

Revisiting α_s extractions from Soft Collinear Effective Theory

Jim Talbert

DAMTP, Cambridge

w/ G. Bell, C. Lee, Y. Makris,
B. Yan, B. Yoon



28 August 2023 || Hellenic (B)SM Workshop || Corfu, GR

The PDG table on α_s

hep-ph/0803.0342 (BS)
 hep-ph/1006.3080 (AFHMS)
 hep-ph/1501.04111 (HKMS)

To be included in the PDG average, a fit must:

- be published in a peer-reviewed journal...
- include $O(\alpha_s^3)$ fixed-order perturbative results...
- include 'reliable' error estimates, including NP effects...

2022 PDG world average:
 $.1179 \pm .0009$

Thrust at N³LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

Riccardo Abbate,¹ Michael Fickinger,² André H. Hoang,³ Vicent Mateu,³ and Iain W. Stewart¹

hep-ph/1006.3080

$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{exp}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$$

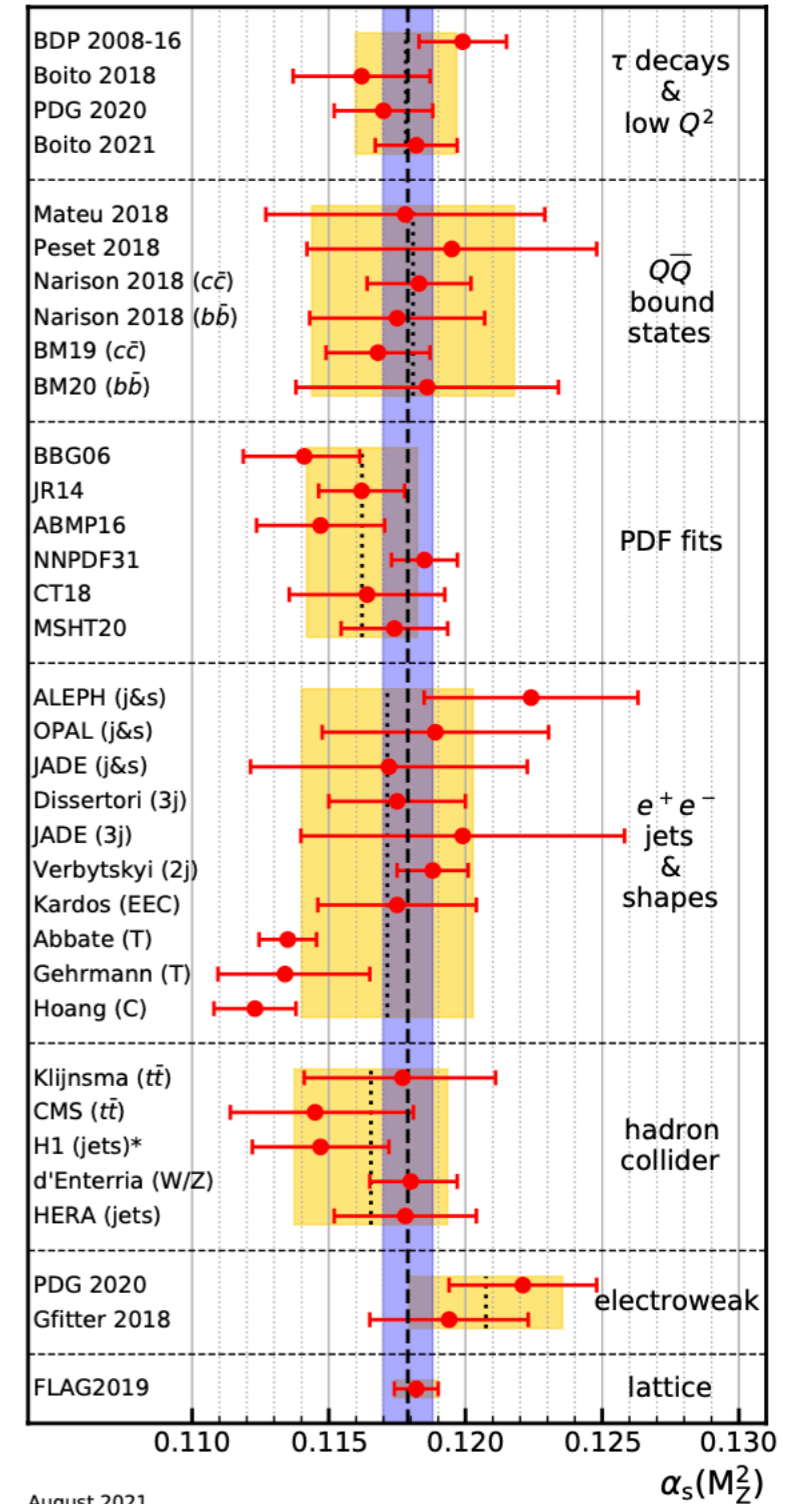
A Precise Determination of α_s from the C-parameter Distribution

André H. Hoang,^{1,2} Daniel W. Kolodrubetz,³ Vicent Mateu,¹ and Iain W. Stewart³

hep-ph/1501.04111

$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\text{exp}} \pm 0.0007_{\text{hadr}} \pm 0.0014_{\text{pert}}$$

- 2015 C-parameter result $\sim 4\sigma$ away from world average...



August 2021

Outline

Event shape distributions and
 α_s in SCET



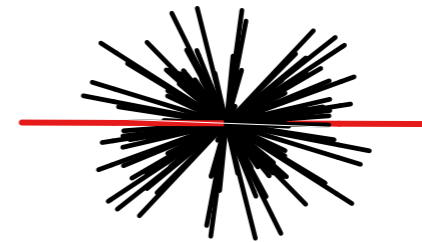
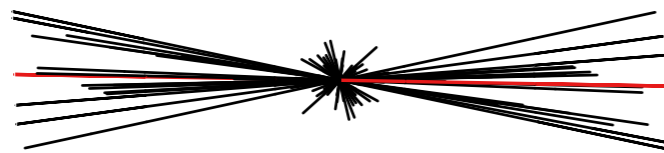
Formalism Revisit(s)

Some Preliminary Fit Results



Event shapes @ e+e- colliders

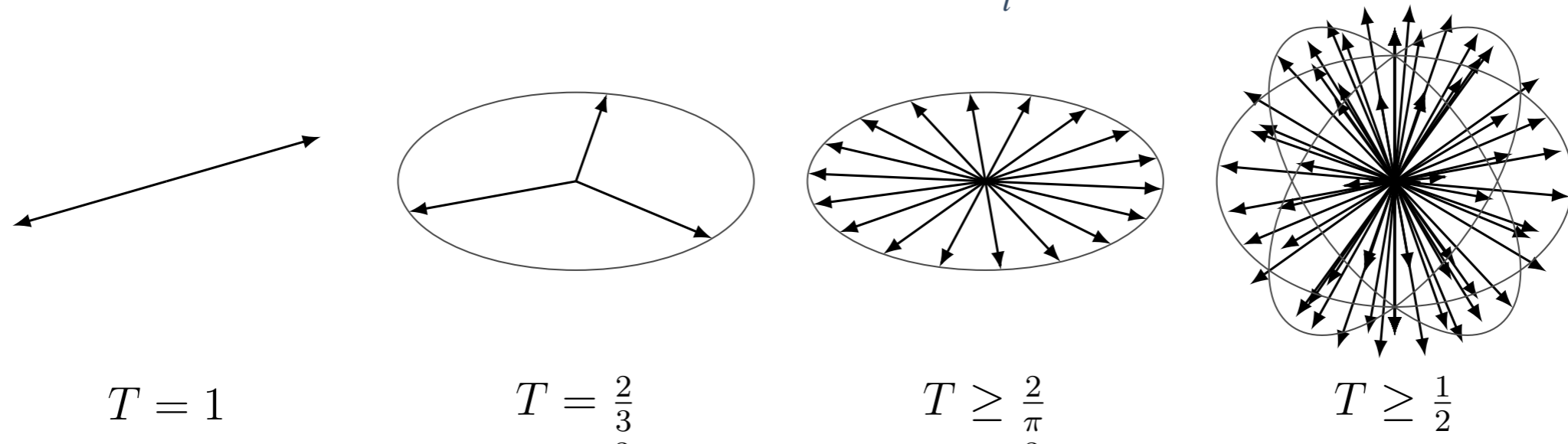
- The strong interaction provides the dominant contribution to collider processes involving colored objects.
- As QCD exhibits **asymptotic freedom**, high-energy colliders probe the interaction at weaker coupling, thereby permitting a perturbative description in QFT.
- However, as an event evolves, **many scales are probed**, including the non-perturbative. As a result, QCD spans a rich array of physical phenomena!
- QCD is 'simpler' in electron-positron collisions, as one avoids the messy **initial state physics** of bound protons (PDFs, ISR, multi-pardon scattering, etc.). This facilitates precision QCD studies, e.g. extractions of the strong coupling constant.
- Today we will focus on **event shapes**, which are geometric observables characterizing the 'shape' of final-state momentum distributions of hadronic objects. The dominant channel is **e+e- -> JJ**, as soft and collinear enhancements via gluon emission primarily lead to dijet events.



- So, a given event shape would assign a number to these geometric configurations, depending on what is actually getting measured...

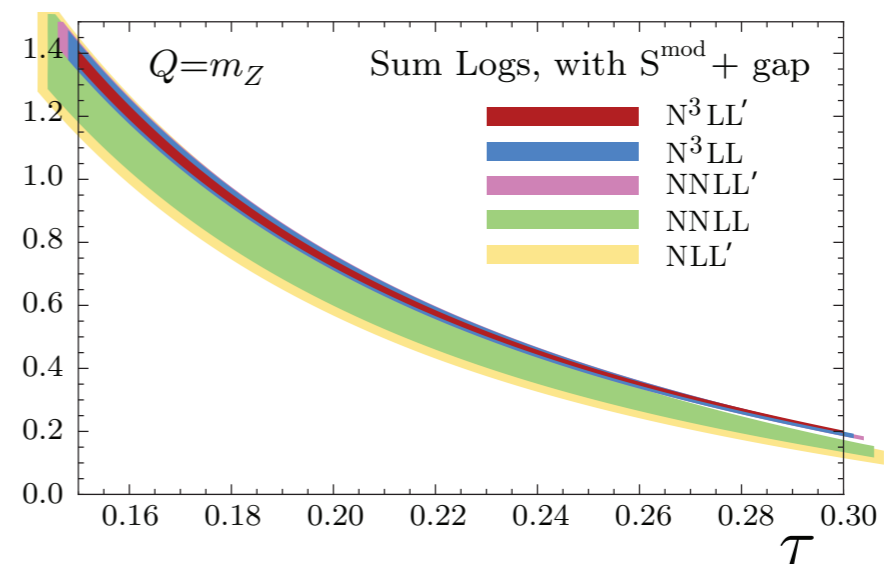
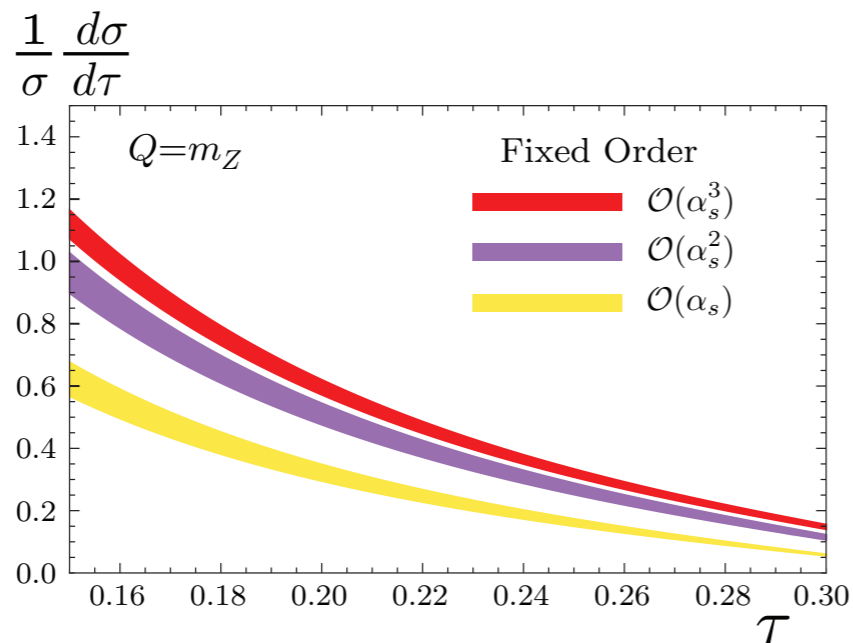
Event shape distributions: thrust

- The classic example is *Thrust*: $\tau \equiv 1 - T = \frac{1}{Q} \sum_i |\mathbf{p}_{\perp}^i| e^{-|\eta_i|}$ [Farhi, PRL 39 (1977)]



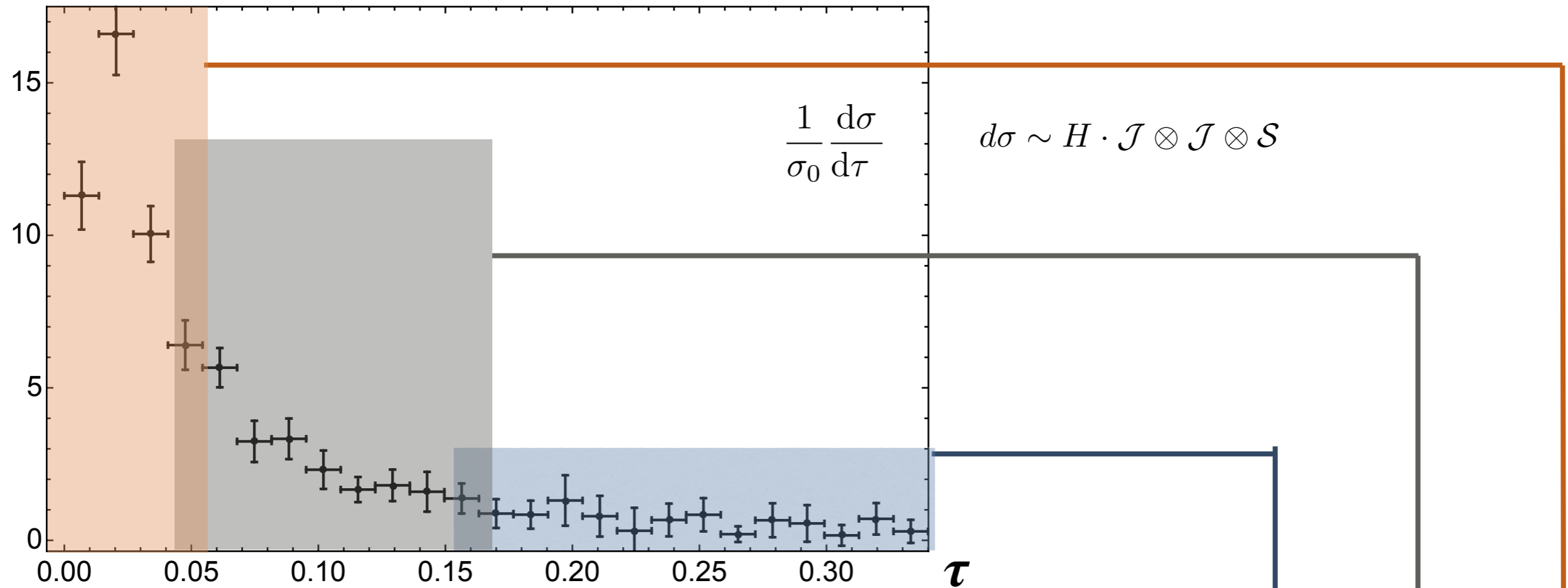
R. Rahn

- The fixed order distribution can readily be computed in QCD, though state of the art is a $N^3LL' + O(\alpha_s^3)$ resummation — readily achieved with **Soft Collinear Effective Theory**.



hep-ph/1006.3080

Dissecting dijets — constructing the curve



'Far Tail' Region: fixed-order, multi-jet region. QCD MATCHING

'Tail' Region: resummation region. PERTURBATIVE SCET PREDICTIONS

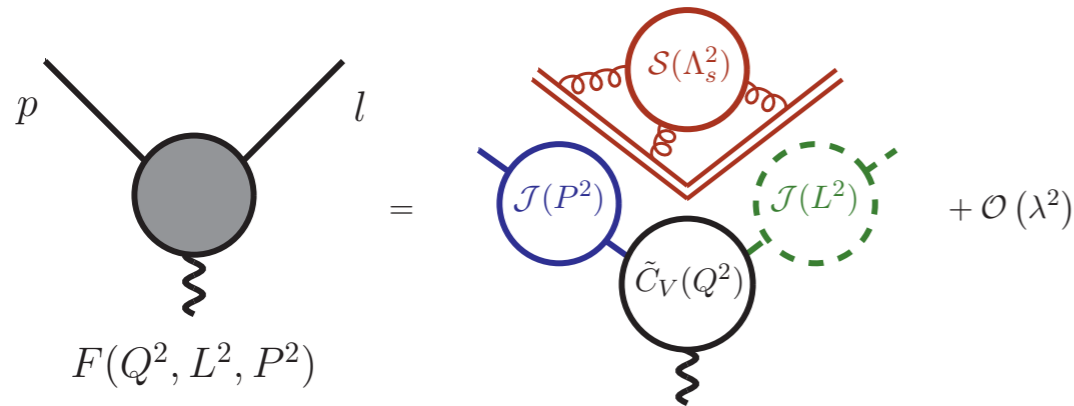
'Peak' Region: non-perturbative, soft region. NON-PERTURBATIVE MODELING

SCETching thrust: perturbative regime

[0801.4569]
 [0901.3780]

- SCET permits all-orders derivations of factorization theorems, with individual components resummed via RG evolution:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \quad \ln \frac{\mu^2}{Q^2}, \quad \ln \frac{\mu^2}{\tau Q^2}, \quad \ln \frac{\mu^2}{\tau^2 Q^2}$$

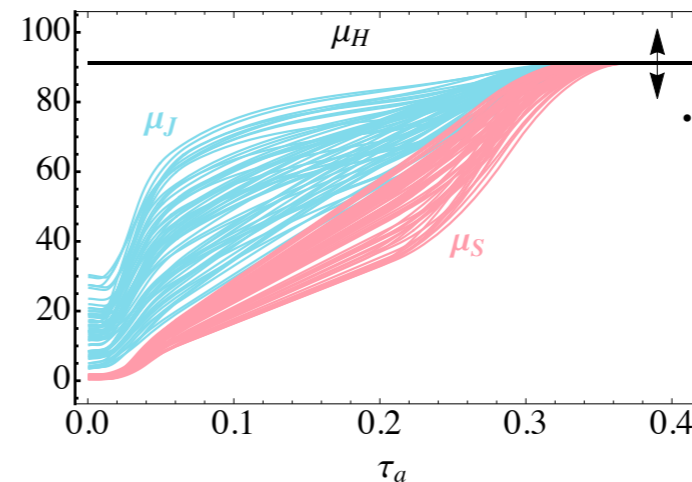


$$\frac{dH(Q^2, \mu)}{d \ln \mu} = \left[2\Gamma_{cusp} \ln\left(\frac{Q^2}{\mu^2}\right) + 4\gamma_H(\alpha_s) \right] H(Q^2, \mu)$$

$$H(Q^2, \mu) = H(Q^2, \mu_h) U_h(\mu_h, \mu)$$

This cookbook changes at 'primed' accuracies, and of course when considering matching to QCD!

Accuracy	Γ_{cusp}	$\gamma_F, \gamma_\Delta^\mu, \gamma_R$	β	$H, \tilde{J}, \tilde{S}, \delta_a$
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
N ³ LL	α_s^4	α_s^3	α_s^4	α_s^2



Note that there also is freedom in scale-setting choices -> 'profiles'

- Results for $O(\alpha_s^{(2,3)})$ matching obtained from **EVENT2 / EERAD3**:

$$\frac{\sigma_c(\tau_a)}{\sigma_0} - \frac{\sigma_{c,sing}(\tau_a)}{\sigma_0} = r_c(\tau_a) = \theta(\tau_a) \left\{ \frac{\alpha_s(Q)}{2\pi} r_c^1(\tau_a) + \left(\frac{\alpha_s(Q)}{2\pi} \right)^2 r_c^2(\tau_a) \right\} +$$



SCETching thrust: non-perturbative regime

- A treatment of **non-perturbative effects** is critical in $e^+e^- \rightarrow \text{hadrons}$...
- When dominant power corrections come from the soft function, NP effects can be parameterized into a shape function f_{mod} :

$$S(k, \mu) = \int dk' S_{\text{PT}}(k - k', \mu) f_{\text{mod}}(k' - 2\bar{\Delta}_a) \quad f_{\text{mod}}(k) = \frac{1}{\lambda} \left[\sum_{n=0}^{\infty} b_n f_n \left(\frac{k}{\lambda} \right) \right]^2$$

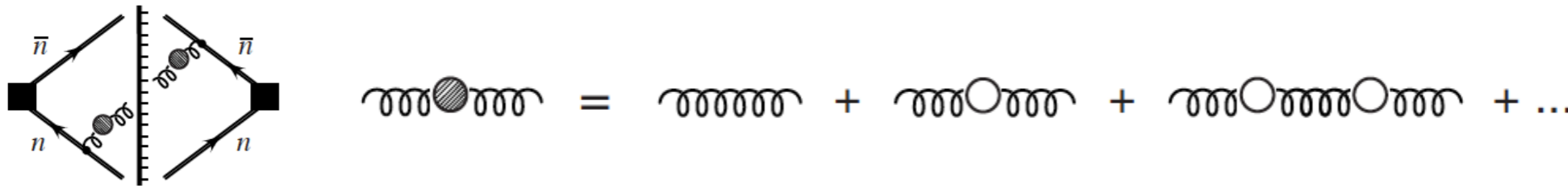
[0709.3519]
[0807.1926]

- The leading impact of this shape function correction is to shift the overall perturbative distribution:

a = 0 (Thrust) $\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow{\text{NP}} \frac{d\sigma}{d\tau_a} \left(\tau_a - c_{\tau_a} \frac{\bar{\Omega}_1}{Q} \right) \quad \frac{2\bar{\Omega}_1}{1-a} = 2\bar{\Delta}_a + \int dk k f_{\text{mod}}(k)$

[9408222]
[9504219]
[9806537]
[9902341]
[0611061]

- However, both the gap parameter Δ_{bar} and the soft function S_{PT} have a renormalon ambiguity!



see e.g.
Beneke
(9807443)

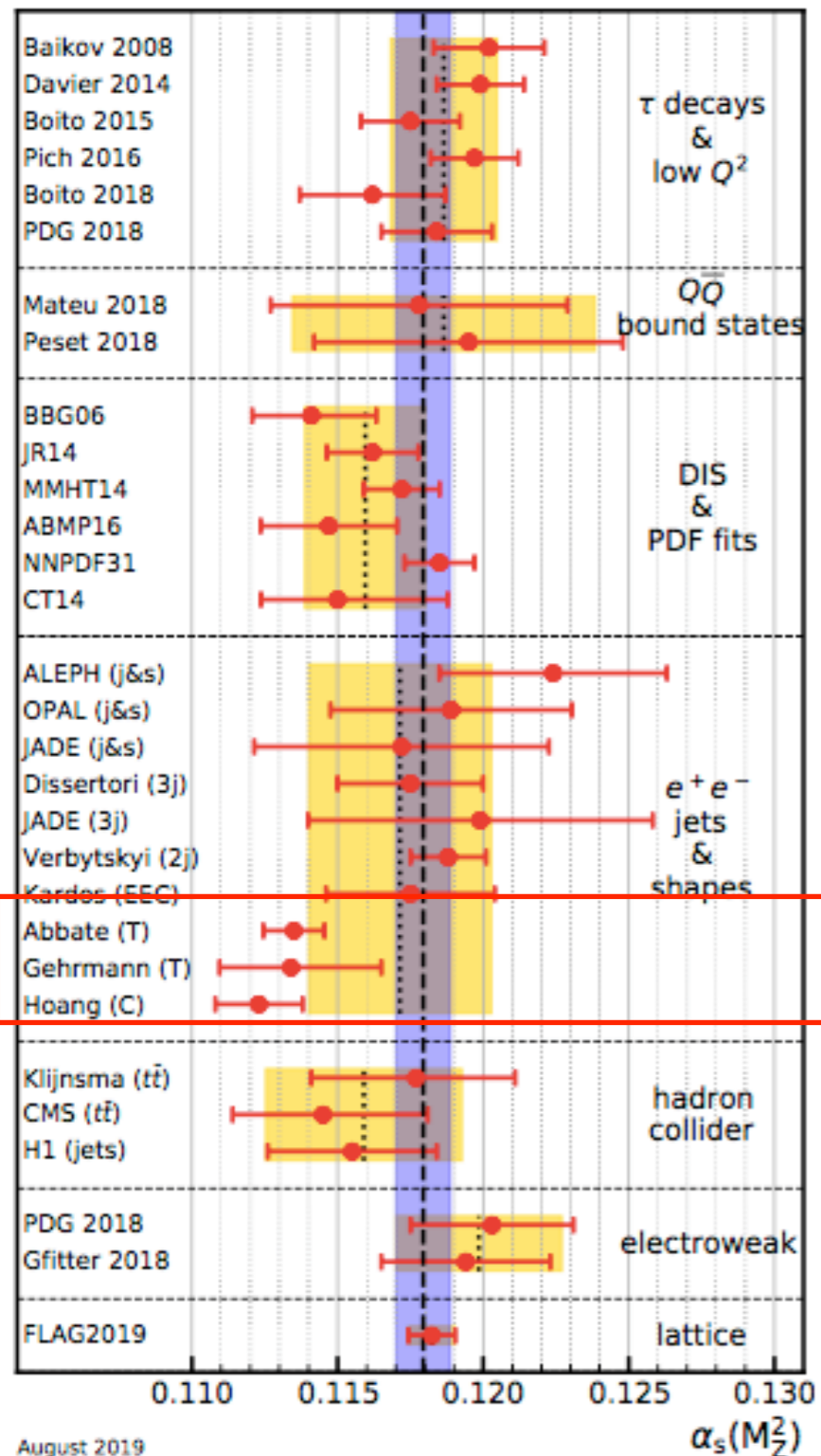
- Solution: subtract a series with a compensating/cancelling ambiguity:

$$\bar{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \quad \longrightarrow \quad \tilde{S}(\nu, \mu) = \left[e^{-2\nu\Delta_a(\mu)} \tilde{f}_{\text{mod}}(\nu) \right] \left[e^{-2\nu\delta_a(\mu)} \tilde{S}_{\text{PT}}(\nu, \mu) \right]$$

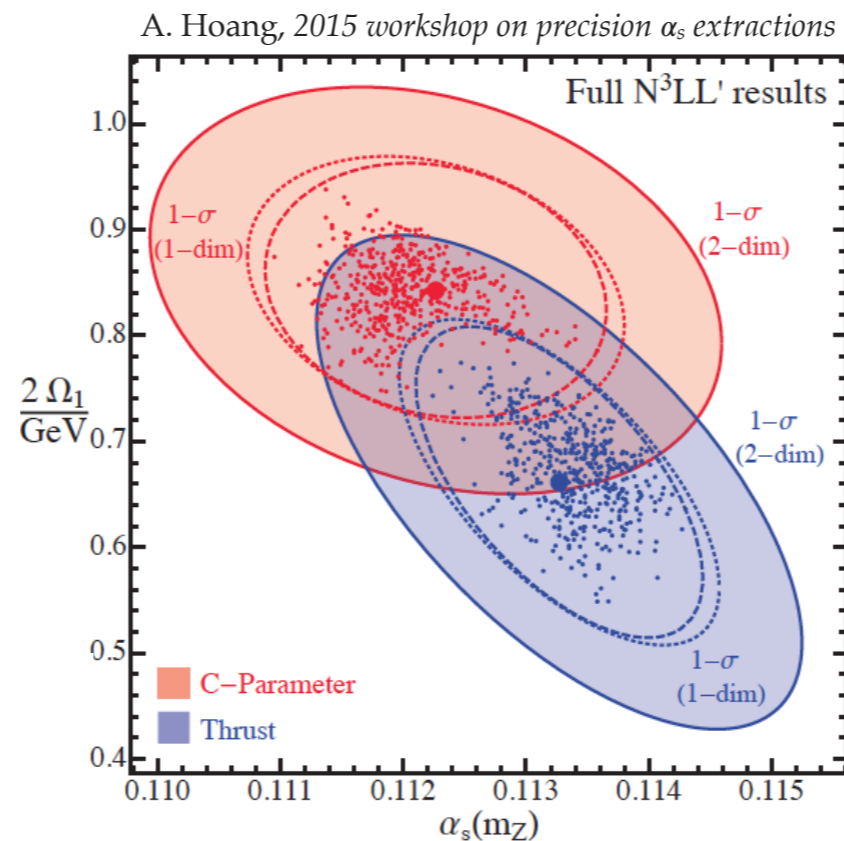
[0003179]
[0709.3519]
[0803.4214]
[0806.3852]
[0801.4743]
[0908.3189]

- The highest precision SCET extractions have done so with a very particular scheme.

SCET extractions @ $N^3LL + O(\alpha^3)$ accuracy



C-parameter versus Thrust Tail Global Fit



2022 PDG world average:
.1179 +- .0009

[hep-ph/0803.0342](https://arxiv.org/abs/hep-ph/0803.0342) (BS)
[hep-ph/1006.3080](https://arxiv.org/abs/hep-ph/1006.3080) (AFHMS)
[hep-ph/1501.04111](https://arxiv.org/abs/hep-ph/1501.04111) (HKMS)

(Q1) Why are SCET results so discrepant with PDG?

(Q2) What can break the $\alpha_s - \Omega$ degeneracy?
(not today, unfortunately)

Revisiting NP Phenomenology

R-Gap scheme

$$\tilde{S}(\nu, \mu) = \left[e^{-2\nu\Delta_a(\mu)} \tilde{f}_{\text{mod}}(\nu) \right] \left[e^{-2\nu\delta_a(\mu)} \tilde{S}_{\text{PT}}(\nu, \mu) \right]$$

- After redefining gap, one can choose the **R-Gap scheme** to cancel the leading renormalon,

$$Re^{\gamma_E} \frac{d}{d \ln \nu} \left[\ln \hat{S}_{\text{PT}}(\nu, \mu) \right]_{\nu=1/(Re^{\gamma_E})} = 0 \longrightarrow \delta_a(\mu, R) = \frac{1}{2} Re^{\gamma_E} \frac{d}{d \ln \nu} \left[\ln \tilde{S}_{\text{PT}}(\nu, \mu) \right]_{\nu=1/(Re^{\gamma_E})},$$

$$\hat{S}_{\text{PT}}(\nu, \mu) = e^{-2\nu\delta_a(\mu)} \tilde{S}_{\text{PT}}(\nu, \mu)$$

All of these objects can be defined perturbatively!

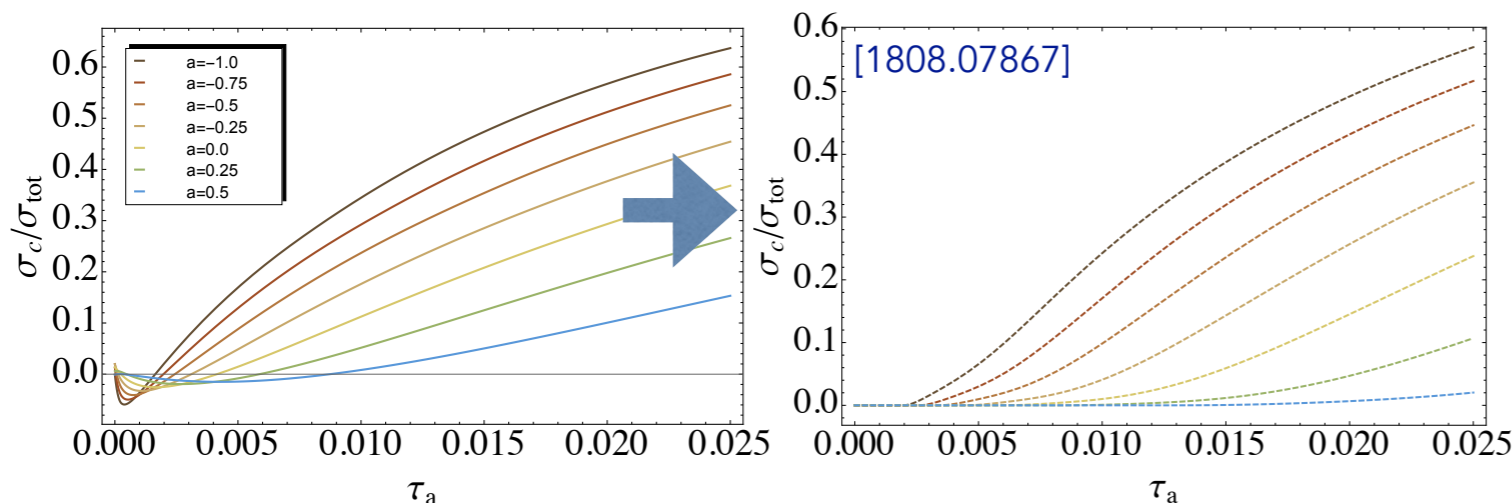
- and accounting for R and μ evolution,

$$\frac{d}{dR} \Delta_a(R, R) = -\frac{d}{dR} \delta_a(R, R) \equiv -\gamma_R[\alpha_s(R)], \quad \mu \frac{d}{d\mu} \Delta_a(\mu, R) = -\mu \frac{d}{d\mu} \delta_a(\mu, R) \equiv \gamma_{\Delta}^{\mu}[\alpha_s(\mu)]$$

- one obtains the final soft function \rightarrow cross section:

Final cross section is expanded order-by-order in bracketed term

$$\frac{1}{\sigma_0} \sigma(\tau_a) = \int dk \sigma_{\text{PT}}\left(\tau_a - \frac{k}{Q}\right) \left[e^{-2\delta_a(\mu_S, R)} \frac{d}{dk} f_{\text{mod}}\left(k - 2\Delta_a(\mu_S, R)\right) \right]$$



Also results in better convergence than shape function alone!

However, fits with this scheme implemented amongst LOWEST in global PDG table...

Effective non-perturbative shifts

- Before considering gapped renormalons, the leading-order NP effect is a constant shift:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow{\text{NP}} \frac{d\sigma}{d\tau_a} \left(\tau_a - c_{\tau_a} \frac{\bar{\Omega}_1}{Q} \right) \quad \frac{2\bar{\Omega}_1}{1-a} = 2\bar{\Delta}_a + \int dk k f_{\text{mod}}(k)$$

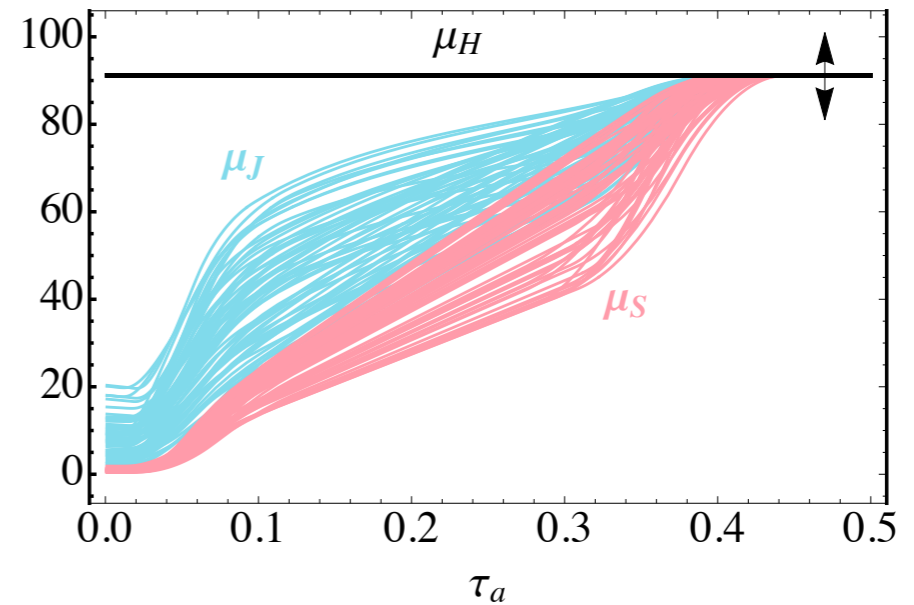
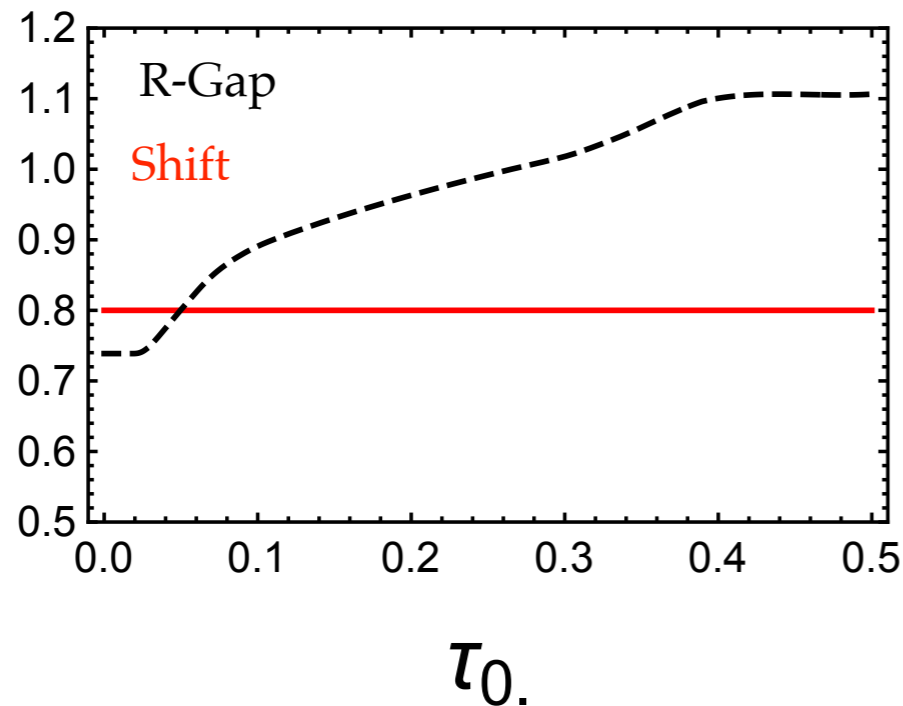
- But what is the 'effective shift' of the distribution in the R-Gap scheme?

$$\int dk k e^{-2\delta_a(\mu_S, R) \frac{d}{dk}} f_{\text{mod}}(k - 2\Delta_a(\mu_S, R)) = \int dk k \left[\sum_i f_{\text{mod}}^{(i)}(k - 2\Delta_a(\mu_S, R)) \right]$$

$$f_{\text{mod}}^{(0)}(k - 2\Delta_a(\mu_S, R)) = f_{\text{mod}}(k - 2\Delta_a(\mu_S, R)),$$

$$f_{\text{mod}}^{(1)}(k - 2\Delta_a(\mu_S, R)) = -\frac{\alpha_s(\mu_S)}{4\pi} 2\delta_a^1(\mu_S, R) R e^{\gamma_E} f'_{\text{mod}}(k - 2\Delta_a(\mu_S, R)),$$

$$f_{\text{mod}}^{(2)}(k - 2\Delta_a(\mu_S, R)) = \left(\frac{\alpha_s(\mu_S)}{4\pi} \right)^2 \left[-2\delta_a^2(\mu_S, R) R e^{\gamma_E} f'_{\text{mod}}(k - 2\Delta_a(\mu_S, R)) + 2(\delta_a^1(\mu_S, R) R e^{\gamma_E})^2 f''_{\text{mod}}(k - 2\Delta_a(\mu_S, R)) \right],$$



Why does this effect grow as one moves toward the fixed-order regime?

R*: a toy scheme

- Generalized renormalon cancellation schemes can be defined: [2012.12304]

$$\delta_a(\mu) = \frac{R}{2\xi} \frac{d^n}{d(\ln v)^n} \ln \tilde{S}(v, \mu) \Big|_{v=\xi/R}$$



$$\delta_a^*(R) = \frac{1}{2} R^* e^{\gamma_E} \frac{d}{d \ln v} \left[\ln S_{\text{PT}}(v, \mu = R^*) \right]_{v=1/(R^* e^{\gamma_E})}$$



we are not forced to set $\mu = \mu_S$ in the subtraction series, we can pick $\mu = R$

R* Scheme:

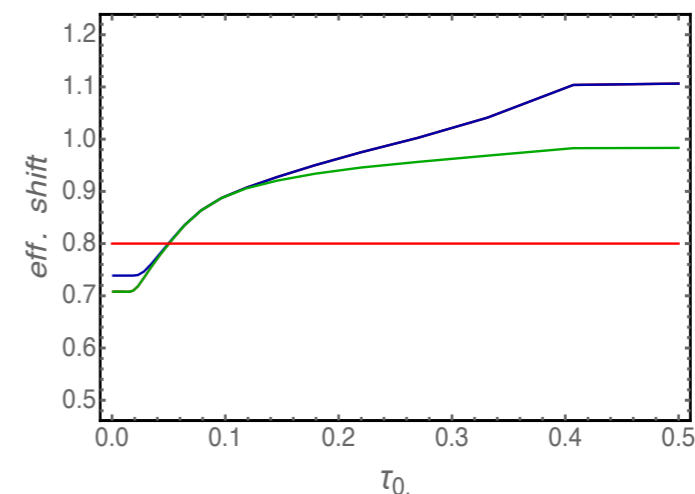
$$(n, \xi, \mu) = (1, \exp(-\gamma_E), R^*)$$

$$R^* \equiv \begin{cases} R & R < R_{\text{max}} \\ R_{\text{max}} & R \geq R_{\text{max}} \end{cases}$$

- Anomalous dimensions, subtractions, turn on at one higher order:

$$\delta_a^*(R^*) = \frac{R^* e^{\gamma_E}}{2} \left[\frac{\alpha_s(R^*)}{4\pi} \cdot 0 + \left(\frac{\alpha_s(R^*)}{4\pi} \right)^2 (\gamma_S^1 + 2c_{\tilde{S}}^1 \beta_0) + \mathcal{O}(\alpha_s^3) \right]$$

$$\gamma_R^* = e^{\gamma_E} \left[\frac{\alpha_s(R^*)}{4\pi} \cdot 0 + \left(\frac{\alpha_s(R^*)}{4\pi} \right)^2 (\gamma_S^1 + 2c_{\tilde{S}}^1 \beta_0) + \mathcal{O}(\alpha_s^3) \right]$$



$R_{\text{max}} = \infty$ (R_{gap})

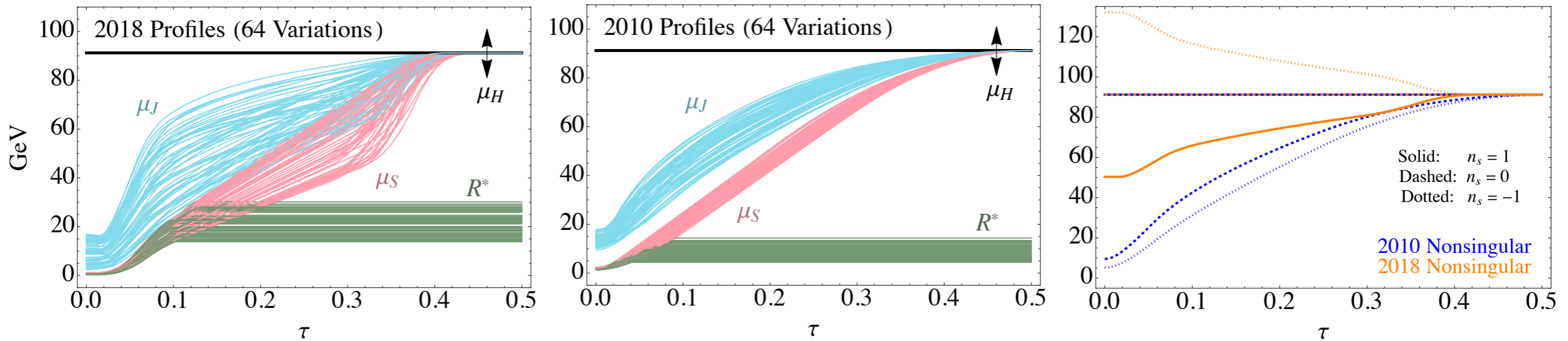
$R_{\text{max}} = R(t_1), \mu_{\text{sub}} = R$

constant shift

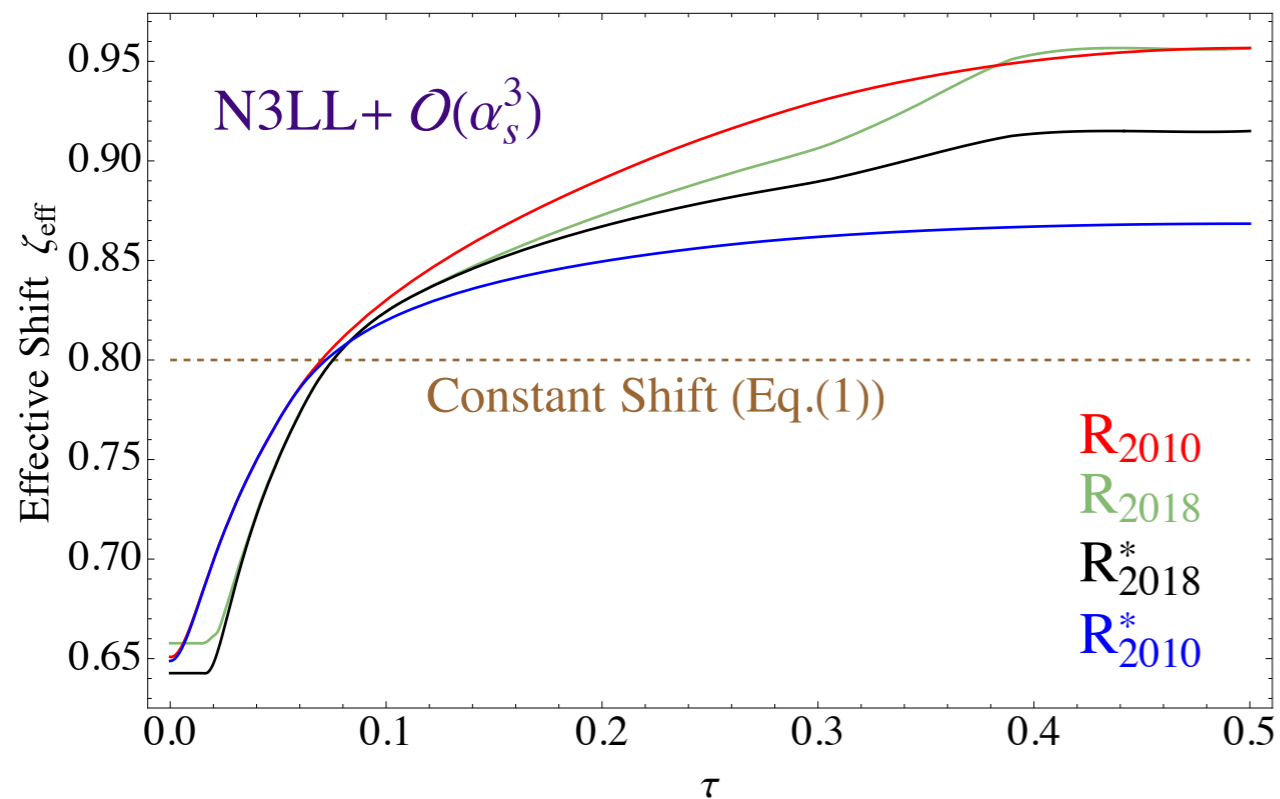
Effective non-perturbative shift flattened, as desired.

What about profiled scale variations?

- Defining a profiled variation scheme is a bit of an art ...



- But the impact on the curve is non-negligible...



I will show results for fits in four 'schemes' (2 renormalon cancellation x 2 profile variations):

R_{2010}

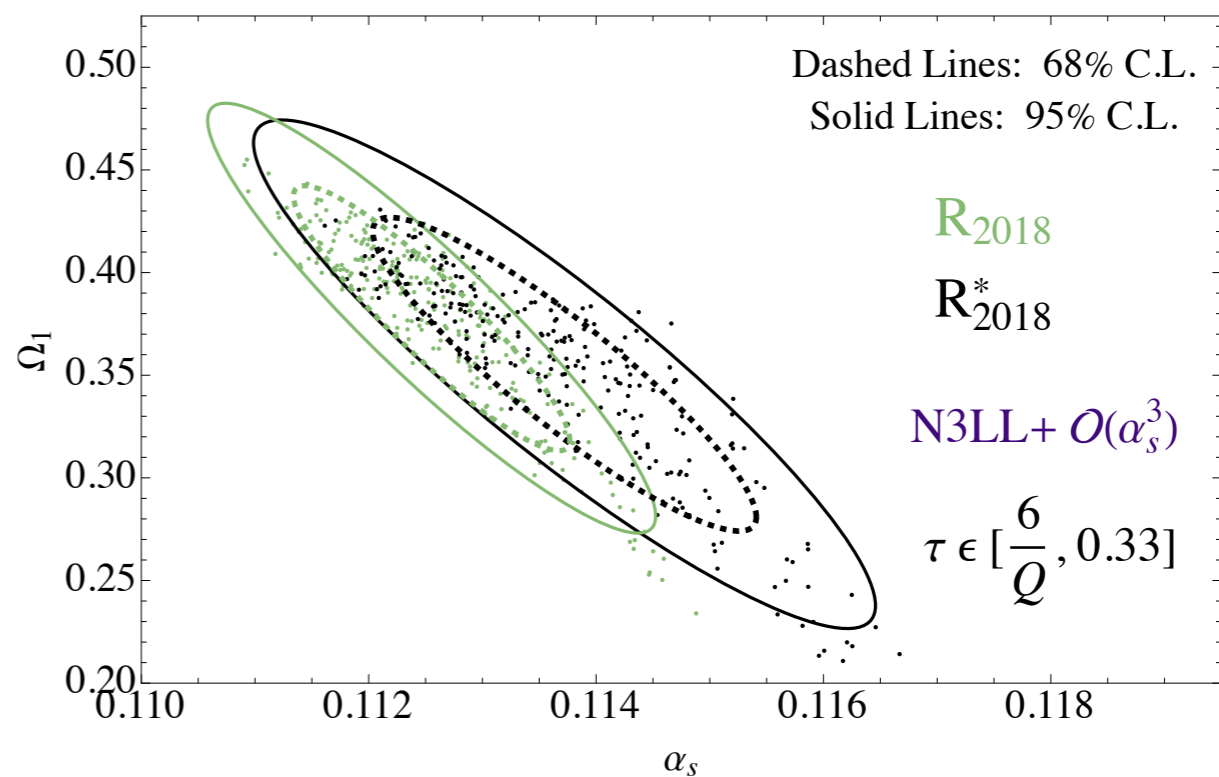
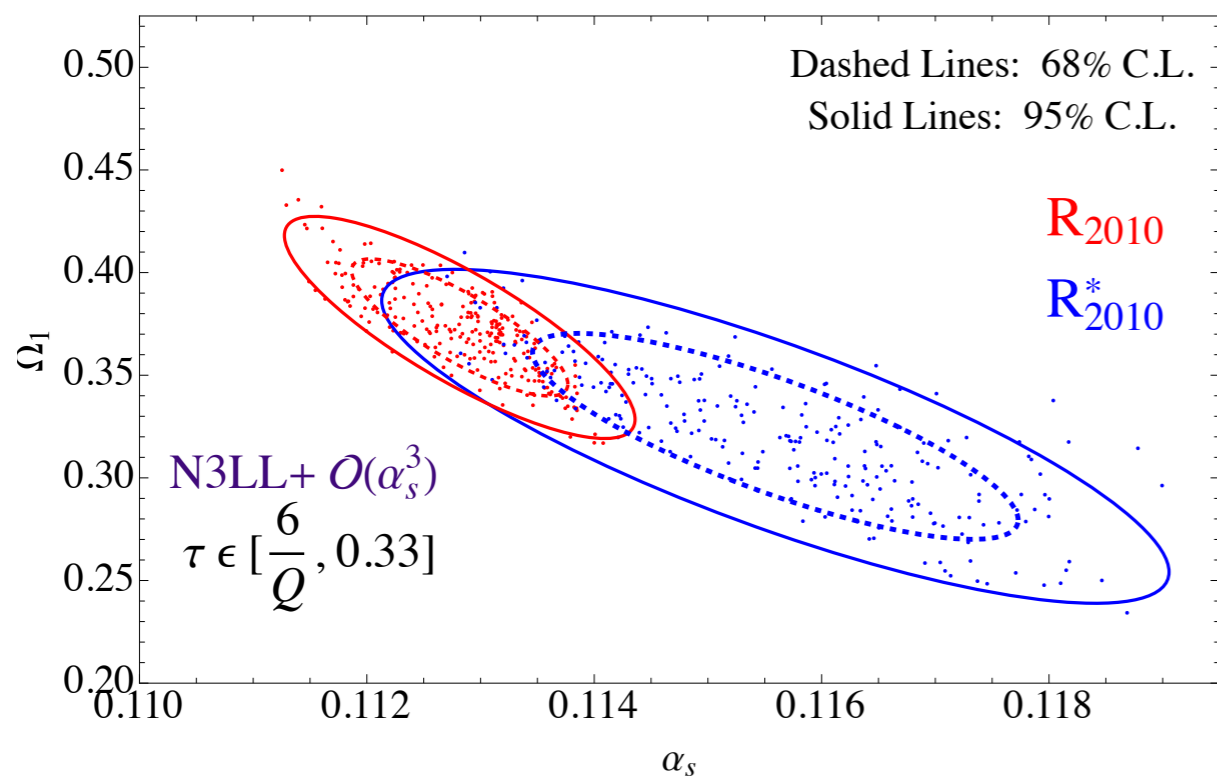
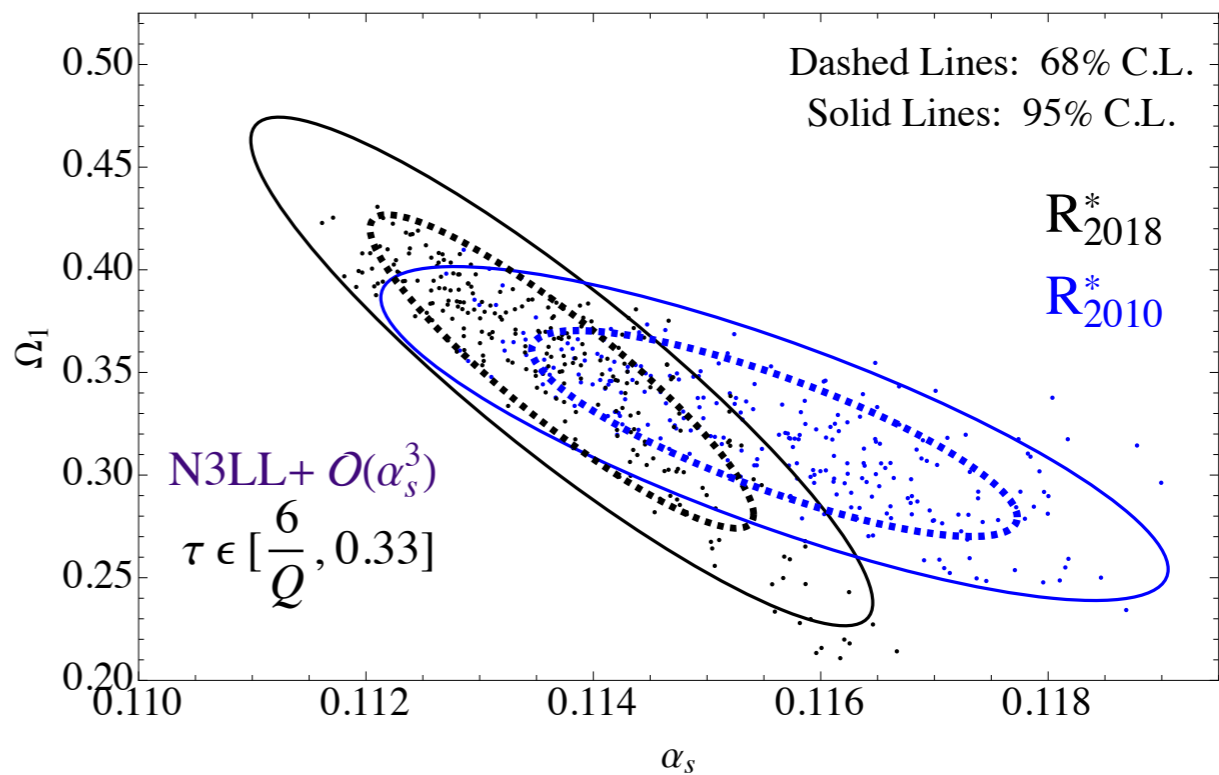
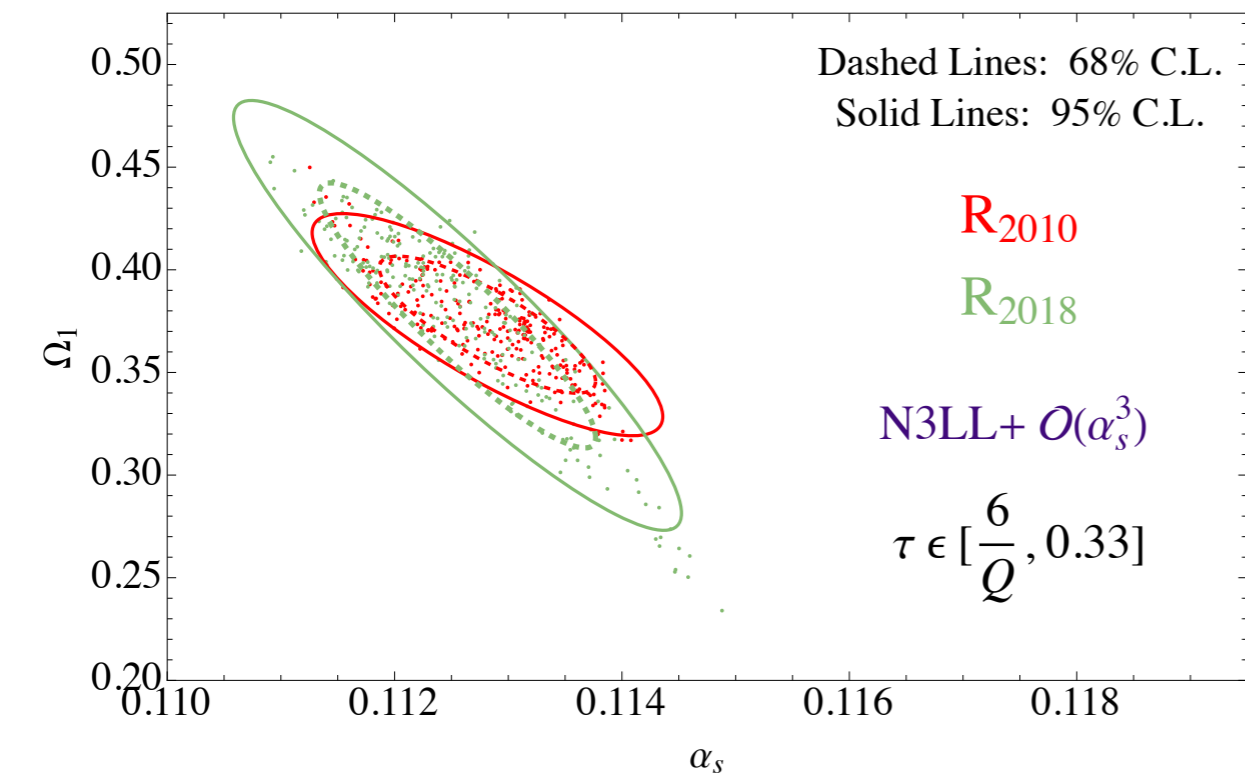
R_{2018}

R_{2010}^*

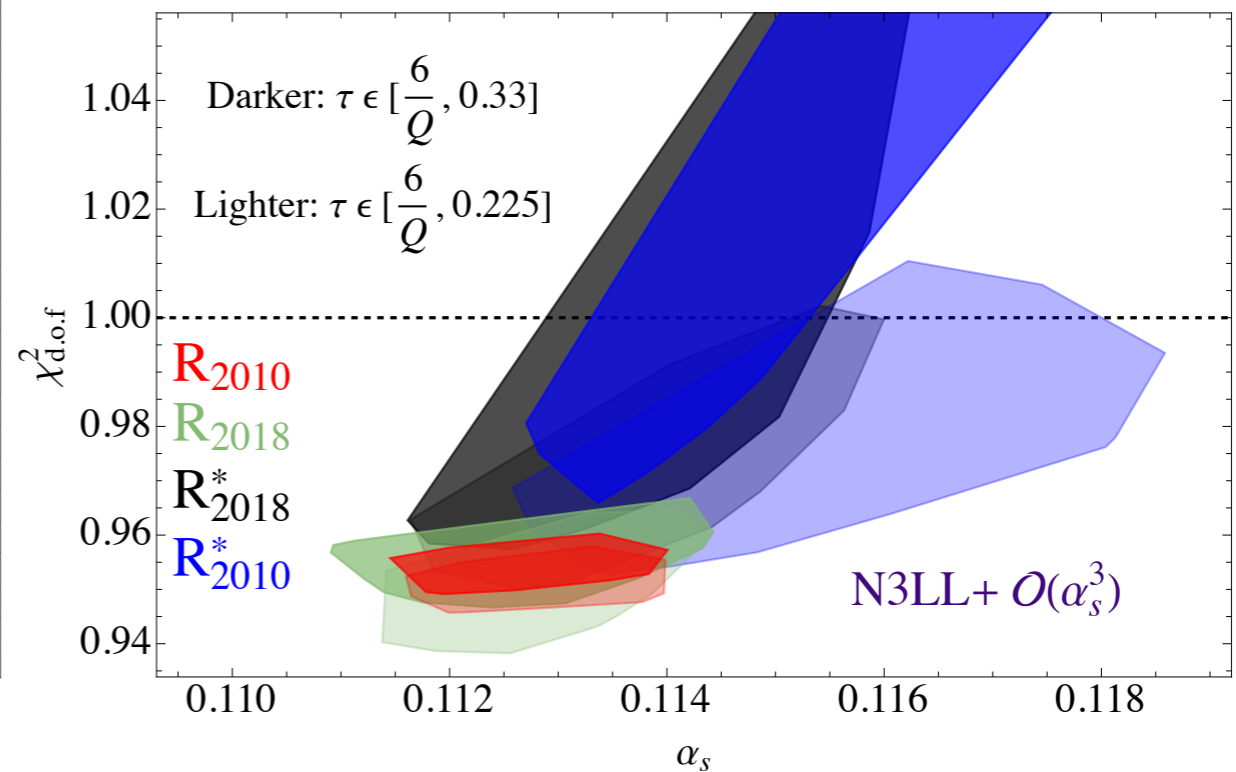
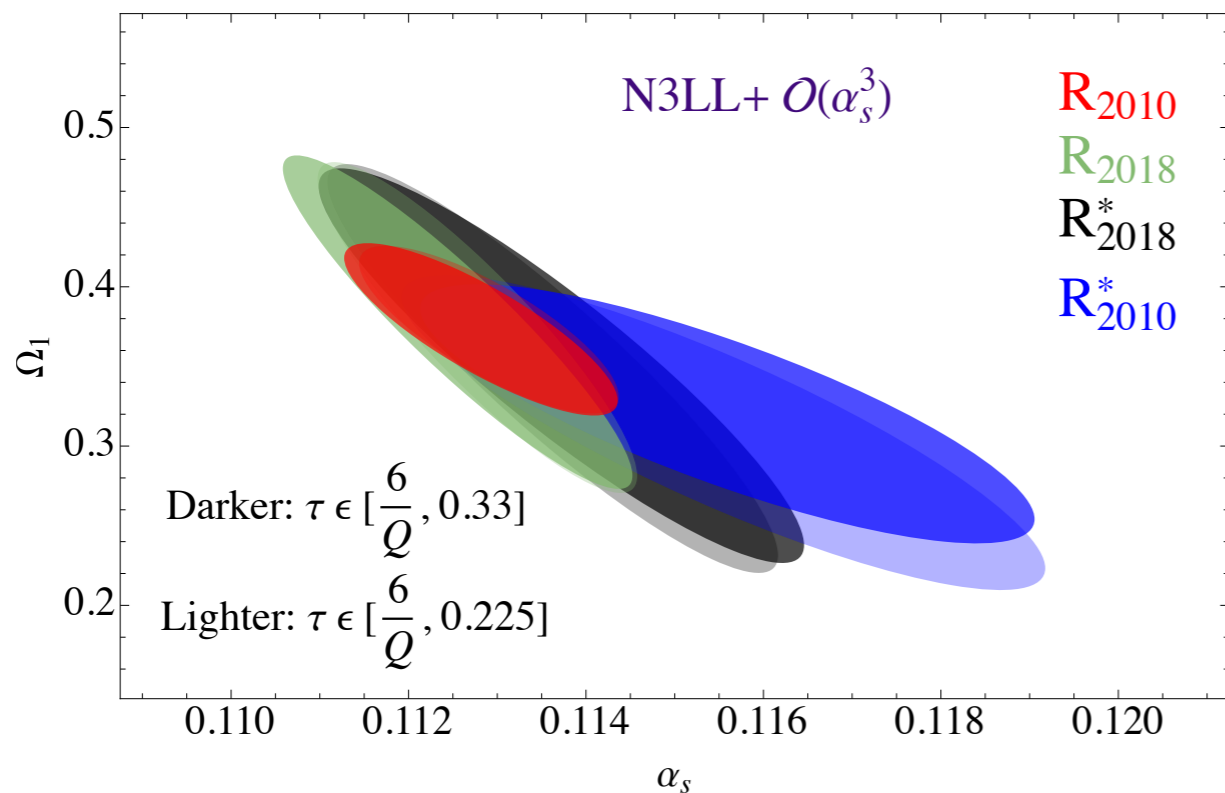
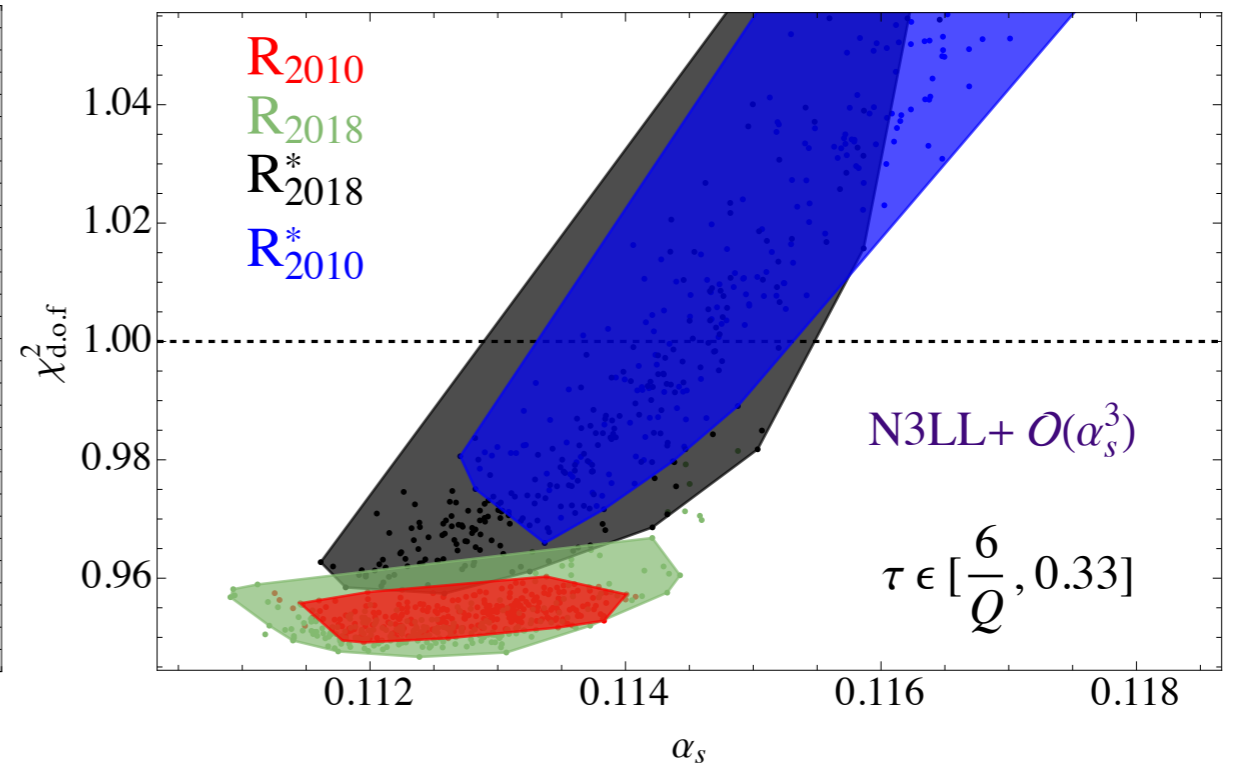
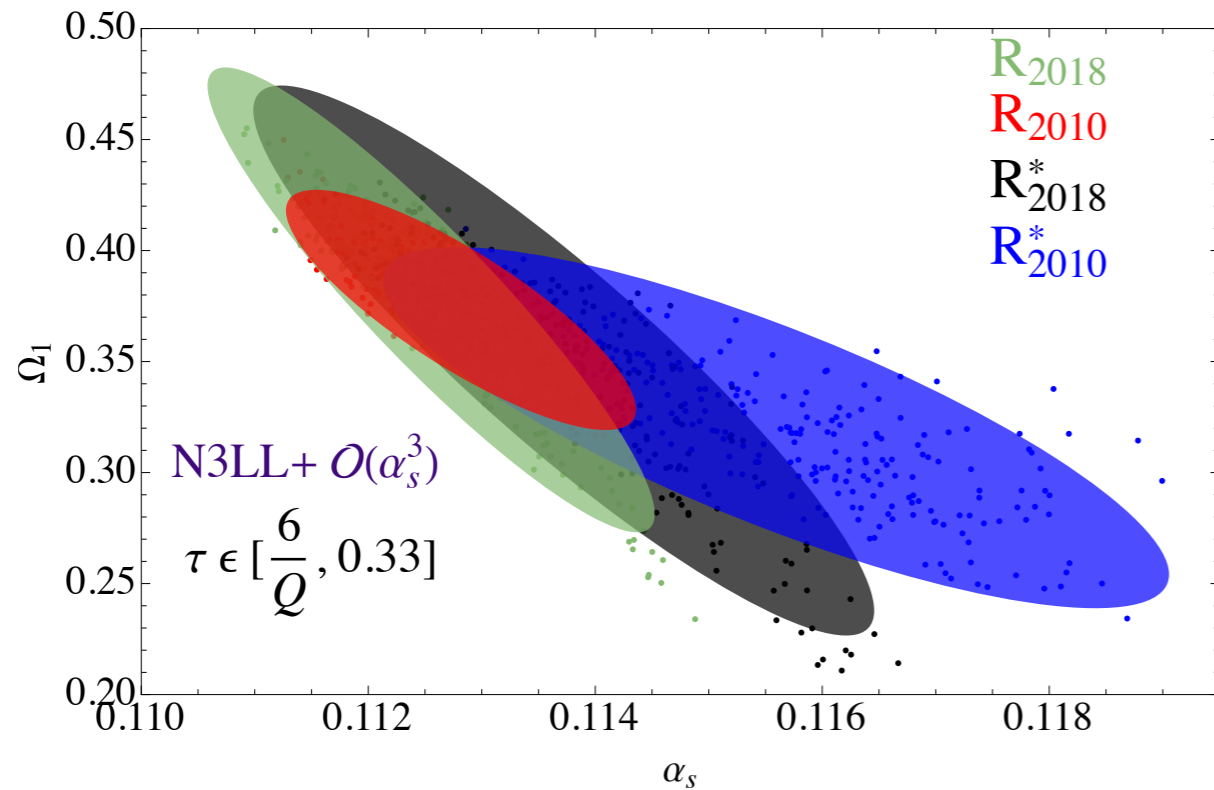
R_{2018}^*

Preliminary Results

N3LL^(*l*) + $O(\alpha_s^3)$ accuracy fits to α_s and Ω_1 *PRELIMINARY!*



The global picture, and fit windows *PRELIMINARY!*



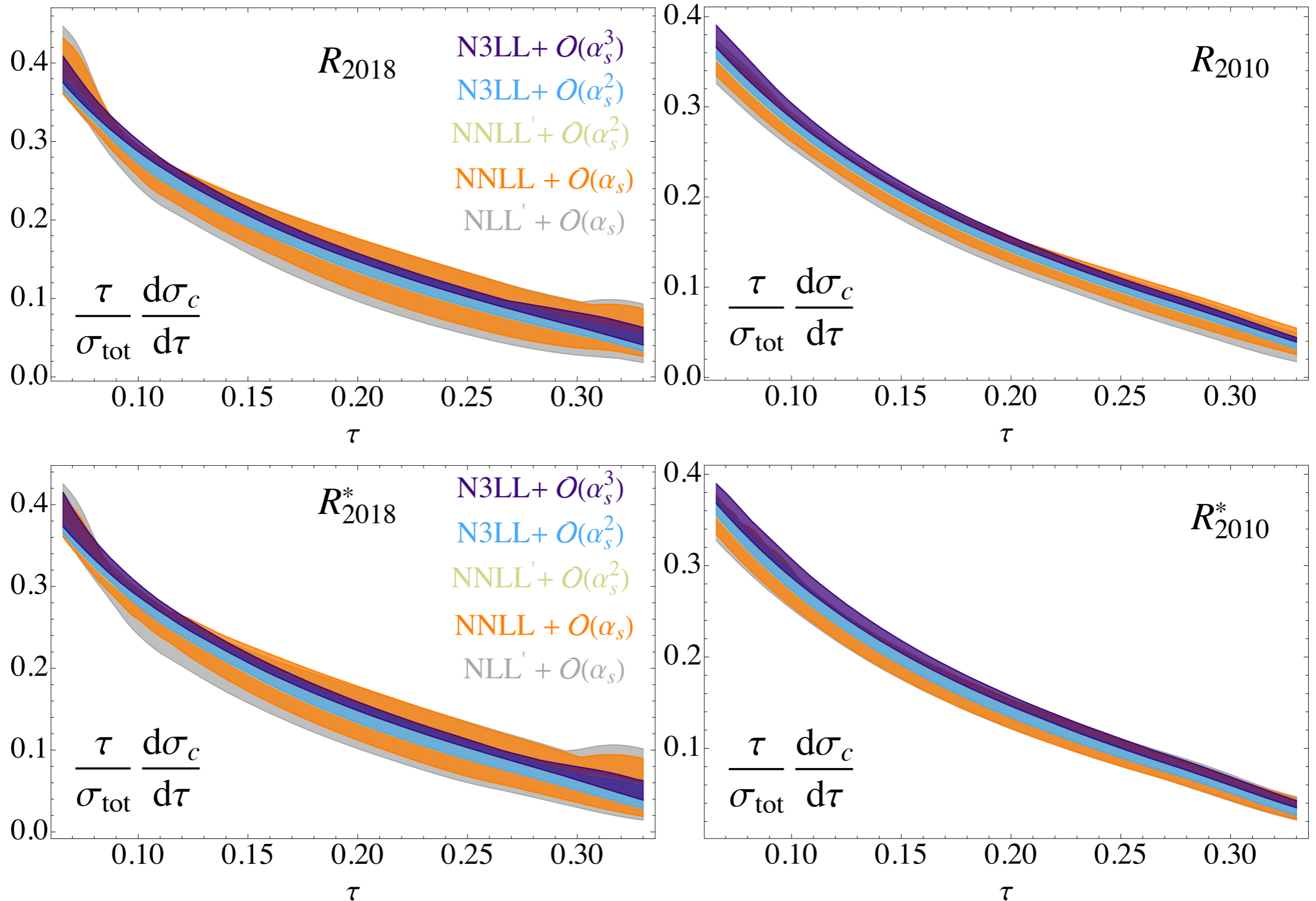
Summary and outlook

- We have presented **preliminary** results demonstrating the impact of **renormalon cancellations** and **profiled scale variations** on global SCET extractions of the strong coupling from the Thrust $e+e-$ event shape.
- Our results are valid at **$N^3LL' + O(\alpha_s^3)$ accuracy**. Further theory improvements can be made.
- We find **general agreement with existing SCET fits**, when comparing 'like for like.' However, the presence of the new **systematic uncertainty** we have identified leads one to error bands that are **much less discrepant** (some schemes are even consistent with...) the PDG average.
- We have also shown how Thrust fit values are sensitive to the fit window chosen to compare theory against data. We observe a **universal improvement in the quality of fit** when we use our dijet factorization theorem to predict data in the dominantly dijet window...
- **Other WIP**: analyzing a more varied and generic set of renormalon cancellation schemes — matching systematics — missing theory ingredients — generalization to angularities....

Thanks!

Backup Slides

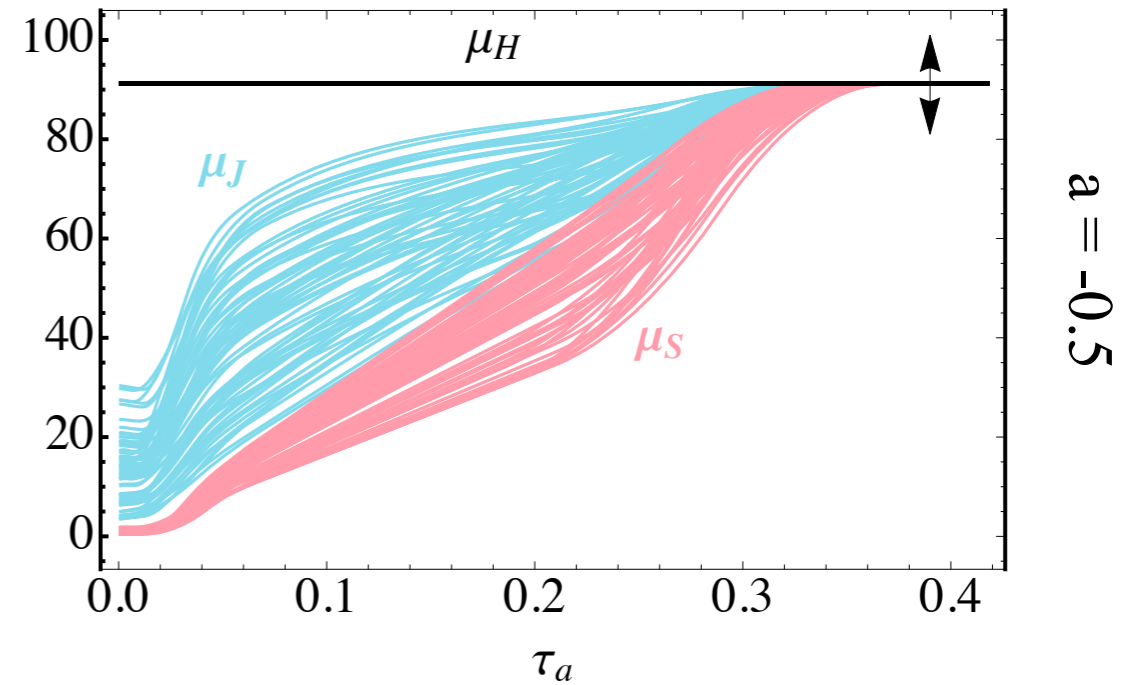
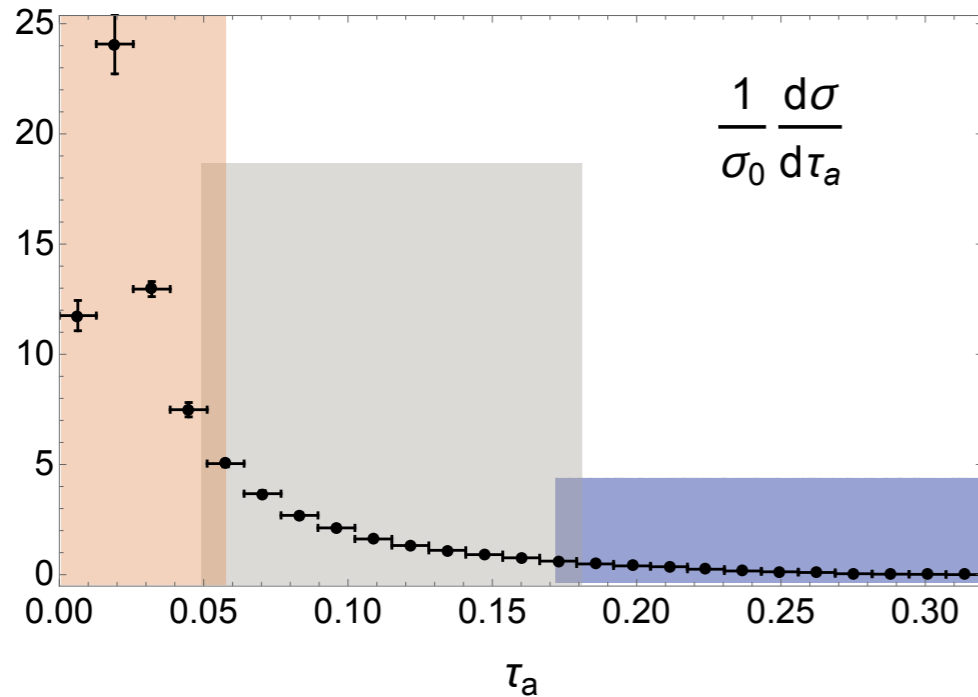
R vs. R* convergence



Profiling a fit window

hep-ph/1808.07867

- How can we identify a region sensitive to \mathcal{A} and α_s , and for which our best theory curves are reliable? Look to the profiles!



- Profiles trace scale hierarchies through different regimes of a given distribution:

Peak	$\mu_H \gg \mu_J \gg \mu_S \sim \Lambda_{QCD}$
Tail	$\mu_H \gg \mu_J \gg \mu_S \gg \Lambda_{QCD}$
Far Tail	$\mu_H = \mu_J = \mu_S \gg \Lambda_{QCD}$

Tracks the peak

$$t_0 = \frac{n_0}{Q} 3^a$$

$$t_1 = \frac{n_1}{Q} 3^a$$

Turns off resummation

$$t_2 = n_2 \times 0.295^{1-0.637 a}$$

$$t_3 = n_3 \tau_a^{\text{sph}}$$

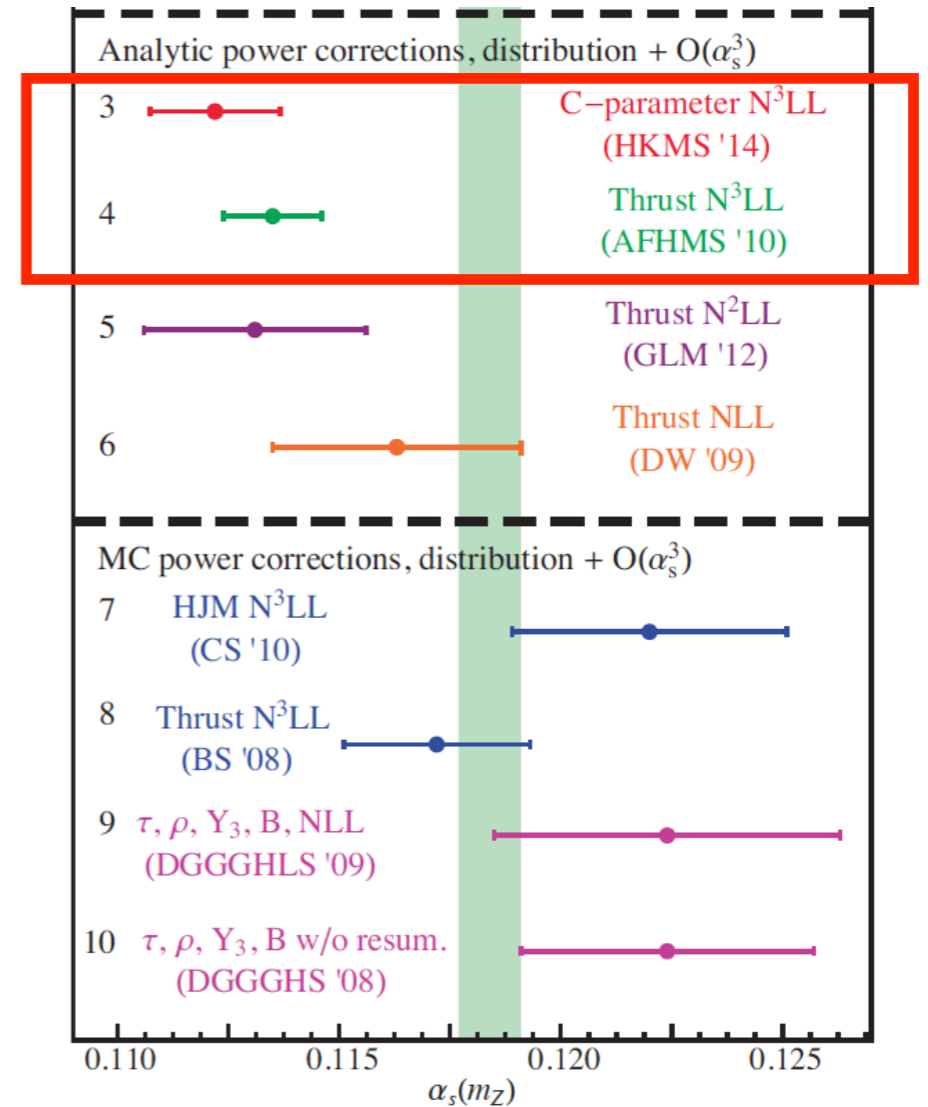
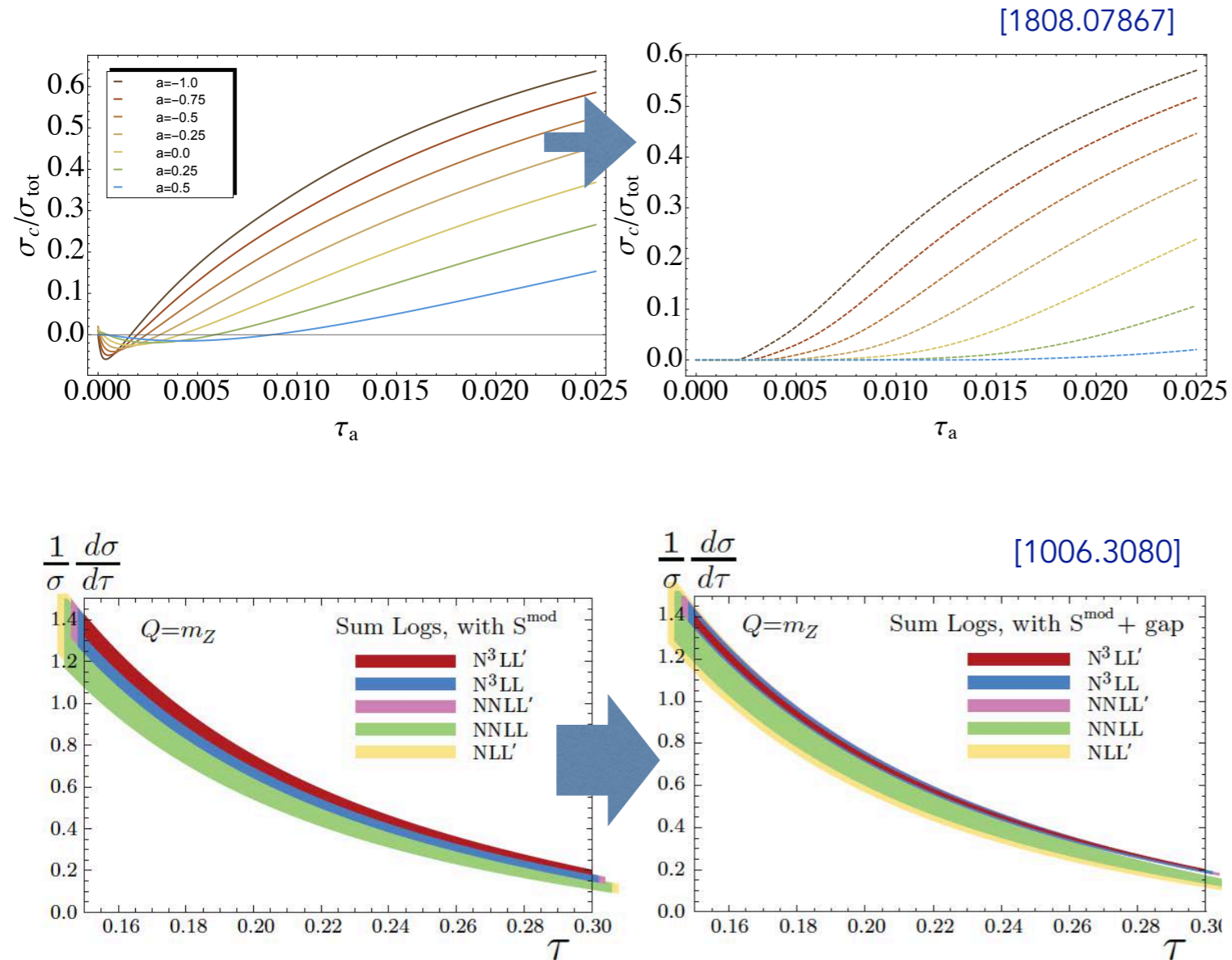
Transitions between NP and PT physics

Reverts to fixed-order perturbation theory

- A default fit window will be between t_1 , and t_2 , which roughly tracks the tail (former) and far-tail (latter) of the distribution.**

R-Gap phenomenology

- R-Gap scheme removes unphysical effects in cross-section predictions and gives good qualitative agreement with data:



see A. Hoang, 2015 workshop on precision α_s

- How non-perturbative effects are implemented (clearly) affects the extraction of the strong coupling!

Fit technique

- We perform a $\chi^2_{d.o.f.}$ analysis at the level of binned theory predictions:

$$\chi^2 \equiv \sum_{i,j} \Delta_i V_{ij}^{-1} \Delta_j$$

$$\bar{\tau} \equiv (\tau_1 + \tau_2)/2$$

$$\Delta_i \equiv \left(\frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau_i) \Big|_{\text{exp}} - \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau_i) \Big|_{\text{th}} \right) \quad \frac{1}{\sigma} \frac{d\sigma}{d\tau}(\tau_i) \Big|_{MP}^{\text{th}} \equiv \frac{1}{\sigma_{tot}} \frac{\sigma_c(\tau_2, \mu_a(\bar{\tau})) - \sigma_c(\tau_1, \mu_a(\bar{\tau}))}{\tau_2 - \tau_1}$$

- Experimental errors (stat. and syst.) accounted for with 'minimal overlap model':

$$V_{ij} \Big|_{\text{MOM}} = (e_i^{\text{stat.}})^2 \delta_{ij} + \min(e_i^{\text{syst.}}, e_j^{\text{syst.}})^2$$

- Theory errors are conveniently parameterized in terms of an error ellipse K :

$$1 = \mathbf{X}^T K_{theory}^{-1} \mathbf{X} \quad \mathbf{X}^T = \{\alpha_s, \Omega_1\} - \{\mu_\alpha, \mu_\Omega\} \quad K_{theory} = \begin{pmatrix} \sigma_\alpha^2 & \rho_{\alpha\Omega} \sigma_\alpha \sigma_\Omega \\ \rho_{\alpha\Omega} \sigma_\alpha \sigma_\Omega & \sigma_\Omega^2 \end{pmatrix}$$

Data sets

■ For thrust:

ALEPH-2004: 133. GeV (7)	L3-2004: 172.3 GeV (12)
ALEPH-2004: 161. GeV (7)	L3-2004: 182.8 GeV (12)
ALEPH-2004: 172. GeV (7)	L3-2004: 188.6 GeV (12)
ALEPH-2004: 183. GeV (7)	L3-2004: 194.4 GeV (12)
ALEPH-2004: 189. GeV (7)	L3-2004: 200. GeV (11)
ALEPH-2004: 200. GeV (6)	L3-2004: 206.2 GeV (12)
ALEPH-2004: 206. GeV (8)	L3-2004: 41.4 GeV (5)
ALEPH-2004: 91.2 GeV (26)	L3-2004: 55.3 GeV (6)
AMY-1990: 55.2 GeV (5)	L3-2004: 65.4 GeV (7)
DELPHI-1999: 133. GeV (7)	L3-2004: 75.7 GeV (7)
DELPHI-1999: 161. GeV (7)	L3-2004: 82.3 GeV (8)
DELPHI-1999: 172. GeV (7)	L3-2004: 85.1 GeV (8)
DELPHI-1999: 89.5 GeV (11)	L3-2004: 91.2 GeV (10)
DELPHI-1999: 93. GeV (12)	OPAL-1997: 161. GeV (7)
DELPHI-2000: 91.2 GeV (12)	OPAL-2000: 172. GeV (8)
DELPHI-2003: 183. GeV (14)	OPAL-2000: 183. GeV (8)
DELPHI-2003: 189. GeV (15)	OPAL-2000: 189. GeV (8)
DELPHI-2003: 192. GeV (15)	OPAL-2005: 133. GeV (6)
DELPHI-2003: 196. GeV (14)	OPAL-2005: 177. GeV (8)
DELPHI-2003: 200. GeV (15)	OPAL-2005: 197. GeV (8)
DELPHI-2003: 202. GeV (15)	OPAL-2005: 91. GeV (5)
DELPHI-2003: 205. GeV (15)	SLD-1995: 91.2 GeV (6)
DELPHI-2003: 207. GeV (15)	TASSO-1998: 35. GeV (4)
DELPHI-2003: 45. GeV (5)	TASSO-1998: 44. GeV (5)
DELPHI-2003: 66. GeV (8)	
DELPHI-2003: 76. GeV (9)	
JADE-1998: 35. GeV (5)	
JADE-1998: 44. GeV (7)	
L3-2004: 130.1 GeV (11)	
L3-2004: 136.1 GeV (10)	
L3-2004: 161.3 GeV (12)	

----- Summary -----

Total: 516
Q > 95 : 345
Q < 88 : 89
Q ~ MZ : 82

■ For angularities:

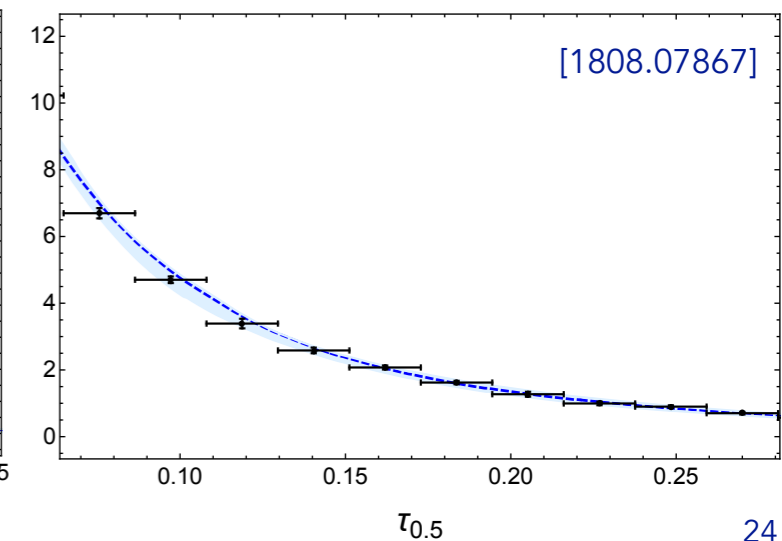
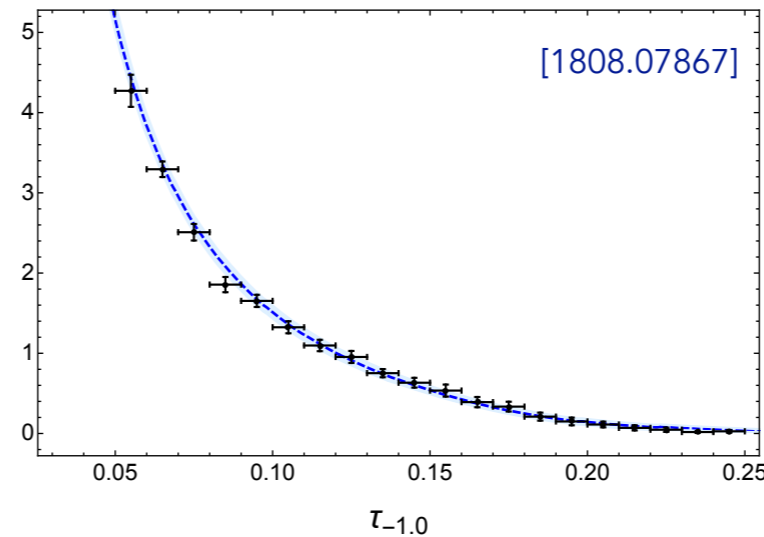
Generalized event shape and energy flow studies in
 e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0$ GeV

L3 Collaboration

JHEP 10 (2011) 143

Also see thesis by Pratima Jindal,
 Panjab University, Chandigarh

- Data for $a = \{-1.0, -0.75, -0.5, -0.25, 0.0, 0.25, 0.5, 0.75\}$ at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = $25 \times 7 = 175$ bins @ $Q = 91.2$ GeV
- e.g. $a = -1$ and 0.5 , $Q = 91.2$ GeV, compared to our NNLL' prediction:



Angularities: from τ to b

- Consider *Angularities*, which can be defined in terms of the of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

IR safe for $a \in \{-\infty, 2\}$!

$$\tau_a = \frac{1}{Q} \sum_i |\mathbf{p}_{\perp}^i| e^{-|\eta_i|(1-a)}$$

$a = 0 \leftrightarrow$ 'Thrust'

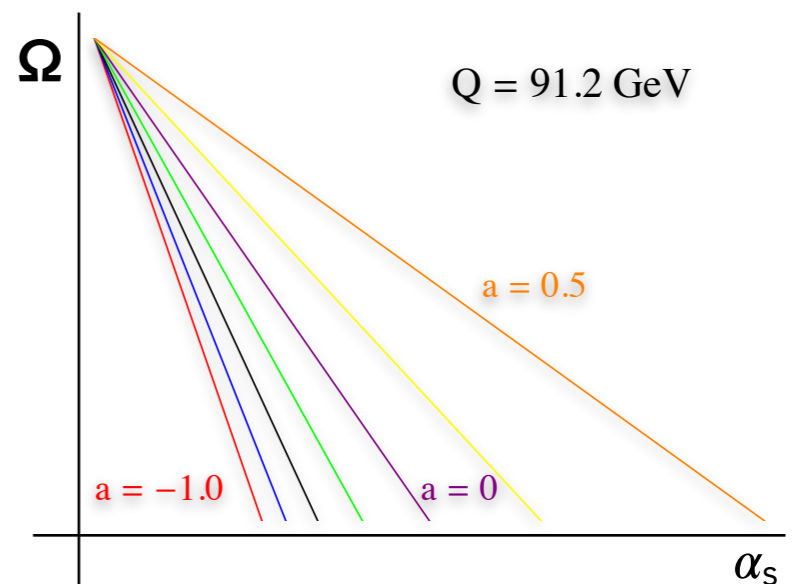
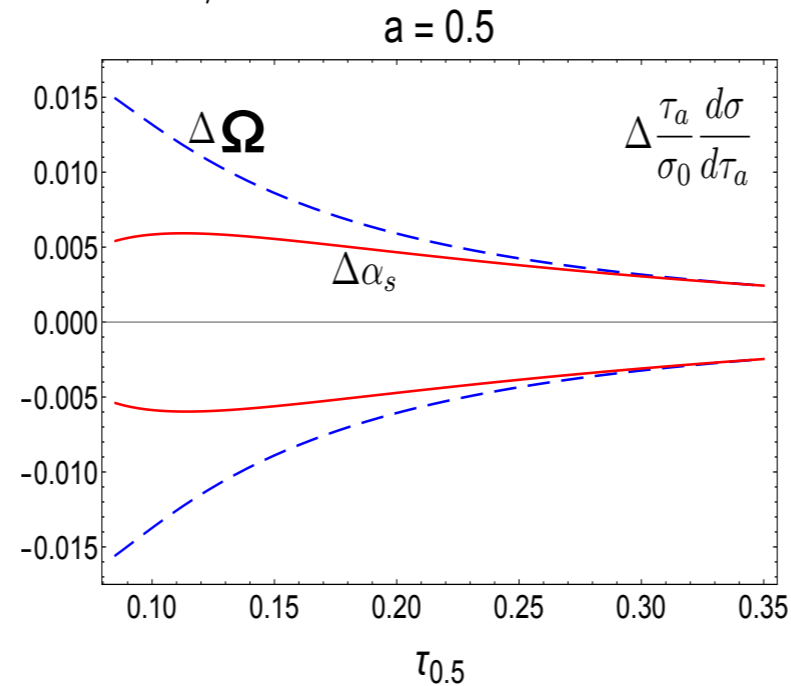
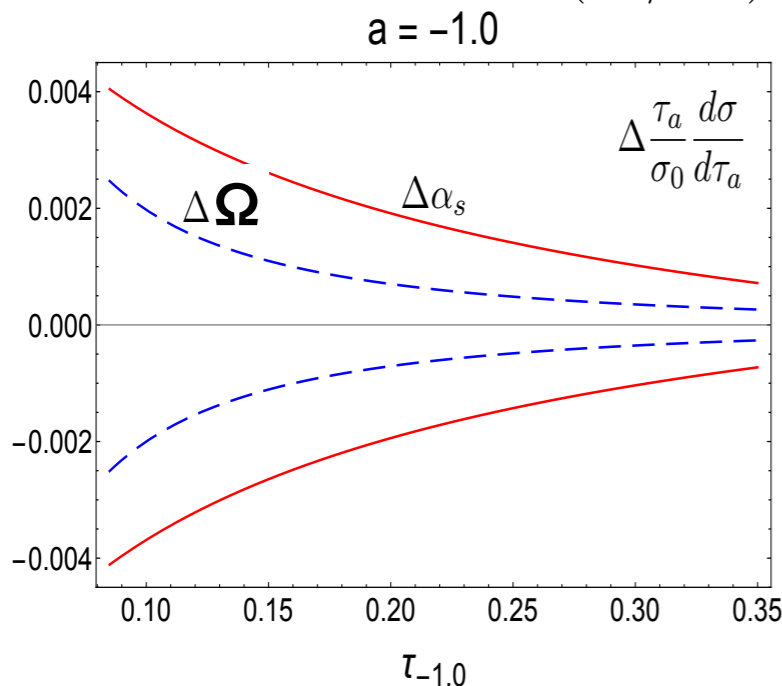
$a = 1 \leftrightarrow$ 'Jet Broadening'

- Leading NP effect is also an (a -dependent (!)) shift of the perturbative distribution:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow{\text{NP}} \frac{d\sigma}{d\tau_a} \left(\tau_a - c_{\tau_a} \frac{\Omega_1}{Q} \right) \quad c_{\tau_a} = \frac{2}{1-a}$$

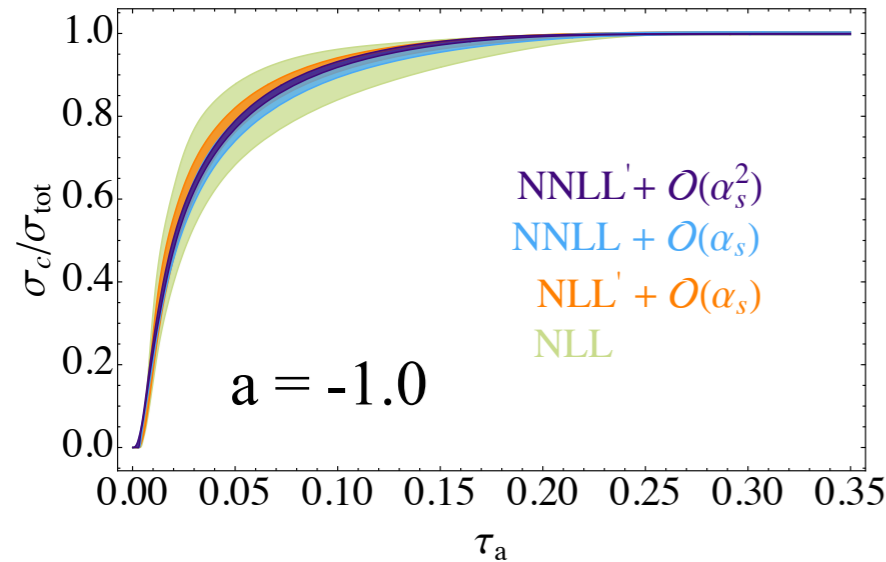
$(d\sigma/d\tau_a)_{\text{central}} - d\sigma/d\tau_a$

$\Delta \text{slope}(\bullet)$



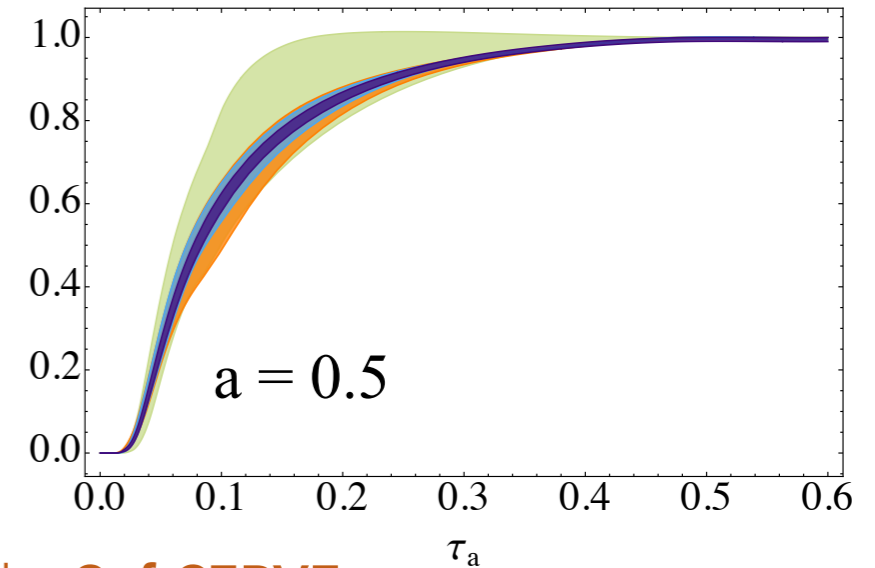
- Varying Ω between 35 and 207 GeV generates same difference as varying $a \in \{-2.0, 0.5\}$ (~ 6)!! 25

2018 progress: NLL' to NNLL'



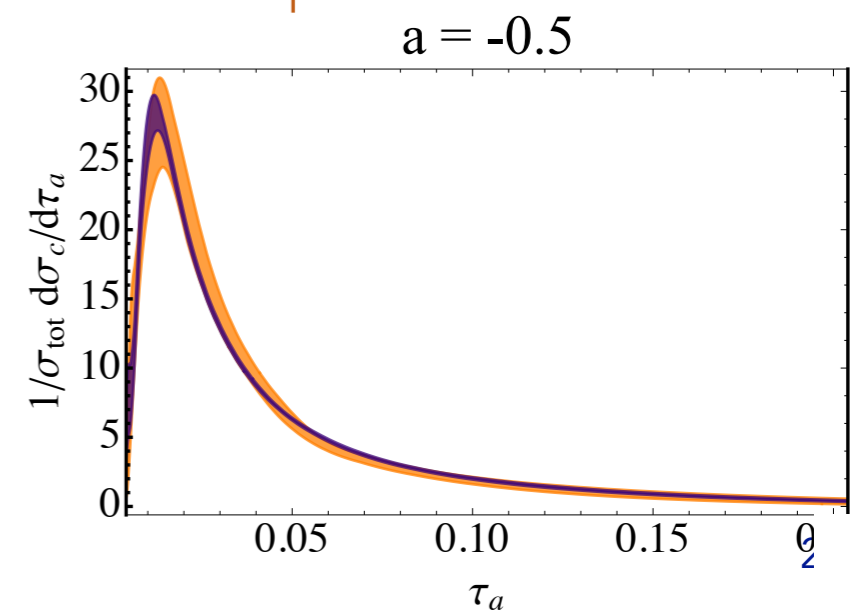
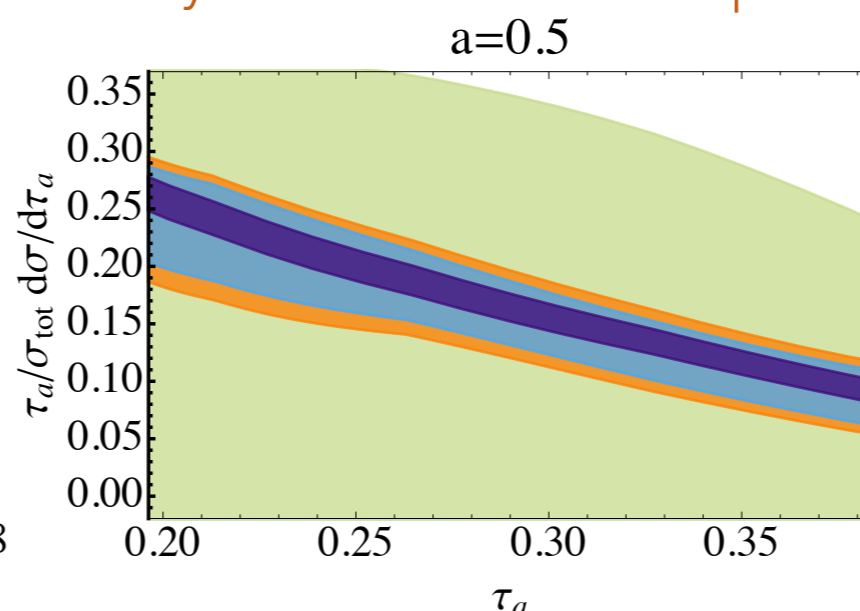
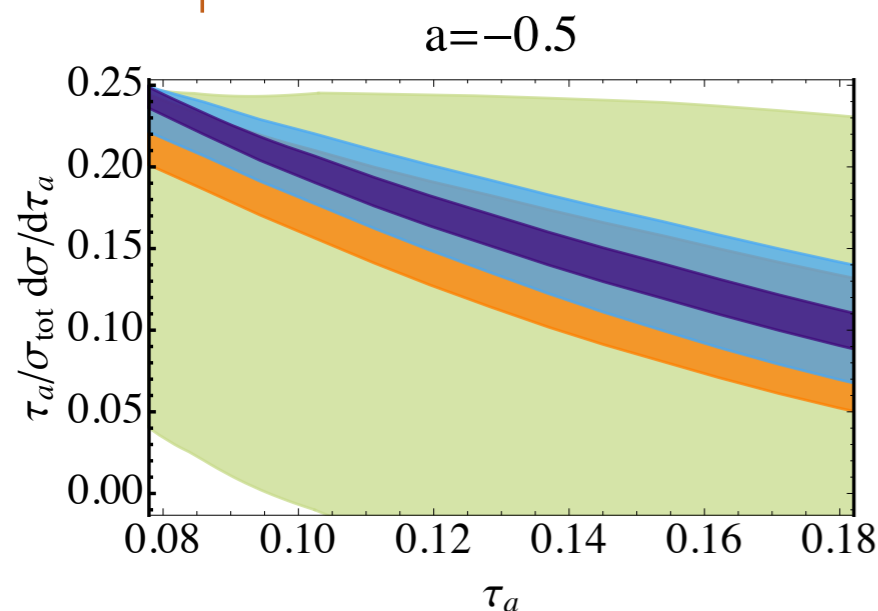
softserve.hepforge.org

Bell, Rahn & Talbert



- Two-loop soft anomalous dimensions and singular constants provided by **SoftSERVE**
- Two-loop jet anomalous dimension obtained from consistency relations
- Two-loop singular jet constants extracted from **EVENT2**
- Matching to QCD at $O(\alpha_s^2)$ extracted from **EVENT2** *
- Includes set of H,J,S, & non-sing. profile scales, tuned for a-dependence, and varied with a random scan over parameters
- Non-perturbative effects accounted for by convolution with RGap—subtracted shape function

Bell, Hornig, Lee & Talbert



The (only) dataset

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0$ GeV

L3 Collaboration

JHEP 10 (2011) 143

RECEIVED: *May 12, 2009*

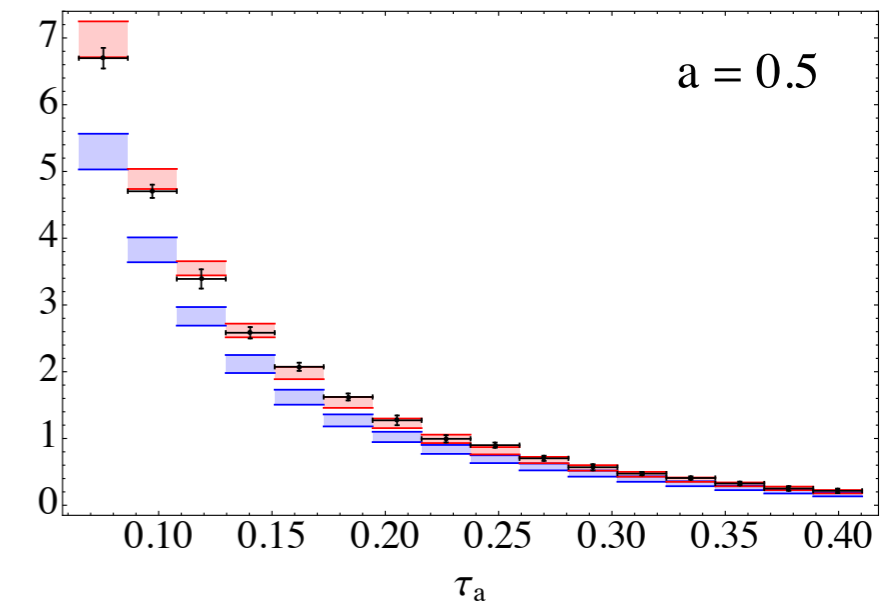
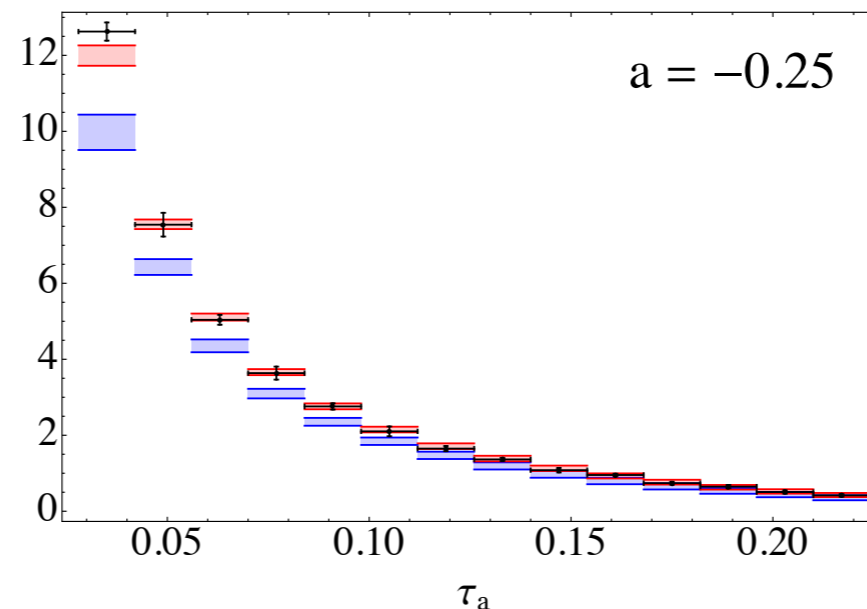
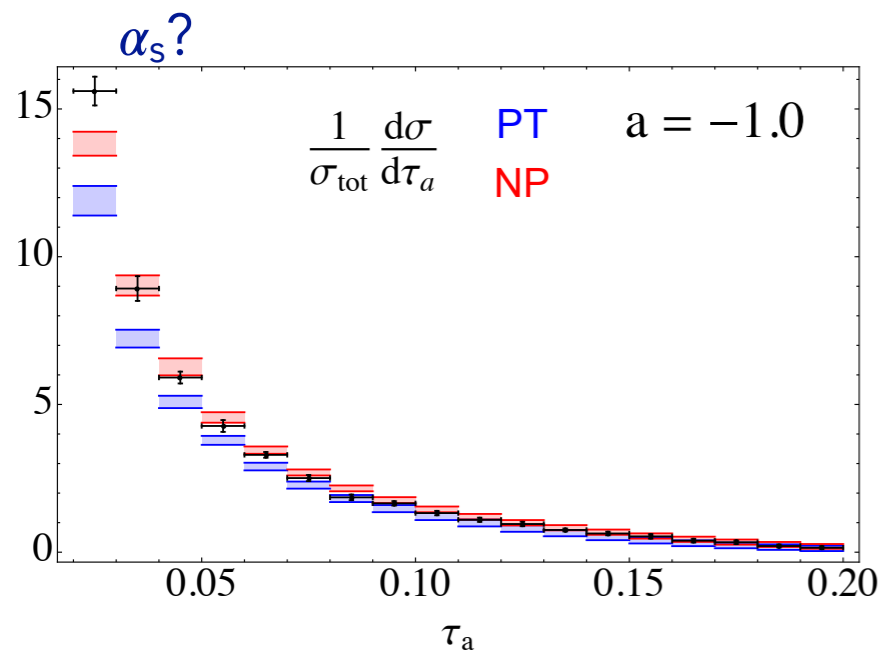
REVISED: *May 3, 2011*

ACCEPTED: *August 24, 2011*

PUBLISHED: *October 31, 2011*

Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for $a = \{-1.0, -0.75, -0.5, -0.25, 0.0, 0.25, 0.5, 0.75\}$ at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ $Q = 91.2$ GeV
- Compare to 404 bins **included** in 2015 C-Parameter fit (across all Q considered)...
- Early theory predictions look good against the data, but what does this translate to for Ω and α_s ?



BLUE: NNLL' + $O(\alpha_s^2)$

RED: NNLL' + $O(\alpha_s^2)$ + NP