NATIONAL SCIENCE CENTRE
POLAND

UNIVERSITY OF WARSAW
Faculty of Physics

Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future high energy lepton colliders

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In collaboration with: M.. Altakach, F. Maltoni, K. Mawatai, Kazuki Sakurai

Workshop on Standard Model and Beyond, Corfu, 1 Sept 2023

Outline of talk

- Introduction of Bell type inequality(already discuss in Prof. Jesus Moreno)
- Define quantum correlations: entanglement, steerability and Bell-nonlocality
- Density Operator for $H \to \tau^+ \tau^-$
- Test of quantum correlations between tau pair at lepton colliders
- Results
- CP-measurement
- Summery

Bohr vs EPR

"Niels Bohr: argued that reality or the state of a particle at the fundamental level was not only unknown but was unknowable until it was measured."

"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exist an element of physical reality corresponding to this physical quantity"

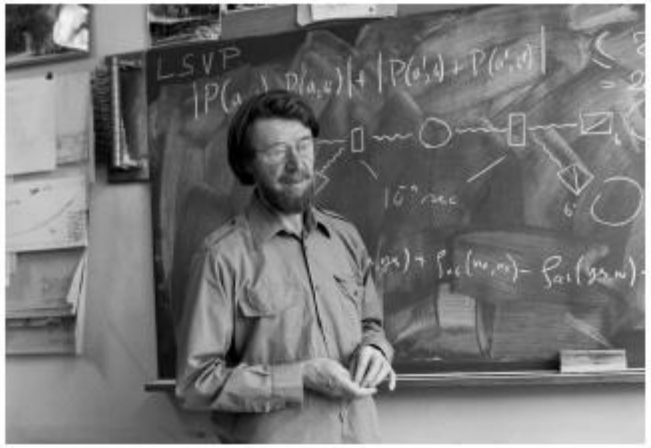
Einstien, Podolski and Rosen, 1935

QM violates both local and real requirements (i.e. entanglement violate Locality).
And QM already tested by Stern Gerlach Experiment.

As per EPR, the QM behavior could be explained by additional variables called Local Hidden variables (LHV). These would restore locality and causality to the theory (and they demonstrated it for the Stern Gerlach experimental observations).

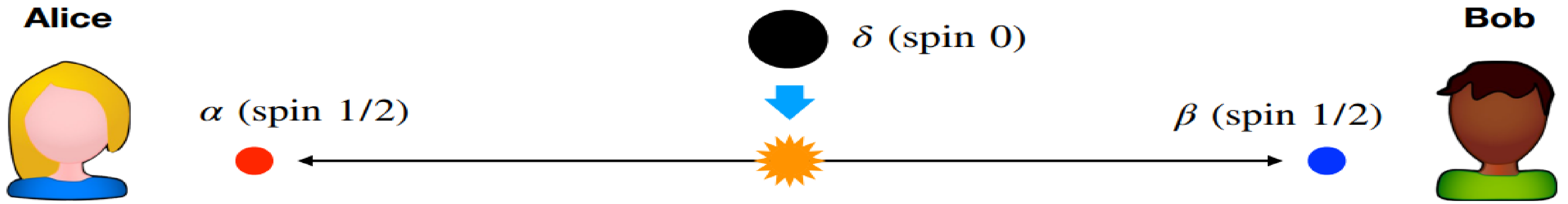
It seems difficult that time to experimentally discriminate QM and general hidden variable theories.

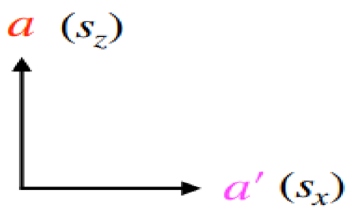
In 1964, John Bell, made a fundamental contribution, showing that no deterministic hidden variable theory can reproduce al the statistical predictions of quantum mechanics(1964) derived simple inequalities that can discriminate QM from any local-real hidden variable theories: Bell inequalities



CHSH *inequality*

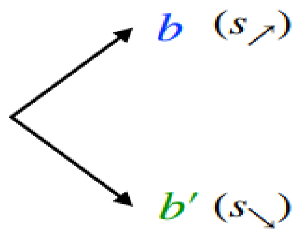
[Clauser, Horne, Shimony, Holt, 1969]





The experiment consists of 4 sessions:

1) Alice and Bob measure $s_a[\alpha]$ and $s_b[\beta]$, respectively. Repeat the measurement many times and calculate $\langle s_a \cdot s_b \rangle$.
2) Repeat (1) but for a and b'.
3) Repeat (1) but for a' and b.
4) Repeat (1) but for a' and b'.



Finally, we construct

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

CHSH *inequality in LHV theories*

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right| \leq 1$$

$$\begin{aligned}
|\langle ab\rangle - \langle ab'\rangle| &= \left|\int d\lambda\,(ab - ab')\,P\right| \\
&= \int d\lambda\,|ab(1 \pm a'b')P - ab'(1 \pm a'b)P| \\
&\leq \int d\lambda\Big(|ab||1 \pm a'b'|P + |ab'||1 \pm a'b|P\Big) \\
&= \int d\lambda\Big[(1 \pm a'b')P + (1 \pm a'b)P\Big] \\
&= 2 \pm (\langle a'b'\rangle + \langle a'b\rangle)
\end{aligned}$$

$\pm\, aba'b'P - (\pm\, aba'b'P) = 0$

$|ab| = |ab'| = 1$

$|1 \pm a'b'|, |1 \pm a'b| \geq 0$

$a = s_a$
$b = s_b$
$\vdots$

$$\tilde{R}_{\text{CHSH}} = \frac{1}{2}\left(|\langle ab\rangle - \langle ab'\rangle| + |\langle a'b\rangle + \langle a'b'\rangle|\right) \leq 1$$

$$\max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}(R_{\text{CHSH}}) = \max_{(\vec{a},\vec{b},\vec{a}',\vec{b}')}\left(\tilde{R}_{\text{CHSH}}\right)$$

$$\boxed{\langle ab\rangle = \int a(\lambda)b(\lambda)P(\lambda)d\lambda}$$

$$\boxed{\int P(\lambda)d\lambda = 1}$$

CHSH *inequality in QM*

- Lets consider an QM wavefunction of singlet state of two spin ½ particles

$$|\psi^{(0,0)}\rangle = \frac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

one can show

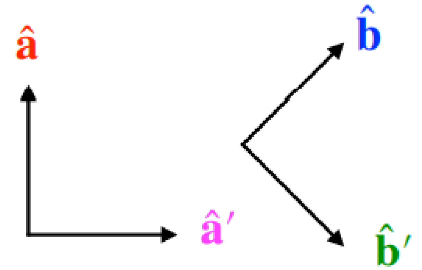
$$\langle s_a s_b \rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)} \rangle = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$$

therefore

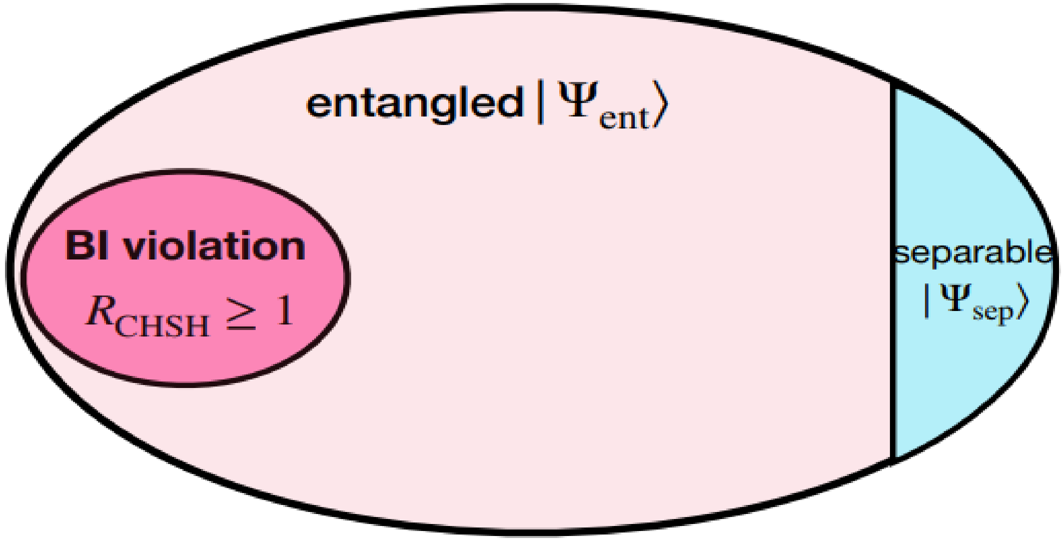
violates the upper bound of hidden variable theories!

$$R_{\mathrm{CHSH}} = \frac{1}{2} \left| \langle s_a s_b \rangle - \langle s_a s_{b'} \rangle + \langle s_{a'} s_b \rangle + \langle s_{a'} s_{b'} \rangle \right|$$

$$= \frac{1}{2} \left| \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}' \cdot \hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}} \right| = \sqrt{2}$$



$$R_{\mathrm{CHSH}} \le \begin{cases} 1 & \text{(HV theories)} \\ \sqrt{2} & \text{(QM)} \end{cases}$$



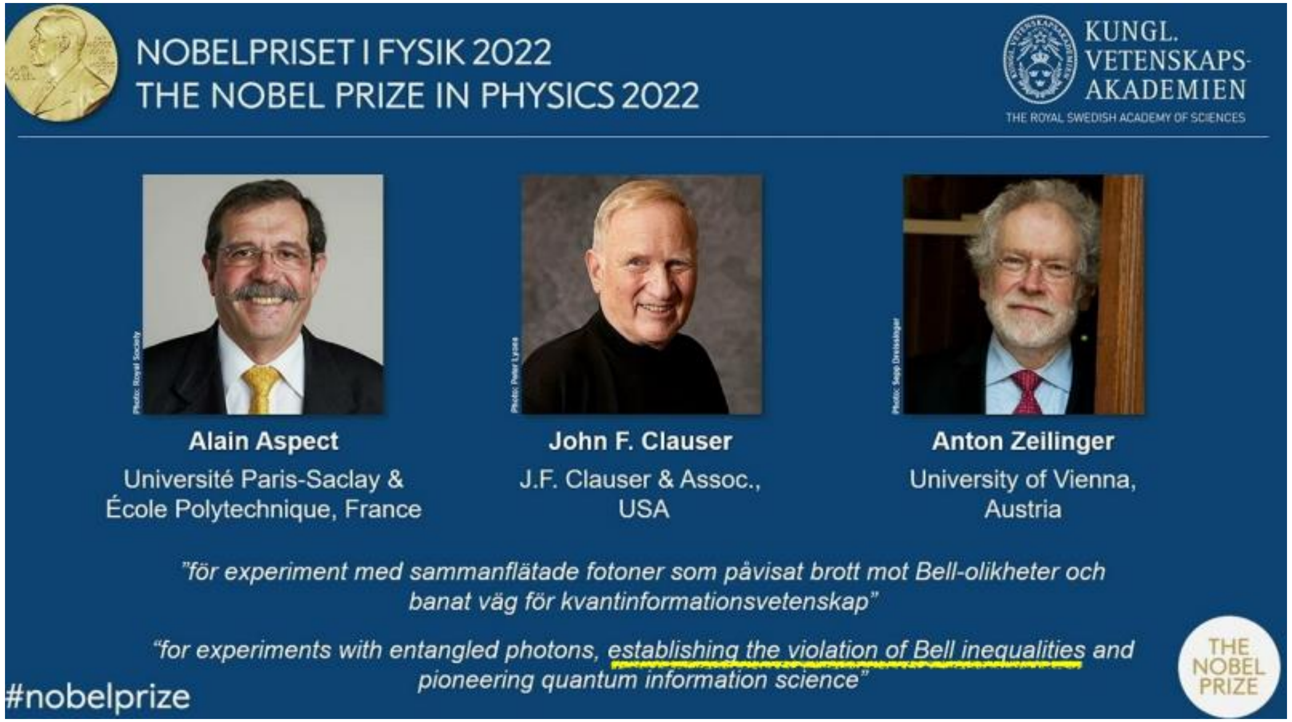
Direction of measurement is play important role.

We already has been observed Bell inequality violation in low energy experiments

- Entangled photon pairs (from decays of Calcium atoms)
  Crauser, Horne, Shimony, Holt (1969), Freedman and Clauser (1972), A. Aspect et. al. (1981, 1982), Y. H. Shih, C. O. Alley (1988), L. K. Shalm et al. (2015) [5σ]

- Entangled proton pairs (from decays of 2_{He})
  M. M. Lamehi-Rachti, W. Mitting (1972), H. Sakai (2006)

- $K^0\overline{K}^0, B^0\overline{B}^0$ flavour oscillation
  CPLEAR (1999), Belle (2004, 2007)



We are interested in Entanglement and Bell-type inequalities in $H \to \tau^+\tau^-$ at the lepton colliders

Vector boson pair
WW,ZZ,$\gamma\gamma$,WW*, ZZ*,…..

Correlation in fermion pair
Top pair, tau pair…….

J.J,Barr, P. Caban, J. Rembielinski, J. A. Aguilar-Saavedra,, A, Bernal, J.A. Casas, J.M. moreno, M, Fabbrichesi, R. Floreanini , E. Gabrielli, R. Aoude, E. Madge, F. Maltoni, L. Mantani, F. Fabbri, J. Howarth, T. Maurin at. all

Y. Afik, J. R. M de nova, M. Fabbrichesi, R. Floreanini, G. Panizzo, C. Severi, CD. E. Boschi, F. Malton, M Sioli, J. A. Aguilar-Saavedra, J. A. Casas,, R. Ashby-Pickering, A. Gabrielli, C. Severi, E. Vryonidou, R. Aoude, E. Madge, F. Maltoni, L. Mantani at all

We will discuss three different level of quantum corrections

❖ *Entanglement*

$$\rho \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^{\alpha} \bigotimes \rho_{\mathbf{k}}^{\beta} \longrightarrow \rho^{\mathbf{T}_\beta} \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^{\alpha} \bigotimes [\rho_{\mathbf{k}}^{\beta}]^{\mathbf{T}}$$

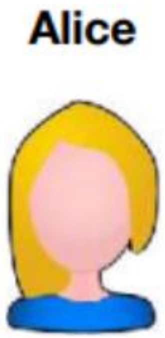
Peres-Horodecki
(1996, 1997)

Non-positive definite → State is entangled

If it is still Physics density matrix with Tr=1 and Positive definite → State is separable

We will discuss three different level of quantum corrections

- ❖ *Entanglement*
- ❖ *Steerability*(first time introduce by the Schrodinger): Alice's ability to affect Bob's state through her choice of measurement basis



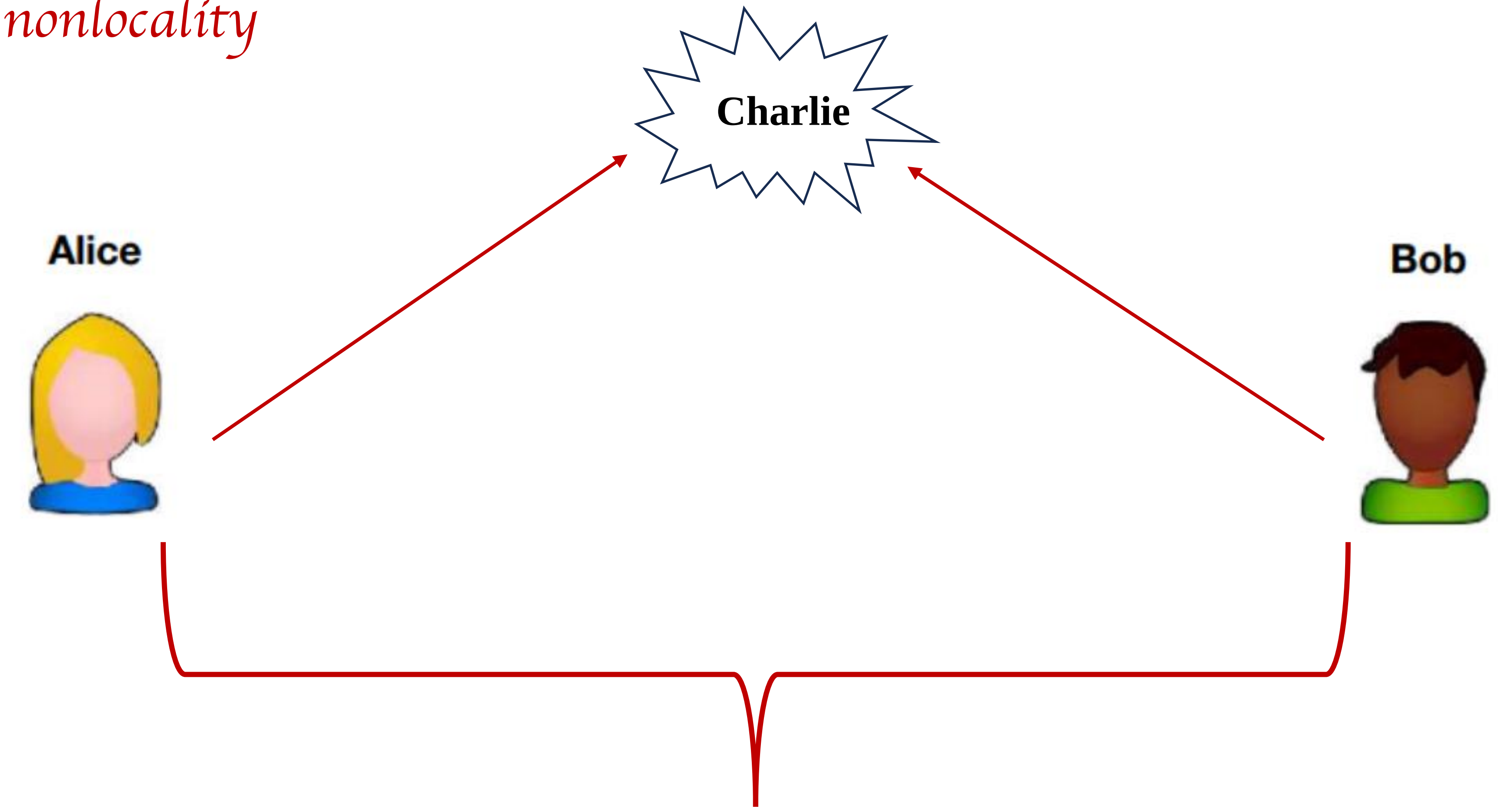
Send one part to Bob

Preparing bipartite quantum state

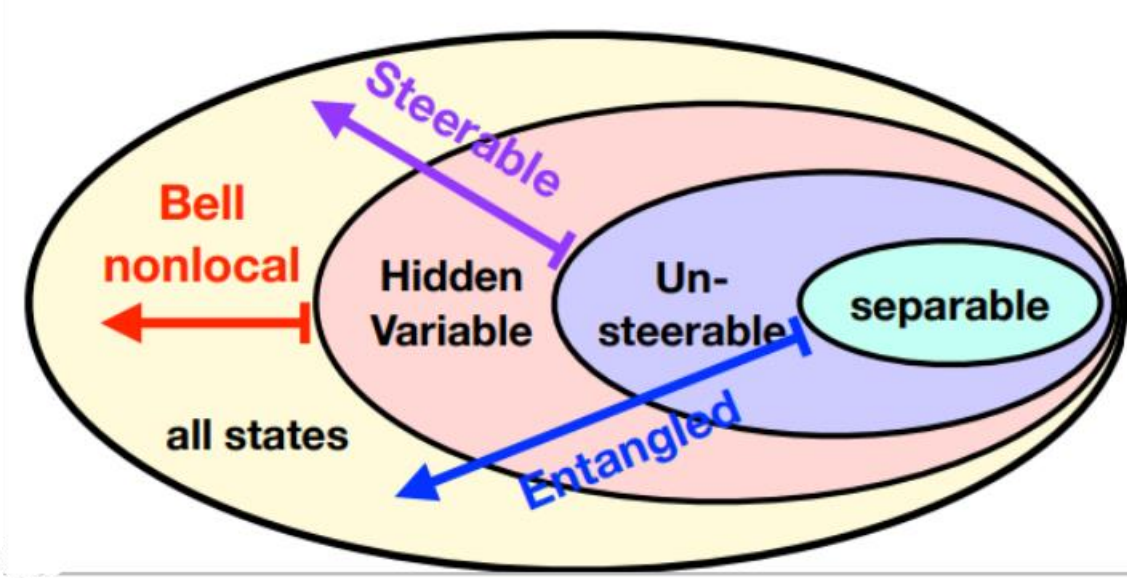
After measurement, Commutating to Alice classically.

We will discuss three different level of quantum corrections

- *Entanglement*
- *Steerability*
- *Bell- nonlocality*



Prepare a shared bipartite state, other then this
any communication is forbidden



To study these quantum correlation, first we have to compute spin correlation matrix

Density Operator

probability of having $|\Psi_1\rangle$

- For a statistical ensemble $\{\{p_1 : |\Psi_1\rangle\}, \{p_2 : |\Psi_2\rangle\}, \{p_3 : |\Psi_3\rangle\}, \cdots\}$, we define the **density operator/matrix**

$$\hat{\rho} \equiv \sum_k p_k |\Psi_k\rangle\langle\Psi_k| \qquad \rho_{ab} \equiv \langle e_a|\hat{\rho}|e_b\rangle$$

$$0 \le p_k \le 1$$

$$\sum_k p_k = 1$$

$$\langle e_a|e_b\rangle = \delta_{ab}$$

- Probability and expectation values:

$$\hat{A}\,|\,a\rangle = a\,|\,a\rangle$$

$P(a\,|\,\hat{A},\hat{\rho}) = \langle a\,|\,\rho\,|\,a\rangle$ Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$

$\langle\hat{A}\rangle_\rho = \mathrm{Tr}\left[\hat{A}\hat{\rho}\right]$ Expectation value for $\hat{A}$ on the state $\hat{\rho}$

Spin ½ biparticle system

- The spin system of α and β particles has 4 independent bases:

$$(\,|e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\,) \;=\; (\,|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\,)$$

- ==> ρ_{ab} is a 4 x 4 matrix (hermitian, Tr=1). It can be expanded as

$$\rho = \frac{1}{4}\left(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1} + \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\right)$$

3x3 matrix ↓

$$B_i, \overline{B}_i, C_{ij} \in \mathbb{R}$$

- For the spin operators $\hat{s}^\alpha$ and $\hat{s}^\beta$,

spin-spin correlation

$$\langle \hat{s}_i^\alpha \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{\rho}\right] = B_i \qquad \langle \hat{s}_i^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\beta \hat{\rho}\right] = \overline{B}_i \qquad \boxed{\langle \hat{s}_i^\alpha \hat{s}_j^\beta \rangle = \mathrm{Tr}\left[\hat{s}_i^\alpha \hat{s}_j^\beta \hat{\rho}\right] = C_{ij}}$$

Bell-nonlocality:

$$\boxed{R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\boxed{\langle s_a s_b \rangle} - \boxed{\langle s_a s_{b'} \rangle} + \boxed{\langle s_{a'} s_b \rangle} + \boxed{\langle s_{a'} s_{b'} \rangle}\right| > 1}$$

[Clauser, Horne, Shimony, Holt, 1969]

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[Clauser, Horne, Shimony, Holt, 1969]

Steerability: (assuming $B_i = \overline{B}_i = 0$)

$$\boxed{\mathcal{S}[\rho] > 1}$$

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi}\int d\Omega_{\mathbf{n}}\sqrt{\mathbf{n}^T \boxed{C^T C}\,\mathbf{n}}$$

[Jevtic, Hall, Anderson, Zwierz, Wiseman 2015]

Bell-nonlocality:

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[Jevtic, Hall, Anderson, Zwierz, Wiseman 2015]

Entanglement:

$$\boxed{\mathrm{E} > 1}$$

$$\mathrm{E} \equiv \boldsymbol{max_i\{|Tr(C) - C_{ii}| - C_{ii}\}}$$

$H \to \tau^+\tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H \bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\,\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$H \to \tau^+\tau^-$

$$\mathcal{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H \bar{\psi}_\tau(\cos\delta + i\gamma_5 \sin\delta)\psi_\tau$$

SM: $(\kappa, \delta) = (1,0)$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}}|\mathcal{M}^{m\bar{m}}|^2}$$

$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5 \sin\delta)v^{\bar{m}}(\bar{p})$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$H \to \tau^+\tau^-$

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$$\rho = \frac{1}{4}\left(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1} + \overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\right)$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$H \to \tau^+\tau^-$

$$\mathcal{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}}\kappa H\,\bar{\psi}_\tau(\cos\delta + i\gamma_5\sin\delta)\,\psi_\tau$$

SM: $(\kappa, \delta) = (1{,}0)$

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$$\mathcal{M}^{m\bar{m}} = c\,\bar{u}^m(p)(\cos\delta + i\gamma_5\sin\delta)v^{\bar{m}}(\bar{p})$$

$$\rho_{mn,\bar{m}\bar{n}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \frac{1}{4}\left(\mathbf{1}_4 + B_i \cdot \sigma_i \otimes \mathbf{1}\right.$$
$$\left.\overline{B}_i \cdot \mathbf{1} \otimes \sigma_i + C_{ij} \cdot \sigma_i \otimes \sigma_j\right)$$

$$B_i = \overline{B}_i = 0$$

$$C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

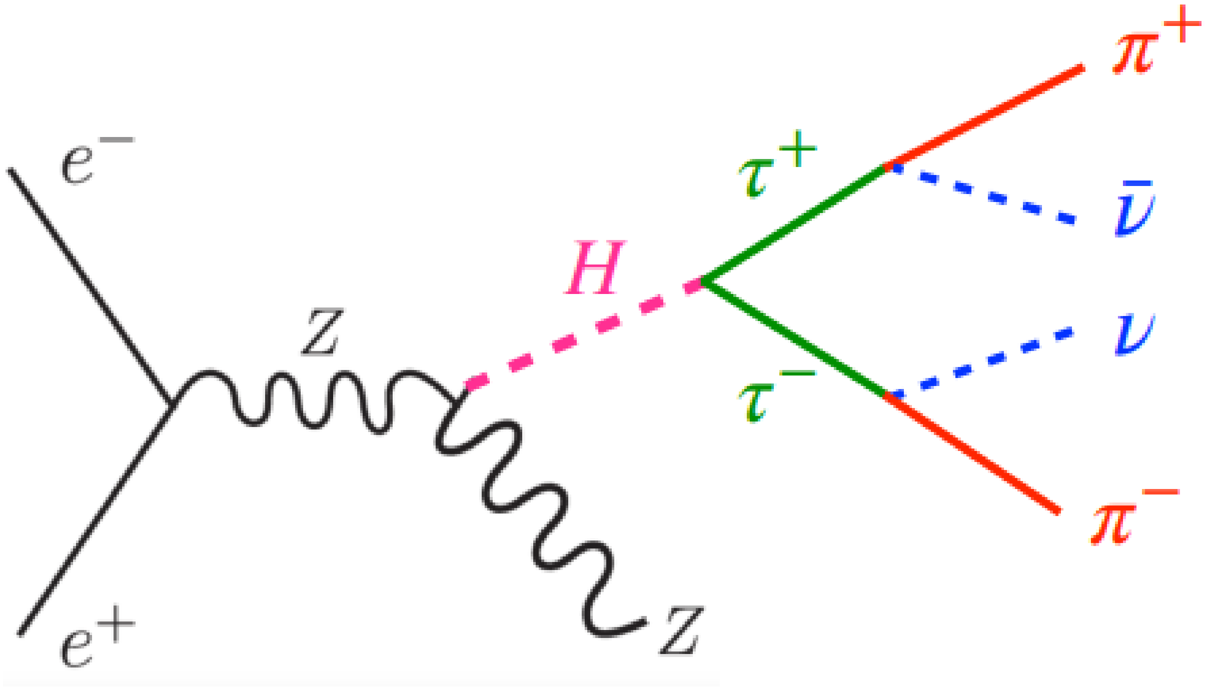
$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$H \to \tau^+\tau^-$

SM values: $C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$

$E = 3$	**Entanglement** $\Longrightarrow E > 1$
$\mathscr{S}[\rho] = 2$	**Steerablity** $\Longrightarrow \mathscr{S}[\rho] > 1$
$R_{\mathrm{CHSH}} = \sqrt{2} \simeq 1.414$	**Bell-nonlocal** $\Longrightarrow R_{\mathrm{CHSH}} > 1$

What we observe at colliders?



Spin correlation of $\tau^-\tau^+$ in term of angular correlation b/w $\tau^-\tau^+$ decay product

- The conditional probability that the decay product, d, takes the direction u (at the rest frame of τ^-) , when the tau spin is polarised into s direction, is given by

$$P(u|s)=1+\boxed{\alpha_{f,d}}\,u.s$$

spin analysing power which is maximum (1 or −1) for $\tau^\pm \to \vartheta\,\pi^\pm$

- Using, join probability P(s| $\bar{s}$) that τ^- and τ^+ are polarized into s and $\bar{s}$, we can write both tau spin correlation and pion momentum correlation as

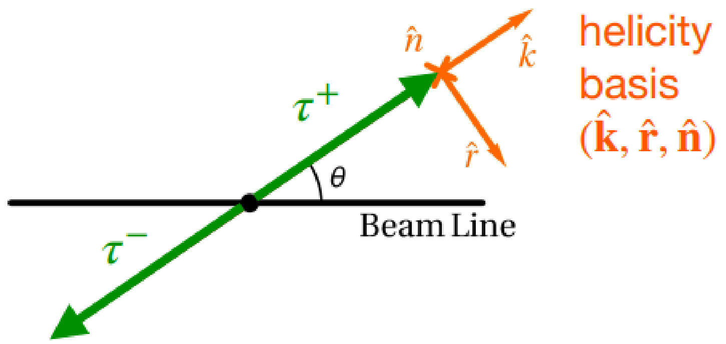
$$\langle s_a \bar{s}_b \rangle = \int \frac{d\Omega_{\mathbf{s}}}{4\pi}\frac{d\Omega_{\bar{\mathbf{s}}}}{4\pi}(\mathbf{a}\cdot\mathbf{s})(\mathbf{b}\cdot\bar{\mathbf{s}})P(\mathbf{s},\bar{\mathbf{s}})$$

$$\langle u_a \bar{u}_b \rangle = \int \frac{d\Omega_{\mathbf{u}}}{4\pi}\frac{d\Omega_{\bar{\mathbf{u}}}}{4\pi}\frac{d\Omega_{\mathbf{s}}}{4\pi}\frac{d\Omega_{\bar{\mathbf{s}}}}{4\pi}(\mathbf{a}\cdot\mathbf{u})(\mathbf{b}\cdot\bar{\mathbf{u}}) \times P(\mathbf{u}|\mathbf{s})P(\bar{\mathbf{u}}|\bar{\mathbf{s}})P(\mathbf{s},\bar{\mathbf{s}})\,.$$

$$\boxed{\langle u_a \bar{u}_b \rangle = \frac{\alpha_{f,d}\alpha_{f',d'}}{9}\langle s_a \bar{s}_b \rangle}$$

Spin-correlation matrix and CHSH in lepton collider

$$\mathbf{R}_{\mathrm{CHSH}} = \frac{1}{2}\left|\langle \hat{\mathbf{s}}^{\mathbf{A}}_{\mathbf{a}}\hat{\mathbf{s}}^{\mathbf{B}}_{\mathbf{b}}\rangle - \langle \hat{\mathbf{s}}^{\mathbf{A}}_{\mathbf{a}}\hat{\mathbf{s}}^{\mathbf{B}}_{\mathbf{b'}}\rangle + \langle \hat{\mathbf{s}}^{\mathbf{A}}_{\mathbf{a'}}\hat{\mathbf{s}}^{\mathbf{B}}_{\mathbf{b}}\rangle + \langle \hat{\mathbf{s}}^{\mathbf{A}}_{\mathbf{a'}}\hat{\mathbf{s}}^{\mathbf{B}}_{\mathbf{b'}}\rangle\right|$$
$$= \frac{9}{2|\alpha_{\mathbf{f},\mathbf{d}}\alpha_{\mathbf{f'},\mathbf{d'}}|} \times \left|\langle \mathbf{u_a}\bar{\mathbf{u}}_{\mathbf{b}}\rangle - \langle \mathbf{u_a}\bar{\mathbf{u}}_{\mathbf{b'}}\rangle + \langle \mathbf{u_{a'}}\bar{\mathbf{u}}_{\mathbf{b}}\rangle + \langle \mathbf{u_{a'}}\bar{\mathbf{u}}_{\mathbf{b'}}\rangle\right|$$



R_{CHSH} can be directly calculated,
once unit vectors $(\hat{a}, \hat{a}', \hat{b}, \hat{b}')$are fixed.

❖ we define helicity basis at the Higgs rest frame.

r ≡ (h − k cos θ)/ sin θ

❖ In tau rest frame, we measure direction of pions and compute R_{CHSH} directly with

$$(\hat{a}, \hat{a}', \hat{b}, \hat{b}') = \left(\hat{k}, \hat{r}, \frac{1}{\sqrt{2}}(\hat{k}+\hat{r}), \frac{1}{\sqrt{2}}(\hat{k}-\hat{r})\right)$$

And measure C_{ij}

Simulation

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	385	663
# of background ($\sigma \cdot \mathrm{BR} \cdot L \cdot \epsilon$)	20	36

❖ Main background

$$e^+e^- \to Z\,\boxed{\tau^+\tau^-} \to \gamma^*/Z^*$$

❖ Event selection

$$|m_{recoil} - M_H| < 5\ GeV$$

❖ Generate the SM events (κ, δ) = (1,0) with **MadGraph5_aMC@NLO.** And use **TauDecay** Package for τ decays.

$$e^+e^- \to H\,Z, \qquad Z \to f\,\bar{f}(f\,\bar{f} = q\bar{q}, e^+e^-, \mu^+\mu^-), \qquad \tau^\pm \to \vartheta\,\pi^\pm\ (Br(\tau^\pm \to \vartheta\,\pi^\pm) = 0.109)$$

❖ Incorporate the detector effects by **smearing energies** of all visible final state particles with

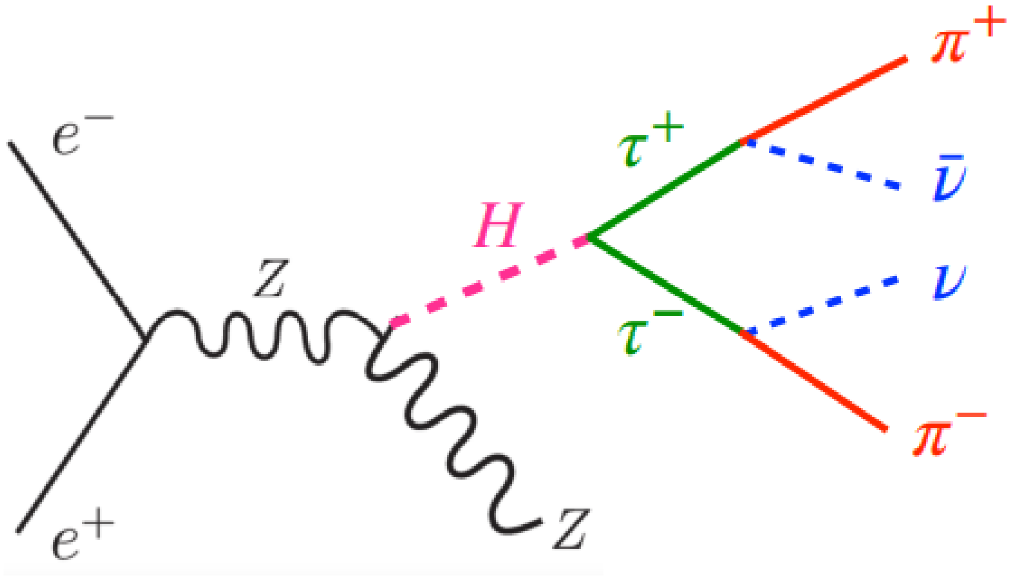
$$E^{true} \to E^{obs} = (1 + \sigma_E.\omega).E^{true}$$

random number from the normal distribution.

Energy resolution $\sigma_E = 0.03$ for both ILC and FCC-ee.

❖ 100 **pseudo-experiments** to estimate the statistical uncertainties.

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu})$, $(p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.



- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^{\nu}, p_y^{\nu}, p_z^{\nu})$, $(p_x^{\bar{\nu}}, p_y^{\bar{\nu}}, p_z^{\bar{\nu}})$.

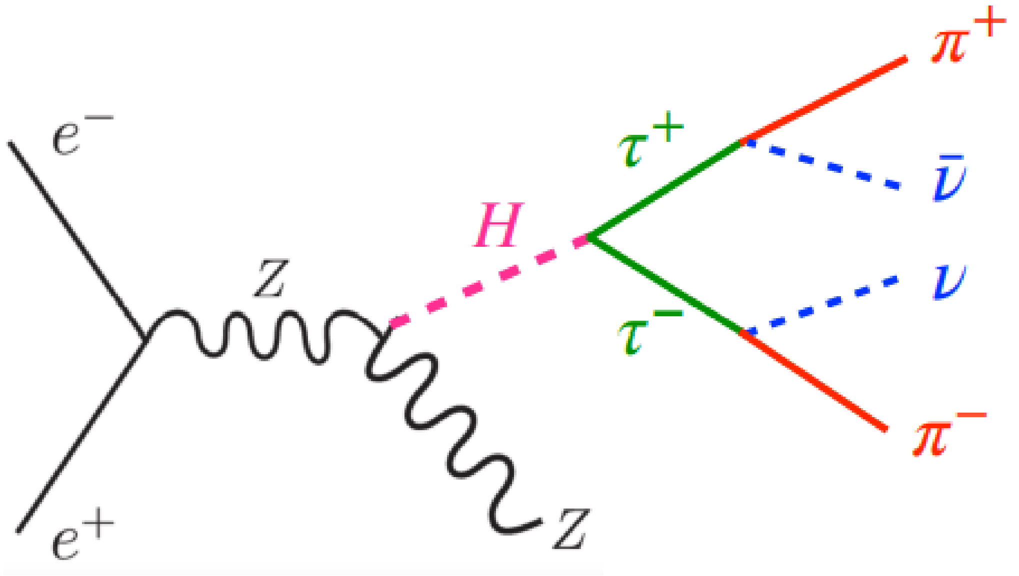
- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

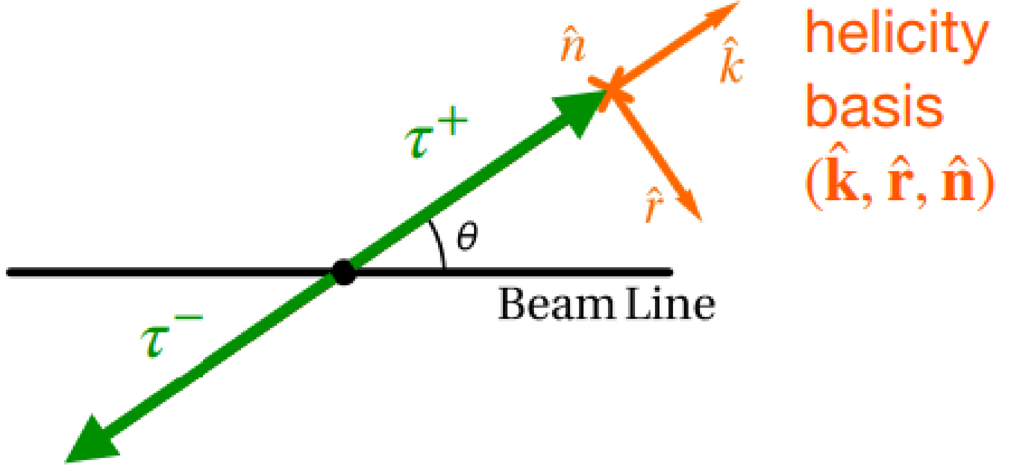
$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar{\nu}})^2$$

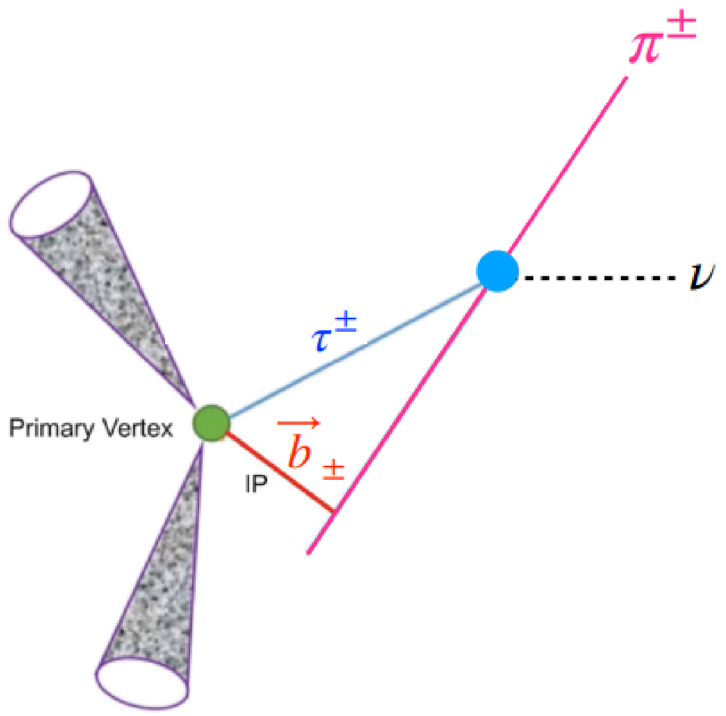
$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar{\nu}})\right]^\mu$$

- With the reconstructed momenta, we define $(\hat{\mathbf{k}}, \hat{\mathbf{r}}, \hat{\mathbf{n}})$ basis at the Higgs rest frame.







Use impact parameter information

- We use the information of impact parameter $\vec{b}_\pm$ measurement of $\pi^\pm$ to "correct" the observed energies of $\tau^\pm$ and Z decay products
- We check whether the reconstructed τ momenta are consistent with the measured impact parameters.
- We construct the likelihood function and search for the most likely τ momenta.

$$\boxed{E_\alpha(\delta_\alpha) = (1 + \sigma_\alpha^E \cdot \delta_\alpha) \cdot E_\alpha^{\mathrm{obs}}}$$

$$\vec{b}_+ = |\vec{b}_+| \left(\sin^{-1}\Theta_+ \cdot \vec{e}_{\tau^+} - \tan^{-1}\Theta_+ \cdot \vec{e}_{\pi^+}\right)$$

$$\vec{\Delta}^i_{b_+}(\{\delta\}) \equiv \vec{b}_+ - |\vec{b}_+| \left(\sin^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}^{\,i}_{\tau^+}(\{\delta\}) - \tan^{-1}\Theta^i_+(\{\delta\}) \cdot \vec{e}_{\pi^+}\right)$$

$$L^i_\pm(\{\delta\}) = \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_x + [\Delta^i_{b_\pm}(\{\delta\})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^i_{b_\pm}(\{\delta\})]^2_z}{\sigma^2_{b_z}}$$

$$\boxed{L^i(\{\delta\}) = L^i_+(\{\delta\}) + L^i_-(\{\delta\})}$$

Results

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	$2.567 \pm 0.279 \sim 5\sigma$	$2.696 \pm 0.215 \sim 5\sigma$
$\mathcal{S}[\rho]$	$1.760 \pm 0.161 \sim 4\sigma$	$1.851 \pm 0.111 \sim 5\sigma$
R^*_{CHSH}	1.103 ± 0.163	$1.276 \pm 0.094 \sim 3\sigma$

SM values: $$C_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$E = 3$ **Entanglement** $\Longrightarrow E > 1$

$\mathcal{S}[\rho] = 2$ **Steerablity** $\Longrightarrow \mathcal{S}[\rho] > 1$

$R_{\text{CHSH}} = \sqrt{2} \simeq 1.414$ **Bell-nonlocal** $\Longrightarrow R_{\text{CHSH}} > 1$

Superiority of FCC-ee over ILC is due to a better beam resolution

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP *measurement*

- Under CP, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$
- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \ \geq 0$$

- Observation of $A \neq 0$ immediately confirms CP violation.
- From our simulation, we observe

$$A = \begin{cases} 0.168 \pm 0.131 & \text{(ILC)} \\ 0.081 \pm 0.060 & \text{(FCC-ee)} \end{cases} \longleftarrow$$

consistent with absence of CPV

- This model independent bounds can be translated to the constraint on the CP-phase δ

$$\mathscr{L}_{\text{int}} \propto H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\, \psi_\tau \Rightarrow C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow A(\delta) = 4\sin^2 2\delta$$

CP *measurement*

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$|\delta| < \begin{cases} 7.9^o & \text{(ILC)} \\ 5.4^o & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$\Delta\delta \sim 11.5^o$ (HL-LHC) [Hagiwara, Ma, Mori 2016]

$\Delta\delta \sim 4.3^o$ (ILC) [Jeans and G. W. Wilson 2018]

Summary

- High energy tests of entanglement and Bell-nonlocality has recently attracted an attention.
- $\tau^+\tau^-$ pairs from $H \to \tau^+\tau^-$ form the EPR triplet state $|\Psi^{(1,0)}> = \frac{|+,-> + |-,+>}{\sqrt{2}}$ which is maximally entangled.
- We investigated feasibility of quantum property tests @ ILC and FCC-ee.
- Quantum test requires to a precise reconstruction of the tau rest frames and impact parameter (IP) information is crucial to achieve this.
- Spin correlation is sensitive to CP-phase and we can measure the CP-phase as a byproduct of the quantum property measurement.

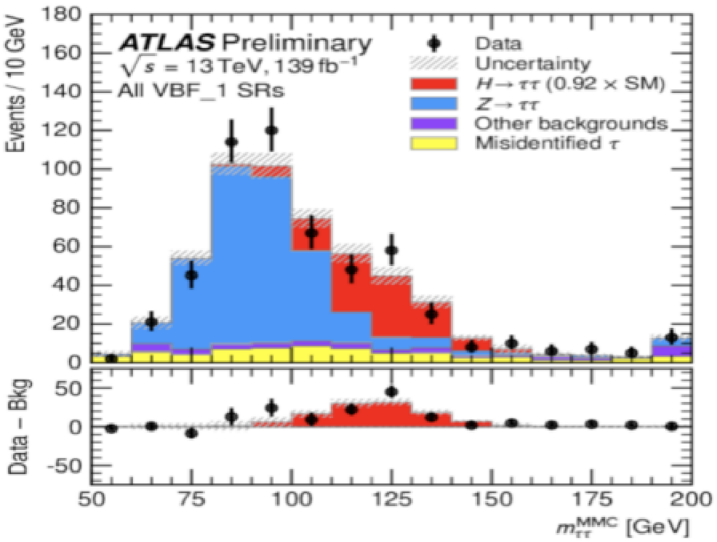
	Entanglement	Steering	Bell-nonlocality	CP-phase
ILC	~ 5σ	~ 4σ		7.9°
FCC-ee	~ 5σ	~ 5σ	~ 3σ	5.4°


Thank you for the attention!

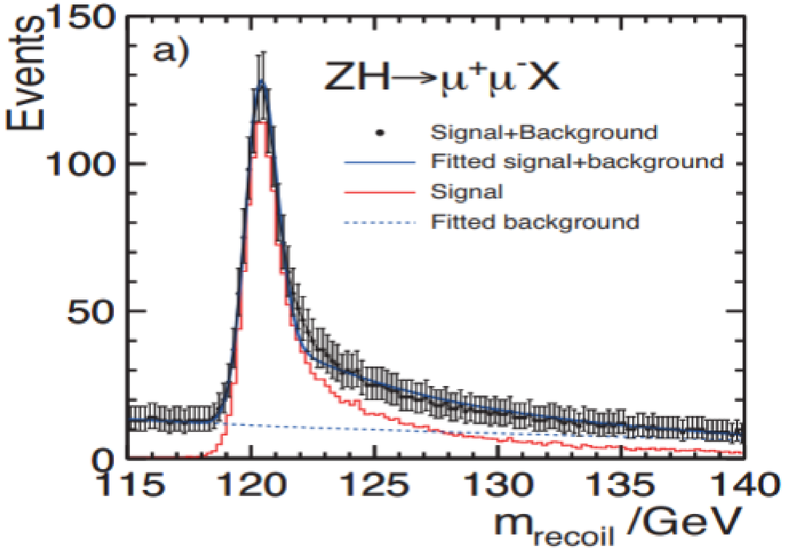
$H \to \tau^+\tau^-$ at lepton colliders

- At LHC, main production mode is $g\, g \to H \to\ \tau^-\tau^+$, which is loop-induced.
- Final state $\tau^-\tau^+$ have large background due to tree-level $q\ \bar{q} \to$ Z* $\to \tau^-\tau^+$.
- The main handle for signal/background is the invariant mass of the visible decay products of two taus, due to neutrinos in tau decays, invariant mass have long tails and therefore signal and background overleap.
- At Lepton colliders, main production channel near threshold is e^-e^+ →ZH, and main background is e^-e^+ →Z $\tau^-\tau^+$, where pair of taus comes from an offshell photon.
- We know initial 4-momentum, can reconstruct Higgs momentum, independent from Higgs decay mode.

LHC



ILC



$0 \le p_k \le 1 \quad \sum_k p_k = 1$

Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_k p_k \langle a\,|\,\rho_k^\alpha\,|\,a\rangle \cdot \langle b\,|\,\rho_k^\beta\,|\,b\rangle \longleftarrow \rho = \sum_k p_k\, \rho_k^\alpha \otimes \rho_k^\beta$$

$$\downarrow$$

$$\rho^{\mathbf{T}_\beta} \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^\alpha \bigotimes [\rho_{\mathbf{k}}^\beta]^{\mathbf{T}}$$

$$\hat{A}\,|\,a\rangle = a\,|\,a\rangle$$

$P(a\,|\,\hat{A},\hat{\rho}) = \langle a\,|\,\rho\,|\,a\rangle$ Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$

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$\downarrow$ If it is still Physics density matrix with Tr=1 and Positive definite

State is separable

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$$\downarrow$$

State is entangled $\xleftarrow{\text{Non-positive definite}}$ $\rho^{\mathbf{T}_\beta} \equiv \sum_{\mathbf{k}} \mathbf{p_k} \rho_{\mathbf{k}}^\alpha \bigotimes [\rho_{\mathbf{k}}^\beta]^{\mathbf{T}}$

$\downarrow$ If it is still Physics density matrix with Tr=1 and Positive definite

State is separable

Peres-Horodecki (1996, 1997)

$$\hat{A}\,|\,a\rangle = a\,|\,a\rangle$$

$$P(a\,|\,\hat{A},\hat{\rho}) = \langle a\,|\,\rho\,|\,a\rangle$$

Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$

$0 \leq p_k \leq 1 \quad \sum_k p_k = 1$

Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_k p_k \langle a\,|\,\rho_k^\alpha\,|\,a\rangle \cdot \langle b\,|\,\rho_k^\beta\,|\,b\rangle \longleftarrow \rho = \sum_k p_k\, \rho_k^\alpha \otimes \rho_k^\beta$$

Un-steerable state (not-steerable by Alice):

[Jones, Wiseman, Doherty 2007]

$$P(a,b\,|\,A,B) = \sum_k p_k\, P_\alpha(a\,|\,A,k) \cdot \langle b\,|\,\rho_k^\beta\,|\,b\rangle \longleftarrow$$

If this description is possible, Alice cannot influence (`steer") Bob's local state

Alice

Send one part to Bob

Bob

Preparing bipartite quantum state

After measurement, Commutating to Alice classically.

$$0 \le p_k \le 1 \qquad \sum_k p_k = 1$$

Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_k p_k \langle a\,|\,\rho_k^\alpha\,|\,a\rangle \cdot \langle b\,|\,\rho_k^\beta\,|\,b\rangle \quad \longleftarrow \quad \rho = \sum_k p_k\, \rho_k^\alpha \otimes \rho_k^\beta$$

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[Jones, Wiseman, Doherty 2007]

$$P(a,b\,|\,A,B) = \sum_k p_k\, P_\alpha(a\,|\,A,k) \cdot \langle b\,|\,\rho_k^\beta\,|\,b\rangle \quad \longleftarrow$$

If this description is possible, Alice cannot influence (\`steer") Bob's local state

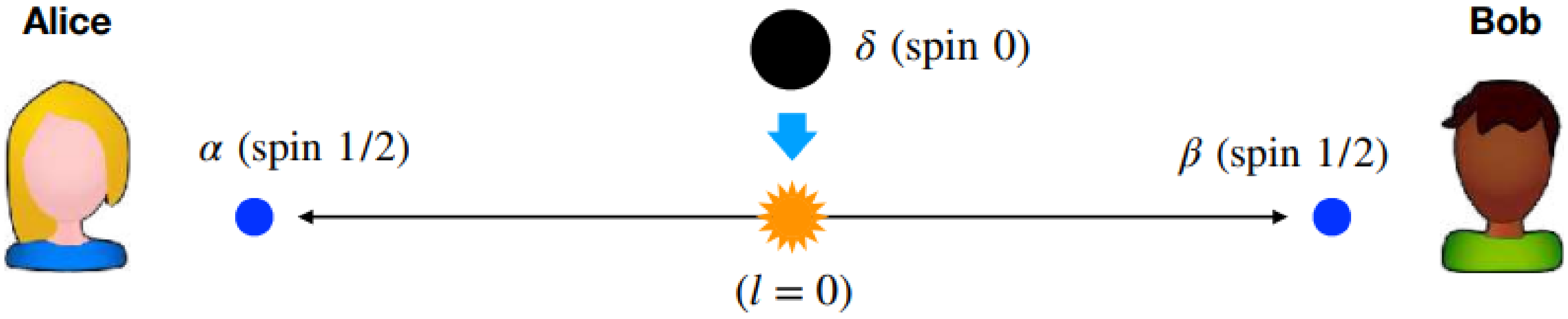
Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b\,|\,A,B) = \sum_\lambda p(\lambda)\, P_\alpha(a\,|\,A,\lambda) \cdot P_\beta(b\,|\,B,\lambda)$$

arbitrary conditional probabilities

$$\hat{A}\,|\,a\rangle = a\,|\,a\rangle$$

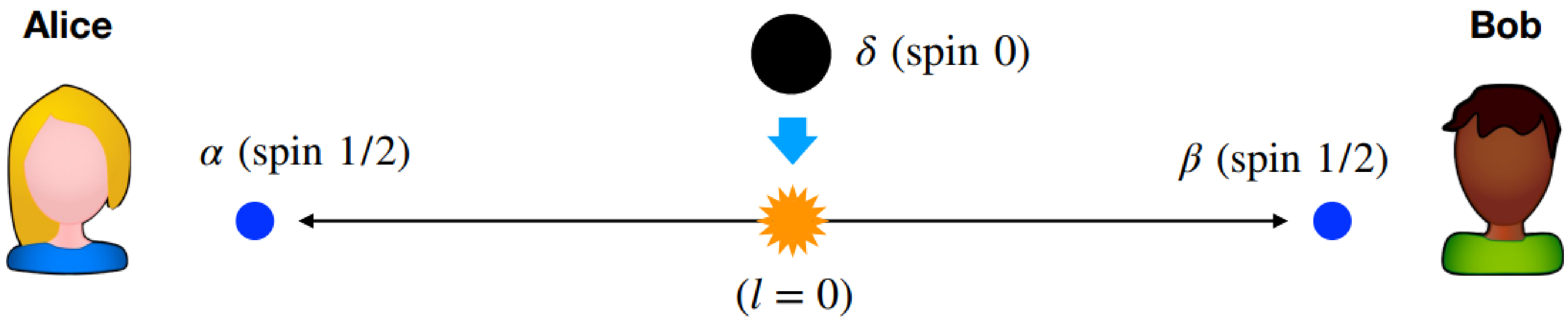
$P(a\,|\,\hat{A},\hat{\rho}) = \langle a\,|\,\rho\,|\,a\rangle$ Probability for outcome a when $\hat{A}$ is measured on the state $\hat{\rho}$



- Alice and Bob receive particles α and β, respectively, and measure the spin z-component of their particles. Repeat the process many times.
- Alice and Bob will find their results are completely random (+1 and -1 50-50%)
- Nevertheless, their result is 100% anti-correlated due to the angular momentum conservation. If Alice's result is +1, Bon's result is always -1 and vice versa.

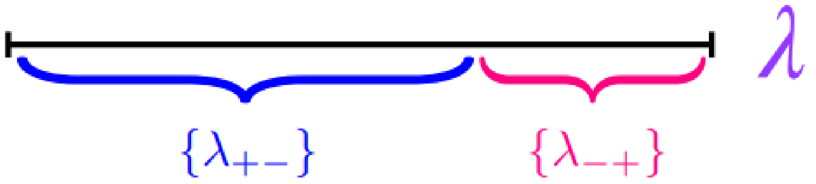
Alice	+	+	-	+	-	-	+	+	+	-	+	-
Bob	-	-	+	-	+	+	-	-	-	+	-	+
$S_z^\alpha \cdot S_z^\beta$	-	-	-	-	-	-	-	-	-	-	-	-

$$\langle S_z^\alpha \cdot S_z^\beta \rangle = -1$$



↑ $\alpha(\lambda \in \{\lambda_{+-}\})$ $\beta(\lambda \in \{\lambda_{+-}\})$ ↓

↓ $\alpha(\lambda \in \{\lambda_{-+}\})$ $\beta(\lambda \in \{\lambda_{-+}\})$ ↑



$$P(\lambda \in \{\lambda_{+-}\}) = P(\lambda \in \{\lambda_{-+}\}) = \frac{1}{2}$$

- Particles have a definite spin-component regardless/prior to the measurement (realism)
- Alice's measurement has no influence on Bob's particle (locality)

The explanation in QM is very different.

Although their outcomes are different in each decay, QM says *the state of the particles are exactly the same for all decays*:

$$|\Psi^{(0,0)}\rangle \doteq \frac{|\overset{\alpha}{+}\overset{\beta}{-}\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

up to a phase $e^{i\theta}$

- Before the measurements, particles have no definite spin. Outcomes are undetermined.

(no realism)

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Although their outcomes are different in each decay, QM says *the state of the particles are exactly the same for all decays*:

$$|\Psi^{(0,0)}\rangle \doteq \frac{|\overset{\alpha}{+}\overset{\beta}{-}\rangle_z - |-+\rangle_z}{\sqrt{2}}$$

up to a phase $e^{i\theta}$

- Before the measurements, particles have no definite spin. Outcomes are undetermined.

(no realism)

- At the moment when Alice makes her measurement, the state collapses into:

$$|\Psi\rangle \xrightarrow{\text{Alice's measurement}} \begin{cases} |+-\rangle_z & \cdots \text{Alice finds } S_z[\alpha] = +1 \\ |-+\rangle_z & \cdots \text{Alice finds } S_z[\alpha] = -1 \end{cases}$$

Bob's outcome is now determined by Alice's measurement

(non-local)

The origin of this bizarre feature is ***entanglement***.

general: $|\Psi\rangle \doteq c_{11}|++\rangle_z + c_{12}|+-\rangle_z + c_{21}|-+\rangle_z + c_{22}|--\rangle_z$

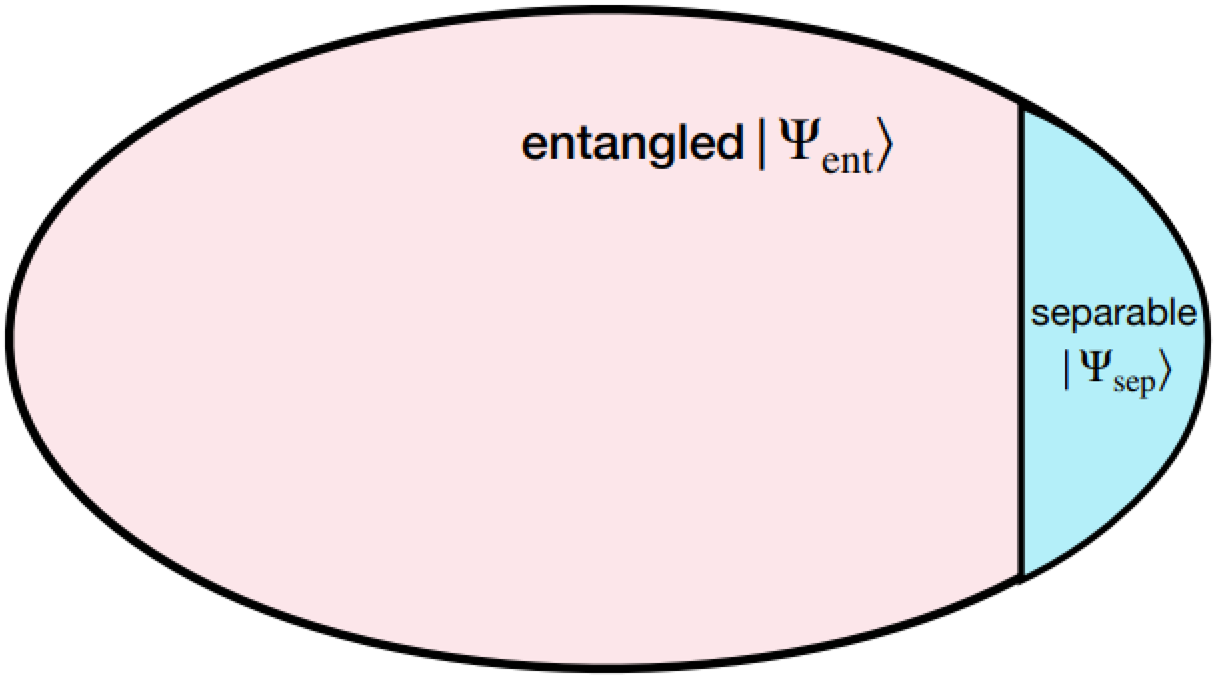
separable: $|\Psi_{\rm sep}\rangle \doteq \boxed{\left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right]} \otimes \underline{\left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]}$

Alice's measurement

Bob's local state is intact

$|+\rangle_z$

$(|-\rangle_z)$



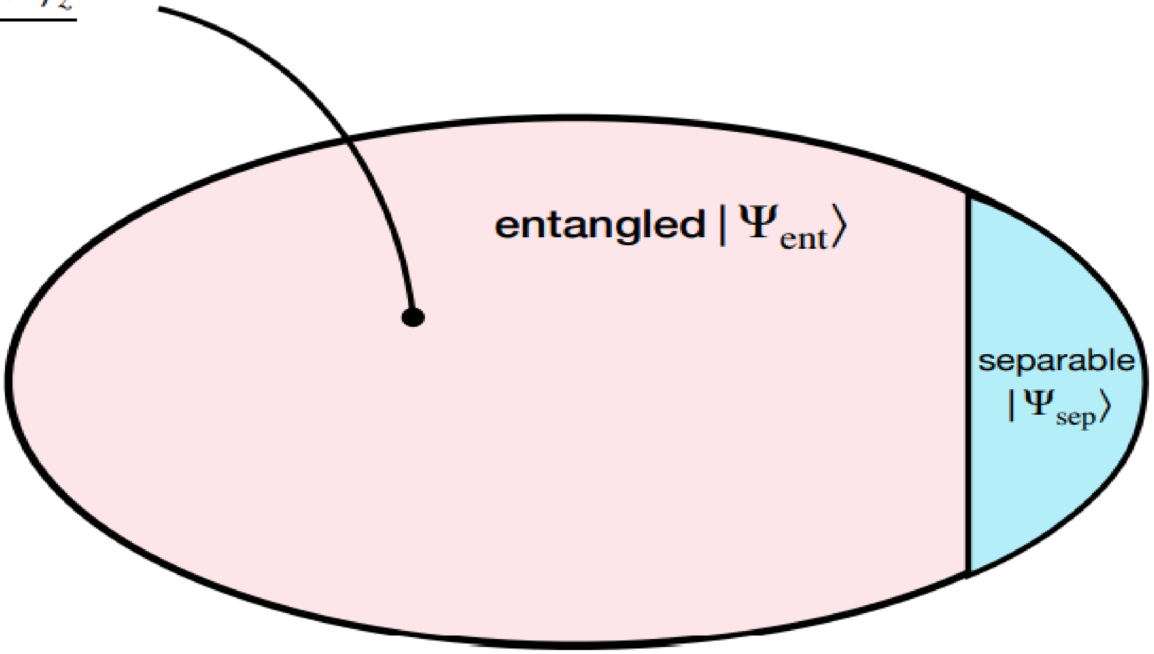
Entanglement

general: $|\Psi\rangle \doteq c_{11}|++\rangle_z + c_{12}|+-\rangle_z + c_{21}|-+\rangle_z + c_{22}|--\rangle_z$

separable: $|\Psi_{\text{sep}}\rangle \doteq \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

entangled: $|\Psi_{\text{ent}}\rangle \cancel{\doteq} \left[c_1^\alpha|+\rangle_z + c_2^\alpha|-\rangle_z\right] \otimes \left[c_1^\beta|+\rangle_z + c_2^\beta|-\rangle_z\right]$

entangled: $|\Psi^{(0,0)}\rangle \doteq \dfrac{|+-\rangle_z - |-+\rangle_z}{\sqrt{2}}$



$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$ (CP even)

$\delta = \pi/2$ (CP odd)

$$|\Psi^{(1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ \boxed{|+-\rangle + |-+\rangle} \\ |--\rangle \end{pmatrix} \qquad |\Psi^{(0,0)}\rangle \propto \boxed{|+-\rangle - |-+\rangle}$$

Parity: $P = (\eta_f \eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f \eta_{\bar{f}} = -1$:

$$J^P = \begin{cases} 0^+ \Longrightarrow -l = s = 1 \\ 0^- \Longrightarrow \quad l = s = 0 \end{cases}$$