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Real–time dynamics of a hot Yang-Mills theory: a numerical analysis*

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We discuss recent results obtained from simulations of high temperature, classical, real time dynamics of $SU(2)$ Yang-Mills theory at temperatures of the order of the electroweak scale. Measurements of gauge covariant and gauge invariant autocorrelations of the fields indicate that the ASY-Bödecker scenario is irrelevant at these temperatures.

High temperature baryon number non conservation in the electroweak theory caused by the anomaly of the baryonic current is important for the understanding of the origin of baryonic matter in the universe. After the work of Ref. [1] it was realized that the baryon number non-conservation, caused by thermal field fluctuations between gauge-equivalent vacua with different winding numbers could be large in the unbroken phase of the electroweak theory. This can be tested by performing real time simulations of hot gauge theories, a task we still do not know how to perform from first principles. It is possible, however, to use *classical* thermal gauge theory in order to address the question [2]. The result is that transitions between vacua with different winding numbers is unsuppressed at high temperatures in the unbroken phase of the electroweak theory [3]. The effect of correctly incorporating ultraviolet thermal fluctuations has been addressed in the the theory of hard thermal loops and the effective small-momentum, low frequency theory of Bödeker (ASY-B) [4]. The soft classical fields (momentum $k \leq gT$) couple to hard currents according to

$$\dot{\mathbf{E}} = \mathbf{D} \times \mathbf{B} - \mathbf{J}_{\mathrm{hard}}, \qquad (1)$$

where $\mathbf{J}_{\mathrm{hard}} = \sigma \mathbf{E} + \boldsymbol{\xi}$, and where the effective noise term $\boldsymbol{\xi}$ is determined by the fluctuation-dissipation theorem $\langle \xi \xi \rangle = 2T\sigma$. In (1) σ denotes the so-called color conductivity, which to a leading log approximation is given by $\sigma = \frac{m^2}{\gamma}$, where m denotes the Debye screening mass and γ the hard gauge fields damping rate. High frequency magnetic modes couple to low frequency ones producing a new time scale for the magnetic fluctuations. Although their length scale is of order $1/g^2T$ (non-perturbative) their lifetime is of order $1/g^4T\ln(g^{-1})$. This picture has been verified in real time computer simulations [5] where hard currents have been implemented in various ways, by measuring primarily the sphaleron rate. The latter is believed to be dominated by classical thermal fluctuations and one can hope that simulations of the classical theory, possibly corrected by ASY-B effective theory, can determine it correctly. It is, however, possible that at the electroweak scale ($\alpha \approx 1/30$, $g \approx 0.65$) and at temperatures close to the electroweak phase transition the ASY-B theory cannot be applied. This can be checked in computer simulations by measuring real time gauge correlators and color conductivity and check whether their long wavelength, low frequency part shows the behavior predicted by ASY-B, this way making direct contact with perturbation theory. If we do not obtain agreement, one could be tempted to conclude that the theory is valid at even higher temperatures and that one should *not* try to match

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the sphaleron rate to formulas based on the validity of the ASY-B theory. Precisely because the sphaleron rate is a non-perturbative quantity, there is no easy way to disentangle the perturbative ASY-B damping from genuine non-perturbative effects. We also note that topology on the lattice is ill-defined and requires special treatment. In Ref. [6] we attempted to address this issue by precisely measuring objects like color conductivity and field autocorrelators. Our finding is that we see no sign of the ASY-B scenario in high-temperature, classical $SU(2)$ theory in a regime roughly corresponding to the electroweak scale. We stress that our results are not in contradiction with the results by Moore and Rummukainen [5] which give zero continuum sphaleron rate, but do not rule out a finite classical rate in the continuum.

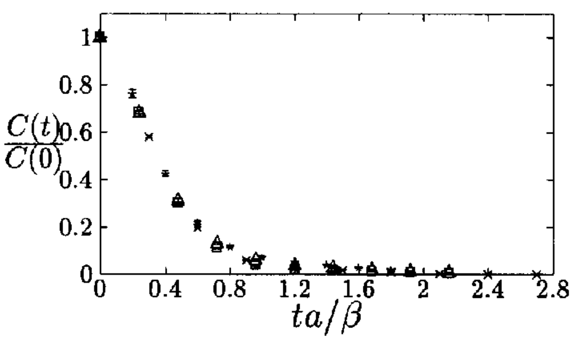

Figure 1. The autocorrelator $\langle \mathbf{D} \times \mathbf{B}(\mathbf{x}, t) \cdot \mathbf{D} \times \mathbf{B}(\mathbf{x}, 0)\rangle$ versus time t in units of $(g^2T)^{-1}$ for $\beta = 8.33$ (crosses), $\beta = 10.0$ (squares), $\beta = 12.5$ (triangles), and $\beta = 15.0$ (stars).

Simulations of the classical theory are performed by generating thermal field configurations for pure $SU(2)$ Yang-Mills theory on a lattice for given temperature T and then lettting the system evolve according to the classical equations of motion [7]. We worked in the temporal gauge where the electric field $\mathbf{E}(\mathbf{x}, t)$ is the conjugate momentum to $\mathbf{A}(\mathbf{x}, t)$. Since the cutoff of the classical theory is the lattice spacing whereas the thermal fluctuations of the full quantum theory are cut

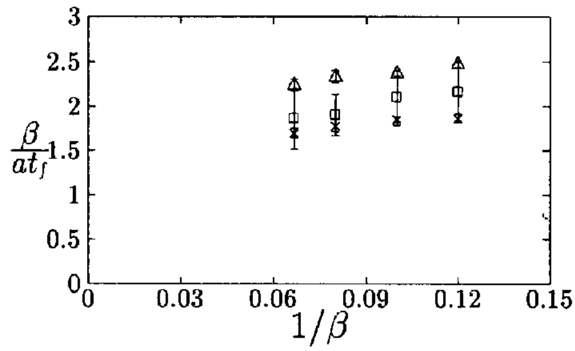

Figure 2. Inverse integral autocorrelation time in units of g^2T plotted against g^2Ta for $\langle \mathbf{D} \times \mathbf{B}(\mathbf{x}, t) \cdot \mathbf{D} \times \mathbf{B}(\mathbf{x}, 0)\rangle$ (triangles), $\langle \mathbf{B}(\mathbf{x}, t \cdot \mathbf{B}(\mathbf{x}, 0)\rangle$ (crosses), and $\langle B_i^2(\mathbf{x}, t) B_i^2(\mathbf{x}, 0)\rangle - \langle B_i^2(\mathbf{x}, t)\rangle\langle B_i^2(\mathbf{x}, 0)\rangle$ (squares).

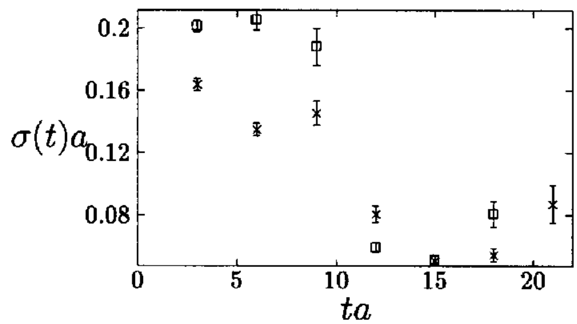

Figure 3. The color conductivity in lattice units versus time t in lattice units for $\beta = 10$ (crosses) and $\beta = 12.0$ (squares).

off at scale T, we need to filter out the large momentum components of the fields by cooling [8], which leads to an exponential decay of high-frequency modes. In our simulations we used cooling times τ such that $(g^2T)^2\tau = 3.84$ (also 2.56 for $\beta = 10$). Variance of τ had very small effect on real time behavior of correlators. The lattice temperature $\beta \equiv 4/(g^2Ta)$ was chosen within a range such that the perturbative Debye mass m_D to g^2T of the classical theory is close to that of the full $SU(2)$ Yang-Mills theory at electroweak temperatures $T \sim 100\text{GeV}$. Since we are interested in the dynamics of fields with momenta of the order of g^2T, the dimensionless combination $L/(\beta a)$ should be large enough in order to avoid finite-size effects. Most of our simulations were performed at $L/(\beta a) = 2.4$. We verified that variations of $L/(\beta a)$ around that value did not have a measurable effect. We measured unequal time correlators of the fields of the form $C_{12}(t) \equiv \frac{1}{V}\int d^3x \ \langle \mathcal{O}_1(\mathbf{x},t)\mathcal{O}_2(\mathbf{x},0)\rangle$ where the $\mathcal{O}_i(\mathbf{x},t)$ are $\mathbf{D}\times\mathbf{B}(\mathbf{x},t)$ or $\mathbf{B}(\mathbf{x},t)$. In particular we measured

$$\sigma(t) \equiv \frac{\int d^3x \ \langle \mathbf{D}\times\mathbf{B}(\mathbf{x},0)\cdot\mathbf{D}\times\mathbf{B}(\mathbf{x},t)\rangle}{\int d^3x \ \langle \mathbf{D}\times\mathbf{B}(\mathbf{x},0)\cdot\mathbf{E}(\mathbf{x},t)\rangle}, \qquad (2)$$

where $\sigma(t) \to \sigma$ for large enough t and σ is the color conductivity. We also studied plasmon excitations by measuring the color charge density $\rho \equiv \mathbf{D}\cdot\mathbf{E}$. An equivalent gauge-invariant definition for correlators of these adjoint objects requires introducing a straight Wilson line connecting $(\mathbf{x},0)$ to $(\mathbf{x},t)$. This Wilson line becomes an identity in the temporal gauge. In order to test the effect of this Wilson line on the characteristic time scale of the correlators we also study a truly gauge-invariant object, $B^2(\mathbf{x},t)$, and we find no difference with gauge covariant quantities. Our definition of the color conductivity is similar to that of Arnold and Yaffe [4], which is also based on the effective theory in the temporal gauge.

In Fig. 1 we show autocorrelators for the B-field, with time measured in units of $(g^2T)^{-1}$. We note that the curves corresponding to different values of β coincide as long as the correlators retain a substantial portion of their original value. We also introduce the integral autocorrelation time defined for an autocorrelator $C(t)$ as $t_\int \equiv (C(0))^{-1}\left(\int_0^\infty C(t)dt\right)$. In Fig. 2 we plot the dimensionless quantity $4/(g^2Tt_\int)$ as a function of $1/\beta = g^2Ta/4$. Remarkably, in all three cases $t_\int$ turns out to be of the order of g^2T and shows little dependence on the lattice spacing throughout the range considered. There is therefore no evidence that in this range of the lattice spacings our cooled autocorrelators follow the ASY-B scenario, wherein the expected behavior is $t_\int \propto 1/(g^4T^2a)$, up to logarithmic corrections. In the case of the color charge autocorrelator $\langle \mathbf{D}\cdot\mathbf{E}(\mathbf{x},t)\mathbf{D}\cdot\mathbf{E}(\mathbf{x},0)\rangle$ the time scale for the color charge correlation is proportional to the lattice spacing and does not depend on g^2T. This result can be contrasted with perturbative predictions. One would expect that the color-charge autocorrelator is dominated by the plasmon mode, whose frequency in the classical theory is of the order $g\sqrt{T/a}$ and whose decay rate is of the order g^2T. We observe none of these properties in the range of lattice spacings considered. Finally, we attempted to determine color conductivity σ. As Figure 3 demonstrates, this attempt failed in two ways. First of all, $\sigma(t)$ does not appear to approach a constant for times in excess of the expected autocorrelation time of the noise term ξ (and far in excess of the measured autocorrelation time of the noise). Secondly, the numerical value of $\sigma(t)$ is very small (less than $0.25/a$) compared to the value expected in the ASY scenario ($a\sigma \approx 15$).

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