The spectral function in a strongly coupled, thermalising CFT

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- Heavy ion collision → formation of Quark-Gluon Plasma
- Behaves as near-ideal Fermi liquid after fast thermalisation
- We want to understand thermalisation process itself

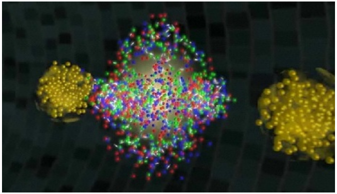


Figure: *Snapshot of RHIC video: "Hot quark soup"*

- Heavy ion collision → formation of Quark-Gluon Plasma
- Behaves as near-ideal Fermi liquid after fast thermalisation
- We want to understand thermalisation process itself
- **Problems:**
  - Strongly coupled dynamics
  - Non equilibrium state

  ⇒ Difficult computation
- **Goal:** Understanding thermalisation process using AdS/CFT correspondence
  - Not just thermalisation time
  - Understand as much as possible

Different probes of thermalisation behaviour:

- Two point functions

  [Abajo-Arrastia, Aparício and López, arXiv:1006.4090] [Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Muller, Schafer, Shigemori and Staessens, arXiv:1103.2683] [Erdmenger, Lin and Ngo, arXiv:1101.5505]
- Spacelike Wilson loops
- Entanglement entropy

  [Albash and Johnson, arXiv:1008.3027]
- Mutual and tripartite information

  [Balasubramanian, Bernamonti, Copland, Craps and Galli, arXiv:1110.0488]

New probe:

- Spectral function: follow time evolution during thermalisation

  [Balasubramanian, Bernamonti, Craps, Keranen, Keski-Vakkuri, Muller, Thorlacius and Vanhoof, *(to appear)*]

- Two point functions in equilibrium determined by
  - Occupation number: $n(k, \omega)$
  - Spectral function: $\rho(k, \omega)$
- Spectral function = "weight" of free propagator in interacting propagator

$$\hat{D}_R(k, \omega) = \int_{-\infty}^{+\infty} \frac{\rho(k, \omega')}{\omega^2 - \omega'^2 + i\epsilon} \frac{\omega' d\omega'}{2\pi} \qquad \text{with} \qquad \int_{-\infty}^{+\infty} \rho(k, \omega') \frac{\omega' d\omega'}{2\pi} = 1$$

  - For a free particle → deltapeak: $\rho(k, \omega) = 4\pi\delta(\omega^2 - \omega_k^2)$
  - For an interacting system → smearing due to interactions
- It can be derived from retarded two point function

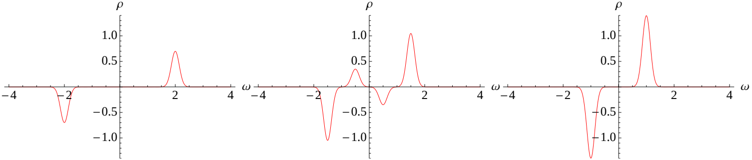
$$iD_R(x, t) = \theta(t)\langle[\mathcal{O}(x, t), \mathcal{O}(0, 0)]\rangle \quad \Rightarrow \quad \rho(k, \omega) = -2\,\text{Im}\hat{D}_R(k, \omega)$$

- We now want to determine this in the strongly coupled regime.

- Time dependence: $\rho(k,\omega,T) = -2\,\mathrm{Im}\hat{D}_R(k,\omega,T)$

$$\begin{cases} t = t_1 - t_2 \\ T = \frac{t_1+t_2}{2} \end{cases} \Leftrightarrow \begin{cases} t_1 = T + \frac{t}{2} \\ t_2 = T - \frac{t}{2} \end{cases}$$

- Compare to quenched harmonic oscillator ($\omega_i = 2$, $\omega_f = 1$)



- Equal-space 2pnt functions $\Rightarrow$ momentum average

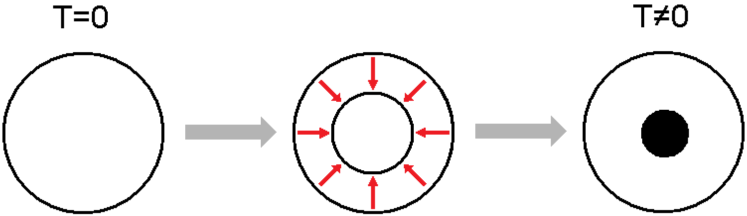
$$\rho_T(\omega) \equiv \int_{-\infty}^{+\infty} \rho_T(k,\omega)\mathrm{d}k = -4\pi\,\mathrm{Im}\left(\int_{-\infty}^{+\infty} \mathrm{d}t\, e^{i\omega t} D_R(x=0,t,T)\right)$$

- **Goal:** Determine equal-space two point functions

- **AdS/CFT:** CFT on boundary ≈ asymptotic AdS spacetime

Equilibrium $(T = 0)$	Injection of energy + thermalisation	Equilibrium $(T \neq 0)$
$\Downarrow$	$\Downarrow$	$\Downarrow$
Empty AdS	Infalling shell of null dust	Black hole $(T = \frac{R}{2\pi})$

- **Vaidya geometry:** thin infalling shell



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- **Vaidya geometry:** thin infalling shell

$$ds^2 = -(r^2 - \theta(v)R^2)dv^2 + 2dvdr + r^2dx^2$$

$$\mathbf{v < 0} \;:\; t = v + \frac{1}{r} \quad \swarrow \qquad \searrow \quad \mathbf{v > 0} \;:\; t = v - \frac{1}{2R}\ln\left|\frac{r-R}{r+R}\right|$$

$$ds^2 = \frac{dr^2}{r^2} - r^2dt^2 + r^2dx^2 \qquad\qquad ds^2 = \frac{dr^2}{r^2 - R^2} - (r^2 - R^2)dt^2 + r^2dx^2$$

AdS_3 (vacuum) BTZ (black hole)

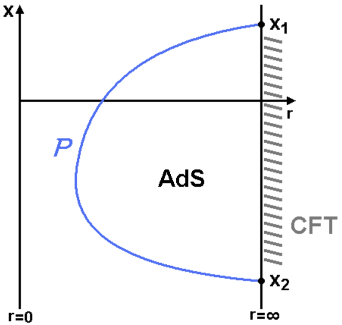
- **AdS/CFT:** two point function = sum over all paths $\mathcal{P}$

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle = \int \mathcal{D}\mathcal{P}\, e^{i\Delta L(\mathcal{P})} \quad \text{with} \quad L(\mathcal{P}) = \int_{\mathcal{P}} \sqrt{-g_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} \mathrm{d}\lambda$$

- Spacelike path $\mathcal{P}$:
  - $\Rightarrow$ $L(\mathcal{P})$ imaginary
  - $\Rightarrow$ Geodesic approximation

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle \approx \sum_{\text{geodesics}} e^{-\Delta L}$$

  - $\Rightarrow$ Better for large Δ



- Implicit expression for equal-time two point functions

[Balasubramanian et al., arXiv:1103.2683]

- Extend this calculation for timelike separated points
- **Problem 1:** Timelike path $\mathcal{P}$:
  - $\Rightarrow$ $L(\mathcal{P})$ real
  - $\Rightarrow$ geodesic approximation?
- **Problem 2:** real timelike geodesics do not extend to boundary
- **Solution:** analytic continuation
  - Wick rotation to Euclidean signature **(!!!)**

$$ds^2 = \frac{dr^2}{r^2} + r^2(-dt^2 + dx^2) \quad \xrightarrow{y=it} \quad ds^2 = \frac{dr^2}{r^2} + r^2(dy^2 + dx^2)$$

  - complexified geodesics

$$\begin{cases} r = r(\lambda) \\ x = x(\lambda) \\ t = t(\lambda) \end{cases} \longrightarrow \begin{cases} r = r(\lambda + i\beta) \\ x = x(\lambda + i\beta) \\ t = t(\lambda + i\beta) \end{cases}$$

- Geodesic approximation for Euclidean two point functions

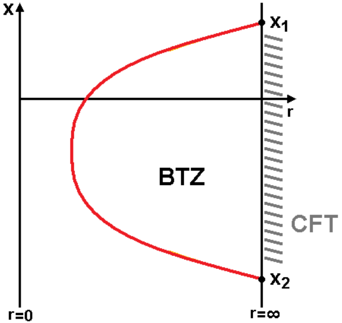
$$D(\mathbf{x}_1, \mathbf{x}_2) = \langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle = \int \mathcal{D}\mathcal{P}\, e^{-\Delta L(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L}$$

- Euclidean BTZ:

$$ds^2 = \frac{dr^2}{r^2 - R^2} + (r^2 - R^2)dy^2 + r^2 dx^2$$

- Euclidean 2pnt function:

$$D_{\text{thermal}}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\left[\frac{4}{R^2}\left(\sinh^2\left(\frac{R\delta x}{2}\right) + \sin^2\left(\frac{R\delta y}{2}\right)\right)\right]^{\Delta}}$$

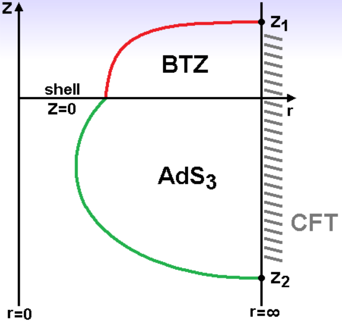


- Limit $R \to 0$ recovers vacuum result

$$D_{\text{vacuum}}(\mathbf{x}_1, \mathbf{x}_2) = \lim_{R\to 0} D_{\text{thermal}}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{(\delta x^2 + \delta y^2)^{\Delta}}$$

- **Problem:** No straightforward continuation of Vaidya metric

$$ds^2 = -(r^2 - \theta(v)R^2)dv^2 + 2dvdr + r^2dx^2$$



- **Solution:**
  - Shell of null matter as limit of spacelike matter $(E \to \infty)$

$$ds^2 = -(r^2 - \theta(v)R^2)dv^2 + \frac{2Edvdr}{\sqrt{r^2 - \theta(v)R^2 + E^2}} + \frac{dr^2}{r^2 - \theta(v)R^2 + E^2} + r^2dx^2$$

  - Double Wick rotation $(z = iv,\ S = iE)$

$$ds^2 = (r^2 - \theta(z)R^2)dz^2 - \frac{2Sdzdr}{\sqrt{r^2 - \theta(z)R^2 - S^2}} + \frac{dr^2}{r^2 - \theta(z)R^2 - S^2} + r^2dx^2$$

Back to Minkowski signature:

- Wick rotation ($y = it$) gives time-ordered two point function

$$D_F(x_1, t_1; x_2, t_2) = iD(x_1, y_1 = it_1; x_2, y_2 = it_2)$$

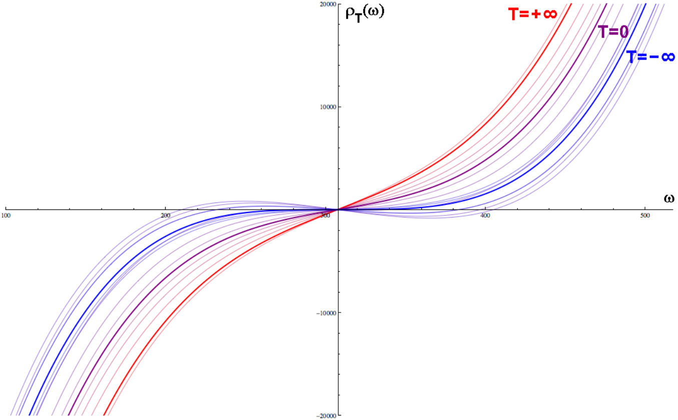
- Retarded two point function can be found from

$$D_R(x_1, t_1; x_2, t_2) = \theta(t_1 - t_2)\left[D_F(x_1, t_1; x_2, t_2) + (D_F(x_1, t_1; x_2, t_2))^*\right]$$

- **Final result:** Explicit expression for equal-space 2pnt function

$$D_R(t_1, t_2) = 2\sin(\pi\Delta)\begin{cases} \frac{1}{|t_1 - t_2|^{2\Delta}} & \text{if } 0 > t_1 > t_2 \\ \frac{1}{\left|\frac{2}{R}\sinh\left(\frac{R}{2}(t_1 - t_2)\right)\right|^{2\Delta}} & \text{if } t_1 > t_2 > 0 \\ \frac{1}{\left|\frac{2}{R}\sinh\left(\frac{Rt_1}{2}\right) - \cosh\left(\frac{Rt_1}{2}\right)t_2\right|^{2\Delta}} & \text{if } t_1 > 0 > t_2 \end{cases}$$

Time evolution of (momentum averaged) spectral function:



$$\rho_T(\omega) \equiv \int_{-\infty}^{+\infty} \rho_T(k,\omega)\mathrm{d}k = -4\pi\,\mathrm{Im}\left(\int_{-\infty}^{+\infty} \mathrm{d}t\, e^{i\omega t} D_R(x=0,t,T)\right)$$

- Use AdS/CFT to probe thermalisation strongly coupled CFT
- We found explicit expression for equal-space 2pnt function
  - Non-standard analytic continuation
  - Complexified geodesics result agrees
- Notion of time dependent spectral function
- Collaborators use different method: results seem to match
- Outlook:
  - (Numerically) find general two-point functions?
  - Occupation numbers?

$$
\begin{aligned}
\rho(k,\omega) &= -2\,\mathrm{Im}\hat{D}_R(k,\omega) \\
(1+2n(k,\omega))\rho(k,\omega) &= -2\,\mathrm{Im}\hat{D}_F(k,\omega)
\end{aligned}
$$

Thank you for your attention!