Higgs boson mass in MSSM with split sfermion masses

Marek Olechowski

Institute of Theoretical Physics, University of Warsaw

Summer School and Workshop on the Standard Model and Beyond
Corfu 8–17 September 2012

based on:

M. Badziak, E. Dudas, M.O. and S. Pokorski, JHEP 1207, 155 (2012) [arXiv:1205.1675]

Outline

- Motivation
- Higgs boson mass in MSSM
  - scale of SUSY breaking
  - stop mixing
  - splitting of two stop masses
- Large stop mixing from RGEs
- Numerical results
- Split sfermion masses and DM
- Conclusions

Motivation

There is some tension between different constraints on the sfermion masses

- In SUSY models FCNC and CP violation problems may be substantially eased when the first two generation sfermions are heavy

- Naturalness arguments suggest that the third generation sfermions should be light

The lower bounds are for the first/second generation sfermions
The upper bounds are for the third generation sfermions

Motivation

⇒ The tension between FCNC constraints and naturalness may be relaxed in Inverted Hierarchy (IH) scenarios, with the first two generations of squarks and sleptons much heavier than the third one

e.g. Pomarol, Tommasini '96, Dvali, Hall '96, . . ., Brümmer et al. '12

Inverted hierarchy of sfermion masses appears in some string models

e.g. Krippendorf et al. '12

and fermion mass models based on horizontal symmetries

e.g. Dudas et al. '95 '96, . . ., Craig et al. '12

Motivation

⇒ The tension between FCNC constraints and naturalness may be relaxed in Inverted Hierarchy (IH) scenarios, with the first two generations of squarks and sleptons much heavier than the third one

e.g. Pomarol, Tommasini '96, Dvali, Hall '96, . . ., Brümmer et al. '12

Inverted hierarchy of sfermion masses appears in some string models

e.g. Krippendorf et al. '12

and fermion mass models based on horizontal symmetries

e.g. Dudas et al. '95 '96, . . ., Craig et al. '12

New tension appeared with the recent LHC results.
Higgs mass of 125-126 GeV is rather big for MSSM
– can not be obtained if stops are too light

What are the predictions for the lightest Higgs boson mass in the MSSM with inverted hierarchy of sfermion masses?

Badziak, Dudas, M.O., Pokorski '12
Baer, Barger, Huang, Tata '12

The Higgs boson mass in the weak scale MSSM

$m_h^2 \approx M_Z^2 \cos^2 2\beta$

The Higgs boson mass in the weak scale MSSM

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left[\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2} \right)\right.$$

The Higgs boson mass in the weak scale MSSM

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right) + \frac{X_t^2}{M_{\rm SUSY}^2} - \frac{1}{12}\,\frac{X_t^4}{M_{\rm SUSY}^4}\right]$$

$$M_{\rm SUSY} \equiv \sqrt{m_{\tilde t_1} m_{\tilde t_2}} \qquad X_t \equiv A_t - \mu/\tan\beta$$

The Higgs boson mass in the weak scale MSSM

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_{\mathrm{SUSY}}^2}{m_t^2}\right) + \frac{X_t^2}{M_{\mathrm{SUSY}}^2} - \frac{1}{12}\,\frac{X_t^4}{M_{\mathrm{SUSY}}^4}\right]$$

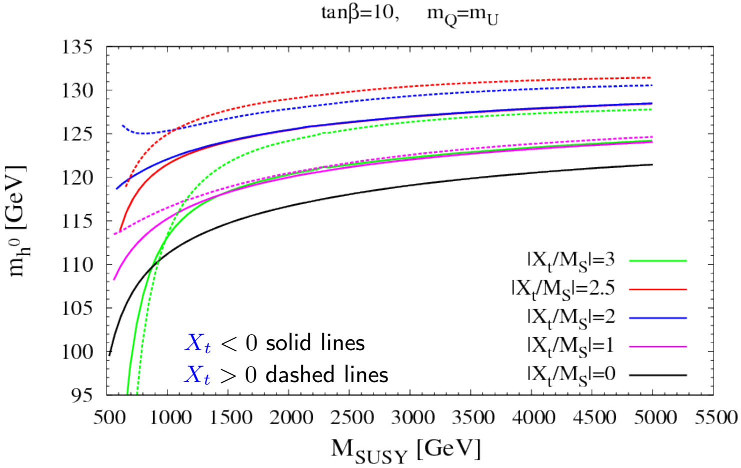
$$M_{\mathrm{SUSY}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \qquad X_t \equiv A_t - \mu/\tan\beta$$

Ways to increase the Higgs boson mass

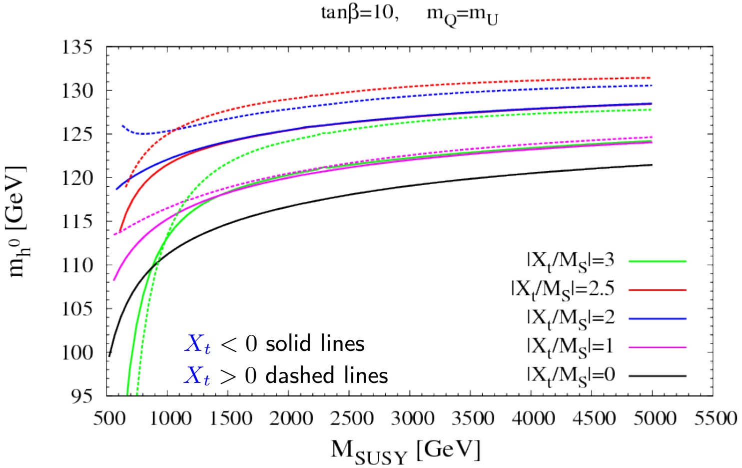
- Bigger values of $\tan\beta$
  - $\cos^2 2\beta > 0.96$ already for $\tan\beta = 10$
- Higher SUSY scale M_{SUSY}
  - not very appealing from the phenomenological (prospects for SUSY discovery) and theoretical (hierarchy problem) points of view
- Stop mixing parameter X_t^2 close to the optimal value
  - in the above approximation the X_t^2-dependent contribution is maximized when

$$\frac{X_t^2}{M_{\mathrm{SUSY}}^2} = 6$$

The Higgs boson mass in the weak scale MSSM



The Higgs boson mass in the weak scale MSSM



The Higgs mass of ~ 125 GeV requires:

- $M_{\rm SUSY} \gtrsim 1$ TeV and large stop-mixing $|X_t|/M_{\rm SUSY} \sim \mathcal{O}(2-2.5)$
- or $M_{\rm SUSY} \gg 1$ TeV

The Higgs boson mass in the weak scale MSSM

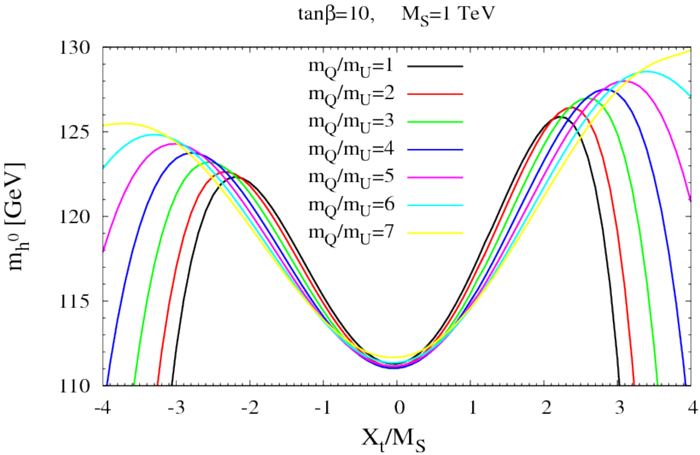
Stop contribution to m_h is modified when $m_U \neq m_Q$

$$\Delta m_h^2 = \frac{3g^2 m_t^2}{8\pi^2 M_W^2}\left[\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right) + \frac{X_t^2}{m_Q^2 - m_U^2}\ln\frac{m_Q^2}{m_U^2} \right.$$
$$\left. + \frac{X_t^4}{\left(m_Q^2 - m_U^2\right)^2}\left(1 - \frac{1}{2}\frac{m_Q^2 + m_U^2}{m_Q^2 - m_U^2}\ln\frac{m_Q^2}{m_U^2}\right)\right]$$

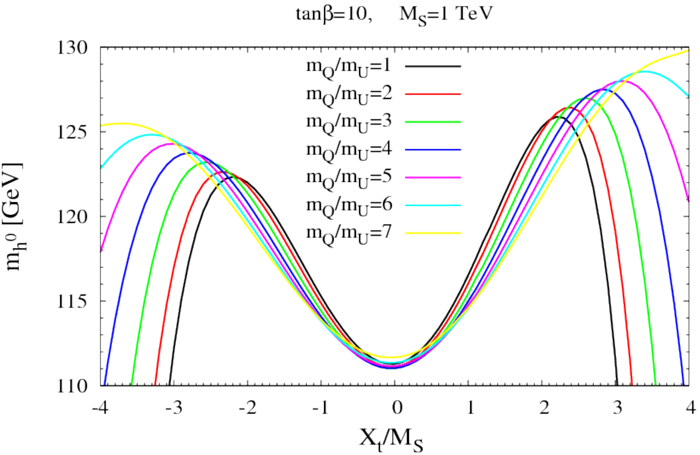
Bigger Higgs boson mass may be obtained if simultaneously

- m_Q is substantially bigger than m_U
- the stop mixing parameter X_t is bigger than $\sim 2.5\, M_{\rm SUSY}$

The Higgs boson mass in the weak scale MSSM



The Higgs boson mass in the weak scale MSSM



Maximal Higgs boson mass increases by:

- 2 GeV if $m_Q/m_U \approx 5$ and $|X_t|/M_{\rm SUSY} \approx 3$
- 4 GeV if $m_Q/m_U \approx 7$ and $|X_t|/M_{\rm SUSY} \approx 4$

Large stop mixing and RGEs

From one-loop RGEs:

$$m_Q^2 \approx 3.1 M_{1/2}^2 + 0.1 A_0 M_{1/2} - 0.04 A_0^2 + 0.65 m_0^2(3)$$
$$m_U^2 \approx 2.3 M_{1/2}^2 + 0.2 A_0 M_{1/2} - 0.07 A_0^2 + 0.35 m_0^2(3)$$
$$A_t \approx -1.6 M_{1/2} + 0.35 A_0$$

How to get $A_t^2 \approx 6\, m_Q\, m_U$?

Large stop mixing and RGEs

From one-loop RGEs:

$$m_Q^2 \approx 3.1 M_{1/2}^2 + 0.1 A_0 M_{1/2} - 0.04 A_0^2 + 0.65 m_0^2(3)$$
$$m_U^2 \approx 2.3 M_{1/2}^2 + 0.2 A_0 M_{1/2} - 0.07 A_0^2 + 0.35 m_0^2(3)$$
$$A_t \approx -1.6 M_{1/2} + 0.35 A_0$$

How to get $A_t^2 \approx 6\, m_Q\, m_U$?

- Large (dominating) $M_{1/2}$ gives $A_t^2 \approx 0.9\, m_Q\, m_U$
- $m_0^2(3)$ contributions even worsen the situation

  ($m_0^2(3)$ is big e.g. in "focus point" scenarios)
- Only large initial value of $|A_0|$ may give $A_t^2/M_{\mathrm{SUSY}}^2 \sim 6$

Optimal stop-mixing requires large initial $|A_0|$

e.g. for $M_{1/2} \approx m_0(3)$: $A_0 \approx -4 M_{1/2}$ or $A_0 \approx 7.7 M_{1/2}$

Large stop mixing and RGEs

1st/2nd generation sfermion masses enter the relevant RGEs at the two-loop level

$$m_Q^2 \approx 3.1M_{1/2}^2 + 0.1A_0M_{1/2} - 0.04A_0^2 + 0.65m_0^2(3) - 0.03m_0^2(1,2)$$
$$m_U^2 \approx 2.3M_{1/2}^2 + 0.2A_0M_{1/2} - 0.07A_0^2 + 0.35m_0^2(3) - 0.02m_0^2(1,2)$$
$$A_t \approx -1.6M_{1/2} + 0.35A_0$$

Large stop mixing and RGEs

1st/2nd generation sfermion masses enter the relevant RGEs at the two-loop level

$$m_Q^2 \approx 3.1M_{1/2}^2 + 0.1A_0M_{1/2} - 0.04A_0^2 + 0.65m_0^2(3) - 0.03m_0^2(1,2)$$
$$m_U^2 \approx 2.3M_{1/2}^2 + 0.2A_0M_{1/2} - 0.07A_0^2 + 0.35m_0^2(3) - 0.02m_0^2(1,2)$$
$$A_t \approx -1.6M_{1/2} + 0.35A_0$$

In the IH scenario, $m_0(1,2) \gg m_0(3)$, RG running of A_t can be disentangled from the running of stop masses.

- $|A_t|$ can be enhanced by gluino contribution
- gluino contribution to the stop masses may be (partially) compensated by negative contributions from $m_0(1,2)$

No large initial A_0 required for optimal stop mixing

Large stop mixing and RGEs

Examples

- $M_{1/2} = m_0(3) \qquad m_0(1,2) = 10\, m_0(3) \qquad A_0 = -m_0(3)$

  $\Rightarrow |X_t| \approx 2.7\, M_{SUSY}$
- $M_{1/2} = m_0(3) \qquad m_0(1,2) = 10\, m_0(3) \qquad A_0 = 0$

  $\Rightarrow |X_t| \approx 1.9\, M_{SUSY}$
- $M_{1/2} = \frac{1}{2} m_0(3) \qquad m_0(1,2) = 5\, m_0(3) \qquad A_0 = 0$

  $\Rightarrow |X_t| \approx 2.2\, M_{SUSY}$

Large stop mixing and RGEs

Examples

- $M_{1/2} = m_0(3) \qquad m_0(1,2) = 10\, m_0(3) \qquad A_0 = -m_0(3)$

  $\Rightarrow |X_t| \approx 2.7\, M_{SUSY}$
- $M_{1/2} = m_0(3) \qquad m_0(1,2) = 10\, m_0(3) \qquad A_0 = 0$

  $\Rightarrow |X_t| \approx 1.9\, M_{SUSY}$
- $M_{1/2} = \frac{1}{2} m_0(3) \qquad m_0(1,2) = 5\, m_0(3) \qquad A_0 = 0$

  $\Rightarrow |X_t| \approx 2.2\, M_{SUSY}$

Stop mixing close to the optimal one
is quite natural in IH scenario

even with vanishing initial A_0

Upper bounds on $m_0(1,2)$

Stop masses:

$$m_Q^2 \approx 3.1M_{1/2}^2 + 0.1A_0M_{1/2} - 0.04A_0^2 + 0.65m_0^2(3) - 0.03m_0^2(1,2)$$

$$m_U^2 \approx 2.3M_{1/2}^2 + 0.2A_0M_{1/2} - 0.07A_0^2 + 0.35m_0^2(3) - 0.02m_0^2(1,2)$$

Proper REWSB:

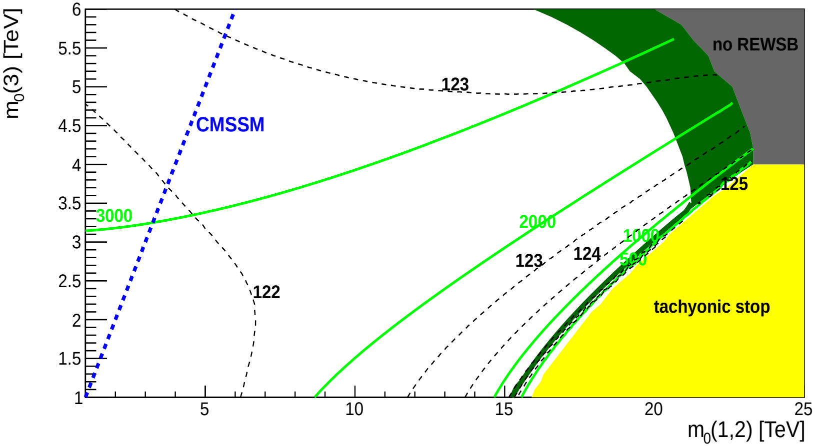
$$\mu^2 \approx 1.3M_{1/2}^2 + 0.1A_0^2 - 0.35M_{1/2}A_0 - 0.01m_0^2(3) - 0.006m_0^2(1,2)$$

For very large $m_0(1,2)$:

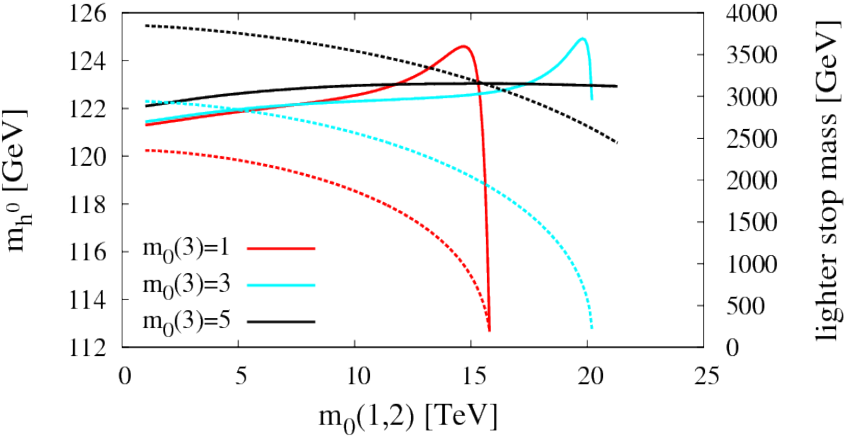
- stops become tachyonic
  maximal $m_0(1,2)/M_{1/2}$ increases with $m_0(3)/M_{1/2}$
- correct REWSB is no longer possible
  maximal $m_0(1,2)/M_{1/2}$ decreases with $m_0(3)/M_{1/2}$

Numerical results

$$M_{1/2} = 1.5 \text{ TeV}, \qquad A_0 = 0, \qquad \tan\beta = 10$$



Numerical results



Numerical results

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_{\rm SUSY}^2}{m_t^2}\right) + \frac{X_t^2}{M_{\rm SUSY}^2}\left(1 - \frac{X_t^2}{12 M_{\rm SUSY}^2}\right)\right]$$

$$m_U^2 \approx 2.3 M_{1/2}^2 + 0.2 A_0 M_{1/2} - 0.07 A_0^2 + 0.35 m_0^2(3) - 0.02 m_0^2(1,2)$$

$$A_t \approx -1.6 M_{1/2} + 0.35 A_0$$

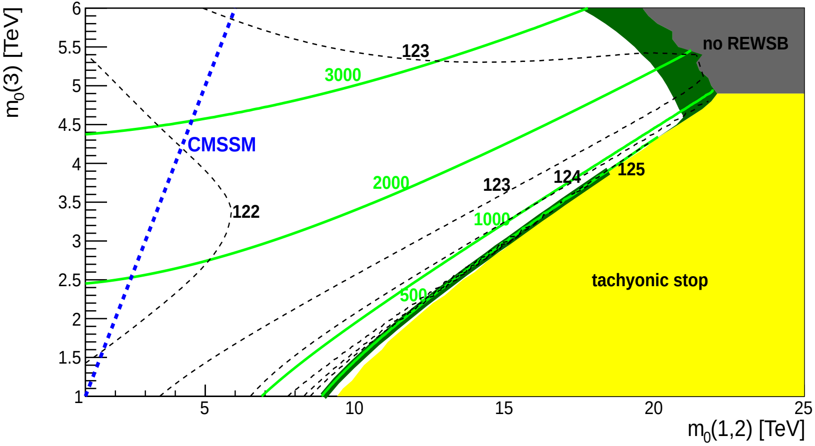
Increasing $m_0^2(1,2) \Rightarrow \begin{cases} \text{increasing } \frac{X_t^2}{M_{\rm SUSY}^2} \text{ (first } m_h \nearrow \text{ then } m_h \searrow \text{)} \\ \text{decreasing } M_{\rm SUSY} \ (m_h \searrow) \end{cases}$

- For small enough $X_t^2/M_{\rm SUSY}^2$ the net result is to increase m_h
- For values of $X_t^2/M_{\rm SUSY}^2$ bigger than the "optimal" one both effects lead to fast decrease of m_h

For big $m_0^2(3)$ problems with correct REWSB appear before the "optimal" $X_t^2/M_{\rm SUSY}^2$ is reached

Numerical results

$M_{1/2} = 1$ TeV, $\quad A_0 = -2$ TeV, $\quad \tan\beta = 10$



Smaller $m_0(1,2)$, $M_{1/2}$ required to obtain a given Higgs mass if $A_0 < 0$

Numerical results

For heavy 1st/2nd generation sfermions the lightest Higgs boson is generically relatively heavy

For instance, $m_0(1,2) > 15$ TeV implies $m_h \gtrsim 122$ GeV

The Higgs boson mass of 125 GeV may be obtained without very big fine tuning

- for $M_{1/2} \gtrsim 1.5$ TeV if $A_0 = 0$
- for $M_{1/2} \gtrsim 1$ TeV if $A_0 = -2$ TeV

m_h higher than 125 GeV may be obtained for bigger $M_{1/2}$ and/or $|A_0|$

Numerical results

For heavy 1st/2nd generation sfermions
the lightest Higgs boson is generically relatively heavy

For instance, $m_0(1,2) > 15$ TeV implies $m_h \gtrsim 122$ GeV

The Higgs boson mass of 125 GeV may be obtained without very big fine tuning

- for $M_{1/2} \gtrsim 1.5$ TeV if $A_0 = 0$
- for $M_{1/2} \gtrsim 1$ TeV if $A_0 = -2$ TeV

m_h higher than 125 GeV may be obtained for bigger $M_{1/2}$ and/or $|A_0|$

There are theoretical uncertainties in the calculation of Higgs boson mass

For example the leading 3-loop contribution is positive (at least in simple models) Kant et al. '10, Kant '11

Numerical results

	Point A	Point B	Point C
$M_{1/2}$	1000	1500	1500
$m_0(3)$	3700	3400	3800
$m_0(1,2)$	17690	21070	22500
A_0	-2000	0	0
$\tan\beta$	10	10	10
μ	888	698	452
m_h	125	125	125.1
m_A	3541	3154	3477
$m_{\tilde{\chi}_1^0}$	444	647	448
$m_{\tilde{\chi}_1^\pm}$	812	700	455
$m_{\tilde{g}}$	2465	3530	3545
$m_{\tilde{t}_{1,2}}$	476, 1801	699, 1581	505, 1632
$m_{\tilde{b}_{1,2}}$	1784, 2926	1555, 2717	1610, 2933
$m_{\tilde{\tau}_1}$	3467	3108	3481
$\Omega_{DM}h^2$	0.111	0.118	0.021

Inverted Hierarchy and Dark Matter

$$\mu^2 \approx 1.3M_{1/2}^2 + 0.1A_0^2 - 0.35M_{1/2}A_0 - 0.01m_0^2(3) - 0.006m_0^2(1,2)$$

The higgsino component of the LSP grows with $m_0^2(3)$ and/or $m_0^2(1,2)$

$$m_U^2 \approx 2.3M_{1/2}^2 + 0.2A_0M_{1/2} - 0.07A_0^2 + 0.35m_0^2(3) - 0.02m_0^2(1,2)$$

Stop may be very light for large enough $m_0^2(1,2)$
if $m_0^2(3)$ is not too big

Inverted Hierarchy and Dark Matter

$$\mu^2 \approx 1.3M_{1/2}^2 + 0.1A_0^2 - 0.35M_{1/2}A_0 - 0.01m_0^2(3) - 0.006m_0^2(1,2)$$

The higgsino component of the LSP grows with $m_0^2(3)$ and/or $m_0^2(1,2)$

$$m_U^2 \approx 2.3M_{1/2}^2 + 0.2A_0M_{1/2} - 0.07A_0^2 + 0.35m_0^2(3) - 0.02m_0^2(1,2)$$

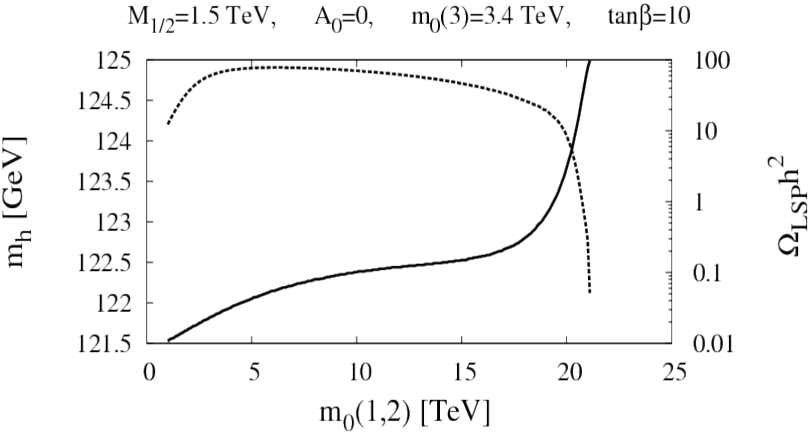
Stop may be very light for large enough $m_0^2(1,2)$
if $m_0^2(3)$ is not too big

In the IH scenario LSP can be a good dark matter candidate.

- Small $m_0(3)$: bino LSP, Stop-coannihilation may give correct $\Omega_{\rm LSP}$
- Intermediate $m_0(3)$: bino-higgsino LSP, $\Omega_{\rm LSP}$ may be correct
- Large $m_0(3)$: higgsino LSP, $\Omega_{\rm LSP}$ typically too small

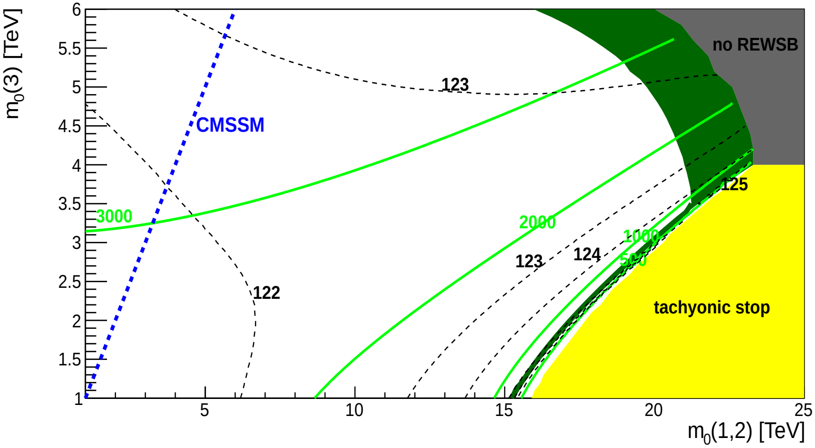
Inverted Hierarchy and Dark Matter

Interesting correlation between Ω_{LSP} and m_h



Numerical results

$$M_{1/2} = 1.5\ \text{TeV}, \qquad A_0 = 0, \qquad \tan\beta = 10$$



Conclusions

Properties of models with heavy sfermions of 1st/2nd generation

- SUSY FCNC and CP violation problems can be substantially eased
- Fine tuning smaller than in CMSSM (“natural SUSY”)
- Large stop mixing is possible without large A_0
- Stop mixing close to the optimal one (giving maximal possible Higgs boson mass) emerges quite naturally from RGE running
- The lightest Higgs is generically heavy, in the vicinity of 125 GeV
- Lighter stop mass $\sim \mathcal{O}(0.5)$ TeV, gluino mass $\sim \mathcal{O}(2-3)$ TeV
- m_h bigger by a few GeV if m_Q is substantially different from m_U
- LSP can be a good dark matter candidate ($m_h - \Omega_{\rm LSP}$ correlation)