The Hidden Power of Modular Flavor Symmetry

Hans Peter Nilles

Bethe Center für Theoretische Physik (bctp)

und Physikalisches Institut,

Universität Bonn

UNIVERSITÄT BONN

bctp Bethe Center for Theoretical Physics

Outline

- The flavor structure of the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Origin of hierarchies for masses and mixing angles
- Specific properties of Modular Symmetry
- UV-IR relation:
  the hidden power of modular flavor symmetry

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Traditional vs Modular Symmetries

So far the flavor symmetries had specific properties and we refer to them as traditional flavor symmetries

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of flavor symmetries are modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- applied recently to lepton sector (Feruglio, 2017)
- modular symmetries are nonlinearly realised!
- Yukawa couplings are modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

String Geometry of extra dimensions

Strings are extended objects and this reflects itself in special aspects of geometry (including winding modes).
We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations lead to modular or symplectic flavor symmetries that cannot be realised linearly in low-energy effective theory.
- They still give restrictions on the low-energy action
- provides constraints from the UV-sector of the theory

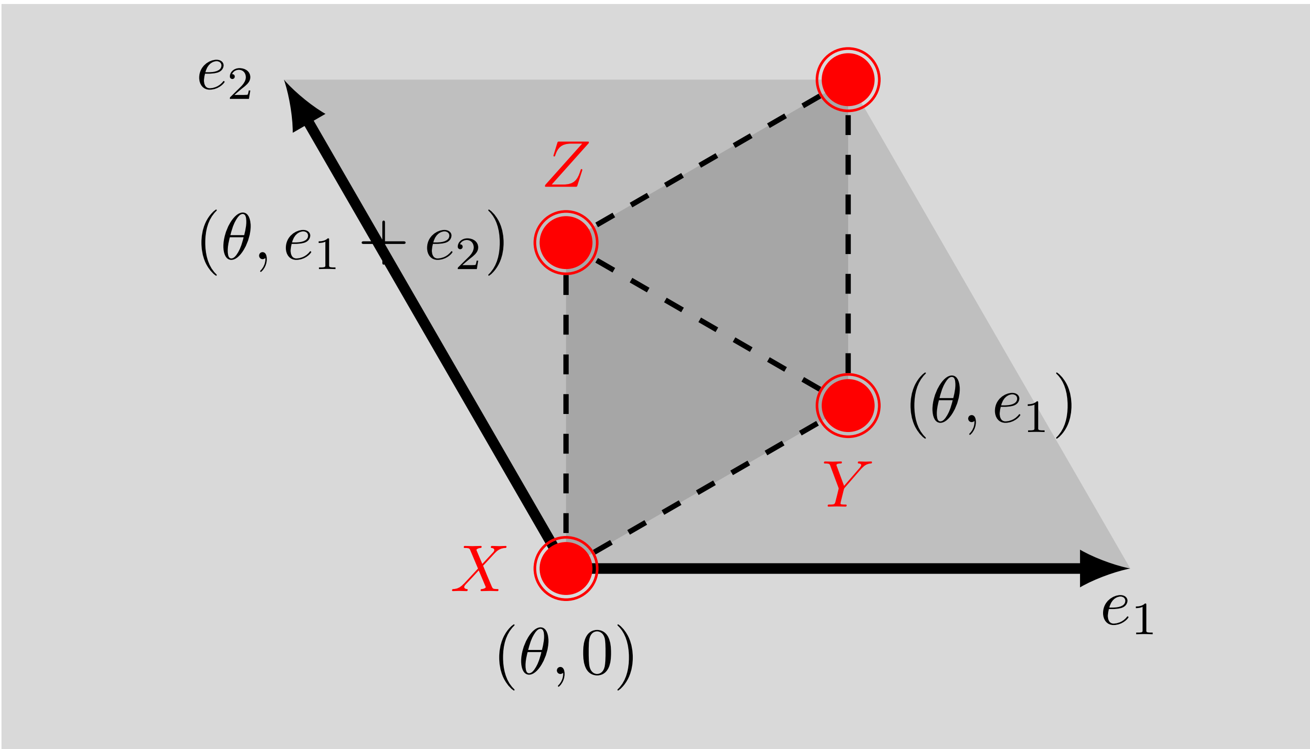
In the following we illustrate with a simple example

- twisted 2D-torus with localized matter fields

Traditional Flavor Symmetries

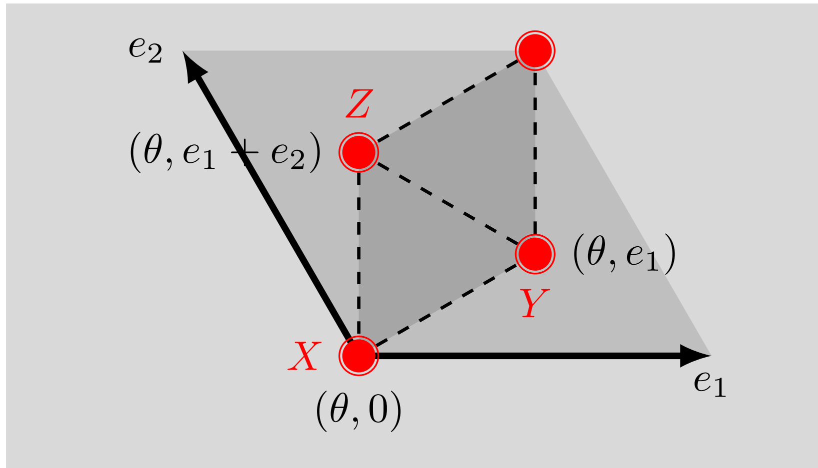
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)_{\text{flavor}}$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory: it is a map not a symmetry

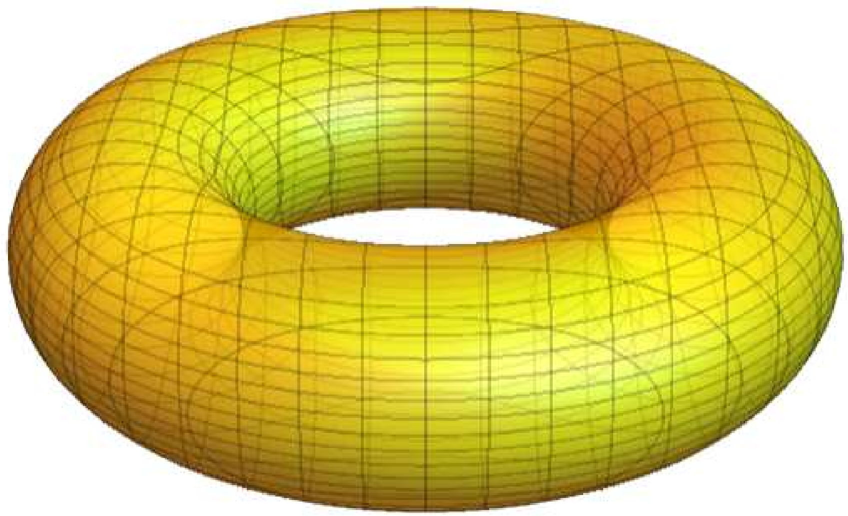
- self-dual point is $R^2 = 1 = \alpha' = 1/M_{\text{string}}^2$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

How does T-duality restrict the low-energy effective theory?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

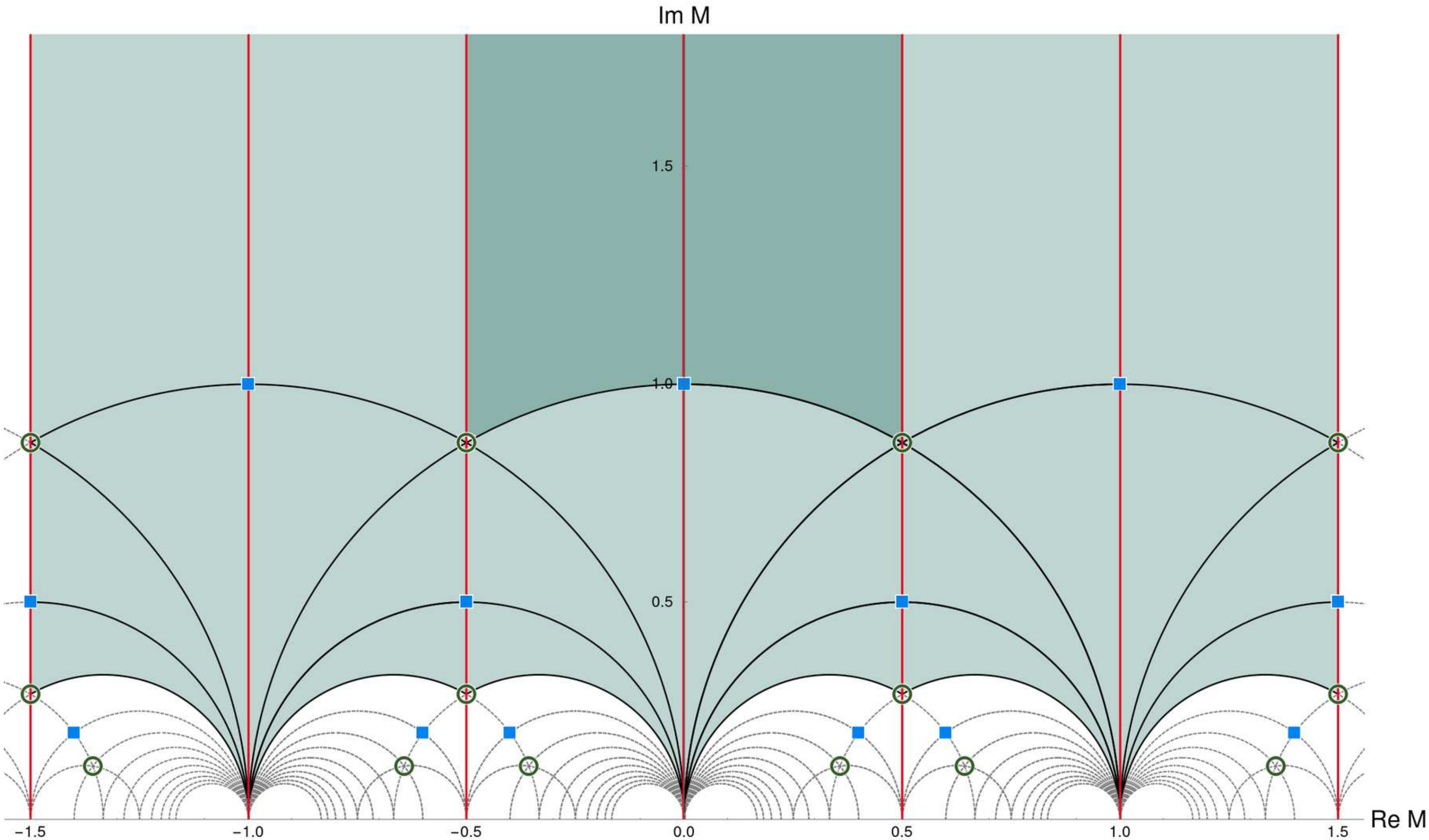
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = (\mathrm{ST})^3 = 1\ \ \text{and}\ \ \mathrm{S}^2\mathrm{T} = \mathrm{TS}^2.$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Forms

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \to \frac{aM+b}{cM+d}$$

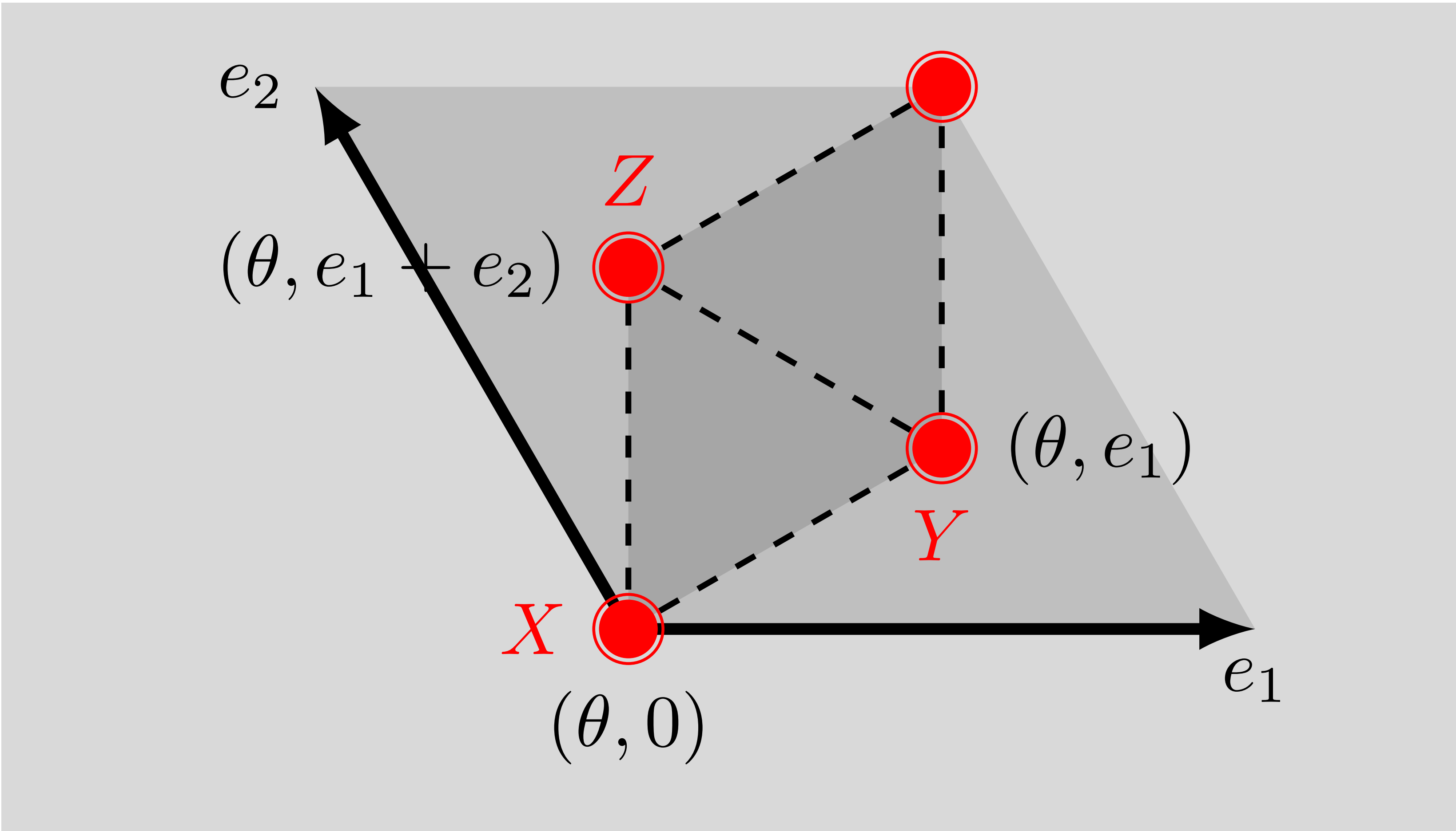
with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma: \quad \phi \to (cM+d)^k \rho(\gamma)\phi \,.$$

Yukawa-couplings transform as modular functions as well. $G = K + \log |W|^2$ must be invariant under T-duality

Orbifold T_2/Z_3



Yukawa Couplings

Yukawa couplings are modular forms that depend nontrivially on the modulus M.

Consider, for example,

- the twisted fields of the T_2/Z_3 orbifold,
- located at the fixed points X, Y and Z.

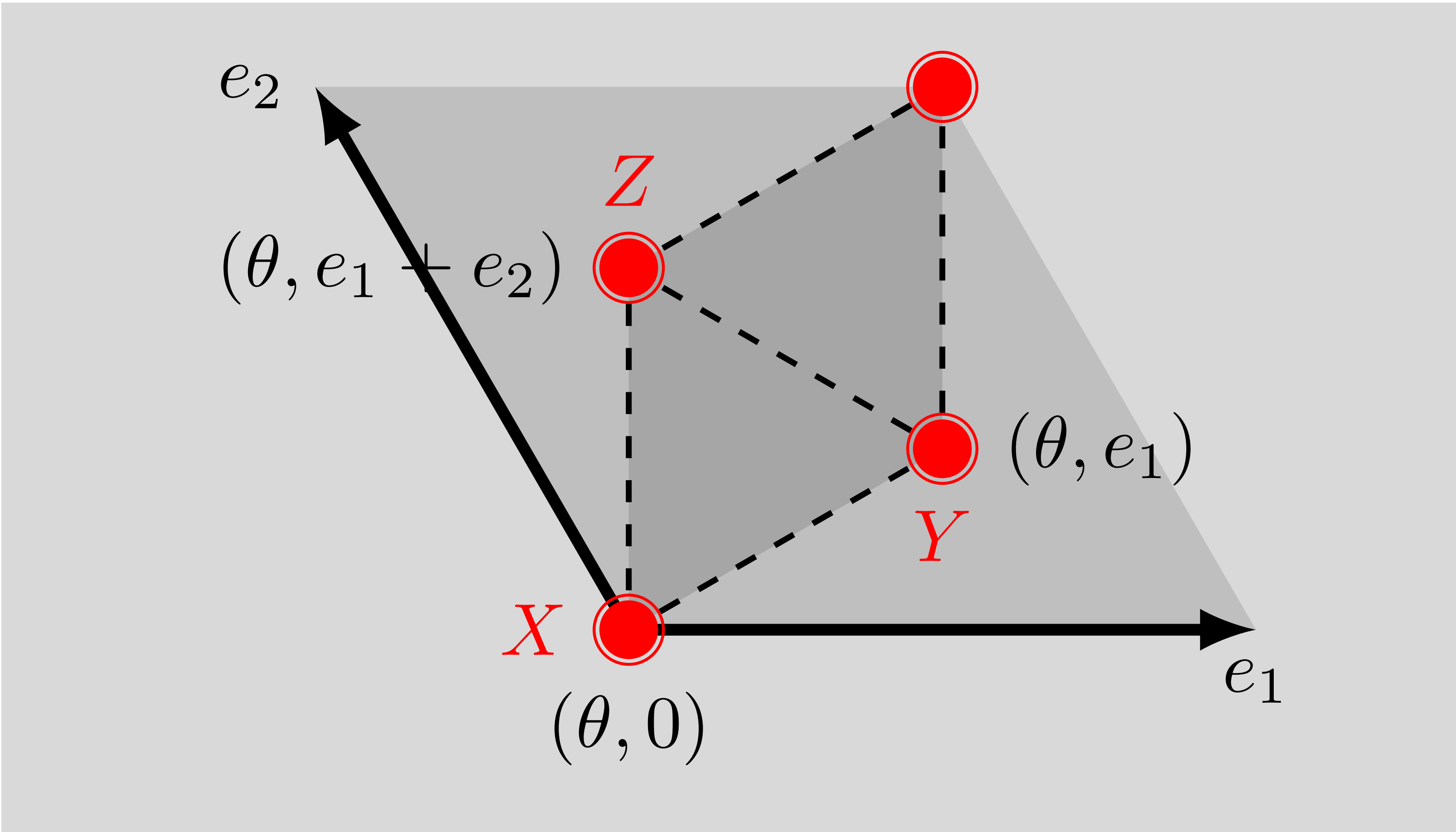
Allowed couplings are:

$$f(M)(X^3 + Y^3 + Z^3) + g(M)XYZ$$

$f(M)$ and $g(M)$ are modular functions of weight k

For large M the coupling $g(M)$ is exponentially suppressed, while $f(M)$ remains finite.

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2,Z)$

- $\Gamma(3) = SL(2,3Z)$ as a mod(3) subgroup of $SL(2,Z)$
- discrete modular flavor group $\Gamma_3' = SL(2,Z)/\Gamma(3)$
- the discrete modular group is $\Gamma_3' = T' \sim SL(2,3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2,3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

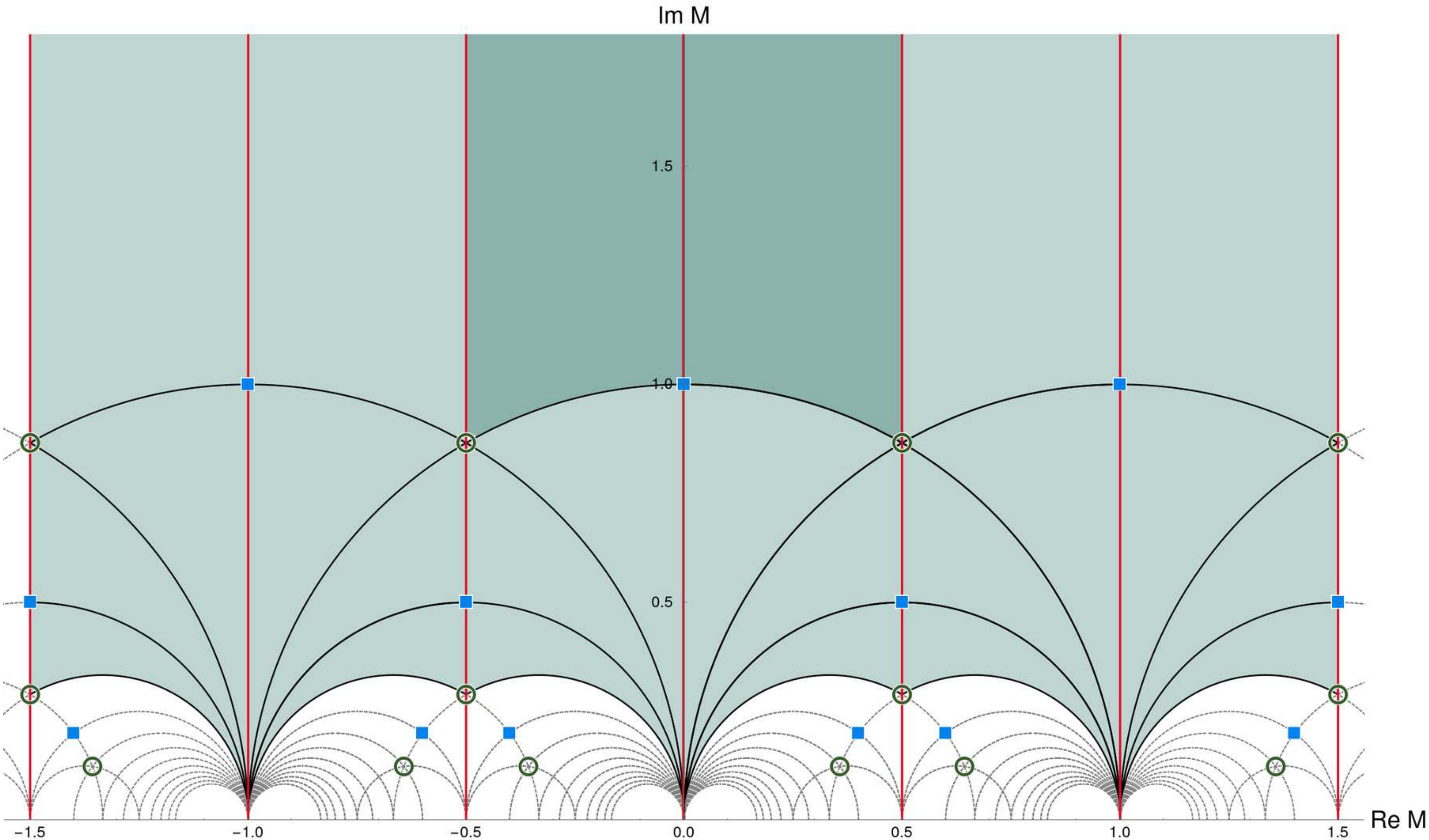
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2, 3)$ including CP

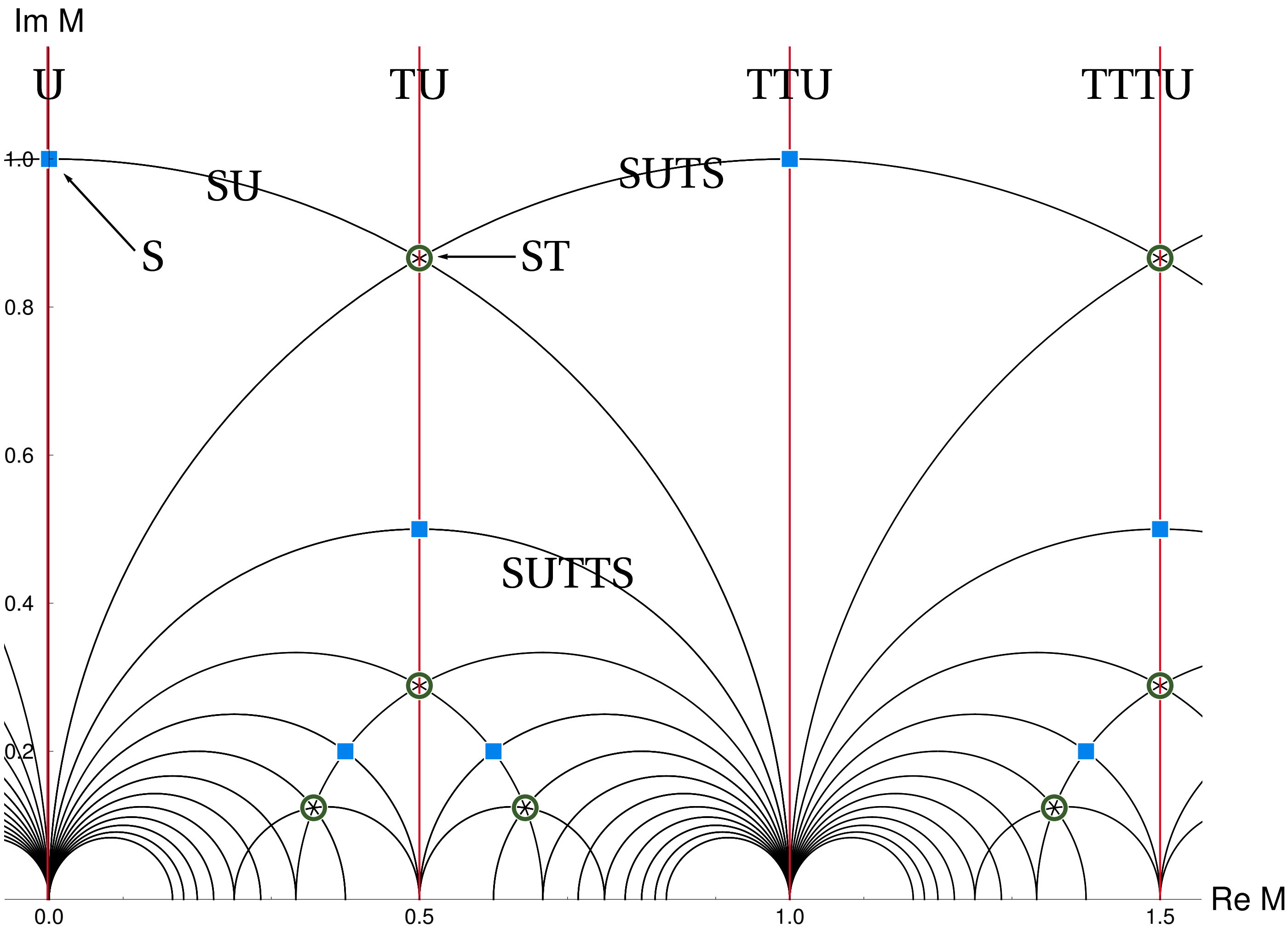
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



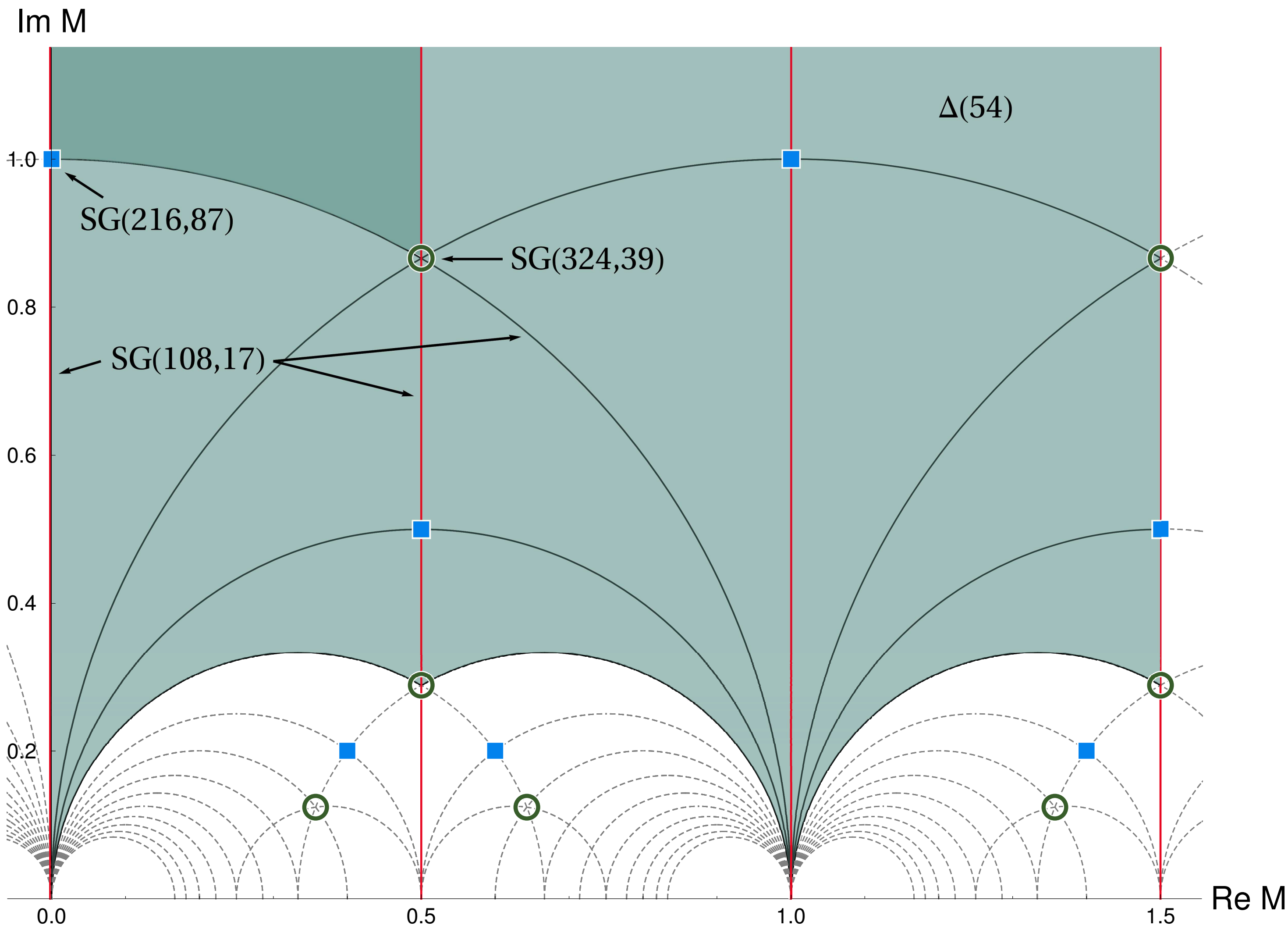
Moduli space of $\Gamma(3)$

Fixed lines and points



$$\mathrm{S}: M \to -\frac{1}{M}, \quad \mathrm{T}: M \to M+1 \quad \text{and} \quad \mathrm{U}: M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Comparison

Traditional and modular flavor symmetries are fundamentally different

- linear versus non-linear realization
- traditional is subgroup of SU(3) (if all Yukawas vanish)
- modular symmetry is not a subgroup of $SU(3)_{\rm flavor}$
- as Yukawa couplings are modular forms (that depend nontrivially on modulus)
- local enhancement at specific locations

This peculiar behaviour of modular flavor symmetry allows a description of the the influence of winding modes in low energy effective theory (and gives a UV-IR connection)

UV-IR connection

String dualities connect winding to momentum modes. Winding modes are heavy. Could there be nonetheless an effect at low energies?

- "Stringy Miracles" and naturalness in string theory – need introduction of "Rule 4" (Font, Ibanez, Nilles, Quevedo, 1988)
- selection rules of CFT lead to vanishing of certain couplings at same fixed point not understood through symmetries of the low energy effective theory
- extended later including "Rule 5" and "Rule 6"

  (Kobayashi, Parameswaran, Ramos-Sanchez, Zavala, 2011)
- these "Stringy Miracles" remained a puzzle till recently

Calculations with eclectic flavor symmetries explain "Rule 4"

(Nilles, Ramos-Sanchez, Vaudrevange, 2020)

Stringy Miracles

Yukawa couplings of twisted fields are modular forms that depend nontrivially on the modulus M.

Consider, for example, the twisted fields of the T_2/Z_3 orbifold, located at the fixed points X, Y and Z.

Ususally the allowed couplings are:

$$f(M)(X^3 + Y^3 + Z^3) + g(M)XYZ$$

with both non-vanishing $f(M)$ and $g(M)$.

Stringy miracles are cases where $f(M) = 0$!

Can we identify the reason for this peculiar situation?

UV-IR connection

String dualities connect winding to momentum modes. Winding modes are heavy. Could there be nonetheless an effect at low energies?

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Modular Flavor

What is the reason for this?

- It is the presence of the discrete modular flavor symmetry and the modular weights.
- modular group $SL(2, Z)$ with $S^4 = 1$ and $S^2 \neq 1$
- $PSL(2, Z)$ with $S^2 = 1$ acts on moduli
- additional Z_2 corresponds to the double cover of finite modular group (originates from CFT selection rules)
- it is also part of the traditional flavor group. It looks "traditional" but it is intrinsically "modular"
- in the string models this Z_2 acts on "twisted" oscillator modes of the underlying string theory

(Work in progress)

Example T_2/Z_3

Superpotential is restricted by the eclectic flavor group

- $SG[648, 533] = \Omega(1)$ from $\Delta(54)$ and T'
- a Z_2 symmetry is common to $\Delta(54)$ and T'
- responsible for double cover T' of A_4
- extends $\Delta(27)$ to $\Delta(54)$
- $\Delta(54)$ contains nontrivial singlet $1'$ as well as two 3-dimensional representations 3_1 and 3_2
- vev of $1'$ breaks $\Delta(54)$ to $\Delta(27)$ with one triplet rep.
- twisted oscillator modes transform as $1'$ rep. of $\Delta(54)$

This Z_2 as part of $\Delta(54)$ together with the action of T' completes the explanation of the "Stringy Miracles".

Messages

The top-down approach to flavor symmetries leads to a

- unification of traditional (discrete) flavor, CP and modular symmetries within an eclectic flavor scheme
- modular flavor symmetry is a prediction of string theory
- traditional flavor symmetry is universal in moduli space
- there are non-universal enhancements (including CP at some places (broken in generic moduli space))
- CP is a natural consequence of the symmetries of the underlying string theory
- the potential flavor groups are large and non-universal
- spontaneous breakdown as motion in moduli space

Open Questions

So far $\Delta(54) \times T'$ seems to be the favourite model

- numerous bottom-up models with these groups
- successful realistic string models from Z_3 orbifolds
- Z_2, Z_4 and Z_6 as alternatives at this level (work in progress)

It has been observed that many of the successful fits are in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space, but AdS-minima (Cvetic, Font, Ibanez, Lüst, Quevedo, 1991)
- uplift moves them slightly away from the boundary

(Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023)

Summary

String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- provides a non-trivial UV-IR relation:
  the hidden power of modular flavor symmetry

Moduli fixing