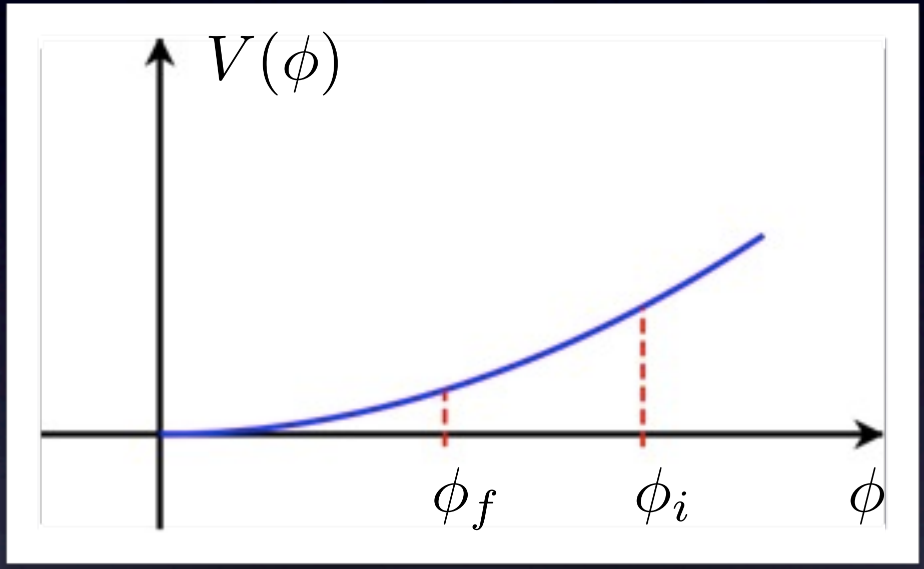
[Guth, 81;
Linde, 82]

# INFLATION

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$



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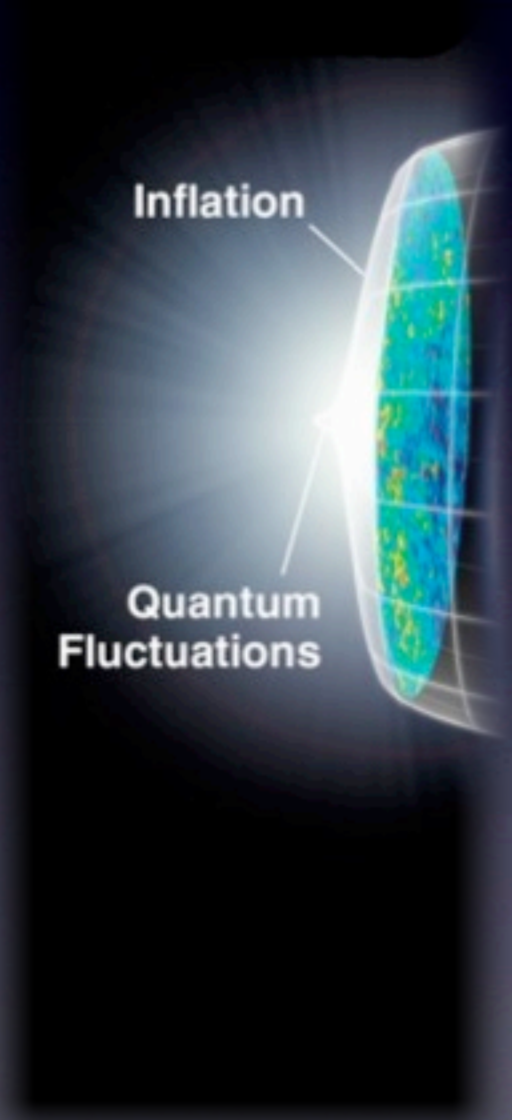


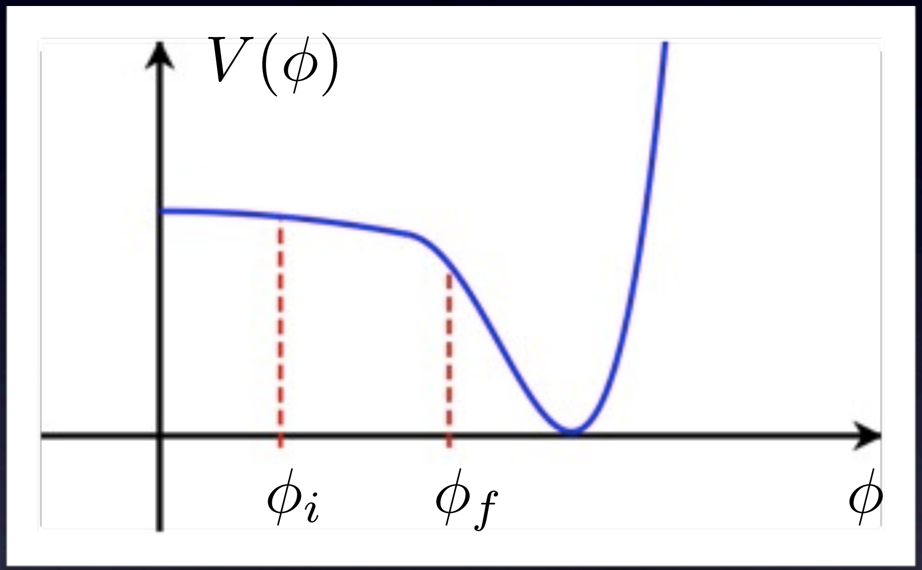


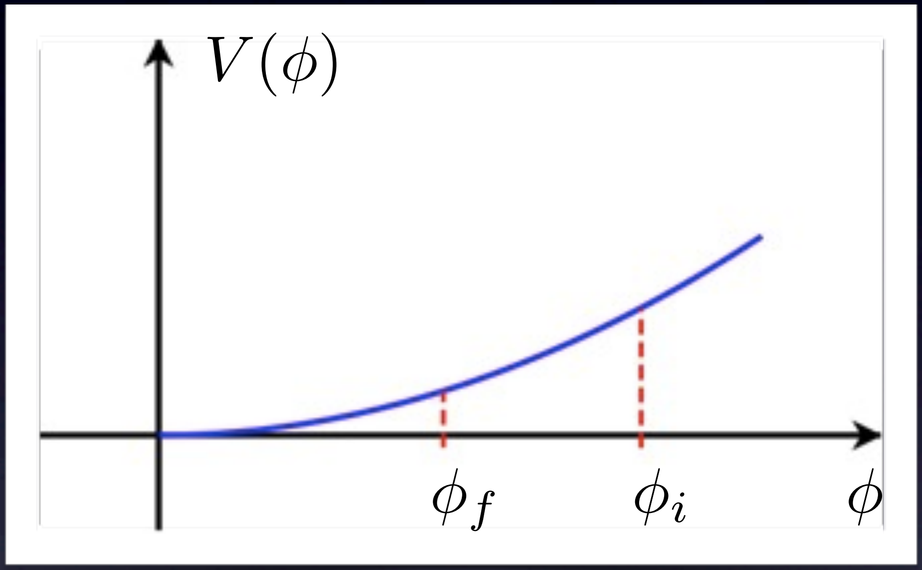


predictions on cosmological observables are
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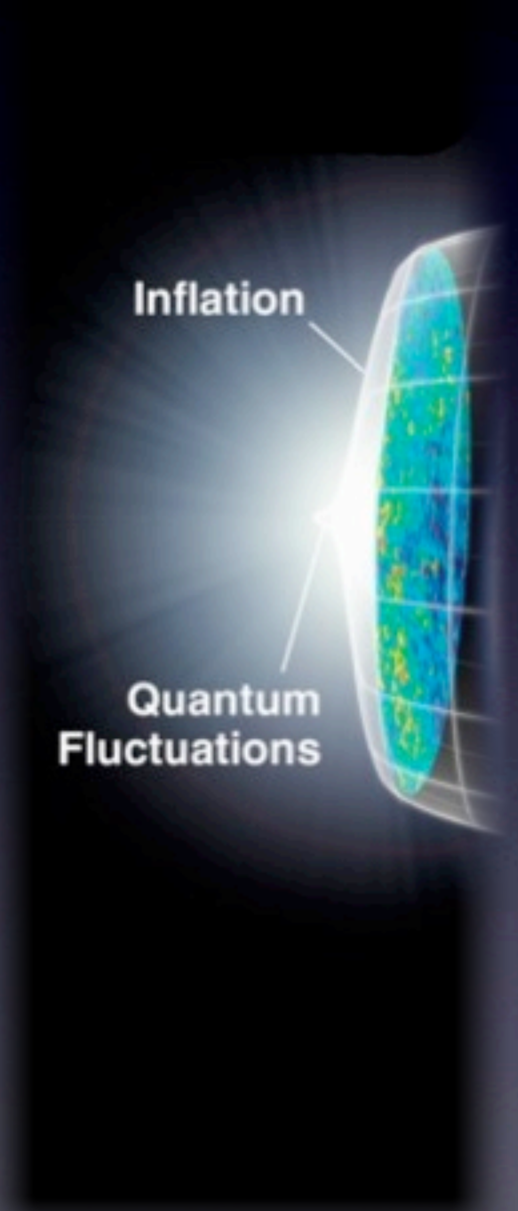


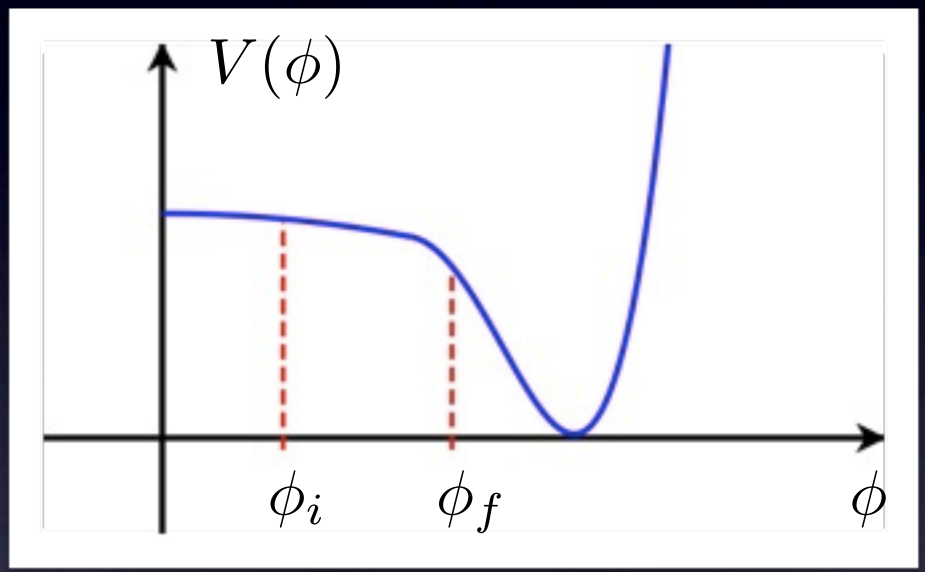


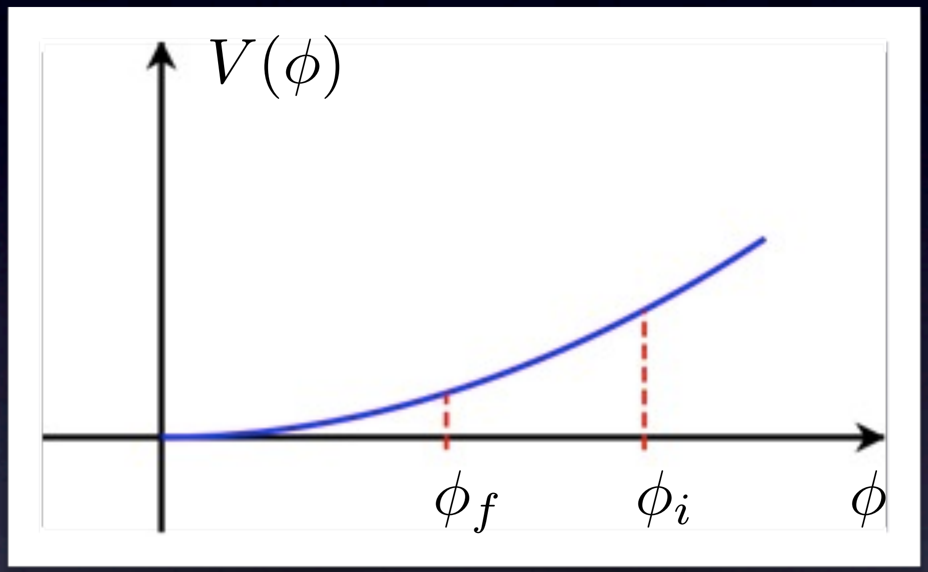


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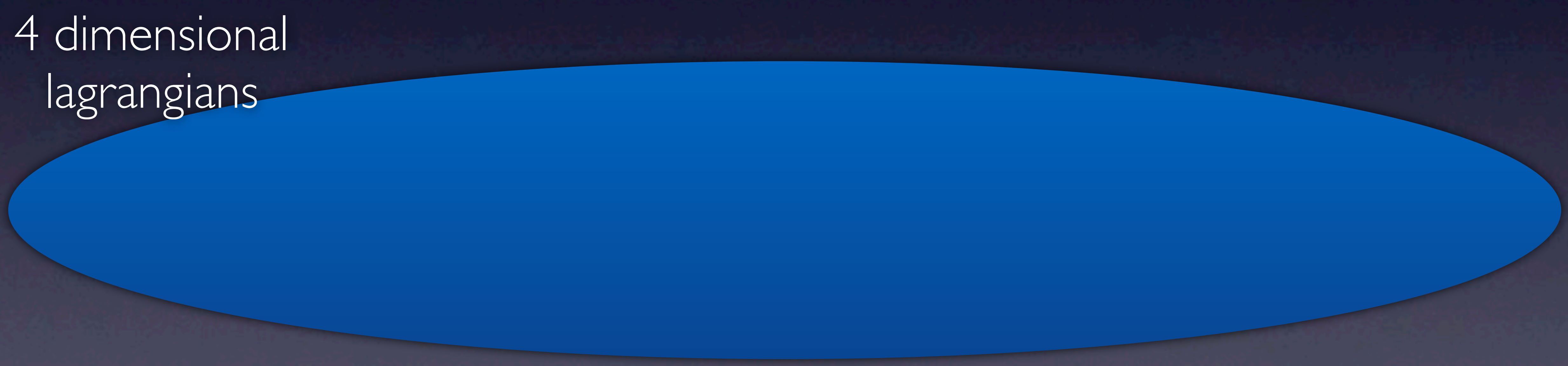

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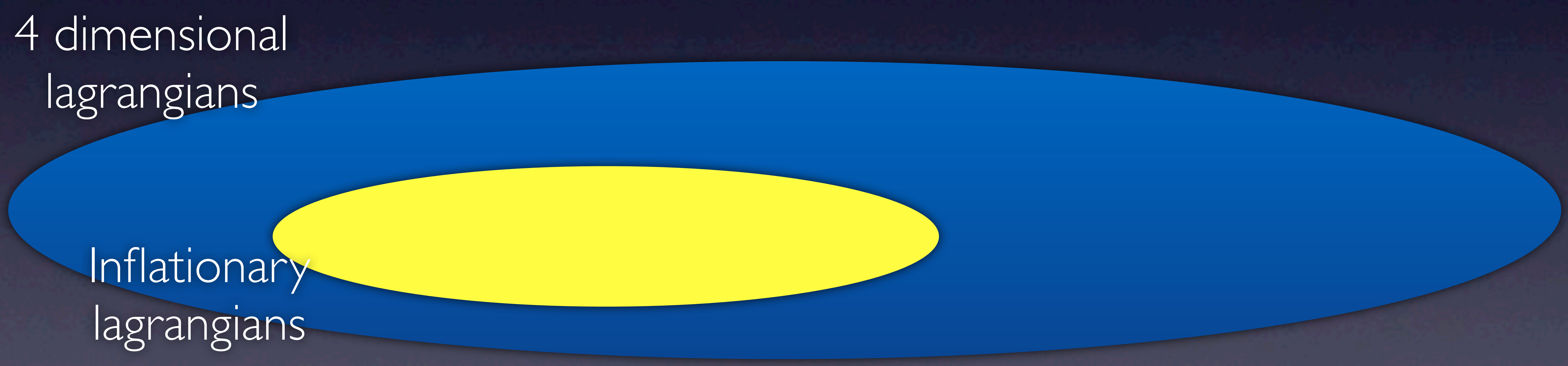

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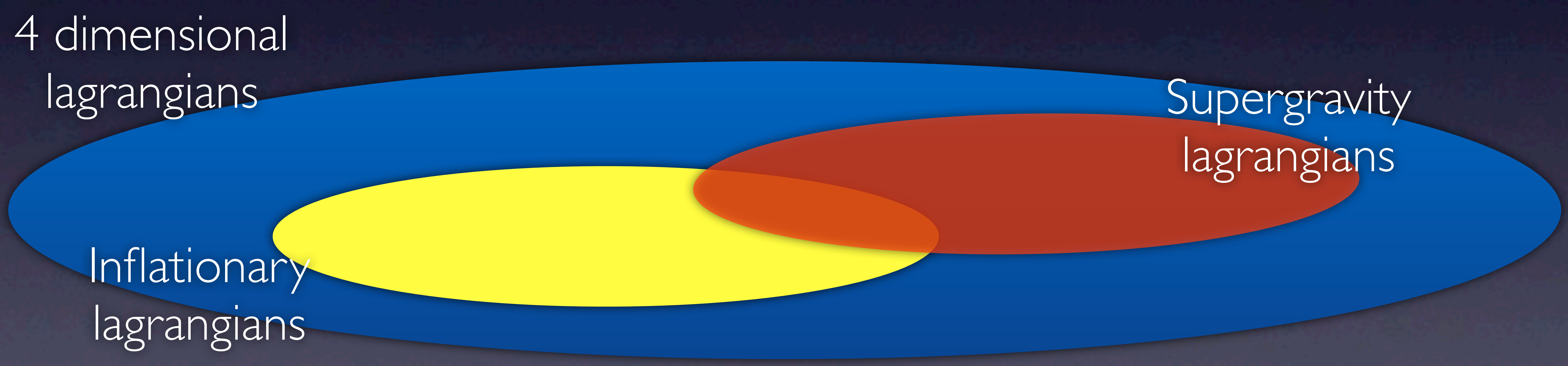

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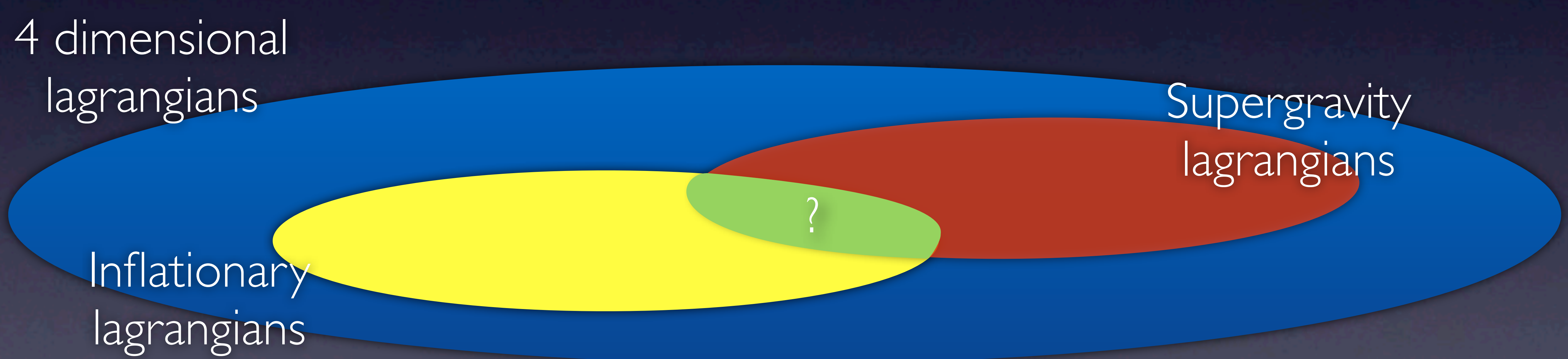


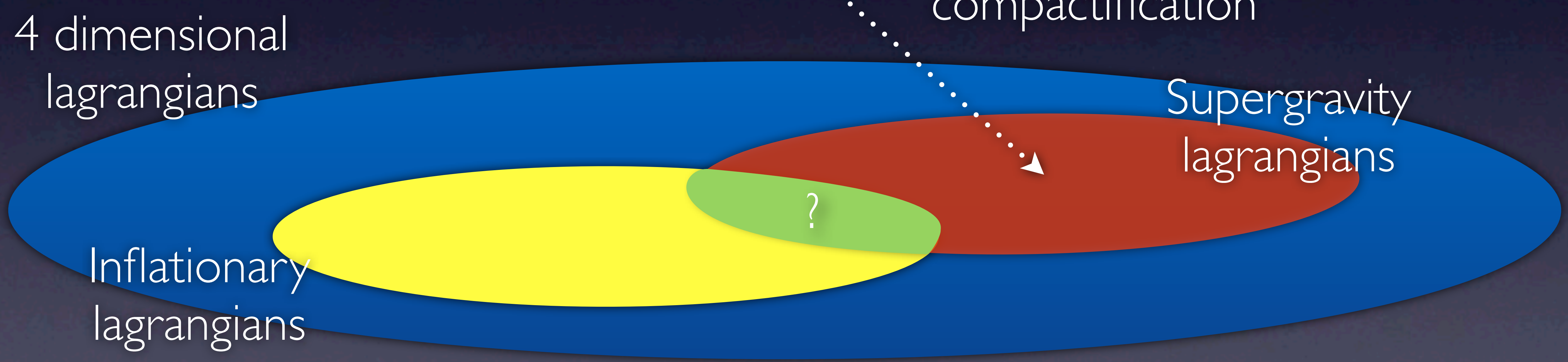
## IS IT POSSIBLE TO EMBED INFLATION IN A UV-COMPLETE THEORY?

STRING THEORY

low energy limit ↓

10 dimensional Supergravity

compactification



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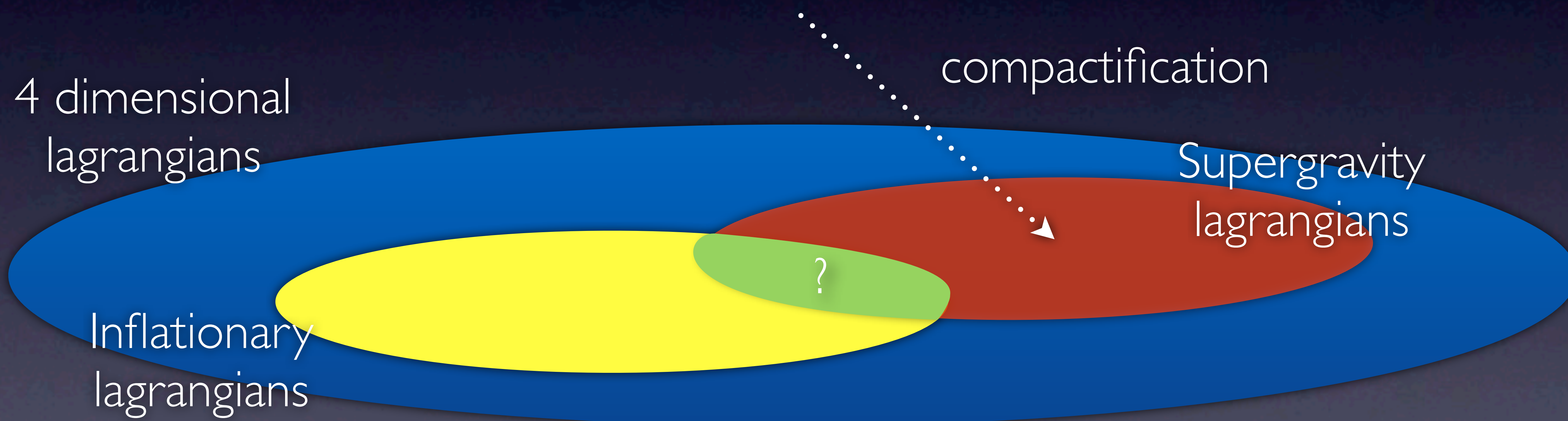


Supergravity is like a bridge between
the EFT of inflation and the UV complete theory

## SUPERGRAVITY SPECTRA

dozens of scalar fields

during inflation we have a deSitter (*dS*) space-time in which SUSY is broken completely

non-supersymmetric configurations high probability of tachyonic directions ( no stable *dS* vacua in $\mathcal{N} = 4, 8$ )

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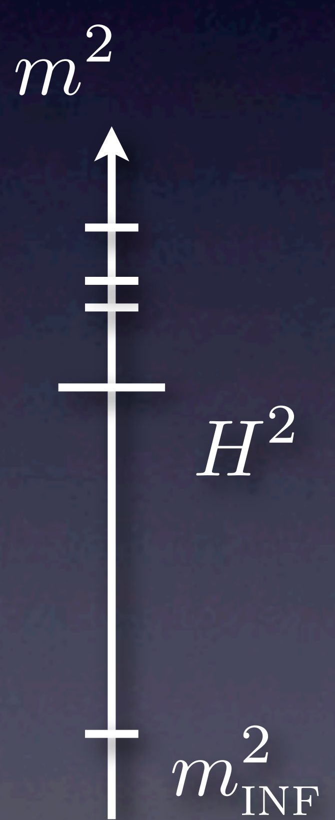
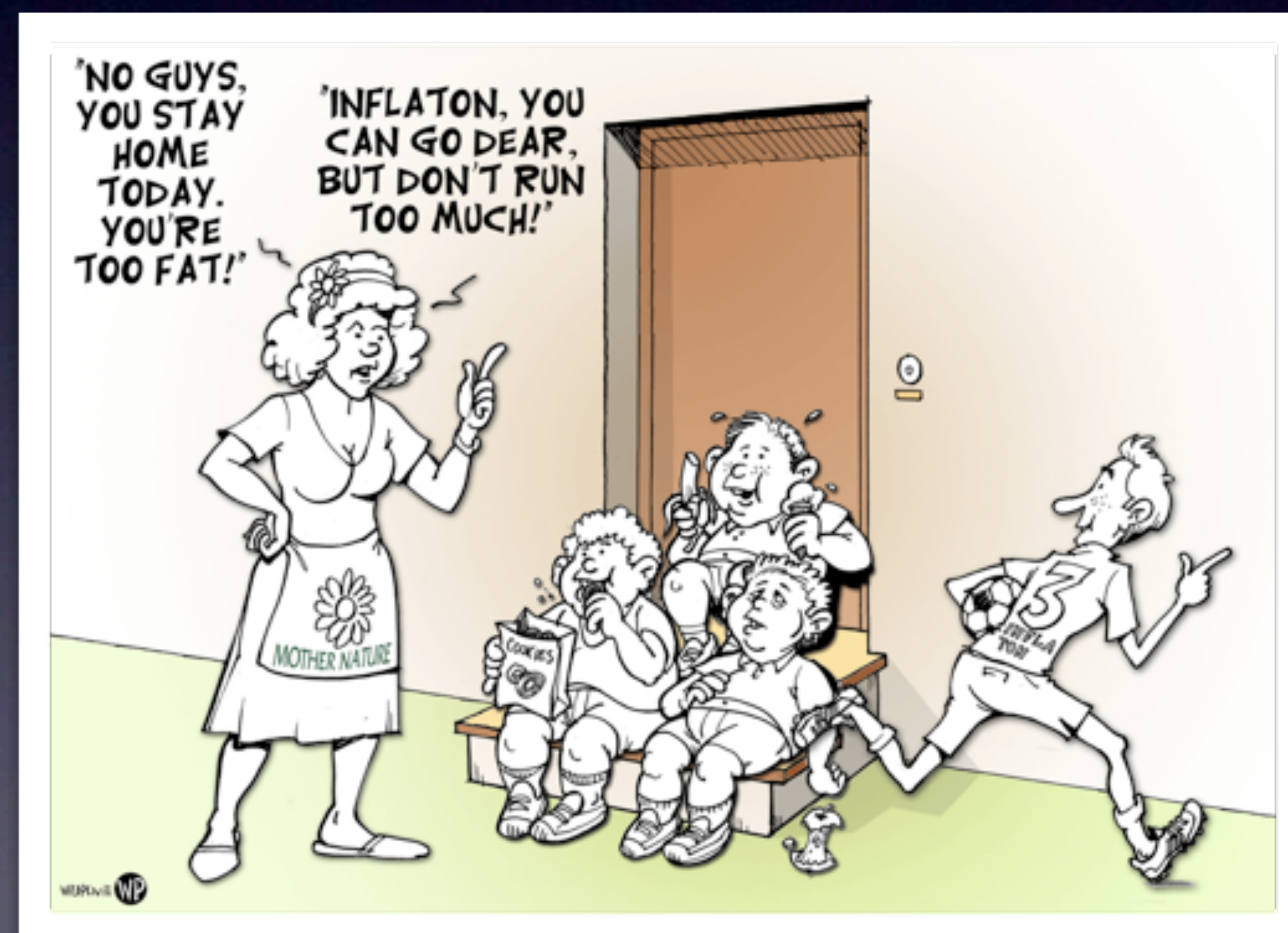



other scalars

Hubble scale (during inflation is given by the value of the scalar potential)

inflaton mass



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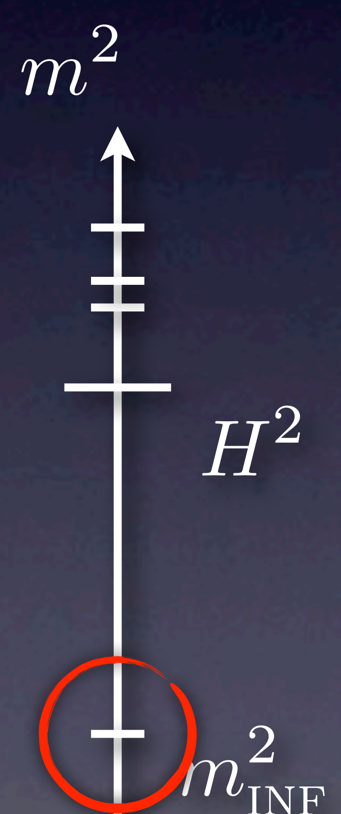


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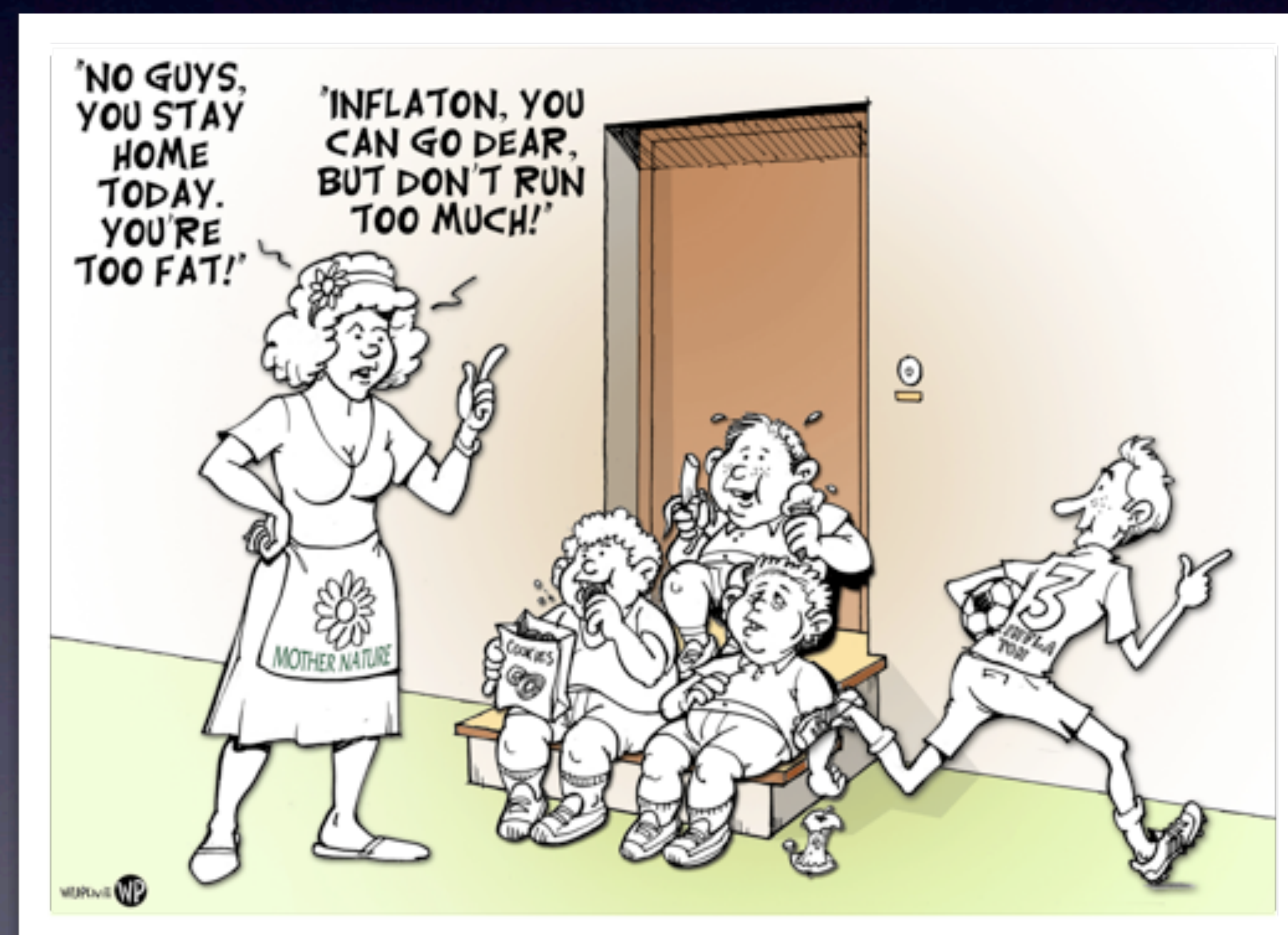

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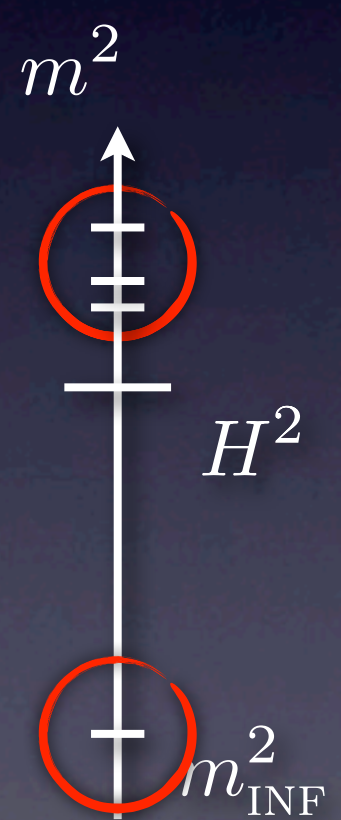


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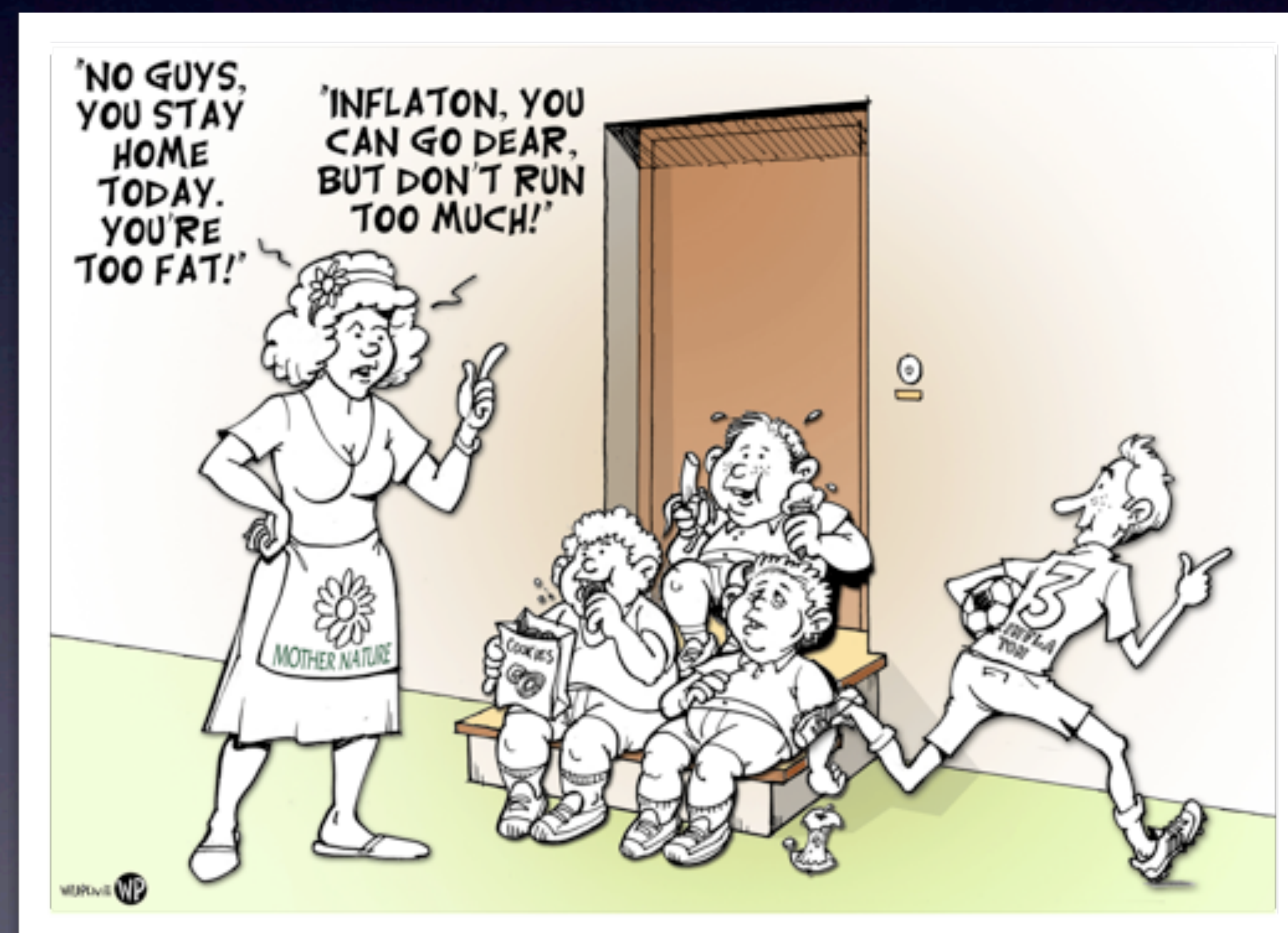
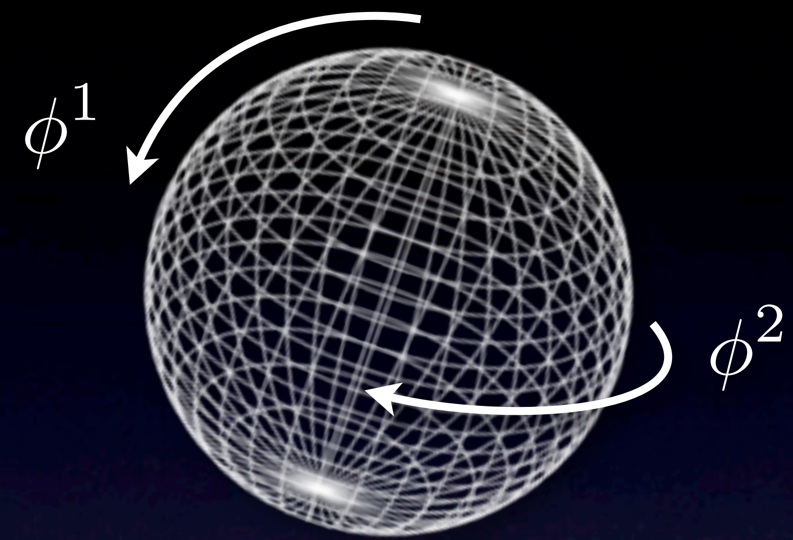


SCALARS ARRANGE THEMSELVES IN MANIFOLDS

for $\mathcal{N} > 2$ they are coset manifolds

for $\mathcal{N} = 1,2$ they are complex manifolds such as Hodge-Kähler, special Kähler or quaternionic-Kähler



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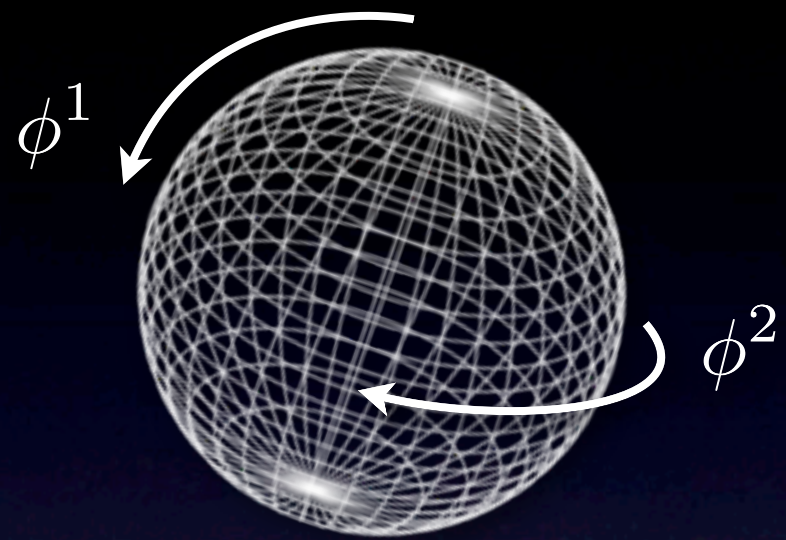


## SGOLDSTINI

Super-Higgs mechanism $=$ Higgs mechanism $\text{“} + 1/2 \text{”}$

For every broken SUSY we have a spin-1/2 field called Goldstino

$$\eta^i \propto N^i{}_a \, \chi^a$$

$i = 1, \ldots, \mathcal{N}$

$a$ labels spin-1/2 fields

the Goldstini are "eaten up" by the gravitini $\psi^i_\mu$
the gravitini eventually become massive

SGOLDSTINI

sGoldstini are the supersymmetric partners of Goldstini

$$\eta^i \;\propto\; N^i{}_a\,\chi^a$$

$$\Big\downarrow\ \varepsilon^j$$

$$\tilde{\eta}^{ij} \;\propto\; N^{ij}{}_\alpha \phi^\alpha$$

$\mathcal{N}^2$ directions in the scalar manifold

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$\mathcal{N} = 1$

one complex direction corresponding to two real scalar d.o.f.

$\mathcal{N} = 2$

4 complex directions
1 anti-symmetric correspond to a gauge direction
and 3 symmetric

$\mathcal{N} = 8$

64 complex directions
28 anti-symmetric correspond to gauge directions
36 symmetric are real scalar d.o.f.

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Used to check perturbative stability of critical points in 4D supergravity

[Gomez-Reino, Scrucca, 06-07;
Gomez-Reino, Louis, Scrucca, 08;
A.B., Roest, 10;
A.B., Linares, Roest, 11]

$\mathcal{N} = 1$ only chiral-multiplets [Covi, Gomez-Reino, Gross Louis, Palma, Scrucca, 08]

theory completely specified by

Kähler potential $\mathcal{K} = \mathcal{K}(\phi^\alpha, \bar{\phi}^{\bar{\alpha}})$

super potential $\mathcal{W} = \mathcal{W}(\phi^\alpha)$

$$V(\underline{\phi}) = e^{\mathcal{K}} \left( -3\,\mathcal{W}\overline{\mathcal{W}} + \mathcal{K}^{\alpha\bar{\beta}}\,\mathcal{D}_\alpha \mathcal{W}\,\mathcal{D}_{\bar{\beta}}\overline{\mathcal{W}} \right)$$

$$N_\alpha = e^{\mathcal{K}/2}\,\mathcal{D}_\alpha \mathcal{W}$$

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$$m^2_{\rm sG} = \frac{1}{2} \left( m_1^2 + m_2^2 \right)$$

$$\eta_{\rm sG} \equiv \frac{m^2_{\rm sG}}{V} \le \frac{2}{3\gamma} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + \frac{\gamma}{1+\gamma}\, \epsilon - \frac{1+\gamma}{\gamma}\, \tilde{\mathcal{R}}$$

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$$\gamma \equiv \frac{V}{3\,|m_{3/2}|^2}$$

Ratio between Hubble scale and gravitino mass

$$\tilde{\mathcal{R}} = \mathcal{R}_{\alpha\bar{\beta}\gamma\bar{\delta}}\,\hat{N}^\alpha\,\hat{N}^{\bar{\beta}}\,\hat{N}^\gamma\,\hat{N}^{\bar{\delta}}$$

sectional curvature related to the plane spanned by sGoldstino directions

$$\epsilon \equiv \frac{\mathcal{K}^{\alpha\bar{\beta}}\,\mathcal{D}_\alpha V\,\mathcal{D}_{\bar{\beta}} V}{2\,V^2}$$

first slow-roll parameter

$$\eta_{\rm sG} \equiv \frac{m_{\rm sG}^2}{V} \leq \frac{2}{3\gamma} + \frac{4}{\sqrt{3}}\frac{1}{\sqrt{1+\gamma}}\sqrt{\epsilon} + \frac{\gamma}{1+\gamma}\,\epsilon - \frac{1+\gamma}{\gamma}\,\tilde{\mathcal{R}}$$

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a similar bound can be obtained
in the case of extended supergravities

[W.I.P.]

$\mathcal{N} = 2$

$$\eta_{\rm sG} \leq c_0 \left(f(\gamma) + g(\gamma)\, \tilde{\mathcal{R}}\right) + c_{1/2} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + c_1 \frac{\gamma}{1+\gamma}\, \epsilon$$

$\mathcal{N} = 8$

## CONCLUSIONS

### VIABILITY OF INFLATION IN F-TERM SUPERGRAVITY

- Geometry is tightly entangled with dynamics of scalar fields
- Constraints on inflationary dynamics: average sGoldstino mass is bounded from above by first slow roll parameter and geometric data
- Similarities in the analysis for minimal and extended supergravity

# THANK YOU!

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faculty of mathematics and natural sciences

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