

SCALAR FIELD EMULATOR VIA ANISOTROPICALLY *DEFORMED* VACUUM ENERGY

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BASED ON THE WORKS IN COLLABORATION WITH
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TENSIONS IN COSMOLOGY IN 2023
SEPTEMBER 6-12 2023
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Inertial mass density $\varrho = \rho + p$

Einstein field equations arises from the twice contracted Bianchi Identity implying

$$\nabla_{\mu} G^{\mu\nu} = 0 \rightarrow \nabla_{\mu} T^{\mu\nu} = 0$$

The EMT can be decomposed relative to u_{μ} in the form

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + p g_{\mu\nu}$$

Projecting parallel and orthogonal to u_{μ} we obtain energy and momentum conservation equations,

$$\dot{\rho} + \Theta\varrho = 0$$

$$D^{\mu}p + (\rho + p)\dot{u}^{\mu} = 0 \text{ Equivalent to the Newton's second law of motion}$$

Local Energy conservation

Momentum conservation

$$\varrho = \gamma\rho_0 \left(\frac{\rho}{\rho_0}\right)^{\lambda}$$

null inertial mass density
usual vacuum energy

$$\gamma = 0$$

Λ CDM

What will the data say?
At which scales?



Simplest (constant)

deviation from null inertial mass density

$$\lambda = 0 \quad \rho + p = \text{const.}$$

non-trivial behaviors

AKARU.BARROW.ESCAMILLA.VAZQUEZ. PRD 101 063528 1912.08751

ACQUAVIVA, AKARU, KATIRCI, VAZQUEZ, PRD 104 023505 2021 2104.02623
BOUHADI-LOPEZ ET. AL., IJMPD 24 1550078 (2015) 1407.2446 2446

$$\rho + p = (1 + w)\rho_0(1 + z)^{3(1+w)} \text{ for } w\text{CDM}$$

$$\begin{aligned} \Theta &= D^{\mu}u_{\mu} \\ \pi_{\mu\nu} &= T_{\langle\mu\nu\rangle} \\ \sigma_{\mu\nu} &= D_{\langle\mu}u_{\nu\rangle} \\ \dot{u}_{\mu} &= u_{\nu}\nabla^{\nu}u_{\mu} \\ \nabla_{\nu}u_{\mu} &= D_{\nu}u_{\mu} - \dot{u}_{\mu}u_{\nu} \\ D_{\nu}u_{\mu} &= \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} \end{aligned}$$

INSPIRED BY
BARROW,
GRADUATED
INFLATIONARY
UNIVERSES, PLB
235 (1990)

Two simplest Λ CDM extensions : Simple graduated DE or curvature

Simple gDE coming from EMSG

ACQUAVIVA, KATIRCI, PDU 38 (2022) 101128 2203.01234

AKARSU ETAL EPJC 2019 1903.11519

Can these, together or separately, successfully realize such a scenario?

$$\frac{H^2}{H_0^2} = \Omega_{ci0} [1 + 3(1 + w_{ci0}) \ln(1 + z)] + \Omega_{k0}(1 + z)^2 + \Omega_{m0}(1 + z)^3 + \Omega_{r0}(1 + z)^4,$$

Simple graduated DE
 $\rho < 0$

$$w_{ci0} < -1, \rho_{ci0} > 0$$

promotes null inertial mass density of conventional vacuum energy to an arbitrary constant.

Reminiscent of PEDE, decreasing with increasing z, yet no extra dof

The spatial curvature, in the case of spatially closed Universe

$$\Omega_{k0} < 0 \quad w = -1/3$$

DI VALENTINO, MELCHIORRI, SILK, NATURE ASTRON. 1911.02087, 2003.04935, HANDLEY, 1908.09139

the de Sitter future of the Λ CDM
 $\dot{H} = 0$

The fact that the Planck data favor positive spatial curvature on top of the Λ CDM model implying such dark energy models

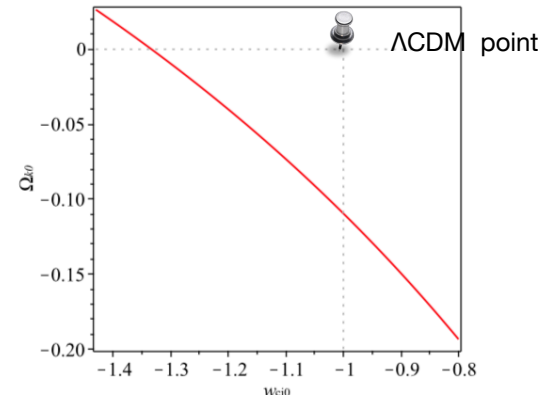
$$\dot{H} = -\frac{1}{2}\rho \neq 0$$

BOUHMADEI-LOPEZ ET.AL., IJMPD 24 1550078 (2015) 1407.2446

it resembles Λ today, alas leading to a future singularity dubbed as the Little Sibling of the Big Rip (LSBR)

$$\rho < 0 \quad w_{ci0} < -1, \rho_{ci0} > 0$$

a finite future bounce $\rho > 0 \quad w_{ci0} > -1, \rho_{ci0} > 0$



Does it compensate to make flat again?

Observational analysis

TABLE I. Constraints (68% CL) on the parameters using the combined BAO+SN+ H and BAO+SN+ H +PLK datasets. Before the two last rows, $-2 \ln \mathcal{L}_{\max}$ is used to compare best fit with respect to the standard Λ CDM model. The last rows contain the Bayesian evidence $\ln \mathcal{Z}$ and the relative Bayesian evidence with respect to the standard Λ CDM model $\Delta \ln \mathcal{Z} = \ln \mathcal{Z} - \ln \mathcal{Z}_{\Lambda\text{CDM}}$.

Dataset	BAO+SN+ H				BAO+SN+ H +PLK			
	Λ CDM	$o\Lambda$ CDM	DE	o DE	Λ CDM	$o\Lambda$ CDM	DE	o DE
Ω_{m0}	0.307 ± 0.014	0.310 ± 0.020	0.304 ± 0.015	0.322 ± 0.022	0.3005 ± 0.0068	0.3009 ± 0.0067	0.3070 ± 0.0088	0.3071 ± 0.0091
$\Omega_{b0} h_0^2$	0.02204 ± 0.00047	0.02204 ± 0.00046	0.02204 ± 0.00047	0.02204 ± 0.00045	0.02245 ± 0.00015	0.02237 ± 0.00017	0.02242 ± 0.00015	0.02241 ± 0.00017
h_0	0.6827 ± 0.0088	0.6862 ± 0.0268	0.6706 ± 0.0202	0.6884 ± 0.0260	0.6829 ± 0.0052	0.6849 ± 0.0067	0.6772 ± 0.0097	0.6773 ± 0.0099
w_{ci0}	-1	-1	-0.937 ± 0.084	-0.872 ± 0.097	-1	-1	-0.948 ± 0.041	-0.951 ± 0.045
Ω_{k0}	—	-0.011 ± 0.077	—	-0.122 ± 0.117	—	0.0012 ± 0.0018	—	-0.0001 ± 0.0019
$\rho_{ci} \times 10^{31} [\text{g cm}^{-3}]$	0	0	3.46 ± 4.76	7.65 ± 5.72	0	0	3.06 ± 2.28	2.85 ± 2.58
Ω_{ci0}	0.693 ± 0.014	0.700 ± 0.064	0.696 ± 0.015	0.800 ± 0.101	0.6994 ± 0.0068	0.6977 ± 0.0065	0.6929 ± 0.0088	0.6929 ± 0.0095
Ω_{kci0}	—	0.690 ± 0.020	—	0.678 ± 0.022	—	0.6991 ± 0.0067	—	0.6928 ± 0.0091
z_{ci*}	—	—	< -0.96 or $\gtrsim 10^7$	< -0.78	—	—	< -0.99	< -0.99
$z_{kci*} (z_{kcc*})$	—	> 1.26	—	> 0.92	—	> 9.62	—	> 6.64
$-2 \ln \mathcal{L}_{\max}$	58.97	58.96	58.28	56.91	60.46	59.27	58.24	58.24
$\ln \mathcal{Z}$	-36.54 ± 0.19	-38.38 ± 0.21	-37.96 ± 0.21	-38.00 ± 0.21	-42.02 ± 0.26	-43.78 ± 0.26	-42.19 ± 0.25	-44.13 ± 0.27
$\Delta \ln \mathcal{Z}$	0	-1.84 ± 0.28	-1.42 ± 0.28	-1.46 ± 0.28	0	-1.76 ± 0.37	-0.17 ± 0.36	-2.11 ± 0.37

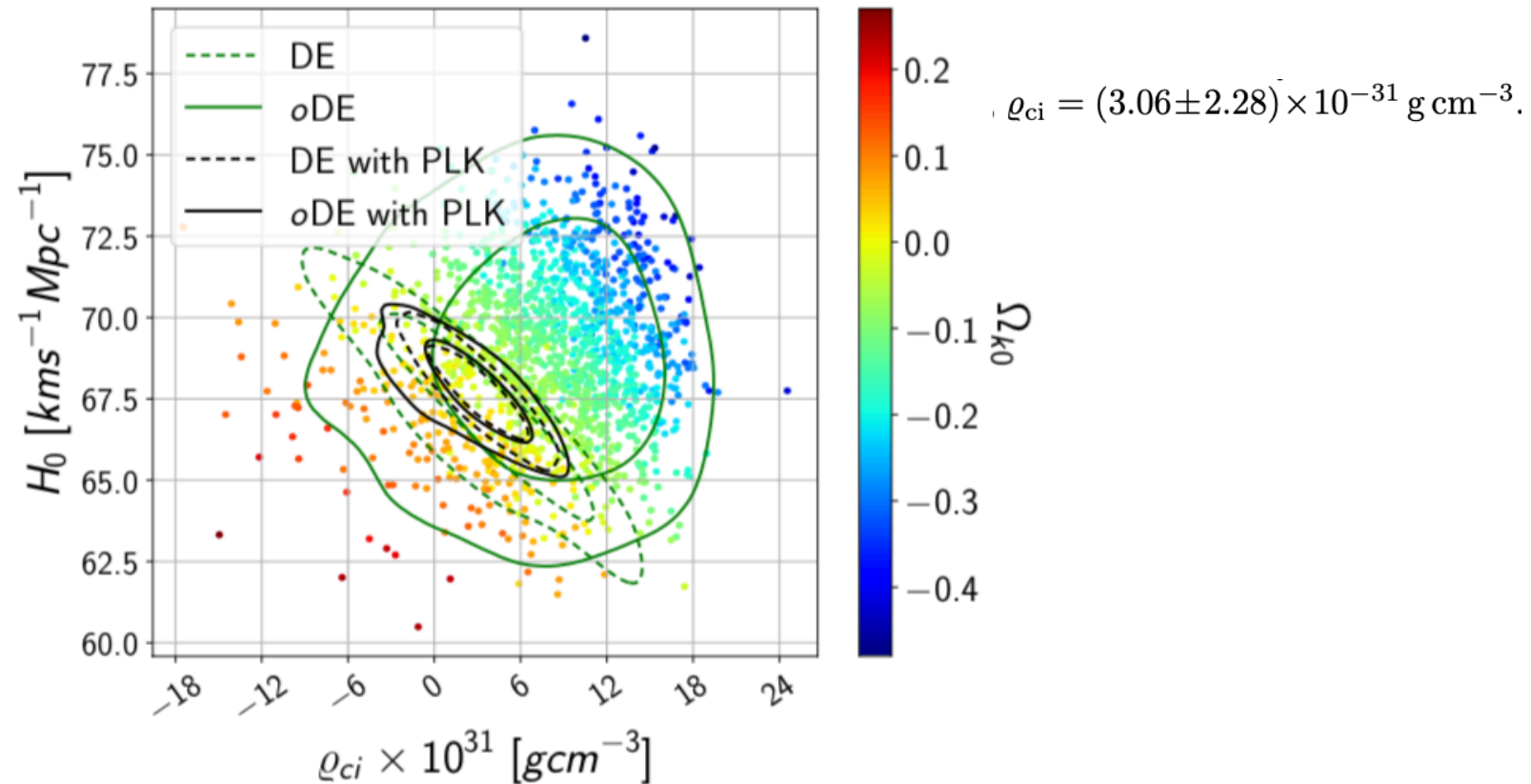
□ Contrary to our initial expectations, the simple-gDE worsens the so-called H_0 tension. The reason is being that the data favor $\rho_{ci} = (3.46 \pm 4.76) \times 10^{-31} \text{ g cm}^{-3}$ ($w_{ci0} = -0.937 \pm 0.084$) rather than a definitely negative inertial mass destiny.

☑ the negative correlation between Ω_{k0} and w_{ci0} .

Simple MC code [1411.1074]

<https://github.com/slosar/april>, version May 2019.

Interplay between H_0 , ρ and Ω_{k0}



The joint data set, including the Planck data, presents no evidence for a deviation from spatial flatness, but almost **the same evidence** for a cosmological constant and the simple-gDE with an inertial mass density of order $O(10-12)\text{eV}^4$.

Vacuum inertial mass density may be a constant of nature, rather than vacuum energy density

Graduated dark energy - a spontaneous sign switch in Λ

AKARSU, BARROW, ESCAMILLA, VAZQUEZ, PRD 101 063528
 AKARSU ETAL PRD 104, 123512 (2021) 2108.09239, PRD 108 023513 (2023), 2211.05742

AKARSU, DI VALENTINO, KUMAR, NUNES, VAZQUEZ, YADAV
 Λ sCDM model, 2307.10899

$$\rho = \rho_0 [1 + 3\gamma(\lambda - 1) \ln a]^{\frac{1}{1-\lambda}} \xrightarrow[m \text{ and } n \text{ are odd integers}]{\frac{1}{1-\lambda} = \frac{m}{n}} \rho = \rho_0 \operatorname{sgn}[1 - \Psi \ln a] |1 - \Psi \ln a|^{\frac{1}{1-\lambda}}$$

$$\Psi \equiv -3\gamma(\lambda - 1) < 0 \quad \lambda < 1 \quad \gamma < 0$$

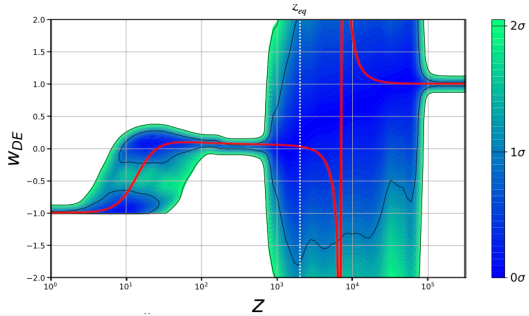
Under these conditions energy density takes negative values in the past and EoS exhibits singularity/pole during its sign change

EoS parameter

$$w = -1 + \frac{\gamma}{1 + 3\gamma(\lambda - 1) \ln a}$$

For large negative values of λ , it creates a phenomenological model described by a smooth function that approximately describes the Λ spontaneously switching sign in the late universe to become positive today.

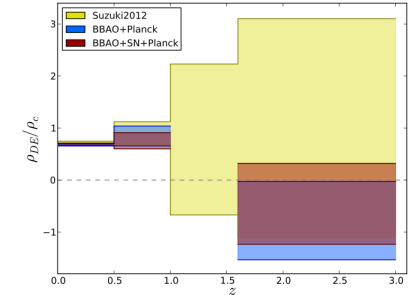
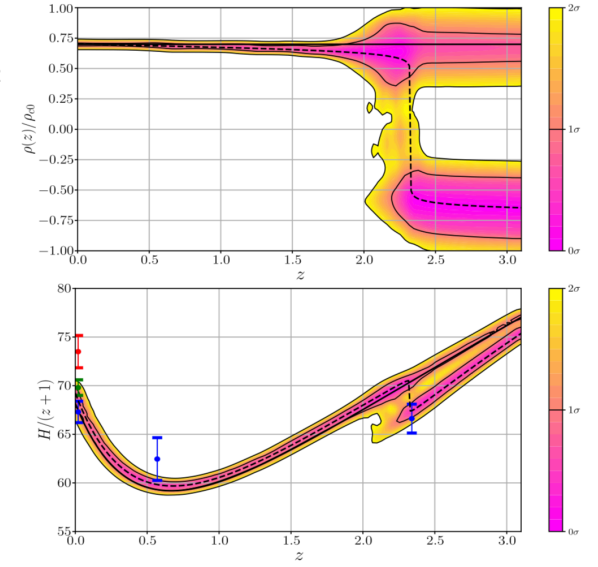
$$\frac{H^2}{H_0^2} = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda_s0} \operatorname{sgn}[z_{\dagger} - z]$$



Massive Brans-Dicke gravity extension of the standard Λ CDM model

$$S_{\text{JBD}} = \int d^4x \sqrt{-g} \left[\frac{\varphi^2}{8} R - \omega \left(\frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi + \frac{1}{2} M^2 \varphi^2 \right) \right] + S_{\text{Matter}},$$

if the transition are not rapid enough, it does not working on solving tensions



Lubourg et al. (BOSS Collab.), PRD (2015), arXiv:1411.1074

Rastall gravity extension of the standard Λ CDM model: Theoretical features and observational constraints, O. Akarsu, N. Katirci, S. Kumar, R.C. Nunes, B. Ozturk and S. Sharma, European Physical Journal C 80, 1050 (2020), arXiv:2004.04074.

Anisotropic massive Brans-Dicke gravity extension of standard Λ CDM model, O. Akarsu, N. Katirci, N. Ozdemir, and J. A. Vazquez, European Physical Journal C 80, 32 (2020), arXiv:1903.06679.

Observations suggesting the presence of a DE source passing below PDL at z around 0.6 with high confidence would imply a strong reason for favoring the BD gravity over GR, or vice versa.

$$\frac{1}{2} \leq z_{\text{PDL}} \leq e^{\frac{1}{2}} - 1 = 0.65 \quad \text{for } \omega \geq 0,$$

General relativity with anisotropy

GEOMETRY: LRS Bianchi type-I metric described by the line element

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 (dy^2 + dz^2),$$

$$ds^2 = -dt^2 + S^2 \left[e^{\frac{4}{\sqrt{6}}\varphi} dx^2 + e^{-\frac{2}{\sqrt{6}}\varphi} (dy^2 + dz^2) \right]$$

shear is time derivative of spatial metric.

$$\dot{\varphi}^2 = \sigma^2 \quad \sigma^2 = \sigma_{ij}\sigma^{ij}$$

MATTER: the most general form of the EMT, accommodated by this metric

$$T_{ab} = \rho u_a u_b + p_{\text{iso}} h_{ab} + \pi_{ab}$$

Trace-free anisotropic pressure



$$\begin{aligned} \dot{\rho} + \Theta(\rho + p_{\text{iso}}) + \sigma^{ab}\pi_{ab} &= 0 \\ D^a p_{\text{iso}} + (\rho + p_{\text{iso}} + \pi_a^a)\dot{u}^a + (\text{div}\pi)^a &= 0 \end{aligned}$$

$$\rho + p = 0$$

Conventional vacuum energy has vanishing inertial mass density,

GENERAL RELATIVITY:

$$\nabla_b G^{ab} = 0 \quad \rightarrow \quad \nabla_b T^{ab} = 0.$$

GR with **anisotropy** + a fluid still has

$$\rho_{\text{inert},x} \equiv \rho + p_{\text{iso}} + \pi_1^1$$

$$\rho_{\text{inert},y(z)} \equiv \rho + p_{\text{iso}} + \pi_2^2 \quad \Rightarrow \quad p_{y(z)} = p_x + \gamma\rho$$

$$\bar{\rho}_{\text{inert}} \equiv \frac{1}{3} (\rho_{\text{inert},x} + 2\rho_{\text{inert},y(z)})$$

$$= \rho + p_{\text{iso}} + \frac{1}{3}\pi_1^1 + \frac{2}{3}\pi_2^2,$$

$$\bar{\rho}_{\text{inert}} = \rho + p_x + \frac{2}{3}\gamma\rho = 0$$

Particular relation with EoS and skewness parameters

$$w_x = -1 - \frac{2\gamma}{3}$$

$$\gamma = w_y - w_x$$

Anisotropic extension of vacuum energy

$$T_{\mu}^{\nu} = \text{diag} \left[-1, -1 - \frac{2}{3}\gamma, -1 + \frac{1}{3}\gamma, -1 + \frac{1}{3}\gamma \right] \rho,$$

Deformed vacuum energy gives, on average, zero inertial mass density

The Einstein field equations in the presence of the deformed vacuum energy described above for the simplest anisotropic background read

$$3\mathcal{H}^2 - \frac{1}{2}\sigma^2 = \rho_{\text{dv}},$$

$$-2\dot{\mathcal{H}} - 3\mathcal{H}^2 - \frac{1}{2}\sigma^2 + 2\sqrt{\frac{1}{6}}(\dot{\sigma} + 3\mathcal{H}\sigma) = -\rho_{\text{dv}} - \frac{2}{3}\gamma\rho_{\text{dv}},$$

$$-2\dot{\mathcal{H}} - 3\mathcal{H}^2 - \frac{1}{2}\sigma^2 - \sqrt{\frac{1}{6}}(\dot{\sigma} + 3\mathcal{H}\sigma) = -\rho_{\text{dv}} + \frac{1}{3}\gamma\rho_{\text{dv}}$$

If we set cosmic triad, then these three resembles usual vacuum energy. Similarly, arbitrary number of them oriented in arbitrary directions would on average lead, stochastically, to the usual vacuum energy

PICON ICAP 07,007 2004 ASTROPH /0405267

GOLOVNEV ET.AL. JCAP 06 009 2008 0802.2068

No correspondence from known anisotropic source (i.e. vector fields, topological defects)

where average Hubble parameter

$$\mathcal{H} = \frac{1}{3}(H_x + H_y + H_z)$$

Shear scalar

$$\sigma^2 = \frac{3}{2}(H_x - \mathcal{H})^2$$

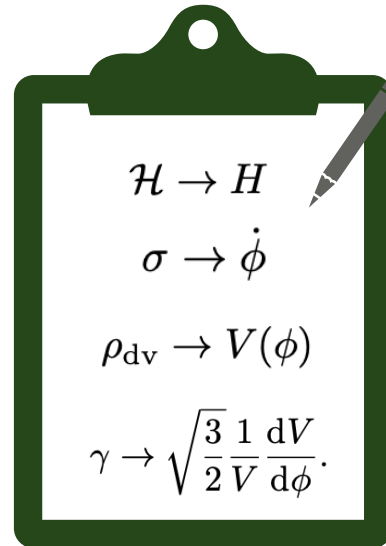
Emulating Scalar (canonical) field with an arbitrary potential

$$\begin{aligned}
 3\mathcal{H}^2 &= \frac{1}{2}\sigma^2 + \rho_{\text{dv}}, \\
 -2\dot{\mathcal{H}} - 3\mathcal{H}^2 &= \frac{1}{2}\sigma^2 - \rho_{\text{dv}}, \\
 \dot{\sigma} + 3\mathcal{H}\sigma &= -\sqrt{\frac{2}{3}}\gamma\rho_{\text{dv}},
 \end{aligned}$$

↗ observable

Shear propagation equation

Einstein field equations
in the presence of deformed vacuum



$$\begin{aligned}
 3H^2 &= \frac{1}{2}\dot{\phi}^2 + V(\phi), \\
 -2\dot{H} - 3H^2 &= \frac{1}{2}\dot{\phi}^2 - V(\phi), \\
 \ddot{\phi} + 3H\dot{\phi} &= -\frac{dV}{d\phi}
 \end{aligned}$$

Einstein field equations
in the presence of scalar field for RW
metric

But physically, they are very different

Under the following transformations:

the cosmologies employing the canonical scalar fields *are mathematically equivalent to* cosmologies with deformed vacuum which means that

you can construct the anisotropic counterpart cosmologies + a bonus

what you can construct cosmologically with SF,

Cosmology with deformed vacuum energy - map

Canonical SF's

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

KG-> continuity eq. for the SF

$$\dot{\rho}_\phi + 3\mathcal{H}\rho_\phi(1 + w_\phi) = 0$$

no-go theorem forbids a single canonical SF with a non-negative potential to cross below the $w=-1$ boundary of the usual vacuum energy, viz., its EoS parameter is confined to the range

$$w_\phi < -\frac{1}{3} \quad -1 \leq w_\phi \leq 1 \quad \dot{\phi}^2 \geq 0$$

$$\dot{\phi}^2 < V \quad V(\phi) \geq 0$$

slow roll parameter for the SF

It is often required a flat potential satisfying

$$\dot{\phi}^2 \ll V \quad \epsilon = \frac{1}{2} \left(\frac{1}{V} \frac{dV}{d\phi} \right)^2$$

Defining the effective quantities,

$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{\sigma^2/2 - \rho_{\text{dv}}}{\sigma^2/2 + \rho_{\text{dv}}}$$

the deformed vacuum energy
+ the shear scalar

Shear propagation equation -> continuity equation for the effective source defined from the cooperation of the deformed vacuum with the shear scalar

$$\dot{\rho}_{\text{eff}} + 3\mathcal{H}\rho_{\text{eff}}(1 + w_{\text{eff}}) = 0,$$

the non-negativity condition on the density of the deformed vacuum energy- along with that the shear scalar is non-negative definite guarantee that

$$w_{\text{eff}} < -\frac{1}{3} \quad -1 \leq w_{\text{eff}} \leq 1 \quad \sigma^2 \geq 0 \quad \rho_{\text{dv}} \geq 0$$

$$\sigma^2 < \rho_{\text{dv}}$$

the role of the flatness of the potential is taken over by the ratio-squared of the rate of change of the energy density of the deformed vacuum to the shear scalar

$$\epsilon \rightarrow \frac{\gamma^2}{3} = \frac{1}{2} \left(\frac{\dot{\rho}_{\text{dv}}}{\rho_{\text{dv}}} \right)^2 \frac{1}{\sigma^2}$$

- if there is no SF any more, we would not suffer from the tension with the string Swampland criterion, the derivative of the SF potential has to satisfy the lower bound

$$\frac{|dV/d\phi|}{V} > c \sim \mathcal{O}(1)$$

as the scalar field does

Some dark energy applications

Λ CDM - **null skewness** $w_{\text{eff}} = -1$

Lowest energy density configuration

no contribution from kinetic energy
+ flat potential (corresponds to Λ)

$$\dot{\phi}^2 \propto (1+z)^6$$

negligible anisotropy, null skewness

$$\sigma^2 \propto (1+z)^6 \quad \gamma = 0$$

$$3\mathcal{H}^2 = \sum_i \rho_{i0} (1+z)^{3(1+w_i)} + \rho_{\text{eff}},$$

$w_{\text{eff}}(z)$ - **varying skewness** **dynamical dark energy models**

Multiplying shear propagation with we obtain

$$\frac{d\rho_{\text{eff}}}{dz} - \frac{6}{1+z} \rho_{\sigma^2} = 0$$

$$\rho_{\text{eff}} = \rho_{\sigma^2} + \rho_{\text{dv}}$$

$$w_{\text{eff}} = \frac{\frac{\sigma^2}{2} - \rho_{\text{dv}}}{\frac{\sigma^2}{2} + \rho_{\text{dv}}} \quad \text{with} \quad \frac{\rho_{\sigma^2}}{\rho_{\text{eff}}} = \frac{1 + w_{\text{eff}}}{2}$$

$$\rho_{\text{eff}} = \rho_{\text{eff},0} \exp \left[\int_0^z 3 [1 + w_{\text{eff}}(z)] d \ln(1+z) \right].$$

$$\gamma = \sqrt{\frac{3(1 - w_{\text{eff}})}{4(1 + w_{\text{eff}})} \frac{\dot{\rho}_{\text{dv}}}{\rho_{\text{dv}}^{3/2}}} = 3 \sqrt{\frac{1 + w_{\text{eff}}}{2}}$$

w CDM - **constant skewness** $w_{\text{eff}} \simeq -1$

shear tracks the vacuum energy deforming it
Anisotropization as the universe expands

$$\begin{aligned} \rho_{\sigma^2} &= \rho_{\text{eff},0} \frac{1 + w_{\text{eff}}}{2} (1+z)^{3(1+w_{\text{eff}})} \\ \rho_{\text{dv}} &= \rho_{\text{eff},0} \frac{1 - 2w_{\text{eff}}}{2} (1+z)^{3(1+w_{\text{eff}})}. \end{aligned}$$

$$\frac{\Omega_{\sigma^2}}{\Omega_{\text{eff}}} = \frac{1 + w_{\text{eff}}}{2} = 0.01 \quad \text{for} \quad w = -0.97$$

Observational constraints of emulation of wcdm model

TABLE I. Equations for An- Λ CDM and dv- w CDM models.

	An- Λ CDM	dv- w CDM
ρ_{eff}	$\rho_{\text{dv}} + \rho_{\sigma^2_0}(1+z)^6$	$\rho_{\text{eff}0}(1+z)^{3(1+w_{\text{eff}})}$
w_{eff}	$\frac{\rho_{\sigma^2_0}(1+z)^6 - \rho_{\text{dv}}}{\rho_{\sigma^2_0}(1+z)^6 + \rho_{\text{dv}}}$	const. ≥ -1
ρ_{σ^2}	$\rho_{\sigma^2_0}(1+z)^6$	$\frac{1}{2}(1+w_{\text{eff}})\rho_{\text{eff}0}(1+z)^{3(1+w_{\text{eff}})}$
ρ_{dv}	const	$\frac{1}{2}(1-w_{\text{eff}})\rho_{\text{eff}0}(1+z)^{3(1+w_{\text{eff}})}$
γ	0	$-3\sqrt{\frac{1+w_{\text{eff}}}{2}} \left[1 + \frac{\rho_{\text{m}0}}{\rho_{\text{eff}0}}(1+z)^{-3w_{\text{eff}}} \right]$

Tight constraints !!!

$$\Omega_{\sigma^2_0} \lesssim 10^{-3}$$

Drastic deviation from the stiff fluid character

AKARSU ETAL, PDU 39 2023, 2112.07807

AKARSU, KUMAR, SHARMA, TEDESCO, PRD 100 2019, 1905.06949

- $\Omega_{\sigma^2_0} \lesssim 10^{-3}$ from Hubble and Pantheon data,
- $\Omega_{\sigma^2_0} \lesssim 10^{-15}$ when the baryonic acoustic oscillations and cosmic microwave background data are included,
- $\Omega_{\sigma^2_0} \lesssim 10^{-23}$ from the standard Big Bang Nucleosynthesis (BBN)

no significant difference between the constraints on $k_{\text{eq}} = H_{\text{eq}}$ (the wavenumber of a mode of density perturbations that enter the horizon at the radiation-matter transition, which is highly sensitive to the modifications to Λ CDM.

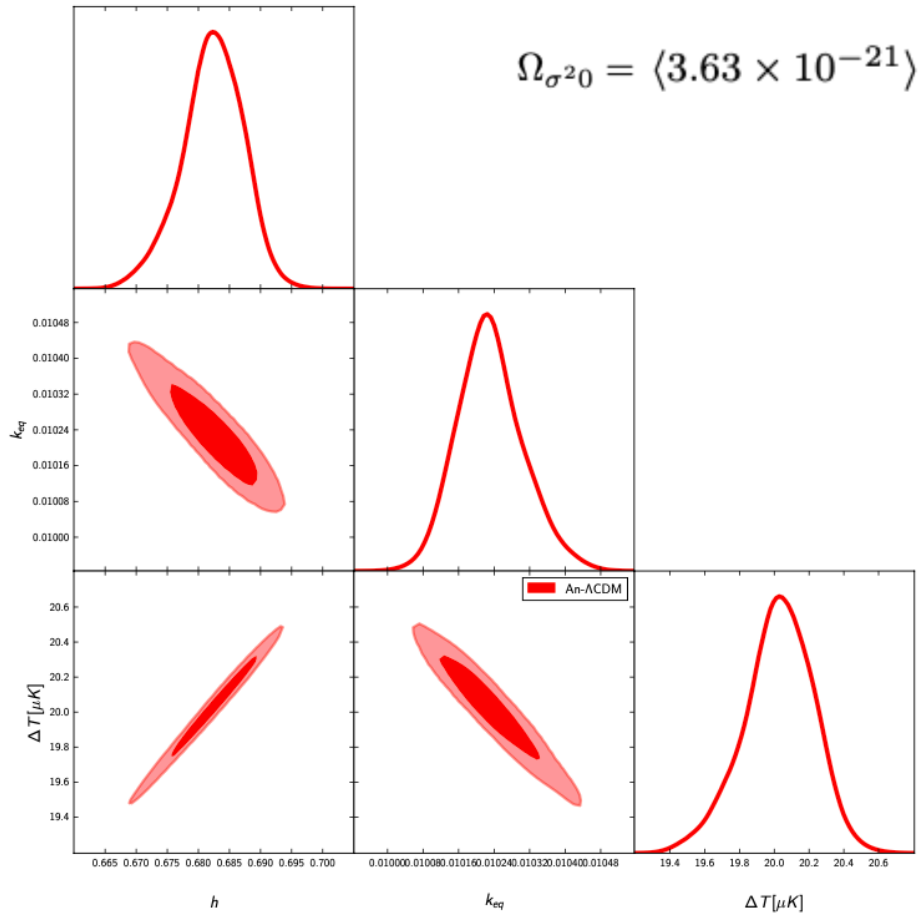
TABLE II. Constraints (68% C.L.) on the parameters using the combined data sets PLK+BAO+SN+ H . Along the analysis, free parameter $w_{\text{eff}} = w_0$ is fixed to a certain value $\langle -1 + 1.83 \times 10^{-11} \rangle$ to restrict our analysis to $\Delta T_{\sigma^2} \sim 20 \mu\text{K}$ region. Derived parameters are labeled with * and the chosen parameters are enclosed in angle brackets.

	Λ CDM	An- Λ CDM	dv- w CDM
H_0 [km s $^{-1}$ Mpc $^{-1}$]	68.33(50)	68.28(50)	68.84(44)
$\Omega_{\text{m}0}$	0.299(6)	0.301(6)	0.298(6)
$\Omega_{\sigma^2_0}$	0	$\langle 3.63 \times 10^{-21} \rangle$	6.42(5) [10 $^{-12}$]
$\Omega_{\text{eff}0}$	0.700(6)	0.699(6)	0.702(6)
$1 + w_{\text{eff}}$	$\langle 0 \rangle$	0	$\langle 1.83 \times 10^{-11} \rangle$
γ_0^*	0	0	-5.92(8) [10 $^{-6}$]
$\Delta T_{\sigma^2}^*$ [μK]	0	20.05(21)	20.00(18)
k_{eq}^* [Mpc $^{-1}$]	0.01022(7)	0.01022(7)	0.01343(7)
$\Omega_{\sigma^2}(z = z_{\text{BBN}})^*$	0	0.727(2)	1.52(3) [10 $^{-41}$]
$-2 \ln \mathcal{L}_{\text{max}}$	526.1	526.1	525.7
$\ln \mathcal{Z}$	-538.11	-538.25	-537.62

the evolution of the comoving volume element [viz., $H(z)$]'s for the An- Λ CDM and dv- w CDM models are observationally indistinguishable from Λ CDM all the way to the matter-radiation transition epoch.

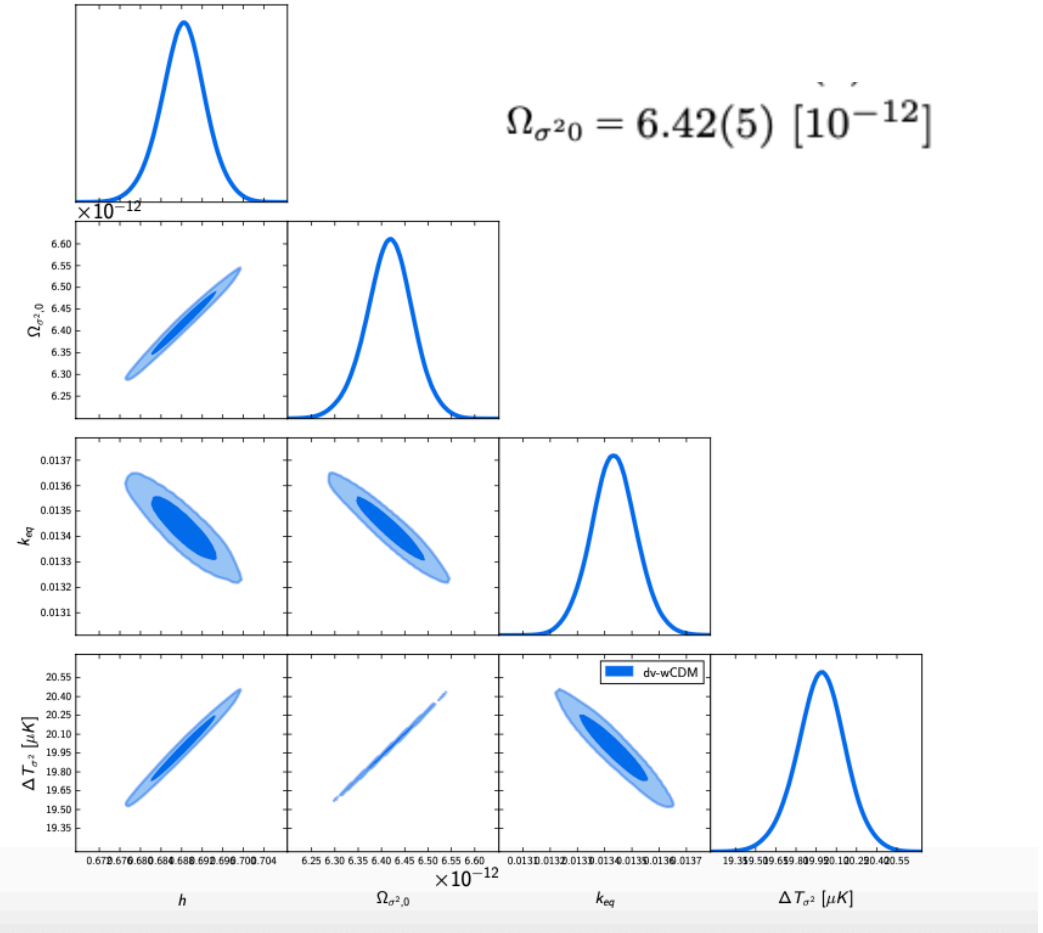
Yet, both these can be distinguished from Λ CDM as they predict $\Delta T_{\sigma^2} \sim 20 \mu\text{K}$, i.e., reduction of $\Delta T_{\text{st}} \approx 34 \mu\text{K}$ in the Λ CDM to the observed value $\Delta T \approx 14 \mu\text{K}$.

An- Λ CDM



dv- w CDM models.

$$\frac{\mathcal{H}^2}{\mathcal{H}_0^2} = \Omega_{\text{eff}0}(1+z)^{3(1+w_{\text{eff}})} + \Omega_{\text{m}0}(1+z)^3 + \Omega_{\text{r}0}(1+z)^4,$$



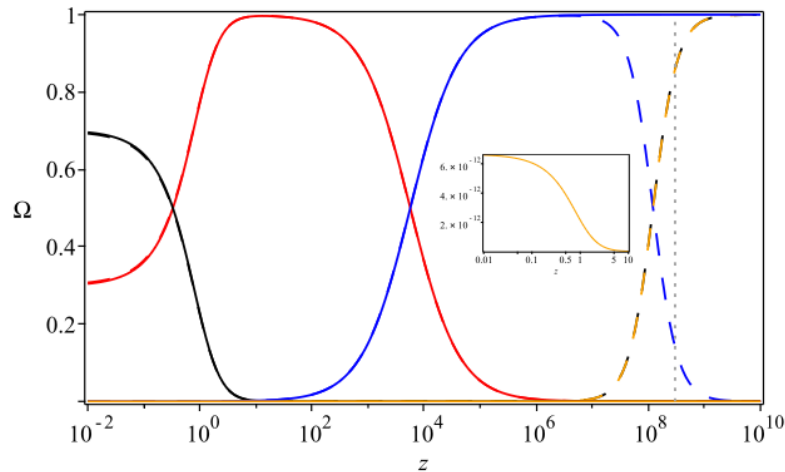


FIG. 2. Ω versus z for Λ CDM $_{\sigma^2}$ (dashed line) and dv - w CDM (solid line) models using the mean values from Table I. Ω_{σ^2} , Ω_{Λ} , Ω_m and Ω_r are colored by orange, black, red and blue, respectively. The vertical line represents the BBN epoch ($z_{\text{BBN}} \sim 3 \times 10^8$).

DI VALENTINO ETAL, ABDALLA ETAL, INTERTWINED PAPERS, 2008.11285, 2008.11286, 2008.11284, 2008.11285, ABDALLA ETAL, INTERTWINED PAPER 2203.06142

PERIVOLAROPOULOS, SKARA, NEW ASTRON. REV. 95 (2022), 101659, 2105.05208

CONCLUSIONS

- In Λ CDM, Universe isotropizes as it expands, eventually the expansion anisotropy dominates over the radiation and spoils the standard BBN.
- On the other hand, in our model, for $w=\text{const.}$, Universe anisotropies as it expands, model approximates the Λ CDM with increasing redshift, leaving BBN unaltered.

It couples to the shear scalar in a unique way, such that they together emulate the canonical scalar field with an arbitrary potential.

Shear scalar is an observable whereas the kinetic term of a scalar field is not.

if there is no SF any more, we would not suffer from the tension with the string Swampland criterion, the derivative of the SF potential has to satisfy the lower bound $\frac{|dV/d\phi|}{V} > c \sim \mathcal{O}(1)$

Deformed vacuum energy emulates the quintessence DE models, is not expected to address the H0 tension through its affect on the average expansion rate of the Universe.

The Cosmological Principle \longleftrightarrow Cosmic expansion determined by single parameter

Implications of the H0 tension may extend beyond Λ CDM to the CP itself.

On the other hand, there are suggestions to address this tension by reanalyzing the cosmological data by breaking down of the RW framework, e.g., allowing anisotropic expansion in the late universe; suggesting, in essence, that the problem in fact is not H₀ itself.

Universe expansion may not be uniform

The Cosmological Principle \longleftrightarrow Cosmic expansion determined by single parameter

Implications of the H_0 tension may extend beyond Λ CDM to the CP itself.

PERIVOLAROPOULOS, 2305.12819, ON THE ISOTROPY OF SNIA ABSOLUTE MAGNITUDES IN THE PANTHEON+ AND SHOES SAMPLES

MIGKAS ET.AL., A&A 649, A151 (2021) COSMOLOGICAL IMPLICATIONS OF THE ANISOTROPY OF TEN GALAXY CLUSTER SCALING RELATIONS ,.....

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KRISHNAN ET AL 2022 PRD105 063514 2106.02532, HINTS OF FLRW BREAKDOWN FROM SUPERNOVAE

KRISHNAN ET AL 2021 CLASS. QUANTUM GRAV. 38 184001, DOES HUBBLE TENSION SIGNAL A BREAKDOWN IN FLRW COSMOLOGY?

Anisotropy is from the Greek: aniso = different, varying; tropos = direction
Nomenclature: in general, we need to use tensors to describe fields and properties, the simplest case of a tensor is a scalar, all we need for isotropic properties : zero rank tensor

Multiwavelength scaling relations of galaxy clusters are an excellent and powerful tool to scrutinize both the H_0 isotropy and the existence of bulk flows at large scales.

MIGKAS ET.AL., A&A 636, A15 (2020)

$A \sim 5.5\sigma$ $LX - T$ anisotropy toward

$(l, b) \sim (280^\circ, -15^\circ)$ that was originally observed in M20. The future eRASS catalogs may help.