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Short communication

On the elastic properties of arteries

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Abstract

A new coefficient of elasticity is proposed that relates to the elastic state of the blood vessels. This measure is proposed as a result of the realization, from personal experience as well as from the international literature, of the difficulty in measuring the thickness of the blood vessels in vivo with acceptable precision. The measurement of E being dependent on the measurement of the thickness of the vessels becomes a highly unreliable proposition. Its relation to E (Young modulus) and to the pulse wave velocity (PWV) is established. We give three examples showing how the proposed coefficient can be measured.

Keywords: Arteries; Mechanical properties elasticity

1. Introduction

It is well known that the elasticity of materials is expressed in Hooke's law as follows: Tensile stress of a solid is proportional to the amount of deformation, provided the deformation is not permanent. Hooke's law applied to an elastic bar of length l can be expressed as

$$P = E\frac{\Delta l}{l}, \tag{1}$$

where E is the coefficient of elasticity of the material (Young modulus), and P is the stress of the bar.

In the case of a thin-walled pipe of internal radius r and thickness h, open at both ends, we have from Eq. (1)

$$\Delta P\frac{r}{h} = E\frac{\Delta r}{r}, \tag{2}$$

where r is the internal radius of the tube and h the thickness, and ΔP the change of internal pressure.

If the ratio h/r is very small then we have that internal and external radii are approximately equal. Although we have reservations in applying the above to arteries because they are not elastic materials in an absolute sense, since they are viscous to a certain degree, and because of the existence of unknown forces from muscular fibers surrounding them, we believe that, by calculating E using Eq. (2), we can obtain important information related to their condition.

The coefficient E can also be calculated from the Moens–Korteweg equation

$$C(\mathrm{PWV}) = \sqrt{\frac{Eh}{2\rho r}} \tag{3}$$

by measuring the velocity C(PWV) and the other parameters.

In both Eqs. (2) and (3) errors are introduced from the geometrical dimensions of the blood vessels, especially the thickness h that cannot be measured with high precision. The error from measurements in vivo can be as high as 100% (Peterson et al., 1960; Rutherford, 1995), a fact that causes propagation of the error in the calculation of E. This is the reason why many researchers use simpler equations that do not employ thickness h (Burton, 1951). So that they either omit from relation (2) the h erroneously, since the resulting units of measure are wrong, or they omit the entire ratio r/h in order to define a new coefficient E_p (Pressure–strain elastic modulus) (Peterson et al., 1960)

$$\Delta P = E_p\frac{\Delta r}{r}, \tag{4}$$

whereas others use only the pulse wave velocity (PWV) as a means to estimate stiffness. Almost all the papers

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Nomenclature

P	stress exercised on a bar
E	Young modulus
l	length of the bar
$r = d/2$	where d is the inner diameter of the artery
h	thickness of the artery wall
C(PWV)	pulse wave velocity
ρ	blood density
E_p	pressure strain elastic modulus
V	volume of the artery segment under control
μ	the coefficient of volume elasticity
d	inner diameter of the artery segment
Y	c_p/c_v
P_s	systolic pressure
P_d	diastolic pressure
d_s	diameter of the artery during the systolic phase
d_d	diameter of the artery during the diastolic phase

presented at the conference (Tenth European Meeting On Hypertension, 2000) refer to PWV as a means for measuring the stiffness of the vessels.

As a consequence of the above, it is impossible to compare results of various researchers in order to draw consistent conclusions.

The Young coefficient E is a very useful measure, dependent only on the structural material constituting the blood vessels, and not at all on their geometry. To use steel as an example, all pipes made of steel, independently of their diameter, have the same E, as long as the ratio of thickness to diameter is a small number. Thus, this is the way the researchers took their measurements, literally handling samples of vessels in the lab (Davison et al., 1995).

2. The proposed coefficient

We believe that in hemodynamics it is also important to know the ease of deformation of blood vessels, which is a function of geometrical dimensions, such as thickness and diameter, and the material (tissue) the blood vessels consist of. In order to study this deformation, we introduce the definition of another elasticity coefficient that we call volume elasticity coefficient μ based on Hooke's law as well, whereby pressure p is proportional to deformations. From now on, we refer to volume deformations, not length deformations

$$\Delta P = \mu \frac{\Delta V}{V}. \tag{5}$$

Of course, here we apply the same constraints on the materials to be elastic, etc. and we have the same reservations we have already mentioned regarding the value of E as applied to blood vessels.

Let us see now in what way the two coefficients μ and E relate to each other. Since the volume V of a blood vessel segment of length l is

$$V = l\pi r^2.$$

And the change of volume is $\Delta V = l2\pi r\Delta r$ and $(\Delta V/V = 2\Delta r/r)$

By also considering Eq. (2), we have

$$\mu = \frac{Eh}{2r}. \tag{6}$$

And now from Eqs. (3) and (6) we have

$$C(\mathrm{PWV}) = \sqrt{\frac{\mu}{\rho}}, \tag{7}$$

where we can see the Moens–Korteweg relation greatly simplified. So in order to measure μ there is no need to measure diameter and thickness of the vessel but all is needed is to measure velocity PWV, raise it to the power of two and multiply by ρ (ρ: blood density).

Here, we would like to emphasize the following facts that make us believe that the coefficient of volume elasticity μ, as defined by Eq. (7), could be a good choice:

(a) The relation is extremely simple.
(b) The propagation velocity of the particular deformation has the classical general form, that is, the propagation velocity of an elastic deformation is equal to the square root of ratio of the corresponding coefficient of elasticity over the density, as shown by the following well-known relations:
$C = \sqrt{E/\rho}$ that is, velocity of longitudinal waves in rod (E: The Young modulus, ρ: density),
$C = \sqrt{G/\rho}$ that is, velocity of shear waves in a solid (G: the shear modulus),
$C(\text{Sound Velosity}) = \sqrt{b_{ad}/\rho}$ that is, velocity of elastic waves in fluids (liquids and gases), where b_{ad} is the adiabatic Bulk modulus.

3. How to measure μ

μ can be measured using methods similar to methods we measure E except that there is no need to measure thickness of the vessel and in many cases we do not even need to measure the other geometric dimensions.

3.1. Method 1

Using a B-scan we take images during the systolic phase and we measure the internal diameter d_s of the vessel. Then we do the same during the diastolic phase and measure the diameter d_d. Then we obtain the difference $\Delta P = P_s - P_d$ and by applying Eq. (5) we find $\mu = \Delta P(V_d/\Delta V)$, where $\Delta V = V_s - V_d$ and finally

$$\mu = (P_s - P_d)\frac{d_d^2}{d_s^2 - d_d^2}. \tag{8}$$

3.2. Method 2

We measure PWV and apply Eq. (7)

$$C(\mathrm{PWV}) = \sqrt{\frac{\mu}{\rho}}.$$

Here we do not need to measure geometrical dimensions.

3.3. Method 3

For the brachial artery we can use a more direct method, by means of an extemporaneous instrument, known in the bibliography as pulse plethysmograph (Rutherford, 1995). With a sphygmomanometer having the original manometer equipped with an additional more sensitive manometer with resolution of, say 0.1 mbar, we measure with Standard Medical Manometer (SM) (Fig. 1) the systolic and diastolic pressures. Then we create a pressure smaller than the diastolic, say 50 mbar, and we see that the instrument Differential Manometer (DM) (Fig. 1) shows a fluctuation Δp_{art} caused by fluctuations in the volume of the artery. Then we insert in the cuff a known volume of air, say $3\,\mathrm{cm}^3$, slowly so that we obtain isothermal compression. The pressure becomes $p + \Delta p$. Since the pressure of the air contained in the cuff changes by Δp, for a change in volume by $3\,\mathrm{cm}^3$, when we have a fluctuation in pressure by $\Delta p_{\mathrm{art}}/\gamma$ what would the change of volume of air in the cuff be. (The division by $\gamma = c_p/c_v = 1.41$ for the air is due to the fact that the compression of the air from the volume fluctuations of the artery is rapid, therefore adiabatic.) What we find is the change in volume ΔV of the artery for the section being covered by the cuff (10 cm). If we also obtain V_d from the B scan we have

$$\mu = \Delta P \frac{V_d}{\Delta V}.$$

Let us present an example of measurements using the methods described above. We apply them to a healthy person and we find.

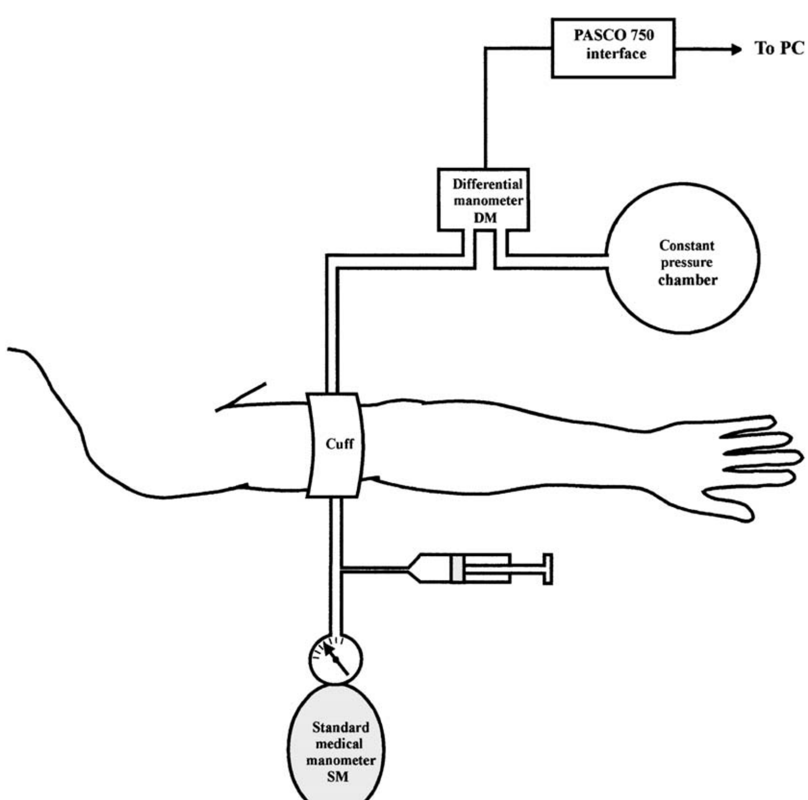

Fig. 1. Diagram of the laboratory setup for the direct measurement of the brachial artery volume fluctuations.

3.4. Method 1 (B-scan)

We measured the μ of the brachial artery. Because this artery is very close to the skin surface, we used 7.5 MHz transducers. The procedure was as follows:

We measured the blood pressure by means of an ordinary sphygmomanometer, with the subject lying in horizontal position. We found

$$\text{systolic pressure}: \quad P_s = 120\,\text{mmHg},$$

$$\text{diastolic pressure}: \quad P_d = 84\,\text{mmHg}.$$

Immediately after, we performed a B-scan. The resolution of the B-Scan was 0.1 mm. In order to have a better perspective of the artery boundary, we also used a Doppler sonography (Fig. 2) so as to obtain a distinct color of the blood in motion. We found

$$\text{diameter during the systolic phase}: \quad d_s = 4.5\,\text{mm},$$

$$\text{diameter during the diastolic phase}: \quad d_d = 4.0\,\text{mm}.$$

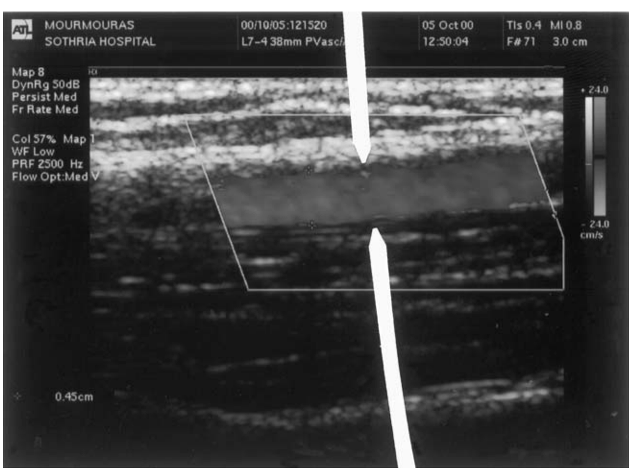

Fig. 2. B-scan illustration of the brachial artery for the systolic phase.

Fig. 2 shows the B-scan for the systolic phase along with the parameters of the measurement. We obtained a similar B-scan for the diastolic phase. By applying the formula, we have

$$\mu_{\text{B-Sc}} = (P_s - P_d)\frac{d_d^2}{d_s^2 - d_d^2} = 135\,\text{mmHg}.$$

3.5. Method 2 (B-scan)

Other researchers generally measure the PWV with automatic instrument such a Complior. Since we were not equipped with such an instrument, we made use of two extemporaneous plethysmographs, as described in Method 3. We positioned them on the upper half of the arm at a predetermined distance, and, from the time difference in the arrival of the cardiac pulses, we calculated the velocity.

We measure PWV and we find $C = 3.8\,\text{m/s}$. From the equation

$$C = \sqrt{\frac{\mu}{\rho}}$$

we have

$$\mu_{\text{PWV}} = C^2\rho = 14,400\ \text{Pa} = 108\,\text{mmHg}.$$

3.6. Method 3 (B-scan)

We measure

$$\text{systolic pressure}: \quad P_s = 120\,\text{mmHg},$$

$$\text{diastolic pressure}: \quad P_d = 84\,\text{mmHg}.$$

We set the pressure inside the cuff to a value of 50 mmHg (Fig. 1). Then we activate the sensitive differential manometer DM and we measure the fluctuation of the pressure Δp_{art} caused by fluctuations in the volume of the artery (Fig. 3)

$$\Delta p_{\text{art}} = 1.2\,\text{mbar}.$$

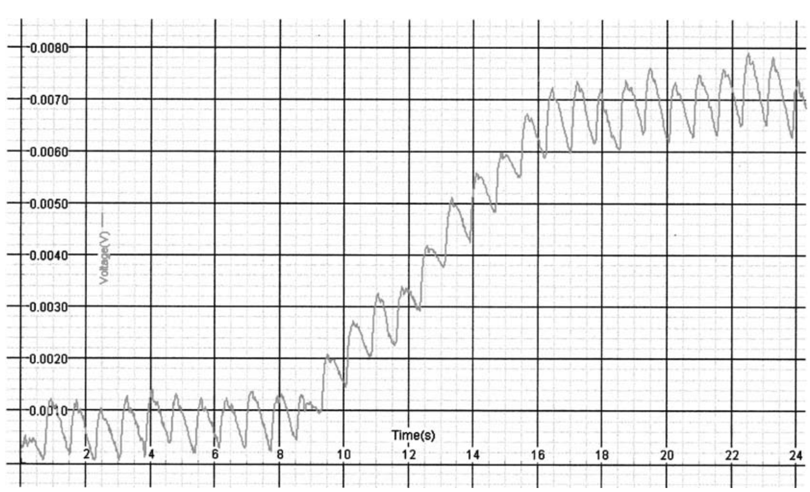

Fig. 3. Pressure fluctuations in the cuff, caused by the pulsations of the artery. 1 mV corresponds to 1 mbar.

The DM has sensitivity of 1 mV/mbar and response time of 100 ns.

Using a syringe, we slowly insert a volume of air equal to $3.0\,\mathrm{cm}^3$ and the change in the pressure of air inside the cuff becomes

$\Delta p = 6.0\,\mathrm{mbar}$.

So, when he have a change in the volume of the cuff equal to

$3.0\,\mathrm{cm}^3$ the pressure rises by $6.0\,\mathrm{mbar}$

what is the change ΔV in the volume when the pressure changes by 1.2/1.41 mbar.

What we find is $\Delta V = (3.0 \times 1.2)/(1.4 \times 6.0) = 0.42\,\mathrm{cm}^3$. Therefore, from Eq. (5)

$$\mu_{pl} = \frac{\Delta P V_\mathrm{d}}{\Delta V} = \frac{36 \times 1.26}{0.42} = 107\,\mathrm{mmHg},$$

where V_d is the volume of a blood vessel with a length of $l = 10\,\mathrm{cm}$

$$\left(V_\mathrm{d} = \pi \frac{d_\mathrm{d}^2}{4} 10 = 1.26\,\mathrm{cm}^3\right).$$

Eventually, from all the above methods, we have

$\mu_{\text{B-Sc}} = 135 \pm 40\,\mathrm{mmHg}$,

$\mu_{\mathrm{PWV}} = 108 \pm 10\,\mathrm{mmHg}$,

$\mu_{pl} = 107 \pm 10\,\mathrm{mmHg}$.

All of the above three values are fully supported (justified, covered) within the limits of the measurement errors of the available equipment. The error in the B-scan method is large because the diameter of the brachial artery is small; therefore the relative error in the measurement of the diameter becomes large.

Appendix A. The estimation of μ error

We consider that in relation (8) $\mu = (P_\mathrm{s} - P_\mathrm{d})(d_\mathrm{d}^2/(d_\mathrm{s}^2 - d_\mathrm{d}^2))$ the error in $(P_\mathrm{s} - P_\mathrm{d}) = 36\,\mathrm{mmHg}$ is very small, whereas the error of d_s and d_d is $\pm 0.1\,\mathrm{mm}$

$$\text{In [1]} := \partial_x \frac{36x^2}{y^2 - x^2},$$

$$\text{Out [1]} = \frac{72x^2}{(-x^2 + y^2)} + \frac{72x}{-x^2 + y^2},$$

$$\text{In (2)} := \partial_y \frac{36x^2}{y^2 - x^2},$$

$$\text{Out [2]} = \frac{72x^2 y}{(-x^2 + y^2)^2},$$

$$\delta(\mu) = \sqrt{\left(\left(\left(\frac{72x^3}{(-x^2 + y^2)} + \frac{72x}{-x^2 + y^2}\right) \mathrm{d}x\right)^2 + \left(-\frac{72x^2 y}{(-x^2 + y^2)^2}\,\mathrm{d}y\right)^2\right)},$$

$\text{In [7]} = x = d_\mathrm{d} = 4,$

$\text{Out [7]} = 4,$

$\text{In (4)} := y = d_\mathrm{s} = 4.5,$

$\text{Out [4]} = 4.5,$

$\mathrm{d}x = \mathrm{d}y = \pm 0.1,$

$$\text{In [5]} := \sqrt{\left(\left(\left(\frac{72x^3}{(-x^2 + y^2)} + \frac{72x}{-x^2 + y^2}\right) 0.1\right)^2 + \left(-\frac{72x^2 y}{(-x^2 + y^2)^2}\,0.1\right)^2\right)},$$

$\text{Out [5]} = 43.1997.$

The error of μ is about 40 mmHg.

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