Dark gravitational sectors on a generalized scalar-tensor vector bundle model

A model for Dark Matter emerging from a modified geometry

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Modified Gravity

Why a modified Gravity?

- Improves Renormalizability
- Offers explanation for the early and late time accelerated phases of the expansion of the universe

Theories of Modified Gravity

- Extensions of Einstein-Hilbert action
- Insertion of Extra Scalar Fields (Scalar-Tensor theories)
- Alternative Geometries beyond Riemannian framework of GR

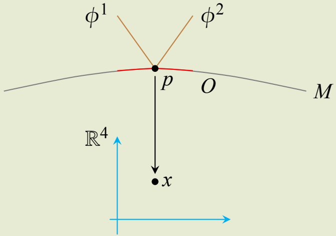
Finsler-like Geometries

- generalised metric structures in a Vector Bundle
- Scalar Fields as fibres (internal variables)

The model

Vector Bundle F^6

- A pseudo-Riemannian 4-dimensional spacetime manifold M of Lorentzian metric, with two scalar fibers: $\phi^1,\ \phi^2$



- local trivialization: $O \times \phi^1 \times \phi^2,\ \{x^\mu, \phi^a\}$
- coordinate transformation: $x^\mu \longrightarrow x'^\mu(x),\quad \phi^a(x) \longrightarrow \phi'^a(x') = \delta^a_b \phi^b(x)$
- The structure group is the trivial group

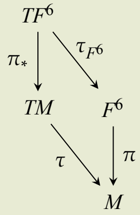
Spyros Konitopoulos, Emmanuel N., Saridakis, P.C. Stavrinos, A. Triantafyllopoulos, *Phys. Rev. D 104, 064018 (2021)*

The model

Vector Bundle F^6

- Vector Bundle: (F^6, π, M)
- Tangent Bundle of F^6: (TF^6, τ, F^6)
- Vertical Space: $\ker \pi_* = (VF^6, \tau_V, F^6) \subset TF^6$
- Connection C: $TF^6 \longrightarrow VF^6$
- Horizontal Space: $\ker C = (HF^6, \tau_H, F^6)$
- Whitney Sum: $TF^6 = HF^6 \oplus VF^6$

The Connection C is, in general, non-linear.



Representation

- $$p \in M\,, \quad V_p = (x^\mu, V^\mu) \in T_pM\,, \quad V = V^\mu \frac{\partial}{\partial x^\mu} \tag{1}$$

- $$u \in F^6\,, \quad X_u = (x^\mu, \phi^a, X^\mu, A^a) \in T_uF^6\,, \quad X = X^\mu \frac{\partial}{\partial x^\mu} + A^a \frac{\partial}{\partial \phi^a} \tag{2}$$

- $$u \in F^6\,, \quad Y_u = (x^\mu, \phi^a, 0, A^a) \in V_uF^6\,, \quad Y = Y^a \frac{\partial}{\partial \phi^a} \tag{3}$$

The model

The Connection C

- Projects onto the vertical space (Not necessarily linear),

$$C: \; TF^6 \longrightarrow VF^6, \;\; (x^\mu, \phi^a, X^\mu, A^a) \stackrel{C}{\longrightarrow} (x^\mu, \phi^a, 0, C^a(x, y, X, A))$$

- Representation

$$C^a = A^a + N^a_\mu(x, \phi) X^\mu$$

Adapted Basis on TF^6

$$X_M = \left(\frac{\delta}{\delta x^\mu}, \frac{\partial}{\partial \phi^a} \right), \quad \frac{\delta}{\delta x^\mu} = \frac{\partial}{\partial x^\mu} - N^a_\mu(x, \phi) \frac{\partial}{\partial \phi^a}$$

- Transforms like a vector,

$$\frac{\delta}{\delta x'^\mu} = \frac{\partial x^\nu}{\partial x'^\mu} \frac{\delta}{\delta x^\nu} \tag{4}$$

- Is non-commutative

$$[X_M, X_N] = \mathcal{W}^L{}_{MN} X_L$$

- Jacobi Identity

$$\circlearrowleft_{M,N,L} X_M \mathcal{W}^R{}_{NL} + \mathcal{W}^R{}_{MS} \mathcal{W}^S{}_{NL} = 0$$

The model

Space Structure

- Covariant Derivatiive (Parallel Transport)

$$\nabla_N X_M = \mathbf{\Gamma}^L_{MN} X_L$$

- Metric

$$\boldsymbol{G} = g_{\mu\nu}(x)\,\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + v_{ab}(x)\,\delta\phi^a \otimes \delta\phi^b, \quad v_{ab}(x) = \delta_{ab}\phi(x)$$

- Metricity Condition

$$\nabla_M \boldsymbol{G} = 0 \Rightarrow \mathbf{\Gamma}^L{}_{MN} = \frac{1}{2} G^{RL} \left(X_M G_{NR} + X_N G_{RM} - X_R G_{MN} \right)$$

Connection Structure

- Connection

$$\mathbf{\Gamma}^L{}_{MN} = \{ L^\lambda{}_{\mu\nu}, L^c{}_{a\nu}, C^c{}_{\mu b}, C^\lambda{}_{ab} \}$$

- Curvature

$$\mathcal{R}^K{}_{LMN} = X_M \mathbf{\Gamma}^K{}_{LN} - X_N \mathbf{\Gamma}^K{}_{LM} + \mathbf{\Gamma}^R{}_{LN} \mathbf{\Gamma}^K{}_{RM} - \mathbf{\Gamma}^R{}_{LM} \mathbf{\Gamma}^K{}_{RN} - \mathcal{W}^R{}_{MN} \mathbf{\Gamma}^K{}_{LR}$$

- Torsion

$$\mathcal{T}^L{}_{MN} = \mathcal{W}^L{}_{MN}$$

- Ricci Tensor

$$\mathcal{R}_{MN} = \mathcal{R}^L{}_{MLN} = X_L \mathbf{\Gamma}^L{}_{MN} - X_N \mathbf{\Gamma}^L{}_{ML} + \mathbf{\Gamma}^L{}_{MN} \mathbf{\Gamma}^R{}_{LR} - \mathbf{\Gamma}^L{}_{MR} \mathbf{\Gamma}^R{}_{LN} + \mathbf{\Gamma}^L{}_{MR} \mathcal{W}^R{}_{NL}$$

Einstein's Equations

Einstein's tensor

$$\mathcal{E}_{MN} = \mathcal{R}_{MN} - \frac{1}{2}\mathcal{R}\mathcal{G}_{MN}$$

Generalised E-H action

$$S = \int_Q d^6 U \sqrt{|\mathcal{G}|}\mathcal{R}$$

Generalised Einstein's Equation

$$\delta S = 0 \Rightarrow \mathcal{E}_{(MN)} + \left(\delta^L{}_{(M}\delta^R{}_{N)} - \mathcal{G}^{LR}\mathcal{G}_{MN}\right)\left(D_L \mathcal{W}^A{}_{RA} - \mathcal{W}^B{}_{LB}\mathcal{W}^C{}_{RC}\right) = \kappa \mathcal{T}_{MN}$$

Sectors of Connections

$$L^{\mu}{}_{\nu\lambda}(x) = \Gamma^{\mu}{}_{\nu\lambda}(x) \ , \quad C^{a}{}_{\mu b} = L^{a}{}_{b\mu} = \delta^{a}_{b}\frac{1}{2\phi}\partial_{\mu}\phi \ , \quad C^{\mu}{}_{ab} = -\frac{1}{2}\delta_{ab}g^{\mu\nu}\partial_{\nu}\phi$$

Sectors of Einstein's tensor

- $$\mathcal{E}_{\mu\nu} = E_{\mu\nu} + E^{(\phi)}{}_{\mu\nu}$$
- $$E^{(\phi)}{}_{\mu\nu} = \frac{1}{\phi}g_{\mu\nu}\left(\Box\phi + \frac{1}{4\phi}\delta^{ab}\partial_{\lambda}\phi C^{\lambda}{}_{ab} + \frac{1}{2}\delta^{ab}C^{\lambda}{}_{ab}W^{c}{}_{\lambda c}\right) - \frac{1}{\phi}D_{\mu}D_{\nu}\phi + \frac{1}{2\phi^2}\partial_{\mu}\phi\partial_{\nu}\phi + \frac{1}{2\phi}\partial_{\mu}\phi W^{a}{}_{\nu a}$$

Sectors of Einstein's Equations

$$\mathcal{E}_{(\mu\nu)} + \left(\delta^{\lambda}_{(\mu}\delta^{\rho}_{\nu)} - g^{\lambda\rho}g_{\mu\nu}\right)\left(D_{\lambda}W^{c}{}_{\rho c} - W^{d}{}_{\lambda d}W^{c}{}_{\rho c}\right) + v^{ab}g_{\mu\nu}C^{\lambda}{}_{ab}W^{c}{}_{\lambda c} = \kappa\mathcal{T}_{\mu\nu}$$

$$\mathcal{E}_{(ab)} - g^{\lambda\rho}v_{ab}\left(D_{\lambda}W^{c}{}_{\rho c} - W^{d}{}_{\lambda d}W^{c}{}_{\rho c}\right) + C^{\lambda}{}_{ab}W^{c}{}_{\lambda c} = \kappa\mathcal{T}_{ab}$$

Effective Dark Matter

The geometrical properties of our model can be viewed as additional terms to the energy momentum tensor and therefore, in the GR framework, interpreted as effective dark matter. Just take extra geometry to the right side.

$$E_{\mu\nu} = \kappa \tilde{\mathcal{T}}_{\mu\nu} = \kappa \left(\mathcal{T}_{\mu\nu} + \mathcal{T}^{(\phi)}{}_{\mu\nu} \right)$$

$$\mathcal{T}^{(\phi)}{}_{\mu\nu} = -\frac{1}{\kappa} \left[E^{\phi}{}_{(\mu\nu)} + \left(\delta^{\kappa}_{(\mu} \delta^{\lambda}_{\nu)} - g^{\kappa\lambda} g_{\mu\nu} \right) \left(D_{\kappa} W^{a}{}_{\lambda a} - W^{b}{}_{\kappa b} W^{c}{}_{\lambda c} \right) + v^{ab} g_{\mu\nu} C^{\lambda}{}_{ab} W^{c}{}_{\lambda c} \right]$$

- Additional degrees of freedom
- anholonomicity of the adapted basis
- FRW metric on F^6:

$$\mathbf{G} = -\mathrm{d}t \otimes \mathrm{d}t + a^2(t)\,(\mathrm{d}x \otimes \mathrm{d}x + \mathrm{d}y \otimes \mathrm{d}y + \mathrm{d}z \otimes \mathrm{d}z) + \phi(t)\left(\delta\phi^{(1)} \otimes \delta\phi^{(1)} + \delta\phi^{(2)} \otimes \delta\phi^{(2)}\right)$$

- matter sector (perfect fluid):

$$\mathcal{T}_{\mu\nu} = (\rho_m + P_m)Y_\mu Y_\nu + P_m g_{\mu\nu}$$

- Friedman's Equations

$$3H^2 + 3H\left(\frac{\dot\phi}{\phi} - W_+\right) - W_+\frac{\dot\phi}{\phi} + \frac{\dot\phi^2}{4\phi^2} = 8\pi G\rho_m$$

$$2\dot H + (W_+)^2 - \dot W_+ - \frac{\dot\phi^2}{2\phi^2} + H\left(W_+ - \frac{\dot\phi}{\phi}\right) - \frac{1}{2\phi}\left(W_+\dot\phi - 2\ddot\phi\right) = -8\pi G(\rho_m + P_m)$$

$$\frac{1}{\phi}\left(\ddot\phi + 3H\dot\phi\right) - \frac{\dot\phi}{2\phi}\left(3W_+ + \frac{\dot\phi}{\phi}\right) + 6\left(\dot H + 2H^2\right) - 6HW_+ + 2(W_+)^2 - 2\dot W_+ = -8\pi G\overset{V}{\mathcal{T}}$$

where, $W_+ = W^a{}_{0a}$

Cosmological Implications

Friedman's Equations

- $$3H^2 = 8\pi G\left(\rho_m + \rho_{eff}\right) \tag{5}$$
$$2\dot{H} = -8\pi G\left(\rho_m + \rho_{eff} + P_m + P_{eff}\right) \tag{6}$$

- Effective Dark Energy

$$\rho_{eff} = \frac{1}{8\pi G}\left[\frac{\dot{\phi}}{\phi}W_+ - \frac{\dot{\phi}^2}{4\phi^2} - 3H\left(\frac{\dot{\phi}}{\phi} - W_+\right)\right] \tag{7}$$

$$P_{eff} = \frac{1}{8\pi G}\left[(W_+)^2 - \dot{W}_+ - 2HW_+ - \frac{\dot{\phi}^2}{4\phi^2} + \frac{1}{2\phi}\left(4H\dot{\phi} - 3W_+\dot{\phi} + 2\ddot{\phi}\right)\right] \tag{8}$$

Equation of State

Scenarios

$$w_{eff} = \frac{P_{eff}}{\rho_{eff}} = \begin{cases} > -1, & \text{Quintessence} \\ < -1, & \text{Phantom regime (Big Rip)} \\ = -1, & \text{Cosmological Constant} \end{cases}$$

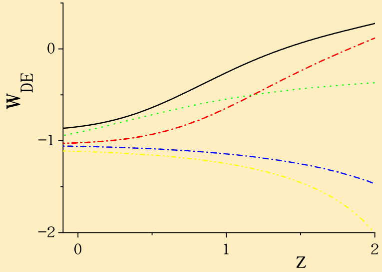


Figure: The evolution of the dark-energy equation-of-state parameter w_{DE} as a function of the redshift z, for various small corrections $\tilde{\mathcal{T}}$. We have imposed the initial conditions $\Omega_{DE}(z=0) \equiv \Omega_{DE0} \approx 0.69$.